PHYSICAL REVIEW C VOLUME 9, NUMBER 2 FEBRUARY 1974

Self-consistent quasiparticle-phonon coupling*

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(Received 28 August 1973)

Using a Green's function formalism, the quasiparticle self-energy in a finite Fermi system is derived from the particle-particle-hole response function with the assumption that particle-particle and three-body correlations are negligible. An approximate calculation of the self-energy includes the usual core-polarization diagram, a backwards exchange diagram with ground-state correlations, and higher-order diagrams which correct for exclusion-principle violations in the propagation of the quasiparticle and the microscopic phonon. Intermediate lines in the self-energy are described self-consistently, thereby including multiple-phonon core excitations. It is shown that the equation for the self-energy may be solved by matrix diagonalization by coupling the even- and odd-nuclear eigenmodes. Renormalized phonons are calculated by taking into account the distribution of quasiparticle strength by quasiparticle-phonon coupling in the solution of the Bethe-Salpeter equation. The self-energy and phonon are calculated self-consistently.

I. INTRODUCTION

It is usual to classify the modes of motion in a spherical nucleus as either single particle or vibrational in nature.[1] In zeroth order, these can be regarded as independent normal modes. However, one frequently observes in odd-mass nuclei that properties associated with the independent particle and with the collective vibration are mixed into the same nuclear state. Thus, if the physical simplicity of concepts like orbiting nucleons and coherent nuclear motions is to be retained, a coupling Hamiltonian must be defined or derived which will mix basis states of the types: $|j\rangle$, a single-particle state with angular momentum j, and $|[j_1, N(R)]\rangle$, a particle with angular momentum j_1 coupled to N vibrational excitations each with angular momentum R. This treatment approximates the nucleus very crudely since, among other effects, it ignores ground-state correlations and higher-energy nuclear modes. However, it produces a very simple matrix diagonalization problem and provides the conceptual basis for all particle-core or particle-vibration coupling treatments, and such calculations are very useful in explaining experimental data. The coupling Hamiltonian is often taken to be the scalar product of the particle's coordinate and the macroscopic coordinate of the vibrating core.[2,3] In other instances,[4-6] the phonon is described microscopically and the coupling Hamiltonian defined as a linear combination of two-body matrix elements. This latter approach is more definitely related to the fundamental many-body problem.

The aim of this paper, and several to follow, is to extend the Fermi liquid theory in a natural way (using the six-point function) to quasiparticle-phonon coupling (QPC)[7] and thereby prescribe unambiguously the inclusion of higher-order nuclear processes into the QPC problem.

The framework of our discussion will be that of Green's functions and diagrams.[8,9] In Sec. II A we introduce intuitively the higher-order diagrams in which we are interested. In Secs. II B and II C the theory is developed. We conclude by comparing our equations to those obtained by other methods. The considerations here will be limited to normal, spherical nuclei. The extension of these ideas to superfluid and deformed nuclei and the calculation of transition probabilities is under way and will be the subject of future installments.

II. THEORY

A. QPC concepts

The usual microscopic intermediate coupling problem (cf. Ref. 4) can be derived by considering the diagram shown in Fig. 1 for the single-nucleon self-energy. The eigenvalues of the odd nucleus are then obtained by inserting this approximation for the self-energy into Dyson's equation, which is shown graphically in Fig. 2.

FIG. 1. The quasiparticle-phonon coupling (QPC) approximation for the mass operator or self-energy. On the left-hand side, the line represents a one-nucleon Green's function. The wiggly line is a phonon, and the dots correspond to QPC vertices. On the right-hand side, the string of bubbles is just the usual microscopic description of the phonon.

The diagram in Fig. 1 describes the core-polarization process and is important to the calculation of the quasiparticle self-energy because, when we put a shell-model particle into the nucleus, there is a good chance that it will scatter into a different configuration. This lowering of probability for stable single-particle propagation is reflected in the distribution of residues of the Green's function. There is single-particle strength at several poles of the odd-mass nucleus because single-particle and core-coupled states mix. Thus, the QPC approximation can explain the satellite peaks in one-nucleon stripping and pickup reactions.[10]

Nevertheless, there are several diagrams, which are only slightly more complicated and contain some important physics, but which are usually neglected in QPC calculations. The diagram in Fig. 3(a) recouples particles and phonons, and takes into account certain violations of the Pauli principle. To see this we construct in Fig. 4 a partially-labeled phonon plus particle. This configuration might be present, for example, as part of the core-polarization diagram in Fig. 1. Note that the particle-hole configuration $|13\rangle$ cannot be excited while the orbital labeled 1 is occupied because this situation is a violation of the Pauli principle. Such violations are usually ignored because the phonon is calculated independently of the quasiparticle. However, the diagram shown in Fig. 3(a) exactly cancels the illegal contribution.

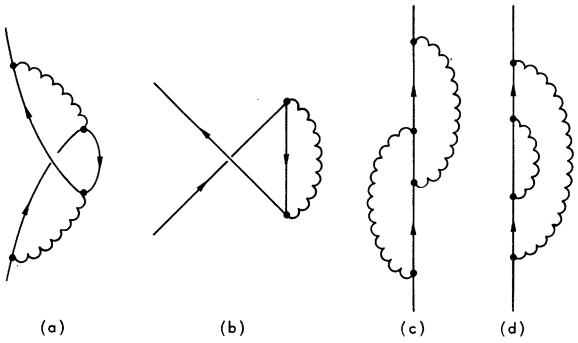

FIG. 3. Higher-order corrections to QPC: (a) corrects for exclusion-principle violations (EPV's) shown in Fig. 4; (b) is the so-called "backwards" diagram which results from ground-state correlations; (c) and (d) are two-phonon contributions to the self-energy; (d) can be obtained by "dressing" the intermediate line in the lowest-order QPC diagram of Fig. 1.

Another Pauli principle violation is shown in Fig. 5. It results from the zero-point core vibration, or ground-state correlation, which coexists with the propagating quasiparticle. Although Goldstone's theorem states that we do not have to evaluate such unlinked diagrams explicitly, the exchange (or backwards) diagram shown in Fig. 3(b) should be computed. It is clearly a repulsive contribution to the self-energy of the quasiparticle because fewer degrees of freedom are allowed for the zero-point motion of the core. One can expect this sort of interference to be important whenever a particle state is occupied with high probability in phonon-type ground-state correlations.

Another higher-order effect is that after the propagating quasiparticle excites a core vibration, the quasiparticle-plus-core excitation may polarize yet another core mode. If the original core excita-

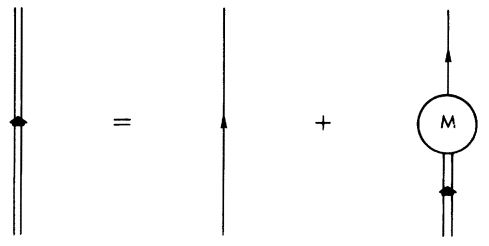

FIG. 2. Dyson's equation for the two-point or one-nucleon Green's function. The double line represents the exact propagator, the single line the unperturbed propagator, and M the mass operator.

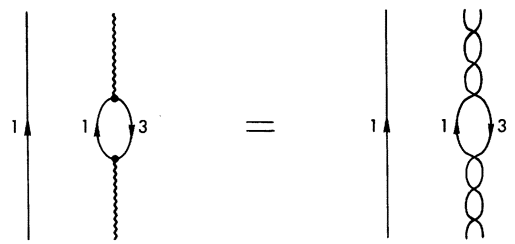

FIG. 4. A violation of the Pauli exclusion principle which occurs when a phonon, calculated separately in the microscopic RPA, coexists with an extra particle.

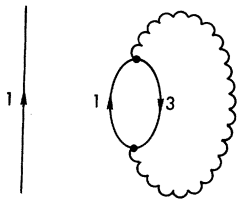

FIG. 5. A violation of the Pauli exclusion principle which occurs when a ground-state correlation, calculated for the even nucleus, coexists with an odd particle.

tion disappears first we obtain the staggered diagram shown in Fig. 3(c), while if the second excitation deexcites first, we obtain the nested diagram shown in Fig. 3(d). Since core polarization fractionates single-particle strength, these higher-order polarizations will further redistribute the strength.

All of the above considerations not only have an effect on the eigenenergies of the odd-mass nucleus but also influence the eigenenergies of the even-mass excited states. This can be seen by inspecting the higher-order random-phase approximation (RPA) graphs in Fig. 6. The descriptions of even and odd nuclei are not independent and should be formulated self-consistently. We shall call such a theory self-consistent quasiparticle-phonon coupling (SCQPC).

B. Self-consistent quasiparticle-phonon coupling

Empirically, it has been demonstrated that QPC provides a qualitative description for low-energy states of many nuclei. The intent of this section is to examine the reasons for the success of this approximation for the self-energy, and to justify the inclusion of higher-order diagrams from the many-body theory. This section is organized as follows: first, we state some preliminary formulas from the theory of finite Fermi systems. Secondly, the particle-particle-hole (pph) or six-point response function is introduced to facilitate calculation of the mass operator. A renormalized integral equation for the (pph) response function is derived in terms of the (ph) response function and the (ph) interaction. Finally, the equations are simplified by making the phonon approximation for the (ph) and (pph) response functions and then the coupled equations for the two- and six-point functions are reduced to a single matrix-diagonalization problem.

1. Preliminary formulas

In order to shorten the development, we first write down a number of results from the theory of finite Fermi systems.[9,11] The one- and two-nucleon Green's functions are defined

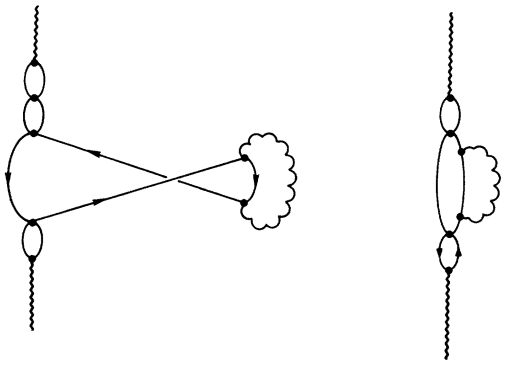

FIG. 6. QPC corrections to the particle-hole bubble approximation, i.e., RPA.

$$G_{12} \equiv -i \langle 0 | T\{\psi_1 \psi_2^\dagger\} | 0 \rangle , \tag{1}$$

$$G_{1234} \equiv -\langle 0 | T\{\psi_1 \psi_2 \psi_4^\dagger \psi_3^\dagger\} | 0 \rangle , \tag{2}$$

where T is the time-ordering operator, and the subscripts represent both the quantum numbers for an arbitrary single-particle basis and the time variable. As usual, $|0\rangle$ is an exact ground state of the even nucleus. The spectral representation of G_{12} is

$$G_{12}(\omega) = \sum_{i(N+1)} \frac{\langle 0 | \psi_1 | i \rangle \langle i | \psi_2^\dagger | 0 \rangle}{\omega - (E_{N+1}^{(i)} - E_N^{(0)}) + i\delta} + \sum_{j(N-1)} \frac{\langle 0 | \psi_2^\dagger | j \rangle \langle j | \psi_1 | 0 \rangle}{\omega + (E_{N-1}^{(j)} - E_N^{(0)}) - i\delta} . \tag{3}$$

The one-nucleon Green's function may be determined exactly from Dyson's equation (see Fig. 2).

$$G_{12} = G_{12}^{(0)} + G_{13}^{(0)} M_{34} G_{42} , \tag{4}$$

where M is the self-energy and contains all Feynman graphs which cannot be separated by cutting one line. An exact expression is given later in this section.

The two-nucleon Green's function or four-point function depends on the scattering matrix, Γ, of two particles in the interacting system

$$G_{1234} = G_{13} G_{24} - G_{14} G_{23} + i G_{15} G_{26} \Gamma_{5678} G_{73} G_{84} . \tag{5}$$

Γ may be reduced in the (pp) or (ph) channel. If in the (ph) channel I represents an irreducible block, a Bethe-Salpeter equation for Γ may be constructed,

$$\Gamma_{1234} = I_{1234} - i I_{1638} G_{76} G_{85} \Gamma_{5274} . \tag{6}$$

A closely related quantity is the linear response function defined by

$$L_{1234} \equiv G_{1234} - G_{13} G_{24} . \tag{7}$$

The Bethe-Salpeter equation for this quantity is

$$L_{1234} = -G_{14}G_{23} - iG_{18}G_{63}I_{8765}L_{5274}\,. \tag{8}$$

Thus Γ and L are related by the equation

$$\Gamma_{1234} = I_{1234} + iI_{1638}L_{8765}I_{5274}\,. \tag{9}$$

The spectral representation of L is given by

$$L_{1234} = -i \sum_{S(N')\neq 0(N)} \left[\frac{\chi^{(S)}_{13}\chi^{(S)*}_{42}}{\bar{\omega} - (E^{(S)}_{N'} - E^{(0)}_{N}) + i\delta} - \frac{\chi^{(S)}_{24}\chi^{(S)*}_{31}}{\bar{\omega} + (E^{(S)}_{N'} - E^{(0)}_{N}) - i\delta} \right], \tag{10}$$

where

$$\bar{\omega} = \omega_1 - \omega_3\,.$$

The spectroscopic amplitudes or generalized density matrix for the excited states may be obtained from $\chi^{(S)}_{13}$ by

$$\frac{-i}{2\pi}\int \chi^{(S)}_{13}\, d\omega_1 = \langle 0|\psi^\dagger_3\psi_1|S\rangle \equiv \rho^{(S)}_{13}\,, \tag{11}$$

where the integral is taken counterclockwise around the upper half plane.

For shell-model nuclei, the one-nucleon Green's function may be separated into a term which is analytic near the Fermi surface and a regular part, that is

$$G = Zg + R'\,, \tag{12}$$

where Z is the residue of the single-particle strength which is close to the Fermi energy. It is customary to assume that G_{12} is diagonal, i.e., $G_{12} = \delta_{12}G_1$. However, the form of Eq. (12) does not exclude the possibility that g should have more than one pole.

The renormalized particle-hole interaction may be defined

$$\begin{aligned} F_{1234} &\equiv (Z_1Z_2Z_3Z_4)^{1/2}\Gamma^{\omega}_{1234} \\ &= (Z_1Z_2Z_3Z_4)^{1/2}(I_{1234} - iI_{1635}B_{56}\Gamma^{\omega}_{5264})\,, \end{aligned} \tag{13}$$

where $G_5G_6 = A_{56} + B_{56}$ is the sum of an analytic term (viz. $A_{56} = Z_5Z_6g_5g_6$) and a regular term.

The renormalized Bethe-Salpeter equation for the response function is

$$\begin{aligned} l_{1234} &\equiv L_{1234}/(Z_1Z_2Z_3Z_4)^{1/2} \\ &= -g_1g_3\delta_{14}\delta_{23} - ig_1g_3F_{1635}l_{5264}\,. \end{aligned} \tag{14}$$

It is also convenient to define the renormalized spectral amplitudes,

$$X^{(S)}_{13} \equiv \rho^{(S)}_{13}/(Z_1Z_3)^{1/2}\,. \tag{15}$$

A homogeneous integral equation for the renormalized spectral amplitudes and the excited state energies, $E_S \equiv E^{(S)}_N - E^{(0)}_N$, is given in Sec. II C.

2. (pph) response function

Recently, Ethofer and Schuck[12] and also Winter[13] have shown that the self-energy may be calculated exactly from the particle-particle-hole (pph) response function, L,

$$M_{44'} = -iV_{414'1'}G_{1'1} + \tfrac{1}{4}V_{43'12}L_{3121'2'3'}V_{1'2'4'3}\,. \tag{16}$$

L is the sum of all contributions to the three-nucleon Green's function (or six-point function) which cannot be separated by cutting a single line. V is the antisymmetrized matrix element of the bare interaction,[14]

$$G_{3121'2'3'} \equiv -i\langle 0|T\{\psi_3\psi_1\psi_2\psi^\dagger_{1'}\psi^\dagger_{2'}\psi^\dagger_{3'}\}|0\rangle\,, \tag{17}$$

$$L_{3121'2'3'} \equiv G_{3121'2'3'} - G_{123'5}G^{-1}_{56}G_{531'2'}\,. \tag{18}$$

The analytical properties of L follow from Eqs. (16) and (18) and the ansatz[13] that

$$M_{44'} = \sum_\gamma \frac{\sigma^\gamma_4(\sigma^\gamma_{4'})^*}{\omega - \omega_\gamma}\,. \tag{19}$$

It can then be shown[13] that

$$\tfrac{1}{4}L_{3121'2'3'}(\omega) = \sum_\gamma \frac{\rho^\gamma_{123'}(\rho^\gamma_{1'2'3})^*}{\omega - \omega_\gamma} \tag{20}$$

with

$$\rho^\gamma_{123'} = -\left[\sum_{i(N+1)} \frac{\langle 0|\psi_1\psi_2\psi^\dagger_{3'}|i\rangle\langle i|\psi^\dagger_4|0\rangle}{E^{(i)} - \omega_\gamma} - \sum_{j(N-1)} \frac{\langle 0|\psi_4|j\rangle\langle j|\psi_1\psi_2\psi^\dagger_{3'}|0\rangle}{E^{(j)} + \omega_\gamma} \right]\sigma^\gamma_4\,, \tag{21}$$

and also

$$\sigma^\gamma_4 = V_{43'12}\rho^\gamma_{123'}\,. \tag{22}$$

In Eqs. (20) and (21), we have assumed that the unnatural energy variables have already been integrated upon, i.e., ρ is obtained by applying to some more general quantity the operator

$$d\omega_1 d\omega_2 d\omega_{3'}\,\delta(\omega_\gamma - \omega_1 - \omega_2 + \omega_{3'})\,.$$

Except where explicitly necessary the Fourier algebra[8,9] for these variables has been omitted.

If an irreducible block, K, in the (pph) channel is introduced, L obeys the integral equation

$$\begin{aligned} L_{3121'2'3'} &= G_{33'}(G_{11'}G_{22'} - G_{12'}G_{21'}) \\ &\quad + G_{7'3'}G_{15}G_{26}K_{7565'6'7'}L_{35'6'1'2'7}\,. \end{aligned} \tag{23}$$

The interaction K contains[12] three-body and particle-particle correlations which we henceforth

neglect except during a brief discussion of the Brueckner model. Thus, we approximate the (pph) interaction by a sum of (ph) interactions, viz.,

$$K_{7565'6'7'} \approx -i(G^{-1}_{66'} I_{755'7'} + G^{-1}_{55'} I_{766'7'}) . \tag{24}$$

This approximation for K when substituted into Eq. (23) gives

$$L_{3121'2'3'} = L^{(21)}_{3121'2'3'} - L^{(21)}_{3122'1'3'} - L^{(21)}_{3211'2'3'} + L^{(21)}_{3212'1'3} - G_{33'}(G_{11'}G_{22'} - G_{21'}G_{12'}), \tag{25}$$

where $L^{(21)}$ obeys an integral equation appropriate for a particle-hole pair and an additional particle,

$$L^{(21)}_{3121'2'3'} = -L_{233'2'}G_{11'} - L_{27'3'6}G_{15}H^{(21)}_{7565'6'7'}L_{35'6'1'2'7} , \tag{26}$$

$$H^{(21)}_{7565'6'7'} = I_{64'7'5'}G_{44'}I_{5746'} + iI_{53'45'}G_{33'}G_{44'}I_{64'7'2}G_{22'}I_{2'736'} + \cdots . \tag{27}$$

The graphical equivalents of these equations are shown in Figs. 7–9. The derivation of Eqs. (25)–(27) may be most easily accomplished using diagrams. Note that the free part (GGG) is seen to correct the lowest-order contribution to $L^{(3)}$ [$L^{(2)} = -GG$ and $H^{(21)} = 0$] for double counting, when summing over the indices of two identical particle lines.

These expressions for the (pph) response function may be substituted into Eq. (16) to obtain the graphs shown in Fig. 10. Note that $L^{(21)}$ is not antisymmetric with respect to exchange of the indices 1 and 2, but that the interaction is. This explains the deletion of the factor of 1/4, also. It has been shown previously[13] that the bare interactions in this figure may be replaced by the Brueckner matrix Γ^{Br}, if the intermediates in the self-energy are restricted to a model space.[15] We further assume that the unperturbed G^0 includes the Brueckner-Hartree-Fock self-energy contribution.

We thus obtain the analytical expression

$$M_{44'} = \Gamma^{Br}_{43'12}(L^{(21)}_{3121'2'3'} - \tfrac{1}{2}G_{33'}G_{11'}G_{22'})\Gamma^{Br}_{1'2'4'3} . \tag{28}$$

In the spirit of the approximation of Migdal[11] and Landau[16] discussed earlier, we separate the self-energy into terms which have poles near the Fermi energy and those which do not,

$$M = M^{(1)} + M^{(R)} , \tag{29}$$

and

$$G^{(1)} \equiv G^{(0)} + G^{(0)}M^{(R)}G^{(1)} \approx Zg^{(1)} + R . \tag{30}$$

With the ansatz

$$G_{12} = (Z_1 Z_2)^{1/2} g_{12} + R' \tag{31}$$

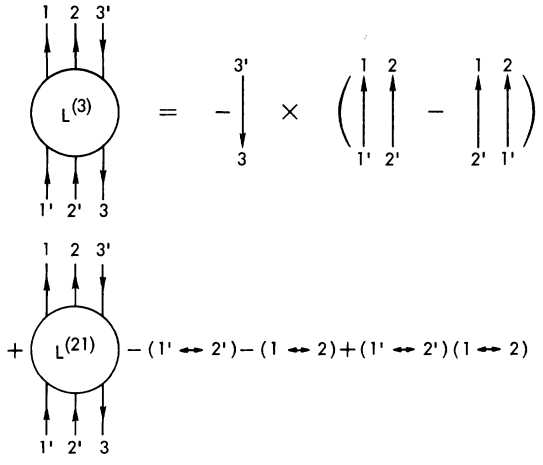

FIG. 7. The particle-particle-hole (pph) response function when only particle-hole correlations are considered. The straight lines represent exact one-nucleon Green's functions. $L^{(21)}$, which obeys the integral equation of Fig. 8, is not antisymmetric with respect to exchange of particle indices. The explicit contributions to $L^{(3)}$ required for antisymmetry are indicated. (Note that the graph of $L^{(3)}$ includes the ingoing and outgoing lines.)

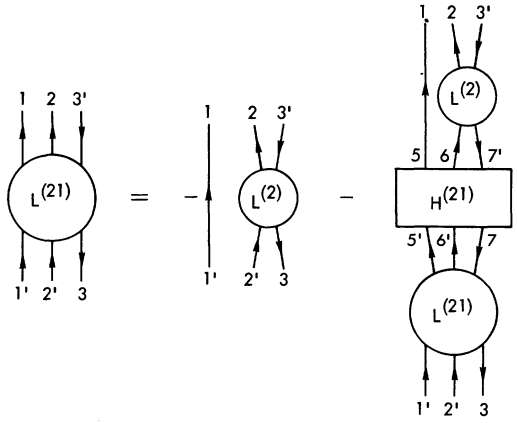

FIG. 8. The integral equation obeyed by the (ph) correlated contribution to the six-point response function. $L^{(2)}$ is just the usual (ph) function of linear response or RPA theory.

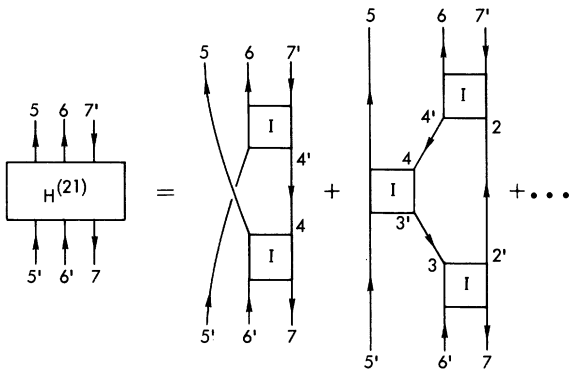

FIG. 9. The "interaction" between the (ph) response function and an extra particle. The ingoing and outgoing "stubs" are not part of the interaction. Intermediate lines labeled 2, 3, 4, etc. are exact or "dressed" propagators. I is the irreducible (ph) interaction. All possible time orderings of the I blocks are implied in this diagram.

we obtain

$$G = G^{(1)} + G^{(1)}M^{(1)}G$$
$$= Z(g^{(1)} + g^{(1)}M^{(1)}G) + (R + RM^{(1)}G),$$

and finally

$$g_{12} = g_{12}^{(1)} + g_{13}^{(1)} m_{34} g_{42}, \tag{32}$$

where

$$m_{34} \equiv (Z_3 Z_4)^{1/2} M_{34}^{(1)}.$$

We shall adopt the approximations that $g^{(1)}$ has only one pole, while g may have more due to effects such as QPC. Thus, we introduce the concepts of "bare" and "dressed" quasiparticles.

Although the above formulation for the self-energy leaves the division of terms into two classes arbitrary, it does clarify exactly the analytical construction of the self-energy contributions which derive from (ph) correlations. We make for simplicity the further approximations that $g^{(1)}$, m, and g are diagonal in the single-particle representation. We thus have the following expressions for $g^{(1)}$ and g:

$$g_1^{(1)}(\omega_1) = \frac{1}{\omega_1 \mp E_1 \pm i\delta} \tag{33a}$$

and

$$g_1(\omega_1) = \sum_{i(N\pm1)} \frac{\zeta_1^{(i)}}{\omega_1 \mp E_1^{(i)} \pm i\delta}. \tag{33b}$$

In the above notation, the E_1 is always positive and is approximated by the shell model as follows

$$E_1 = \pm(\epsilon_1 - \lambda), \tag{34a}$$

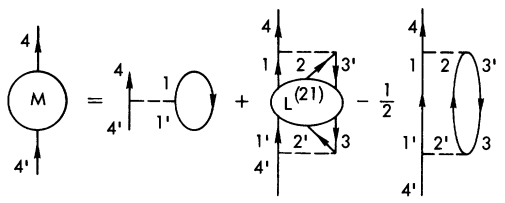

FIG. 10. The equation for the mass operator when all (ph) correlations are included. The dotted horizontal lines represent the "bare" interaction. The first graph on the right-hand side is the Hartree-Fock term, henceforth presumed in the zeroth-order propagator. The last term is neglected for the QPC approximation which follows.

while the exact energies

$$E_1^{(i)} = E_{N\pm1}^{(i)} - E_N^{(0)} \mp \lambda. \tag{34b}$$

The ζ are reduced residues since the quasiparticle Green's function already includes only the percentage of single-particle strength which is of analytic importance for $\omega \approx \lambda$.

The equation for m may be formulated entirely in terms of renormalized quantities, viz.,

$$T_{1234} \approx (Z_1 Z_2 Z_3 Z_4)^{1/2} \Gamma_{1234}^{\mathrm{Br}}, \tag{35}$$

and the renormalized (ph) interaction, F, and response function, l, defined previously.

The renormalized expressions for the self-energy are as follows:

$$m_4 = \tfrac{1}{4} T_{43'12} l_{3121'2'3'} T_{1'2'43}$$
$$= T_{43'12}[l^{(21)}_{3121'2'3'} - \tfrac{1}{2} g_{33'} g_{11'} g_{22'}] T_{1'2'43}, \tag{28'}$$

$$m_4 = \sum_\gamma \frac{(\bar\sigma_4^\gamma)(\bar\sigma_4^\gamma)^*}{\omega - \omega_\gamma}, \tag{19'}$$

$$\tfrac{1}{4} l_{3121'2'3'} = \sum_\gamma \frac{(\bar\rho^\gamma_{123'})(\bar\rho^\gamma_{1'2'3})^*}{\omega - \omega_\gamma}, \tag{20'}$$

$$\bar\rho^\gamma_{123'} = \rho^\gamma_{123'}/(Z_1 Z_2 Z_{3'})^{1/2}, \tag{21'}$$

$$\bar\sigma_4^\gamma = T_{43'12} \bar\rho^\gamma_{123'}, \tag{22'}$$

$$l^{(21)}_{3121'2'3'} = -l_{233'2'} g_{11'} - l_{27'3'6} g_{15} h^{(21)}_{7565'6'7'} l^{(21)}_{35'6'1'2'7}, \tag{26'}$$

and

$$h^{(21)}_{7565'6'7'} = F_{64'7'5'} g_{44'} F_{5746'} + iF_{53'45'} g_{33'} g_{44'} F_{64'7'2} g_{22'} F_{2'736'} + \cdots. \tag{27'}$$

3. *Phonon approximation*

The four-point response function, when written in the energy formalism, usually contains a singularity corresponding to a collective state, com-

monly called a phonon. In the case of 2+ and 3− states, the phonon is the lowest in energy and most likely to affect the low-energy structure of odd nuclei by quasiparticle-phonon coupling. Further, the spectroscopic amplitudes for the phonon are coherent, i.e., if all the matrix elements are attractive, all amplitudes are positive. This means that the QPC interaction, which, as we shall see, is a linear combination of matrix elements times amplitudes, is particularly large for the collective state.

Thus we are motivated to make the following approximations: First of all, we consider only the collective pole(s) of the four-point function,

$$l_{1234}(\omega)\approx\frac{X_{13}^{(S)}X_{42}^{(S)}}{\omega-E_S}-\frac{X_{31}^{(S)}X_{24}^{(S)}}{\omega+E_S}\,. \tag{36}$$

Secondly, the (pph) amplitude is factored into a phonon and a quasiparticle contribution.

This is accomplished by setting

$$\langle 0|\psi_1\psi_2\psi_{3'}^{\dagger}|i\rangle=-\sum_{S\neq 0}\langle 0|\psi_{3'}^{\dagger}\psi_2|S\rangle\langle S|\psi_1|i\rangle\,. \tag{37}$$

In fact, Eq. (37) is exact. In anticommuting the creation operator and excluding $|0\rangle=|S\rangle$, we have used Eq. (2.24) of Ref. 13. Of course, this factorization is useful only if we limit our attention to (ph) correlations. The approximation is to truncate the set of states $|S\rangle$ to the important collective state(s). (We leave the summation over more than one such phonon implicit.)

$$\langle 0|\psi_1\psi_2\psi_{3'}^{\dagger}|i\rangle\approx -X_{23'}^{(S)}X_{1S}^{(i)}(Z_1Z_2Z_{3'})^{1/2}, \tag{38a}$$

where we have used the definitions (15) and

$$X_{1S}^{(i)}\equiv\langle S|\psi_1|i\rangle/\sqrt{Z_1}\,. \tag{39}$$

Similarly, we may factor

$$\begin{aligned}\langle j|\psi_1\psi_2\psi_{3'}^{\dagger}|0\rangle&\approx-\langle j|\psi_1|\bar{S}\rangle\langle\bar{S}|\psi_{3'}^{\dagger}\psi_2|0\rangle\\&\equiv -X_{1S}^{(j)}X_{23'}^{(S)}(Z_1Z_2Z_{3'})^{1/2}.\end{aligned} \tag{38b}$$

The complete set of states, $|\bar{S}\rangle$, must correspond to the negative energies of RPA since the coupling is to a hole rather than a particle. [See Eq. (45) for the contour integration which requires this.] In Eq. (38b) we have used the analytic connection between $|S\rangle$ and $|\bar{S}\rangle$:

$$X_{3'2}^{(\bar{S})*}=X_{23'}^{(S)}\,. \tag{40}$$

Equation (21) may now be written

$$\begin{aligned}\bar{\rho}_{123'}^{\gamma}&\approx X_{23'}^{(S)}\bar{\sigma}_4^{\gamma}\Big[\sum_{i(N+1)}\frac{X_{1S}^{(i)}X_4^{(i)}}{E^{(i)}-\omega_\gamma}-\sum_{j(N-1)}\frac{X_{1S}^{(j)}X_4^{(j)}}{E^{(j)}+\omega_\gamma}\Big]\\&\equiv X_{23'}^{(S)}\bar{\rho}_{1S}^{\gamma}\,,\end{aligned} \tag{41}$$

where

$$X_4^{(i)}\equiv\langle i|\psi_4^{\dagger}|0\rangle/\sqrt{Z_4}\,.$$

Equation (20) may be rewritten

$$\tfrac{1}{4}l_{3121'2'3'}=X_{23'}^{(S)}l_{1S,1'S'}X_{2'3}^{(S')*}, \tag{42}$$

where

$$l_{1S,1'S'}\equiv\sum_{\gamma}\frac{\bar{\rho}_{1S}^{\gamma}(\bar{\rho}_{1'S'}^{\gamma})^*}{\omega-\omega_\gamma}\,.$$

The physical significance of Eq. (42) is that both the (ph) and (pph) response functions have been approximated by the product of two (ph) amplitudes and a phonon part.

The final step to obtaining a phonon approximation to the six-point function is to identify the analytical expression for l with $l^{(21)}$, viz.,

$$\tfrac{1}{4}l=\sum_{\gamma}\frac{X\bar{\rho}\bar{\rho}X}{\omega-\omega_\gamma} \tag{43}$$

replaces $l^{(21)}$ in Eq. (26′). If we assume that $h^{(21)}$ does not depend on the unnatural energy variables (which will be the case if F is completely energy-independent), this yields a homogeneous Bethe-Salpeter equation for the amplitudes, $\bar{\rho}_{1S}^{\gamma}$,

$$\bar{\rho}_{1S}^{\gamma}=l_{1S}^{\mathrm{QPC}}(h_{\mathrm{eff}})_{1S,1'S'}\bar{\rho}_{1'S'}^{\gamma}\,, \tag{44}$$

where

$$\begin{aligned}l_{1S}^{\mathrm{QPC}}&=\frac{i}{2\pi}\int g_1(\omega_1)\Big[\frac{1}{\omega-E_S+i\delta}-\frac{1}{\omega+E_S-i\delta}\Big]\\&\quad\times\delta(\omega_\gamma-\omega-\omega_1)d\omega d\omega_1\end{aligned} \tag{45}$$

$$=\sum_{k(N+1)}\frac{\xi_1^{(k)}}{\omega_\gamma-E_1^{(k)}-E_S}+\sum_{l(N-1)}\frac{\xi_1^{(l)}}{\omega_\gamma+E_1^{(l)}+E_S}\,, \tag{46}$$

and

$$(h_{\mathrm{eff}})_{1S,1'S'}=X_{23'}^{(S)}h_{3121'2'3'}^{(21)}X_{2'3}^{(S')*}\,. \tag{47}$$

Equation (43) presumes that the free part (ggg) of l is unimportant. This is a reasonable assumption for bound-state problems, since the energy of three quasiparticles is several MeV greater than that of a phonon-quasiparticle configuration. It also assumes that the quasiparticle and phonon are distinguishable: Even though $Tl^{(21)}T$ is antisymmetric and $l^{(21)}$ includes exclusion-principle violation (EPV) corrections via $h^{(21)}$, there will be double counting and consequently normalization difficulties, when more than one phonon is considered. A simple example is that the configuration $1=a$, $S=bc+\cdots$ has nonzero overlap with $1'=b$, $S'=ac+\cdots$. We shall have to assume, however, that this difficulty can be overlooked.

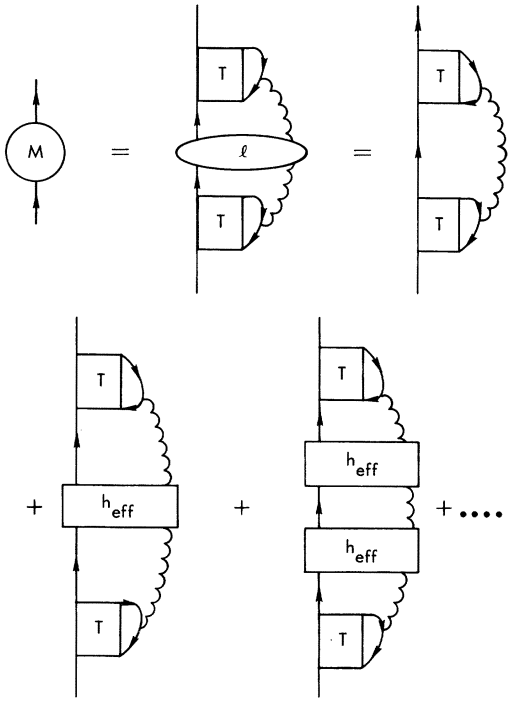

FIG. 11. The phonon approximation for the mass operator after Hartree-Fock and Landau renormalization. T is the Brueckner matrix multiplied by the Landau-Migdal residues. l, which includes the incoming and outgoing straight and wiggly lines, is the phonon approximation for the (pph) response function. h_{eff} is the effective interaction between quasiparticle-phonon configurations. See Fig. 12.

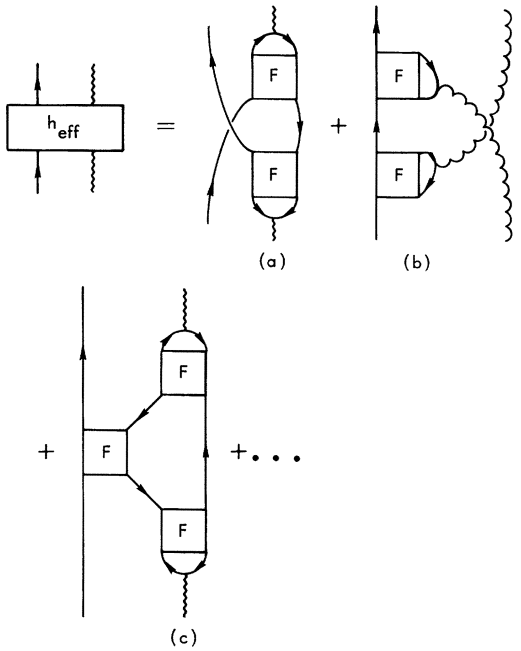

FIG. 12. The effective interaction between quasiparticle-phonon configurations. The incoming and outgoing lines are not part of the interaction. The two different time orderings of the second-order contribution are explicitly shown.

4. Diagonalization

Equations (42) and (44) constitute a QPC prescription for obtaining the six-point function. Equation (19′) and the relation

$$\begin{aligned}\bar{\sigma}_4^{\gamma} &= T_{43'12}\bar{\rho}_{123'}^{\gamma} = T_{43'12}X_{23'}^{(S)}\bar{\rho}_{1S}^{\gamma} \\ &\equiv T_{4,1S}\bar{\rho}_{1S}^{\gamma} \qquad (22'')\end{aligned}$$

define the mass operator in terms of the six-point function. (See Figs. 11 and 12.) We obtain a single diagonalization problem for the poles and residues of the two-point function in several steps: First, Eq. (44) is reduced to a matrix problem. Then Dyson's equation is combined with Eq. (19′) and (22″) to obtain a subsequent matrix problem. Finally, the two matrix diagonalizations are combined into one.

If we define

$$\rho_{kS}^{\gamma} \equiv \frac{\sqrt{\zeta^{(k)}}\,\bar{\rho}_{1S}^{\gamma}}{(\omega_{\gamma} - E_1^{(k)} - E_S)\,l_{1S}^{\text{QPC}}} \qquad (48a)$$

and

$$\rho_{lS}^{\gamma} \equiv \frac{\sqrt{\zeta_1^{(l)}}\,\bar{\rho}_{1S}^{\gamma}}{(\omega_{\gamma} + E_1^{(l)} + E_S)\,l_{1S}^{\text{QPC}}} \,. \qquad (48b)$$

Equations (44) and (46) may then be rewritten

$$\begin{aligned}(\omega_{\gamma} - E_1^{(k)} - E_S)\rho_{kS}^{\gamma} &= [\zeta_1^{(k)}\zeta_{1'}^{(k')}]^{1/2}(h_{\text{eff}})_{1S,1'S'}\rho_{k'S'}^{\gamma} + [\zeta_{1k}^{(k)}\zeta_{1'}^{(l')}]^{1/2}(h_{\text{eff}})_{1S,1'S'}\rho_{l'S'}^{\gamma} \\ (\omega_{\gamma} + E_1^{(l)} + E_S)\rho_{lS}^{\gamma} &= [\zeta_1^{(l)}\zeta_{1'}^{(k')}]^{1/2}(h_{\text{eff}})_{1S,1'S'}\rho_{k'S'}^{\gamma} + [\zeta_1^{(l)}\zeta_{1'}^{(l')}]^{1/2}(h_{\text{eff}})_{1S,1'S'}\rho_{l'S'}^{\gamma} \,. \qquad (49)\end{aligned}$$

The normalization of Eq. (49) is simply that

$$\sum_{kS} |\rho_{kS}^{\gamma}|^2 + \sum_{lS} |\rho_{lS}^{\gamma}|^2 = 1 \,. \qquad (50)$$

This may be obtained from Eqs. (43), (26′), (44)–(46), and (48) using the method for deriving normalizations outlined in the Appendix. If we further take h_{eff} to be independent of the natural energy

variable (a drastic approximation since the graphs in Fig. 12 are clearly retarded) and make use of the fact that

$$(h_{\text{eff}})_{1S,1'S'} = (h_{\text{eff}})_{1'S',1S},$$

it is clear that the Hamiltonian matrix in Eq. (49) is Hermitian and as a result the following unitary transformation holds

$$UHU^{-1} = \delta_{\gamma\gamma'}\omega_\gamma, \tag{51}$$

where

$$U^{-1} = U^{+} = \begin{pmatrix} \rho_{kS}^{\gamma} & \rho_{kS}^{\gamma'} & \cdots \\ \rho_{k'S}^{\gamma} & \rho_{k'S}^{\gamma'} & \cdots \\ \vdots & \vdots & \\ \rho_{lS}^{\gamma} & \rho_{lS}^{\gamma'} & \cdots \\ \vdots & \vdots & \end{pmatrix}$$

and

$$\langle kS|H|k'S'\rangle = \delta_{kk'}\delta_{SS'}(E_1^{(k)} + E_S) + [\zeta_1^{(k)}\zeta_{1'}^{(k')}]^{1/2}(h_{\text{eff}})_{1S,1'S'},$$

$$\langle kS|H|l'S'\rangle = [\zeta_1^{(k)}\zeta_{1'}^{(l')}]^{1/2}(h_{\text{eff}})_{1S,1'S'},$$

and

$$\langle lS|H|l'S'\rangle = \delta_{ll'}\delta_{SS'}(-E_1^{(l)} - E_S) + [\zeta_1^{(l)}\zeta_{1'}^{(l')}]^{1/2}(h_{\text{eff}})_{1S,1'S'}.$$

We may also reduce Dyson's equation (32) to a matrix diagonalization. With the assumption that m is diagonal, one obtains at poles of g_4 with $\omega_4 = E_4^{(i)}$ or $-E_4^{(j)}$, the eigenenergies of the $N \pm 1$ nuclei, respectively,

$$1/g_4^{(1)} = (\omega_4 \mp E_4) = m_4 = \sum_\gamma \frac{(\bar{\sigma}_4^\gamma)(\bar{\sigma}_4^\gamma)^*}{\omega_4 - \omega_\gamma}. \tag{52}$$

Multiplying both sides by C_4 and defining

$$(\sigma_4^\gamma)^* C_4 \equiv (\omega_4 - \omega_\gamma)C_\gamma$$

we obtain the diagonalization problem:

$$VSV^{-1} = \delta_{ii'}\omega_4, \tag{53}$$

where

$$V^{-1} = \begin{pmatrix} C_4^{(i)} & C_4^{(i')} \cdots C_4^{(j)} \cdots \\ C_\gamma^{(i)} & C_\gamma^{(i')} \cdots C_\gamma^{(j)} \cdots \\ C_{\gamma'}^{(i)} & C_{\gamma'}^{(i')} \cdots C_{\gamma'}^{(j)} \cdots \end{pmatrix},$$

$$S_{44} = \pm E_4,$$

$$S_{4\gamma} = S_{\gamma 4} = \bar{\sigma}_4^\gamma,$$

and

$$S_{\gamma\gamma'} = \delta_{\gamma\gamma'}\omega_\gamma.$$

If $\bar{\sigma}_4^\gamma$ is energy-independent, then V is unitary. The physical significance of the C numbers is established from the theorem that[9]

$$\zeta_4^{(i)} = \left(1 - \frac{dM}{d\omega}\right)^{-1}.$$

Hence $C_4^{(i)} = X_4^{(i)}$ and, for the $N+1$ nucleus, we have

$$\zeta_4^{(i)} = |X_4^{(i)}|^2 = |\langle i|\psi_4^\dagger|0\rangle|^2/Z_4 \tag{54a}$$

and, for the $N-1$ nucleus,

$$\zeta_4^{(j)} = |X_4^{(j)}|^2 = |\langle j|\psi_4|0\rangle|^2/Z_4. \tag{54b}$$

Equations (51) and (53) constitute matrix diagonalizations for the coupling of quasiparticle-phonon configurations and for the subsequent coupling to quasiparticles. The two problems may be combined by defining H^{QPC}

$$H^{\text{QPC}} = \begin{pmatrix} \pm E_4 & \langle 4|H|kS\rangle & \cdots \langle 4|H|lS\rangle & \cdots \\ \langle k'S'|H|4\rangle & \langle k'S'|H|kS\rangle & \cdots \langle k'S'|H|lS\rangle & \cdots \\ \vdots & \vdots & & \\ \langle l'S'|H|4\rangle & \langle l'S'|H|kS\rangle & \cdots \langle l'S'|H|lS\rangle & \cdots \\ \vdots & \vdots & & \end{pmatrix}, \tag{55}$$

where

$$\langle 4|H|kS\rangle = \sqrt{\zeta_1^{(k)}}\, T_{4,1S}, \tag{56}$$

and $\langle k'S'|H|kS\rangle, \langle l'S'|H|kS\rangle$ were defined after Eq. (51). H^{QPC} is diagonalized by

$$W(H^{\text{QPC}})W^{-1} = \delta_{ii'}\omega_4,$$

where $W = VU$.

Because of the analytical properties of $\bar{p}_{lS}^{\gamma}$ [see Eq. (41)] we may identify the elements of W with the the quasiparticle and quasiparticle-phonon spectroscopic amplitudes defined previously, i.e.,

$$W^{-1} = \begin{pmatrix} X_4^{(i)} \cdots X_4^{(j)} \cdots \\ X_{kS}^{(i)} \cdots X_{kS}^{(j)} \cdots \\ X_{l'S}^{(i)}, \cdots X_{l'S}^{(j)}, \cdots \end{pmatrix}, \tag{57}$$

where

$$\sum_k \sqrt{\zeta_1^{(k)}}\, X_{kS}^{(i)} + \sum_i \sqrt{\zeta_1^{(i)}}\, X_{iS}^{(i)} = X_{1S}^{(i)} = \langle i | \psi_1^\dagger | S\rangle / \sqrt{Z_1}$$

and a similar expression exists for $X_{1S}^{(j)}$. The normalization of the amplitudes $X^{(i)}$ follows from the unitarity of U and V, hence

$$\sum_{kS} |X_{kS}^{(i)}|^2 + \sum_{i'S'} |X_{i'S'}^{(i)}|^2 + |X_4^{(i)}|^2 = 1 . \tag{58}$$

Equations (54)–(58) constitute the odd-mass part of the self-consistent quasiparticle-phonon coupling problem (SCQPC). The resultant energies and residues are also inputs to the same problem. In addition, they depend on the phonon energies and the amplitudes of the even nucleus which depend on the odd nucleus. Thus, there are two self-consistency problems.

C. Renormalized RPA equations

In this section, we present the renormalized equations for the (ph) phonon in the even-even nucleus. In particular, we wish to illustrate the manner in which the splitting of the single-particle strength alters the appearance of the usual RPA equations. Our derivation is similar in spirit to that of Zawischa and Werner[17]; however, the prescription for taking the backwards residues into account will be made explicit.

At the poles ($\omega \approx E_N^{(S)} - E_N^{(0)}$) of the renormalized (ph) response function [see Eq. (14)], we have the following homogeneous Bethe-Salpeter equation

$$l_{1234} = -ig_1 g_3 F_{1635} l_{5264} .$$

Substituting the analytical expression for the unrenormalized response function, L, given in Eq. (10) and canceling the common terms, yields

$$\chi_{13}^{(S)}/(Z_1 Z_3)^{1/2} = -ig_1 g_3 F_{1635} \chi_{56}^{(S)}/(Z_5 Z_6)^{1/2} . \tag{60}$$

In Eq. (60), the repeated indices imply integration over the unnatural energy variables as well as summation over the single-particle labels. If we assume F_{1635} is unretarded, i.e., no energy dependence, the integral may be performed above the axis to yield

$$\chi_{13}^{(S)}/(Z_1 Z_3)^{1/2} = g_1 g_3 \sum_{56} F_{1635} X_{56}^{(S)} , \tag{61}$$

where $X_{56}^{(S)} \equiv \rho_{56}^{(S)}/(Z_5 Z_6)^{1/2}$ as defined previously. The 1, 3 energy variables may also be integrated to give

$$X_{13}^{(S)} = A_{13}(E_S) \sum_{56} F_{1635} X_{56}^{(S)} , \tag{62}$$

where

$$A_{13}(E_S) \equiv \frac{-i}{2\pi} \int g_1(\omega_1) g_3(\omega_1 - E_S) d\omega_1 . \tag{63}$$

If we substitute Eq. (33b) for the g_1 and g_3 we obtain

$$A_{13}(\omega) = \sum_{\substack{i(N+1) \\ j(N-1)}} \frac{\zeta_1^{(i)} \zeta_3^{(j)}}{\omega - E_1^{(i)} - E_3^{(j)}} - \sum_{\substack{k(N-1) \\ l(N+1)}} \frac{\zeta_1^{(k)} \zeta_3^{(l)}}{\omega + E_1^{(k)} + E_3^{(l)}} . \tag{64}$$

Clearly $A_{13}(E_S) = A_{31}(-E_S)$.

It is usual in (ph) RPA problems to divide the Eq. (62) into two coupled equations for the amplitudes, X_{ph} and X_{hp} (usually called Y_{ph}) where p and h refer to those levels whose unperturbed energies are above and below the Fermi surface. In this manner, each configuration, (ph), needs to be considered only once in the summation. If backwards amplitudes are included in the analytic expression for the renormalzied two-point function, the configuration (pp′) has a finite amount of particle-hole strength. In this most general case, Eq. (62) reduces to the following matrix problem:

$$\begin{aligned} P_{1234} X_{42} + Q_{1234} Y_{42} &= X_{13}/A_{13}(\omega) , \\ Q_{1234} X_{42} + P_{1234} Y_{42} &= Y_{13}/A_{13}(-\omega) , \end{aligned} \tag{65}$$

where

$$P_{1234} = \frac{F_{1234}}{(1+\delta_{13})^{1/2}(1+\delta_{42})^{1/2}} = Q_{1432} \text{ and } Y_{42} = X_{24} ,$$

and the configurations (13) include the following three types: $(p_1 h_3)$, $(p_1 p_3)$, $(h_1 h_3)$ with $1 \leq 3$. The matrices P and Q are symmetric.

The normalization condition for the amplitudes X_{13}, Y_{13} follows from Eqs. (10), (14), and (65) and may be derived as in the Appendix. The result (which differs from that of Ref. 17) is

$$\sum_{13} [\, |X_{13}^{(S)}|^2 W_{13}(E_S) - |Y_{13}^{(S)}|^2 W_{13}(-E_S)] = 1 , \tag{66}$$

where the weighting matrix W_{13} is defined

$$W_{13}(E_S) = \left. \frac{dA_{13}^{-1}}{d\omega} \right|_{\omega = E_S} .$$

Thus, the inclusion of more than one pole in the one-quasiparticle Green's functions results in two essential changes in the (ph) RPA matrix problem. First, the usual $(\omega - E_1 - E_3)$ factor is replaced by $1/A_{13}(\omega)$, where A_{13} is the distribution of particle-hole strength defined in Eq. (64). Thus the solution cannot be obtained by simple diagonalization.

Secondly, (pp) and (hh) configurations must be included, considerably increasing the dimensions of the problem. An additional point of interest is that since

$$I \neq \frac{dM}{dG},$$

i.e., I (or F) is energy-independent while M is complicated by QPC and thus energy-dependent, the even-even equations proposed here are not conserving,[18] and spurious states are not isolated as in the usual RPA.

III. DISCUSSION OF SCQPC AND COMPARISON TO OTHER THEORIES

The equations which we have given prescribe a method for calculating both even and odd nuclei with quasiparticle-phonon coupling included. In the odd nucleus, the symmetric matrix problem includes exclusion-principle corrections due to ground-state correlations (see Fig. 5) by coupling the eigenstates of the $N \pm 1$ nuclei. Many-phonon contributions to the mass operator [see Fig. 3(d)] are included by using dressed quasiparticle intermediates. The higher-order diagrams, such as those of Fig. 3(a) and (c), are calculated by defining an h_{eff} between quasiparticle-phonon configurations, i.e., the off-diagonal elements in all but the first row or column. In particular, these correct for Pauli principle violations in the propagation of a quasiparticle and quasiparticle-hole pair. [Note that Figs. 12(a) and 3(a) are identical.] In the even problem, the phonon is improved by including QPC anharmonicities. The backwards QPC dressing pushes the phonon higher, because of exchange with ground-state clusters, while frontwards QPC lowers the energy by mixing in certain 2p-2h configurations.

The odd and even problems we have derived are conceptually and mathematically equivalent to a theory in which basis states are formed by coupling eigenstates of the even and odd systems. For example, in the odd nucleus one might define a set of quasifermions $b_\alpha^\dagger$, b_β such that[19]

$$b_4^\dagger = \psi_4^\dagger,$$

where $\psi_4^\dagger$ creates quasiparticles, and

$$b_{kS}^\dagger = \psi_k^\dagger Q_S^\dagger,$$

where $\psi_k^\dagger$ creates the kth eigenstate containing an appreciable fraction of the single-particle strength, 1, and $Q_S^\dagger$ creates a phonon. The SCQPC equations may be derived by considering the dynamical equation for

$$L_{\alpha\beta} \equiv -i\langle 0 | T\{ b_\alpha(t) b_\beta^\dagger(0)\} | 0\rangle . \tag{67}$$

The notion of an operator which creates a fermion eigenstate (we might call it a spectron) is also useful for the even nuclei. The renormalized RPA equations of Sec. II C are identical to the equations we might derive by considering the quasiboson

$$A_{ij}^\dagger \equiv \psi_i^\dagger \psi_j ,$$

Thus, in the odd nucleus, we can form basis states by coupling exact eigenstates of the odd system to exact eigenstates of the even system; and, in the even nucleus, we can form basis states by coupling two exact eigenstates of the odd system. The idea of coupling known physical states to one another to obtain the most important parts of a more complicated physical state is not new. However, its equivalence to Green's functions techniques has not been pointed out before this time.

From the point of view of many-body theory we have employed a different prescription for the self-energy than is usual. Here, we have

$$M = \tfrac{1}{4} V L^{(3)} V . \tag{16}$$

Normally,

$$M = \tfrac{1}{2} VGGG\Gamma .^{9,11,14} \tag{68}$$

Equation (68) may be rewritten by using Eqs. (8) and (9)

$$M = \tfrac{1}{2} V L^{(2)} G I . \tag{69}$$

One may replace V by I, as is done without justification in Refs. 11 and 20, if the ladder approximation is adopted, viz., V, I become $T = F$ after renormalization. Note such a radical approximation [e.g., F must now be antisymmetric, unlike Migdal's phenomologically based (ph) interaction] is the only way to enforce symmetry between the two "one-three" vertices (i.e., those which connect the self-energy to the propagator line). All of the complicated contributions to $L^{(3)}$ of Eq. (16) have been incorporated into the I block of Eq. (69) and then approximated away. In addition Eq. (69) requires a factor of $\frac{1}{2}$ which is normally thrown away with no justification, other than its absence in a theory where bosons and fermions are coupled.[21]

The approximation

$$M = F(L^{(2)}G)F \tag{70}$$

has been employed in several calculations.[17,22] Their theory differs from ours in three respects: First, the "one-three" vertices are the same as the (ph) interaction. Secondly, backwards correlations are not included. Finally, higher-order diagrams, for example EPV corrections, are not included.

Brown, Evans, and Thouless[4] have performed a coupled calculation of the $N \pm 1$ nuclei for $N = {}^{16}$O, ^{40}Ca. In that work, self-energy intermediates are

not dressed, self-consistency with the even nucleus is neglected, and higher-order diagrams are not calculated. However, this is the only previous calculation to our knowledge, which correctly includes the backwards correlations.

Our matrix diagonalization problem (including backwards diagrams) is similar in spirit to the spectral decomposition theories of Goswami, Nalcioglu, and Sherwood[23] and also those of Dreiss *et al.*[24] and Malov,[25] when pairing correlations are neglected. In each of these studies, the even and odd nuclei are calculated self-consistently, although in the even nucleus the amplitudes and phonon energy are solved for by spectral decomposition. Several of the third-order contributions to h_{eff} between quasiparticle-phonon configurations are included, although all of the second-order terms are neglected. Also intermediates in the self-energy are not dressed. The main difference when compared to the SCQPC, presented in this paper, is in the calculation of the backwards amplitudes. In our work, the backwards amplitude for ^{209}Pb, for example, is $\psi_h|S\rangle$, a ^{207}Pb configuration, with diagonal energy $= -E_h - E_S = -(\lambda - \epsilon_h) - E_S$. In Refs. 23–25 the amplitude is $\psi_h^\dagger|S\rangle$. Thus, the $N \pm 1$ nuclei are not coupled, but rather the $N+1$ nucleus is coupled with a redundant set of $N+1$ basis states. This redundancy arises because $\psi_p^\dagger|0\rangle$ and $\psi_h^\dagger|S\rangle$ have an overlap which is proportional to the RPA amplitude, X_{ph}. This difference manifests itself algebraically in the diagonal matrix element for this configuration: $-E_h + E_S$. This peculiarity of the spectral decomposition theories has three undesirable consequences. One is that it is difficult to separate "physical" odd-mass states; secondly, since $(-E_h + E_S)$ can become positive in energy and even comparable to E_p, the "pushing up effect" will be exaggerated. This explains the unrealistically large effect found in Ref. 26, which seems to be reduced when proper orthogonalization procedures are developed.[27] Finally, it is difficult to interpret these backwards amplitudes, in contrast to those of Green's function theories. Nalcioglu[28] has speculated that they correct for higher-order EPV processes, e.g., Fig. 3(a), although this is not self-evident.

The weak coupling equations derived by Arita and Horie[29] from the shell model are similar to those given here but also include configurations formed by coupling a pairing vibration to a hole. The second- and third-order interactions given by Figs. 12(a) and 12(c) are approximately calculated, and the overlap between configurations, neglected here, is taken into account. However, backwards correlations and self-consistency of the intermediate particle are not included. In the work of Bes and Broglia,[30] only the 2p-1h states are described. The basis set consists of particles coupled to phonons and also pairing vibrations coupled to holes. The coupled particles are not explicitly dressed, although the experimental single-particle energies may be taken as such. Coupling to seniority-one degrees of freedom is included by generalizing the effective interaction. The Hamiltonian which acts between the basis states is second order in the QPC coupling interaction, e.g., Fig. 12(a). In a recent paper by Schuck, Villars, and Ring[31] the generalized equations for the 2p-1h problem (RPA-like shell model for $N \pm 1$ nuclei) are derived using the memory function approach. The (pp) and (ph) RPA normalization formulas are inserted in the problem to obtain a scheme which embraces both "pairing vibration + hole" and "(ph) vibration + particle" configurations. The effective interaction is second order. The coupling to seniority-one basis states is neglected.

ACKNOWLEDGMENT

The authors wish to thank Dr. J. Winter and Dr. O. Nalcioglu for valuable comments and discussion. One of us (J.D.I.) expresses his gratitude to Dr. M. Weigel for the hospitality extended to him by the Theoretical Physics Institute at Munich.

APPENDIX

In this section, we outline the procedure for normalizing the homogeneous Bethe-Salpeter equations, which arise both for even- and odd-mass nuclei. The normalization derives essentially from the inhomogeneous term; that is, the correlated part of the equation cannot be multiplied by an arbitrarily large constant without taking account of the uncorrelated part.

The Eqs. (A1) to (A3) provide the starting point. The variables have been collected into two single indices which represent all the quantum numbers relevant to the "before" and "after" configurations of one or more particles. The "unnatural" energy variables have been integrated out, and the interactions are assumed to be independent of energy. Near a pole, the Green's (or response) function may be written

$$L_{12} = L_{12}^R + \frac{X_1 X_2}{\omega - E_S} . \tag{A1}$$

The inhomogeneous Bethe-Salpeter equation for the same function is

$$L_{12} = A_1 \delta_{12} + A_1 I_{13} L_{32} , \tag{A2}$$

and the homogeneous Bethe-Salpeter equation is

$$\frac{X_1}{A_1(E_S)} = I_{13} X_3 . \tag{A3}$$

Substituting (A1) into (A2), one obtains the inhomogeneous equation which is valid in the vicinity of a pole,

$$L^R_{12}+\frac{X_1X_2}{\omega-E_S}=A_1\delta_{12}+A_1I_{13}L^R_{32}+A_1I_{13}\frac{X_3X_2}{\omega-E_S}\,. \tag{A4}$$

Since the A_1 term also contains an energy dependence, we expand in the vicinity of the poles of L,

$$A_1(\omega)=A_1(E_S)+(\omega-E_S)\frac{dA_1}{d\omega}\bigg|_{\omega=E_S}\,. \tag{A5}$$

Equation (A4) becomes

$$L^R_{12}+\frac{X_1X_2}{\omega-E_S}=A_1\delta_{12}+A_1I_{13}L^R_{32}+A_1(E_S)I_{13}\frac{X_3X_2}{\omega-E_S}+\frac{dA_1}{d\omega}\bigg|_{\omega=E_S}I_{13}X_3X_2\,.$$

Equation (A3) permits the cancellation of the pole terms to obtain

$$L^R_{12}=A_1\delta_{12}+A_1I_{13}L^R_{32}+\frac{dA_1}{d\omega}\frac{X_1}{A_1}X_2\,.$$

Multiplying by X_1/A_1 (summation over repeated indices is implied), and noting that

$$\frac{X_1}{A_1}A_1I_{13}L^R_{32}=\frac{X_3}{A_3}L^R_{32}=\frac{X_1}{A_1}L^R_{12}$$

we obtain finally

$$0=X_2+\sum_1\frac{X_1}{A_1}\frac{dA_1}{d\omega}\frac{X_1}{A_1}X_2\,.$$

Hence

$$1=\sum_1|X_1|^2\frac{dA_1^{-1}}{d\omega}\bigg|_{\omega=E_S}\,. \tag{A6}$$

In the simplest cases, e.g., Tamm-Dancoff approximation, $A_1^{-1}=\omega-E_1$ and the derivative is unity. In RPA, the derivative is ±1 depending on whether the configuration is "frontwards" or "backwards."

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*This work was supported in part by the U. S. Atomic Energy Commission and by the Bundesministerium für Bildung und Wissenschaft.

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