# Coupled-channel-Born-approximation study of $(p, t)$ reactions leading to two-phonon states of vibrational nuclei*

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(Received 2 July 1973)

A coupled-channel-Born-approximation (CCBA) study is made of $(p, t)$ reactions leading to two-phonon states of vibrational nuclei. A detailed numerical analysis is performed, particularly for the $^{116}$Cd$(p, t)$ $^{114}$Cd reaction with the incident energy $E_p = 28$ MeV. It is shown that many features observed in the reactions, which cannot be explained by the distorted-wave Born approximation (DWBA), can successfully be accounted for by CCBA. It is also shown that three-step processes play an essential role in the successful explanation of the observed data, particularly the cross sections of the $0^+$ and $4^+$ two-phonon states.

$$\left[ \text{NUCLEAR REACTIONS } {}^{116}\text{Cd}(p, t), \ E = 28 \text{ MeV; calculated } \sigma(\Theta). \right]$$

## I. INTRODUCTION

The two-nucleon transfer reaction has proved to be a very useful tool in obtaining nuclear structure information, particularly on the correlations of the transferred pair in the nucleus and also collective modes associated with these pair correlations.[1] Actually, by utilizing this reaction, many pairing-vibrational states were identified in near-magic nuclei, which may be one of the most important successes of this new reaction method. [1] Of course the reaction can be, and indeed has been, applied to study the nuclear structure of many other classes of nuclear excitation.

So far most of these studies of the two-nucleon transfer reactions were based on the distorted-wave Born approximation (DWBA),[2] which assumes a direct, single-step mechanism for the two-nucleon transfer process. Recent experimental evidence, however, suggests that the single-step mechanism is not necessarily the dominant reaction mechanism of the two-nucleon transfer reaction. For example, angular distributions of the $(p, t)$ reaction leading to the first excited $2^+$ ($2_1^+$) state of rotational[3] and vibrational[4] nuclei are often found to show an angular pattern which is quite different from that of an $L = 2$ DWBA calculation. Unnatural-parity states, which are not allowed to be excited via direct, single-step mechanism, are also found to be excited rather strongly.[5] These observations demonstrate that a multistep process plays an important role in the two-nucleon transfer reaction.

Theoretically, it has become feasible recently to calculate multistep processes arising from the inelastic scattering processes within the framework of the coupled-channel-Born-approximation (CCBA)

formalism.[6,7] Thus, Glendenning and his collaborators[8] have performed CCBA calculations of $(p, t)$ reactions on some of vibrational nuclei, demonstrating that the effects are quite important. The same authors[9] and also Tamura et al.[10] then made CCBA analysis of actual observed data of the $(p, t)$ reactions on deformed nuclei,[3] explaining successfully the observed anomaly of the $2_1^+$ angular distribution mentioned above. Yagi et al.[4] also made a similar analysis of the $(p, t)$ reaction on vibrational nuclei, obtaining the same success. Further, it was shown that the excitation of unnatural-parity states in the $^{22}$Ne$(p, t)^{20}$Ne reaction[5] can also be explained well by CCBA.[11]

In the present work, we extend such CCBA studies of the $(p, t)$ reaction to those leading to the two-phonon states of vibrational nuclei. The experimental data to be used have recently become available due to Comfort et al.,[12] who did a very accurate measurement of the cross section of these two-phonon states for three cases, $^{112, 114, 116}$Cd$(p, t)^{110, 112, 114}$Cd.

The analysis of the above data for the two-phonon states is particularly interesting since it provides a further detailed test of CCBA; indeed, as is expected and also will later be shown, three-step processes play a very essential role in the excitation of the two-phonon states. This was not so in the cases studied before, where the two-step processes are the dominant processes among the various conceivable multistep processes. Therefore, one can test up to the next higher order in the CCBA amplitude in the present case.

In addition to interest from the point of view of reaction mechanism, the study of the reaction is also interesting from the nuclear structure point of view. The structure of the two-phonon states,

particularly their detailed dynamical nature, is
not well known yet, in spite of the fact that a great
number of papers have so far been devoted to the
study of this subject. This might partly be due to
the fact that we have not enough experimental
means to explore these states; indeed, the dynam-
ical nature of the states has so far been studied
only by means of inelastic scattering[13] and Cou-
lomb excitation,[14] which tell us only the particle-
hole (p-h) correlation aspect of the state. The
two-nucleon transfer reactions can provide sup-
plemental information which is thus very valuable.
However, in order that such a spectroscopic study
of the $(p, t)$ reaction become feasible, one must
have a reaction theory which can really describe
the reaction mechanism. It is the aim of the
present work to show that CCBA, together with a
phenomenological microscopic model of nuclear
structure, can indeed describe fairly well the
observed data of the $(p, t)$ reaction.

In Sec. II we first summarize the experimental
data that are to be analyzed in the present work.
A few remarks are given on the data, particularly
on those aspects which cannot be explained by
DWBA. In Sec. III a discussion is given of the
form factors that are used in the present calcu-
lations. We introduce a rather phenomenological
treatment of the form factors used in the calcula-
tion, which is given in detail in this section, to-
gether with the results of the actual evaluation of
the form factors. Sec. IV is devoted to a discus-
sion of the optical-potential parameters used in
the present calculations. Results of actual nu-
merical calculations are then shown in Sec. V,
where the results are also compared with experi-
ment. Finally, in Sec. VI concluding remarks are
given.

## II. EXPERIMENTAL DATA

We summarize in this section the experimental
materials that are to be analyzed here, and give
a brief discussion on them. As was already men-
tioned in the introduction, we utilize the data
taken by Comfort et al.,[12] who measured the cross
sections of $(p, t)$ reactions from three Cd target
nuclei, $^{112, 114, 116}Cd$. The incident energy used was
$E_p = 27.9$ MeV. In the actual experiments the mea-
surement of the cross sections was made for
various final states up to 3–4-MeV excitation
energy. In the present work, however, we are only
interested in the cross sections of the ground
$0^+$ $(0_g^+)$, the one-phonon $2^+$ $(2_1^+)$, and the two-phonon
$0^+$ $(0_1^+)$, $2^+$ $(2_2^+)$, and $4^+$ $(4_1^+)$ states.

The experimental cross sections of these states
obtained for three cases of target nuclei are all
summarized in Figs. 1 and 2, separately for each

final spin state. In Fig. 1, the cross sections of
the $0_g^+$, $2_1^+$, $2_2^+$, $4_1^+$ states are presented. Now,
we observe one remarkable feature in Fig. 1,
which is that if the final spin state is fixed all
the cross sections taken for these different target
nuclei have almost the same magnitude as well as
the shape. There is one exception to this, which
is seen in the $2_2^+$ cross section of the $^{112}Cd(p, t)$-
$^{110}Cd$ reaction. In this case the cross section is
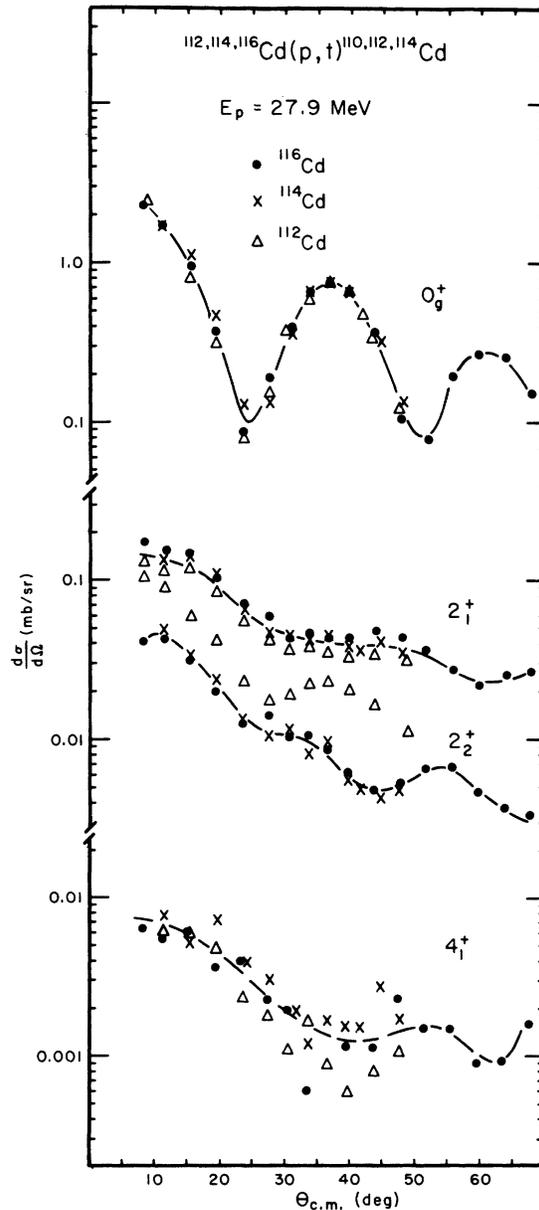


FIG. 1. Experimental differential cross sections for
$(p, t)$ reactions leading to the $0_g^+$, $2_1^+$, $2_2^+$, and $4_1^+$ states
in $^{114-110}Cd$.

somewhat larger than the two other $2_2^+$ cross sections. This might, however, be due to the fact that the $2_2^+$ state is a doublet with the $0_1^+$ state[15] and therefore the observed cross section would involve some amount of the $0_1^+$ cross section. If one really assumes that this is the case, i.e., that the $2_2^+$ cross section of $^{110}$Cd is the same as those of the two other $2_2^+$ states, one can estimate the $0_1^+$ cross section by subtracting from the observed data an average $2_2^+$ cross section of the neighboring nuclei. The $0_1^+$ cross section for $^{110}$Cd given in Fig. 2 is estimated in this way.

Now, the similarity observed in the $0_g^+$, $2_1^+$, $2_2^+$, and $4_1^+$ cross sections and demonstrated clearly in Fig. 1 seem to indicate that the wave functions of these states are more or less very similar for all the Cd isotopes concerned. This is also consistent with evidence obtained from other experimental data, such as the energies and also the various $E2$ transition rates.[14] The above similarity further suggests that it is enough, for the purpose of the present work, to carry out the calculations only for one example of the reactions, particularly for $0_g^+$, $2_1^+$, $2_2^+$, and $4_1^+$ states because if a reasonable explanation is established for one



FIG. 2. Experimental differential cross sections for $(p, t)$ reactions leading to $0_1^+$ states of $^{114-110}$Cd. The $0_1^+$ cross section of the $^{110}$Cd was extracted from the doublet peak of the observed data in a manner explained in the text.

case, it can be applied to other cases literally. In the present work, we actually perform the calculations specifically for the $^{116}$Cd$(p, t)^{114}$Cd reaction.

In contrast to the $0_g^+$, $2_1^+$, $2_2^+$, and $4_1^+$ cross sections, the $0_1^+$ cross sections shown in Fig. 2 exhibit a rather dramatic but regular change in both their shape and magnitude with the target mass number. The angular pattern of the cross sections shifts to forward angles, accompanying an increase of the magnitude with decreasing mass number. The observed change in the cross section, particularly the change of the shape with the mass number, cannot of course be explained by DWBA.

In connection with this it may be interesting to note that the position of the deep minimum observed at about $34°$ in the $0_1^+$ cross section of the $^{116}$Cd$(p, t)^{114}$Cd reaction is very close to the maximum (observed at about $36°$) of the $0_g^+$ cross section. This implies that the angular distribution of the $0_1^+$ cross sections looks almost out of phase to that of $0_g^+$. This feature also cannot be explained by DWBA.

Besides the points discussed above, there are at least three other remarkable features that are also difficult to explain by DWBA: (i) There is a marked difference between the two $2^+$ ($2_1^+$ and $2_2^+$) cross sections; (ii) the angular distribution of the $4_1^+$ state is very different from that of the $L = 4$ DWBA (which are calculated for instance in Ref. 16), and (iii) the $2_1^+$ cross section also shows an angular pattern which is different from an $L = 2$ DWBA.[12] We shall show later that all these features in the observed cross sections are explained very well by CCBA.

While not directly related to the difficulty with which DWBA meets, it might be worthwhile to note here that the two-phonon $2^+$ cross section is larger by about a factor of 5 than those of other two-phonon $0^+$ and $4^+$ states. We shall try to give a possible explanation of this feature of the observed data.

## III. FORM FACTORS

### A. Preliminary remarks

As was already mentioned in the introduction, we introduce a rather phenomenological treatment of the form factors used in the present CCBA calculations. Thus, we first outline it before going into details of the actual numerical calculations, which are performed in the following subsection (III B). From consideration of the angular momentum and parity conservation, we expect to have 45 nonvanishing form factors in all between the ground, one-phonon, and two-phonon states of the initial target and final residual nuclei. At the
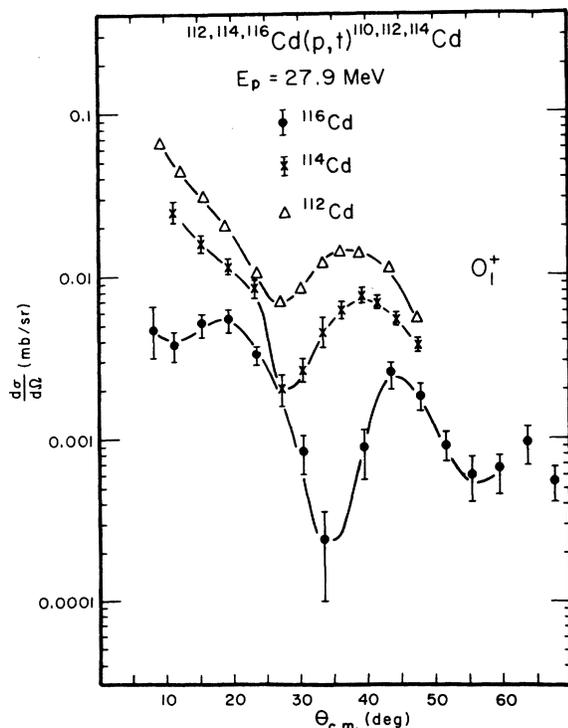
present moment, it is very difficult to derive all these form factors unambiguously, particularly those related to the two-phonon states, since the wave functions of these states are not well known.

Thus, we are more or less forced to introduce an approximation for the treatment of the form factors. The approximation that we particularly use in this work is that we start with the following set of 7 form factors (we call it conveniently set I), i.e., 5 $L=0$ form factors of transitions between the same $I_n^+$ states of both the initial and final nuclei, and 2 $L=2$ form factors of the $0_g^+$-$2_1^+$ and $2_1^+$-$0_g^+$ transitions. Other form factors are then treated as phenomenological, to be fixed from experiments; namely, additional form factors are introduced only when they are required in order to obtain a better fit to experiment.

As will be shown later, however, most of the experimental data can be reproduced very nicely without introducing any additional form factor. Thus, most of the calculations in the present work are performed by using only the form factors of the set I. When an additional form factor is introduced, however, it will be explained at the stage when it is actually used.

The above choice of starting form factors was made based on the fact that these are the ones expected to be very important. Particularly, this is so for the $L=0$ form factors for the reason that the neutrons in Cd isotopes considered here are in a superconductive state.[1,17] These form factors may then be calculated by using the BCS wave functions derived from a simple pairing-force model, which enjoyed some success to explain observed collective features of $(p, t)$ reactions.[1] As will be discussed later (Sec. III B), we indeed calculate the $L=0$ form factors with a wave function which is essentially the same as that of BCS.

Before proceeding, we want to remark on the $L=2$ form factors considered above, particularly on the form factor of the direct, $0_g^+$-$2_1^+$ transition. As will be shown later, the calculated form factor turned out to be very small, implying that the direct $2_1^+$ excitation is very weak. Also, the calculation involves a fairly large theoretical uncertainty. Due to this, we show also in the present work calculations which include all the form factors of the set I, except the direct, $0_g^+$-$2_1^+$ transition form factor; we call this set of form factors, which exclude the direct $0_g^+$-$2_1^+$ form factor, set II.

Another remark that might be worthwhile to make here is that the smallness of the direct, $0_g^+$-$2_1^+$ form factor justifies the present approximate treatment of the form factors. The direct $2_1^+$ excitation is the process that creates a phonon during the $(p, t)$ processes. The smallness of the

form factor of this phonon-creation process simply means that the form factor for the one-phonon-to-two-phonon-state transitions will also be small. Further, it is physically reasonable to consider that the direct excitation of the two-phonon states from the target ground state by the $(p, t)$ process should be small.[8,12]

### B. Calculations

In order to calculate the form factor, use is made of a formula [2]

$$F_L(I_n^+ - I_m^+; r) = \begin{cases} \sum_N G_N u_{NL}(r) & r \leq r_m \\ \\ h_L(r) & r > r_m, \end{cases} \quad (1)$$

where $u_{NL}(r)$ is the harmonic-oscillator (H.O.) wave function, $h_L(r)$ the Hankel function, and $G_N$ the structure function that carries all the nuclear structure information, and is calculated from nuclear wave functions. As is seen, the form factor is constructed first with help of the H.O. wave function in the interior region of the nucleus, which is then joined smoothly onto a Hankel function at appropriate radius $r_m$.

In order to calculate the $G_N$ factor, we assume in this work wave functions of the $0_g^+$ and $2_1^+$ states which are very similar to those used before[18] to describe those states in $^{140}$Nd which have two neutron holes. The nucleus considered now has two proton holes, instead of the two neutron holes, but the wave functions may be given in a very similar manner as[19]

$$|0_g^+\rangle = [\alpha_1 B_0^\dagger + \alpha_2 (B_2^\dagger C_2^\dagger)_0 + \alpha_3 B_0^\dagger (C_2^\dagger C_2^\dagger)_0]|0\rangle,$$

$$(2a)$$

$$|2_1^+\rangle = [\gamma_1 B_2^\dagger + \gamma_2 B_0^\dagger C_2^\dagger + \gamma_3 (B_2^\dagger C_2^\dagger)_2]|0\rangle. \quad (2b)$$

Here $B_0^\dagger$ and $B_2^\dagger$ are, respectively, the creation operators of the $0^+$ and $2^+$ pairing-vibrational phonons that describe the excitations of the two-proton holes, while $C_2^\dagger$ is the creation operator of the collective, p-h vibrational phonon. This p-h vibrational phonon represents the excitation of the core, the rest of the nucleus other than the two-proton holes. Since the proton shell is closed, the excitation of the core is of course mainly due to that of the neutron in the open shell. The p-h phonon, $C_2^\dagger$, is thus mainly made of the neutron two-quasi-particle states. $|0\rangle$ in Eq. (2) is a vacuum state of the phonons and describes the ground state of the core. Except for the ground-states correlations created by our use of random-phase approximation (RPA) in constructing the phonon operators (see the discussion given below), $|0\rangle$ essentially describes the proton closed-shell

TABLE I. Single-particle states and their energies used in the wave function and form factor calculations. The pairing-force strengths of the monopole and quadrupole types, $(G)$, as well as the $Q$-$Q$ forces, $(F)$ are also listed in the table.

| | Proton | | Neutron | |
|---|---|---|---|---|
| $l_j$ | $E_{1j}$ (MeV) | | $l_j$ | $E_{1j}$ (MeV) |
| $0f_{5/2}$ | $-5.41$ | | $0f_{5/2}$ | $-5.53$ |
| $1p_{3/2}$ | $-3.01$ | | $1p_{3/2}$ | $-5.10$ |
| $1p_{1/2}$ | $-2.71$ | | $1p_{3/2}$ | $-4.80$ |
| $0g_{9/2}$ | $-2.41$ | | $0g_{9/2}$ | $-4.50$ |
| $1d_{5/2}$ | $2.95$ | | $1d_{5/2}$ | $0.0$ |
| $0h_{11/2}$ | $4.35$ | | $0g_{7/2}$ | $0.25$ |
| $1d_{5/2}$ | $5.05$ | | $2s_{1/2}$ | $2.18$ |
| $2s_{1/2}$ | $5.37$ | | $0h_{11/2}$ | $2.24$ |
| | | | $1d_{3/2}$ | $2.50$ |
| | | | $1f_{7/2}$ | $6.40$ |
| | | | $2p_{3/2}$ | $7.30$ |
| | | | $0h_{9/2}$ | $8.30$ |
| | | | $1f_{5/2}$ | $10.20$ |
| | | | $2p_{3/2}$ | $12.50$ |
| | | | $0i_{13/2}$ | $15.50$ |
| $G_0$ (MeV) | $0.180$ | | $0.128$ ($^{114}$Cd) | $0.135$ ($^{116}$Cd) |
| | | | $0.135$ ($^{116}$Cd) | |
| $G_2$ (MeV) | $0.096$ | | $0.045$ | |
| $F_2$ (MeV) | $0.050$ | | $0.050$ | |
| $F_2^{np}$ (MeV) | | $0.060$ | | |

core and the neutrons in BCS vacuum state.

All the phonon operators are obtained by using RPA, together with the simple quadrupole-quadrupole $(Q$-$Q)$ and the (monopole and quadrupole) pairing-force model. The mixing of various components in the wave functions of Eqs. (2a) and (2b) is then derived by diagonalizing the $H_{31}$ term of the $Q$-$Q$ interaction, which has not been taken into account in constructing the phonon operators, $C_2^\dagger$, $B_0^\dagger$, and $B_2^\dagger$.[18] With these approximations, and also with the help of the parameters listed in Table I, the coefficients in Eq. (2) were found to be $\alpha_1 = 0.82$, $\alpha_2 = 0.40$, $\alpha_3 = 0.18$, $\gamma_1 = 0.63$, $\gamma_2 = 0.64$, and $\gamma_3 = 0.16$. The structure factors of the $0_g^+$-$0_g^+$, $0_g^+$-$2_1^+$, and $2_1^+$-$0_g^+$ transitions obtained by using the above wave functions are presented in Table II. The form factors of these transitions will then be calculated by using Eq. (1).

The form factors of other $L = 0$ transitions (between one-phonon and also two-phonon states) may be obtained from the $0_g^+$-$0_g^+$ transition form factor

[we denote it by $F_0(r)$] by making a correction due to the blocking effect, which originates from the presence of the quasiparticles in the excited phonon states, reducing somewhat the magnitude of the form factor. The importance of the blocking effects in the $(p, t)$ processes was recently noted in $(p, t)$ on odd-$A$ nuclei.[20] The form factors may then be written by

$$F_0(I_n^+ - I_n^+; r) = b_n(I_n^+)F_0(r),   (3)$$

where $b$ describes the blocking effect. We made a rough estimation of the $b$ value for the $2_1^+$ state by using the $2_1^+$-state wave function of Eq. (2b). We then get $b(2_1^+) = 0.85$. For the case of the two-phonon states the $b$ value might be somewhat smaller, but here we simply assume that the $b$ values are the same as that of the one-phonon state.

The following remarks may be worthwhile here: (i) The calculated $L = 2$, $G_N$ factors are rather small, particularly for the $0_g^+$-$2_1^+$ transition. This means that the strength of the direct $2_1^+$ excitation is weak; indeed we shall show later that the DWBA predicts a too small cross section to explain the observed cross section. This weakness of the direct $2_1^+$ excitation results from the fact that the effects of the ground state correlations contribute destructively for the $(p, t)$ transition processes. This is in contrast to the effect on the $E2$ transition rate.[17] Another reason for the reduction is that the $2_1^+$ state has significant mixing of the proton configuration [the first term in Eq. (2b)] which does not contribute to the $(p, t)$ transition. (ii) For reasons discussed above, the form factor for the direct transition strength to the $2_1^+$ state becomes very small. Moreover, it is rather sensitive to the details of the parameters involved in the calculations. Thus, for instance, if one changes the position of the $h_{11/2}$ single-particle state by about 300 keV, the form factor changes its value by about 15%. The form factor is also sensitive to the values of the strength parameters of the $Q$-$Q$ force or quadrupole pairing force. Because of this we performed for this work calculations which assume another set of form factors (we call it the set II), which neglect the form factor of the direct $2_1^+$-excitation form factors of the set I, as mentioned earlier.

TABLE II. Calculated structure factors $G_N$ for $L = 0$, $0_g^+ \to 0_g^+$ and $L = 2$, $0_g^+ \to 2_1^+$ and $2_1^+ \to 0_g^+$ transitions.

| $N$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $0_g^+ \to 0_g^+$ | 0.0064 | $-0.0184$ | 0.0681 | $-0.2497$ | 0.6308 | $-0.1953$ | 0.002 62 |
| $0_g^+ \to 2_1^+$ | 0.0008 | $-0.0039$ | 0.0165 | $-0.0669$ | 0.0008 | | |
| $2_1^+ \to 0_g^+$ | $-0.0018$ | 0.0061 | $-0.0231$ | 0.1676 | $-0.0321$ | | |

## IV. OPTICAL POTENTIAL PARAMETERS

In the present CCBA calculation, use is made of a deformed-vibrational optical-potential model[13] to generate the CC distorted waves. As is usually done, we use insofar as possible optical-potential parameters, which are determined from the analysis of the elastic and inelastic scattering data.[21-25] The parameters actually used in the present calculations are summarized in Table III.

Strictly speaking, however, the procedure of fixing the optical-potential parameters by the elastic and inelastic scattering data can be justified only for the case of reactions in which the angular momentum mismatch is comparable with the actual angular momentum transfer so that the partial waves well defined by the elastic and inelastic scattering data dominantly contribute to the reaction cross sections.

Unfortunately this is not the case for the present reaction. To demonstrate this, we present in Fig. 3 a diagram of the absolute magnitude of the reflection coefficient,[26] $|\eta_l|$, as a function of the orbital angular momentum, $l$, for both the proton and triton, together with the overlap integral, $|I_l|$, multiplied by $(2l+1)^{1/2}$ for the $L=0$, $0_g^+$-$0_g^+$ transition. As is seen from Fig. 3, the value reaches its half value at $l \simeq 7$ for the proton, and $l \simeq 12$ for

the triton, respectively. The angular momentum mismatch is thus found to be 5, which is much larger than the angular momentum transfer for the $L=0$ or $L=2$ transitions. Due to this, the overlap integral $|I_l|$ has its maximum value for triton partial waves which are not well defined by the elastic scattering, although proton partial waves that give the maximum overlap integral are still in the region well defined by the elastic scattering. This feature is also seen in Fig. 3.

The above result shows that the triton waves that contribute to the reactions cannot be well defined by the elastic and inelastic scattering data, and therefore there is an ambiguity for the choice of the parameters. Actually, we treated the deformation parameter of $0_g^+$-$2_1^+$ inelastic coupling as an adjustable parameter and determined the value in such a way that the calculated CCBA $2_1^+$ cross section fits the observed data. The values determined in this way are $\beta_2 = 0.09$ and 0.13 according to the choice of the form factor set I and II, respectively.

Finally, we remark here that we use a fixed value of the zero range parameter, $D_0^2 = 4.8 \times 10^4$

TABLE III. Optical-potential parameters used in the present CCBA calculations. The notation used in this table is the same as that of Refs. 13 and 22.

|  | $P$ | $t$ |
|---|---|---|
| $V$ | 56.3 | 170.0 |
| $W$ | 2.9 | 19.0 |
| $W_D$ | 6.7 | $\cdots$ |
| $r$ | 1.22 (1.17) | 1.15 |
| $r_I$ | 1.32 | 1.52 |
| $r_D$ | 1.32 | $\cdots$ |
| $r_c$ | 1.25 | 1.40 |
| $a$ | 0.75 | 0.74 |
| $a_I$ | 0.62 | 0.76 |
| $a_D$ | 0.62 | $\cdots$ |
| $W_c^{(1)}$ | 0.8 | 0.8 |
| $W_c^{(2)}$ | 1.0 or 0.8 [a] | 1.0 or 0.8 [a] |
| $W_c^{(3)}$ | 1.0 | 1.0 |
| $\beta_{02}$ | 0.17 | 0.09 [b] or 0.13 [c] |
| $\beta_{20}$ | +0.125 | 0.07 |
| $\beta_{00}^{11}$ | −0.006 | −0.012 |
| $\beta_{22}$ | +0.13 | 0.09 |
| $\beta_{02}^{11}$ | 0.05 | 0.05 |
| $\beta_{24}$ | 0.13 | 0.070 |
| $\beta_{04}^{11}$ | 0.02 | 0.00 |

[a] Used for $0_g^+$-$2_1^+$-$I_n^+$ coupling calculation.
[b] Set I.
[c] Set II.



FIG. 3. Reflection coefficients of proton and triton elastic scattering. The DWBA radial-overlap integrals are also shown.

MeV$^2$ fm$^3$, for all calculations performed here. This value was actually determined by fitting the calculated CCBA $0_g^+$ cross section to experiments as we shall discuss later in Sec. V A.

## V. RESULTS OF CCBA CALCULATIONS AND COMPARISON WITH EXPERIMENTAL DATA

We discuss in this section the CCBA calculations and compare the results with experiments. A computer code MARS is used for this purpose.[27] The inelastic coupling to the two phonon states is neglected in computing the cross section to the $0_g^+$ and $2_1^+$ states, while a $0_g^+$-$2_1^+$-$I_n^+$ coupling is used when the cross sections of the two-phonon spin-I state are calculated.

### A. Ground-state cross section

In Fig. 4, we show the results of the CCBA calculations for the $0_g^+$ state, together with the experimental data. As is seen, the shape of the experimental cross section is well reproduced by the calculations. The absolute magnitude is also well reproduced with the help of the zero-range parameter, $D_0^2 = 4.8 \times 10^4$ MeV$^2$ fm$^3$ assumed before. This zero-range parameter is used for all the calculations discussed henceforth.

It might be worthwhile to note here that the

CCBA $0_g^+$ cross section differs very little from that of DWBA. This means that the two-step and multistep processes are not important for the $0_g^+$ cross section. The reason for this lies in the fact that the $L = 2$ form factors that are responsible for the lowest multistep processes are rather small as compared with that of the direct, $L = 0$ transition.

### B. $2_1^+$ cross section

In Fig. 5 we present various CCBA cross sections corresponding to various choices of form factors, together with that of DWBA as well as experiment. $\beta_2 = 0.13$ is used for the triton-channel deformation parameter. Some remarks may be given on the results obtained: (i) The DWBA cross section is smaller by about a factor 6 than the observed cross section. The fit of the shape of the calculated cross section to experiment is also not good, thus DWBA fails to explain the observed data; (ii) the CCBA cross sections that are obtained by including only one form factor of either the $0_g^+$-$0_g^+$ or the $2_1^+$-$2_1^+$ transition can account almost for the entire observed cross section, although the fit is not very good; (iii) the CCBA cross section which includes all of the form fac-



FIG. 4. Calculated CCBA cross section of the $0_g^+$ state, compared with experiment.
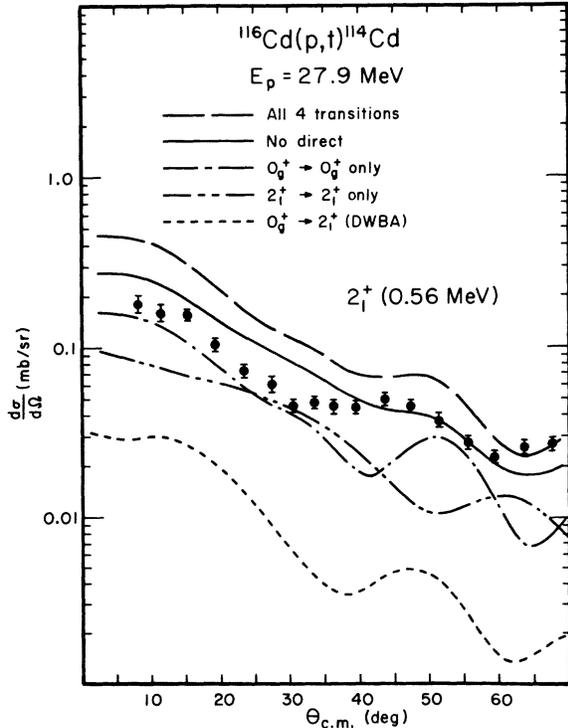


FIG. 5. Various calculated CCBA cross sections for the $2_1^+$ state. $\beta_2 = 0.13$ is used for the triton optical-potential-deformation parameter. The calculated results are compared with experiment.

tors of set II, which ignores the direct transition form factor, reproduces very well the observed cross section. This implies that the $2_1^+$ cross section can be explained only in terms of the multistep processes; (iv) if we take into account the direct transition, i.e., if one uses the set I form factors, keeping the deformation parameter to be the same, the calculated CCBA cross section becomes slightly too large as compared with experiment.

This discrepancy, however, is remedied if use is made of $\beta_2 = 0.09$ (the set I) for the $0_g^+$-$2_1^+$ inelastic transition in the triton channel. To show this, we present in Fig. 6 the CCBA cross section obtained by using the form factor and the deformation parameter set I-I, together with that of II-II. We see that the calculated CCBA cross sections for both sets can reproduce almost equally the observed cross section.

As is clearly seen in the results of Fig. 5, the interference between the direct and indirect multistep processes is constructive. A similar constructive interference was also observed in the $2_1^+$ cross section of the $^{142}Nd(p, t)^{140}Nd$ reaction.[18] On the other hand, we found a destructive interference in the case of $^{144}Nd(p, t)^{142}Nd$. These observations, together with the help of the RPA wave functions, enable us to understand the rather general behavior of the interference phenomena, which will be discussed in the next section.

## C. $0_1^+$ cross section

The CCBA calculations of the $0_1^+$ were performed for both sets of the parameters I-I and II-II which reproduce the observed $2_1^+$ cross section equally well. The results were presented in Fig. 7, together with the experimental data. The angular pattern of the observed cross section, which is quite different from that of $0_g^+$, is reproduced very well in the calculations of both sets of parameters. The fit is, however, slightly better for the set I-I. Also, it is remarkable that the observed magnitude of the cross section is explained very well by the calculations.

Since the above CCBA calculations include no direct $(p, t)$ excitation, the calculated cross section comes entirely from the multistep processes. We want now to show that the major contribution to the cross section comes from the three-step and higher order multistep processes. To do this, we tried to calculate the cross section, neglecting all possible simple two-step processes, namely the $0_g^+$-$2_1^+$ form factor as well as direct $0_g^+$-$0_1^+$ inelastic excitations in both the incident and final nucleus channels. The results obtained in this way are shown in Fig. 8 and compared with the full CCBA cross sections obtained before. As is seen, the full CCBA cross section and the cross section



FIG. 6. Calculated CCBA cross sections of the $2_1^+$ state for the form factor and the deformation parameter set I-I and II-II, compared with experiments.
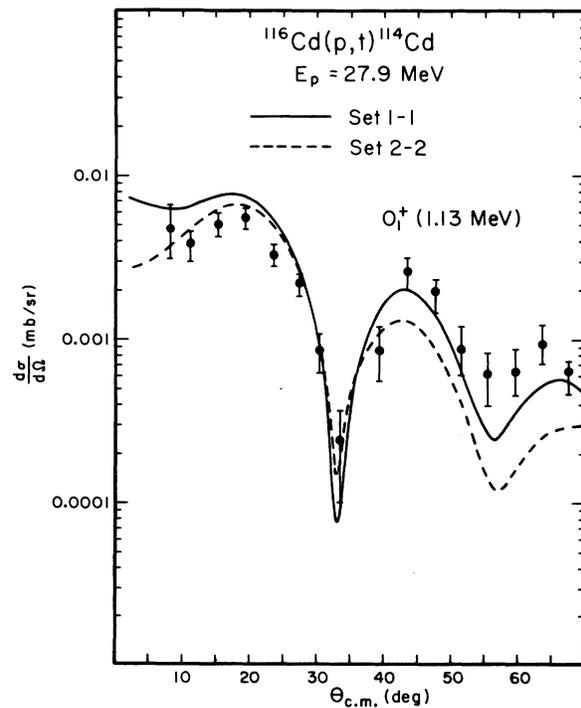


FIG. 7. Calculated CCBA cross sections of the $0_1^+$ states for the two sets of form factors and deformation parameters. The results are compared with experiment.

in which the two-step processes are excluded dif-
fer very little from each other, indicating that
the $0_1^+$ cross section mainly comes from the three-
step and higher multistep processes.

There are three important three-step processes,
which come from the $0_g^+$-$0_g^+$, $2_1^+$-$2_1^+$, and $0_1^+$-$0_1^+$ $(p, t)$
transitions. We studied contributions arising from
each of the processes separately, finding that all
these contributions are almost equally important:
Thus, the neglect of any one of them destroys the
fit that we obtained already, particularly the fit
to the angular distributions.

As was already pointed out in Sec. II, the 3 $0_1^+$
cross sections involving 3 different target nuclei
differ markedly from each other in both the shape
and magnitude. This seems to suggest that the
details of the $0_1^+$ wave function change with the
mass number. Phenomenologically this change in
the wave function may be described by modifying
some of the basic form factors and/or introducing
some other form factors which are not in the basic
set. Indeed, the observed change in the cross sec-
tions can rather easily be interpreted if we intro-
duce the direct, $0_g^+$-$0_1^+$ $(p, t)$ transition, which was
neglected in the earlier calculations of the $^{116}$Cd-
$(p, t)^{114}$Cd reaction. To demonstrate this, we did

a CCBA calculation for the $^{114}$Cd$(p, t)^{112}$Cd re-
action including a direct transition, which is 10%
of the $0_g^+$-$0_g^+$ $(p, t)$ transition. The results ob-
tained in this way are shown in Fig. 9, together
with those for the $^{116}$Cd$(p, t)^{114}$Cd reaction, as well
as experiment. As is clearly seen in the figure,
the inclusion of the direct transition mentioned
above explains nicely the observed cross section
for the $^{114}$Cd$(p, t)^{112}$Cd reaction, and thus the ob-
served change in the cross sections between the
different targets.

It might be of value to note here that the above
success is obtained since we assumed the same
sign of the form factor as that of the $0_g^+$-$0_g^+$ tran-
sition. If the sign is reversed, the resultant CCBA
cross section becomes very different from what
we obtained before and thus the observed features
can no longer be explained.

D. $2_2^+$ cross section

We present in Fig. 10 the calculated CCBA cross
sections obtained for the two sets of form factors
and the deformation parameters used before. They
are also compared with experiments. For the sake
of reference, we plotted in this figure also the



FIG. 8. Comparison of 2 CCBA cross sections for the
$0_1^+$ states; one is the full CCBA cross section while the
other is the CCBA cross section obtained by including
only the three-step processes; i.e., $\beta'$ and $\beta''$ were set
zero for both incident-proton and final-triton channels.
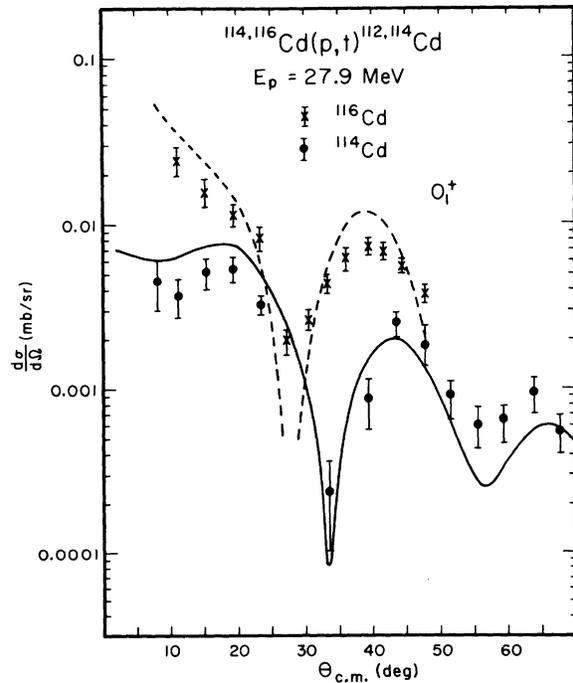
FIG. 9. Calculated CCBA cross sections of the $0_1^+$
states. The full curve is that of Fig. 7, while the dotted
curve is that obtained by including the form factor for
the direct $0_1^+ \rightarrow 0_1^+$ transition. The assumed form factor
is 10% of the form factor of the $0_g^+ \rightarrow 0_g^+$ transition. Rele-
vant data are also shown.

cross sections for the $2_1^+$ state, already given in Fig. 5. Both the shape and the magnitude of the observed cross section are seen to be reproduced very well in both types of calculations. It is particularly remarkable that the very marked difference between the shapes of the observed cross sections of the $2_1^+$ and $2_2^+$ are explained very well. Of course, the above difference between the two $2^+$ cross sections cannot be explained by DWBA.

As was remarked in the last paragraph in Sec. II, the $2_2^+$ state has a cross section which is much larger (by about a factor 5) than the cross sections of other two two-phonon states. We observe a similar situation in the inelastic scatterings, in which the observed $2_2^+$ cross section is larger, say by about a factor 2, than those of other two two-phonon states.[22]

This particular large cross section for the $2_2^+$ state was explained in the inelastic scattering case as resulting from a rather strong direct $0_g^+$-$2_2^+$ inelastic excitation.[22] Now, we want to show that this is also the case for the present $(p, t)$ reaction; indeed, if we do the CCBA calculation without including the direct inelastic excitation mentioned above we can hardly explain the large cross section observed any more. To show this, we present in Fig. 11 the calculated CCBA cross section obtained by neglecting the above direct inelastic scattering processes, and also

by including only the direct inelastic scattering process, together with the full CCBA cross section of Fig. 10. It is seen that the CCBA cross section, which neglects the direct inelastic scattering processes is smaller by about a factor 6 than that of full CCBA: This implies that the direct excitation effect is amplified in the $(p, t)$ reaction case. This is due to a coherent effect of the direct excitations in both the incident-proton and exit-triton channels.

The CCBA calculation which includes only the direct inelastic excitation and neglects the $0_g^+$-$2_1^+$ and $2_1^+$-$2_2^+$ inelastic scattering processes explains almost the entire cross section observed. Thus, in contrast to the $0_1^+$ state, the cross section of this $2_2^+$ state comes mainly from the two-step processes.

### E. $4_1^+$ cross section

Three types of CCBA calculations were made for the $4_1^+$ cross section, assuming the set I-I and II-II and also the set I-I plus a small form factor for the direct transition, $F_4(0_g^+$-$4_1^+)$, respectively. The form factor for the direct transition was obtained by using the $G_N$ factor calculated for $^{142}$Nd-$(p, t)^{140}$Nd leading to the 1.8-MeV $4^+$ state.[28]

The cross sections thus obtained are plotted in Fig. 12, together with the experimental data. We plotted two experimental cross sections, one is



FIG. 10. Calculated CCBA cross sections of $2_2^+$ states. The results are compared with experiment, as well as the calculated and observed $2_1^+$ cross sections.
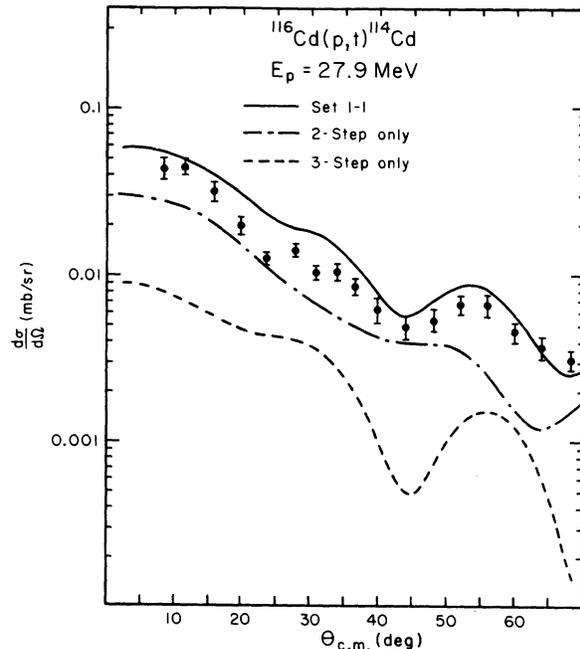


FIG. 11. Comparison of three CCBA cross sections of the $2_2^+$ state; one is the full CCBA cross section, and the other two are those obtained by including only the three-step and the two-step processes, respectively.

that of $^{116}$Cd($p$, $t$)$^{114}$Cd and the other is an average
of the cross sections taken for 3 different target
nuclei, since the $4_1^+$ state is actually a doublet and
the observed cross section involves a contribution
from the excitation of another doublet ($0^+$) state.
The observed cross section, particularly the
magnitude, is well reproduced in all of the calcu-
lations of Fig. 12.

However, the fit of the calculated angular distri-
bution with experiment is not so good as compared
with the fit we obtained for other two-phonon
states. Note, however, that the fit is significantly
improved if the direct process is included in the
calculation. This might suggest that there is a
possibility that the fit will be further improved by
introducing other form factors and/or adjusting
some of the parameters involved in the calcula-
tions.

We made a similar study of the effects of the
two-step and three-step contributions as that made
for the $0_1^+$ and $2_2^+$ states, finding that for this case
the main contribution comes from three-step pro-
cesses. The situation is the same as that of the
$0_1^+$ state. This is consistent with the fact that the
observed magnitude of the cross sections are al-
most the same for both cases.

## VI. FINAL REMARKS

A CCBA analysis is made of the $^{116}$Cd($p$, $t$)$^{114}$Cd
reaction, leading to the ground, one-phonon, and
two-phonon states of the final residual nucleus.
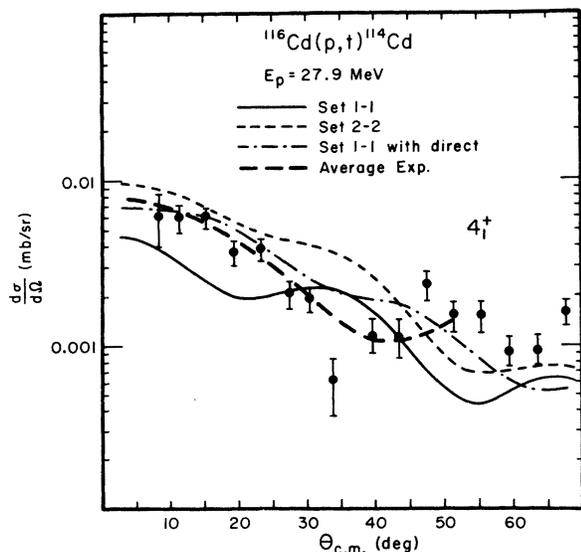


FIG. 12. Various calculated CCBA cross sections of
the $4_1^+$ state, compared with experiment. The average
cross section is that obtained by taking the average of
the $4_1^+$ cross sections observed in 3 reactions.

Perhaps the most important conclusion obtained
in this work is that CCBA does describe very well
the ($p$, $t$) excitation of the two-phonon states. In
fact we have seen that a number of the observed
facts, which defy explanation by DWBA, can
readily be explained by CCBA. It was also shown
that the dominant contribution to the two-phonon
cross sections comes from the three-step pro-
cesses for the $0_1^+$ and $4_1^+$ states, but the two-step
process is the most important for the $2_2^+$ state.

In the present calculations, we neglected almost
all the form factors associated with the two-pho-
non states, i.e., the form factors for the direct
($p$, $t$) excitations from the ground and the one-
phonon states to the two-phonon states. We have
found, however, that we can reproduce the ob-
served data consistently this way. This may show
that the form factors neglected in the present
calculations are indeed quite small.

As was already remarked, this smallness of the
form factors can be considered to result from two
conditions: (i) The two-phonon state is essentially
a four-quasiparticle state, so that the direct
transition from the ground to the two-phonon
state is forbidden[8,12]; (ii) the ($p$, $t$) creation of a
phonon is very unlikely for reactions such as
those considered here. Under such circumstances
it would be natural to expect that the ($p$, $t$) transi-
tion from one-phonon state is also weak.

It should be remarked, however, that the above
two conditions may sometimes be violated. In-
deed we needed, as was shown above, to intro-
duce the direct $0_g^+$-$0_1^+$ ($p$, $t$) transition to explain
the observed $0_1^+$ cross section of the $^{114}$Cd($p$, $t$)-
$^{112}$Cd reaction, while such was not needed in
order to explain a similar $0_1^+$ cross section of
$^{116}$Cd($p$, $t$)$^{114}$Cd. The direct $0_g^+$-$0_1^+$ ($p$, $t$) transition
may possibly arise if the BCS vacuum state or a
two-quasiparticle state (such as a pairing-vibra-
tional state) is mixed into the two-phonon state.
This is certainly a violation of the condition (i),
mentioned above.

The condition (ii) might be violated for target
nuclei with neutron numbers which are somewhat
different from what we considered in this work,
so that the pairing and particle-hole correlations
interfere more strongly. In such cases, the one-
phonon to two-phonon ($p$, $t$) transitions may not
be negligible anymore. It may thus be very in-
teresting to perform a similar analysis for other
cases of reactions involving different target nu-
clei.

Though the main objective of the present anal-
ysis was to test whether CCBA can successfully
describe the ($p$, $t$) excitation of the two-phonon
states of vibrational nuclei, we summarize in
the rest of this section the results of the calcula-

tions on the ground and one-phonon states.

The ground-state cross section was very well reproduced by the DWBA as well as CCBA calculations. The zero-range constant $D_0$ needed to reproduce the observed cross section was found to be $D_0^2 = 4.8 \times 10^4$ MeV$^2$ fm$^3$, which is smaller than the value determined before.[29] This, however, seems to be reasonable since in the work of Ref. 29 the single-particle states in only one major shell were considered.

The one-phonon $2_1^+$ cross section was also reproduced well by CCBA, but not by DWBA; the DWBA cross section was found to be too small to explain the observed cross section. This implies that the $2_1^+$ cross section comes mainly from a multistep process, rather than the direct process; indeed it was shown that the $2_1^+$ cross sections were explained without taking into account any direct contribution.

The interference between the direct and indirect multistep processes was found to be constructive in the present cases studied. A similar interference was also found in $^{142}$Nd($p, t$)$^{140}$Nd (see Ref. 18), while a destructive interference was found in $^{144}$Nd($p, t$)$^{142}$Nd, as was already noted.[4] Let us keep in mind these observations and also assume that the dependence of the interference on kinematic terms such as the incident energy, the reaction $Q$ value, the target mass, and so on is rather weak, so that if the dynamical condition is the same, we obtain the same interference. Then, we can draw a rather general conclusion about the occupation-number dependence of the interference. Namely, the interference is destructive for $(p, t)$ reactions from target nuclei in the beginning of the closed shell [$^{144}$Nd($p, t$)$^{142}$Nd is such a case], and by increasing the number of neutrons in the open shell, the interference becomes constructive

for some number of neutrons and then the. constructive nature continues as the shell is closed.

The dynamical element that determines the constructive or destructive nature of the interference is the relative sign of the $L = 0$, $0_g^+$-$0_g^+$ and $L = 2$, $0_g^+$-$2_1^+$ form factors (in the asymptotic region) under a fixed-phase convention. The phase of the wave function is determined in such a way that we always have a positive (transition) deformation parameter and also a positive value of $L = 0$ form factor in the asymptotic region. Then, utilizing the RPA wave function, it is easy to see that the sign changes at some neutron number in the middle of the closed shell region. This change of the sign results from a competition between two contributions coming from the forward and backward amplitudes in the RPA wave functions. At the beginning of the closed shell, the backward amplitude contribution exceeds that from the forward amplitude, while at the end of the closed shell the situation reverses. The backward amplitude describes the effect of the ground-state correlation, which contributes constructively to the $E2$ transitions, resulting in a strong enhancement. In the case of a $(p, t)$ or a $(t, p)$ reaction the effect is just opposite, contributing destructively to the main term.[30]

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*Supported in part by U. S. Atomic Energy Commission.

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