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Real-well depth of the optical potential

D. C. Agrawal and P. C. Sood

Nuclear Physics Section, Physics Department, Banaras Hindu University, Varanasi 221005, India

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An examination of the results of the optical-model analyses of the experimental data obtained by the scattering of protons with energies varying from 9.8 to 61.4 MeV from target nuclei in the mass range from 40 to 208 is carried out to look for the linear dependence of of the real potential-well depth on the target mass number A which had been unexpectedly revealed in the analysis of 11-MeV proton scattering from nuclei having a mass between 45 and 70. It is found that such an A dependence cannot be extrapolated as such to heavier nuclei at any energy. An assumed linear variation over the whole mass region yields a coefficient which is an order of magnitude smaller than that deduced for the restricted mass region and which has a much wider scatter from linear dependence. Such a behavior could possibly be simulated in part by the isospin-dependent potential, the Coulomb correction term in the potential, and the closed-shell effects.

[NUCLEAR REACTIONS Proton elastic scattering, optical-model analyses, well depth search, studied A dependence for $E_p = 9.8-61.4$ MeV; target $A = 40-208$.]

The characterization of the optical-model potential has been usually attempted through multiparameter search procedures for fitting the angular distributions and the polarization data obtained from the nucleon-nucleus elastic scattering experiments. The number of parameters is generally restricted by adopting an average geometry, the search being then made for the various well depths, and systematic variations of these well depths are then investigated to extract their dependence on specific factors. One of the earliest and most exhaustive investigations of proton scattering data undertaken by Perey[1] ten years ago resulted in a potential whose real-well depth V_s specifically includes a Coulomb correction term arising from the velocity dependence of the interaction, a linear dependence on incident projectile energy, and a symmetry dependence similar to that deduced for the shell-model potential earlier[2,3] and interpreted[4] as the consequence of the isospin dependence of the potential directly verified through quasielastic (p,n) reactions.[5] The last mentioned dependence has since been firmly established, at least for the proton potential, on the basis of a very large number of studies.[6,7] However, in the optical-model analysis of 11-MeV proton scattering data Perey *et al.*[8] concluded that for nuclei within the mass range $A = 45$ to 70 the real-well depths fail to follow the expected isospin dependence but instead exhibit very smooth variation as a function of mass number A. This study[8] raised the following interesting questions about the deduced A dependence which, somehow, have remained unanswered so far: (a) its adequacy outside the range of nuclei considered and at other energies; (b) the feature said to "also appear with the 14.5-MeV data" and "though not firmly established, similar effects may have been seen with 18.6-MeV polarization data"; (c) while "ruling out our geometry as the cause for this effect" they could not offer any physical explanation for it.

Our present study is directed to find an answer to these questions. In such a task one faces certain insurmountable difficulties if an attempt is to be made to find a quantitative measure of some of these hyperfine characteristics of the optical potential. Even if all the analyses at each energy were carried out using the same geometrical parameters of the real potential, certain inherent ambiguities exist due to interrelationships of the real, imaginary, and spin-orbit potential terms, and due to various types of errors associated with the experimental uncertainties.[9] In our case, fortunately, no quantitative evaluation is being sought as an answer to the three questions mentioned above. Accordingly we use the real-well depths obtained through search procedures by various investigators[10-17] at different energies within the range 9.8 to 61.4 MeV for a fixed geometry in each case. The results of these investigations[18] are discussed below.

In order to determine the validity of A dependence found in the mass range 45 to 70 for 11-MeV protons outside the range of the nuclei considered and at other energies, we examine the results of optical-model analyses at various energies, listed in Table I, wherein the real-well depths V_s have been determined by a search procedure. These

TABLE I. A dependence of the optical-model potential.

Energy (MeV)	Reference	No. of nuclei	Mass range	Real well Radius r_s (fm)	Real well Diffuseness a_s (fm)	V_s (MeV)
9.8	Greenlees *et al.* (Ref. 10)	7	54–120	1.21	0.72	$50.32+0.059(1\pm0.170)A$
14.5	Rosen *et al.* (Ref. 11)	28	45–120	1.25	0.65	$44.54+0.056(1\pm0.170)A$
14.5	Lind *et al.* (Ref. 12)	21	54–120	1.25	0.65	$45.45+0.060(1\pm0.183)A$
18.6	Kossanyi-Demay *et al.* (Ref. 13)	10	48–92	1.24	0.65	$47.44+0.047(1\pm0.276)A$
30	Greenlees and Pyle (Ref. 14)	8	40–208	1.20	0.70	$44.65+0.045(1\pm0.156)A$
40	Fricke *et al.* (Ref. 15)	9	40–208	1.16	0.75	$42.53+0.051(1\pm0.255)A$
50	G. S. Mani *et al.* (Ref. 16)	19	42–208	1.17	0.75	$41.13+0.040(1\pm0.375)A$
61.4	Fulmer *et al.* (Ref. 17)	6	40–208	1.16	0.75	$37.84+0.048(1\pm0.417)A$

well depths have been plotted as a function of the mass number A in Figs. 1(a) and 1(b). Assuming a linear A dependence through the relation $V_s = V_0 + \alpha A$ we have made a least-squares fit to the data at each energy to determine V_0 and α, which are listed in the last column in Table I. This straight-line fit in each case is shown in Fig. 1 which also includes the dashed line for $\alpha = 0.13$ deduced by Perey *et al.*[8] from 11-MeV data. It is seen that the extrapolation with their α to heavier

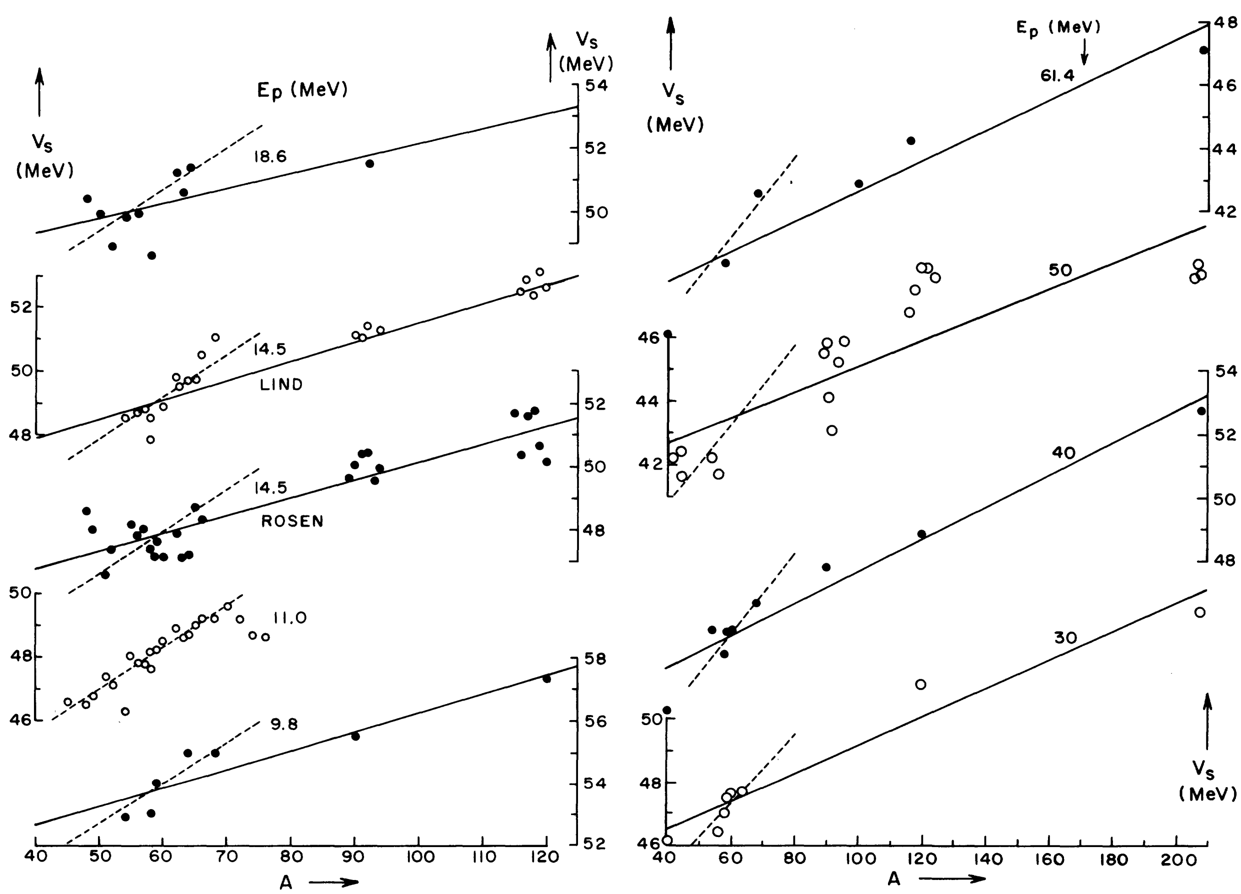

FIG. 1. The real-well depths V_s plotted as a function of the mass number A for different energies. The dashed straight lines correspond to the A dependence deduced by Perey *et al.* (Ref. 8) and the full lines show the A dependence obtained in each case (see Table I) by a least-squares fit over the whole mass region investigated.

mass region is not at all satisfactory for any energy. In the whole mass region and over the whole energy range the coefficient α is found to have the value 0.05 ± 0.01 with rather large uncertainty at each energy.

Another way to demonstrate this feature discussed by Perey *et al.*[8] was to examine the plot of V_s as a function of the symmetry parameter $(N-Z)/A$ shown in our Fig. 2(a). It was said[8] that "the analysis of the 18.6-MeV polarization data of Kossanyi-Demay *et al.*[19] seems to suggest that similar effects are seen up to that energy; however, this cannot be firmly established"; on the other hand, analysis of 14.5-MeV data[11,12] for $T_Z = 2$ and 3 with two different geometries was said to give similar results. We have examined the revised results of 18.6-MeV analysis[13] and also the well depths obtained by Rosen *et al.*[11] obtained by search of V_s only from 14.5-MeV data analysis. The results at these two energies are shown in Figs. 2(b) and 2(c) in comparison with those at 11 MeV. It is evident that T_Z dependence observed for 11 MeV is not appearing at the other energies and, if at all discernible, is of opposite sign to that deduced for 11 MeV. One cannot possibly ascribe this reversed characteristic appearing for the 14.5-MeV data to the fact that it includes only the polarization data, since Perey *et al.*[8] have concluded that "the analysis of polarization data only will most likely yield a real-well depth within 0.5 MeV of that which would be obtained if both differential cross section and polarization data were available." Thus we fail to verify the claim of such a T_Z dependence appearing at 14.5 and 18.6 MeV.

Perey *et al.*[8] did not offer any physical explanation of the deduced A dependence. Greenlees, Pyle, and Tang[20] attempted to explain this feature on the basis that the volume integral per particle of the potential, rather than the potential well depth, is the more physically significant quantity, and concluded that the observed variation of V_s with A may "be simply interpreted as properties of the particular parametrization chosen." However, Perey *et al.*[8] and Satchler[6] do not find this explanation acceptable. We proceed to offer some physical arguments for this feature as elucidated by the points established in our above discussion.

In view of the well-established[6,7] $[(N-Z)/A]$-dependent term of the proton potentials with its coefficient[6] $V_1 = 25 \pm 10$ MeV obtained by assuming a Coulomb correction term of $0.4(Z/A^{1/3})$, one may examine what possible A dependence such a term would simulate. For nuclei along the line of β stability the semiempirical mass formula yields the relation

$$Z/A = (2 + 0.015A^{2/3})^{-1},$$

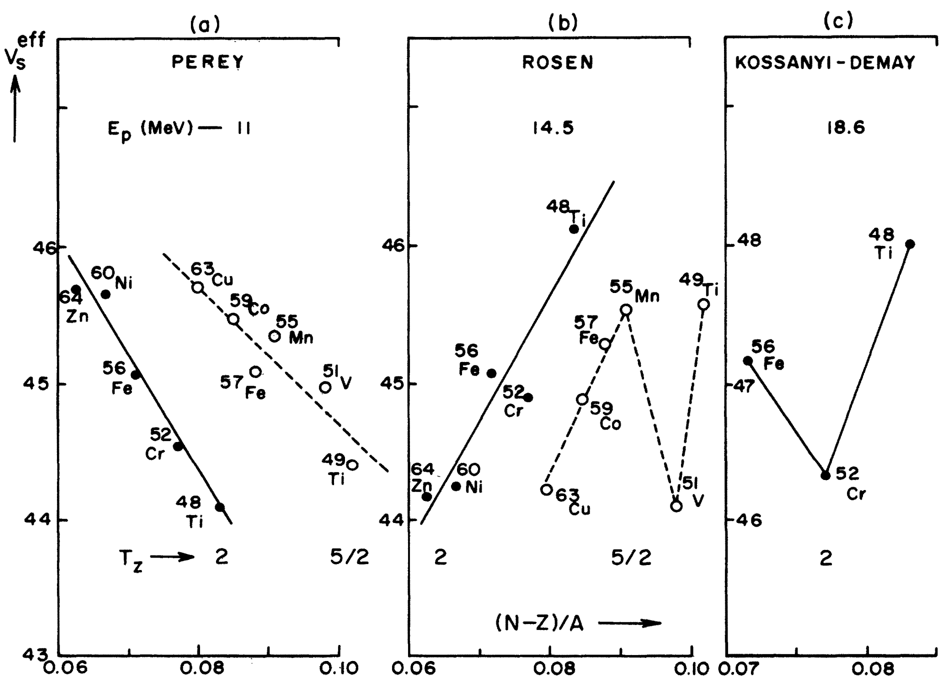

FIG. 2. Plot of the real-well depth (corrected for the Coulomb term) as a function of the symmetry parameter $(N-Z)/A$ for three separate energies. The data points with the same value of $T_Z = (N-Z)/2$ have been joined in each case.

such that we get

$$\frac{V_1(N-Z)}{A}=\left(1-\frac{2Z}{A}\right)V_1=\left(\frac{0.015\,V_1}{2A^{1/3}+0.015A}\right)A$$

and

$$\frac{0.4Z}{A^{1/3}}=\left(\frac{0.4}{2A^{1/3}+0.015A}\right)A\,.$$

Thus for A around 55 these two terms combined would yield an approximately linear dependence with coefficient of about 0.1. Over the whole mass region the coefficient may average around 0.06 with appreciable scatter around this value. These conclusions are quite consistent with the above mentioned points.

In order to examine this explanation in a specific case we take the results of the analysis of 61.4-MeV data by Fulmer *et al.*[17] shown in Fig. 3. It is seen that over the restricted mass region 40–70 a linear dependence with coefficient 0.14 is observed whereas for $A \geqslant 68$ the coefficient obtained is 0.035. If we correct the well depths V_s with the Coulomb correction term βV_c and the symmetry term V_τ the A dependence disappears for $A > 40$.

Although at other energies or for other analyses such a clear demonstration is not likely, there is no denying the fact that the symmetry term and the Coulomb correction term can give a simulated linear A dependence very similar to the one deduced in our Table I.

One other physical consideration of interest is the shell effects. Correlations of the plots of the optical-potential real-well depths as a function of A with the approximate positions where neutron and proton shell closures take place provide the indication that a somewhat deeper well is needed for magic nuclei. Thus restricting ourselves to the vicinity of such regions a steep increase in depth with A may be expected in specific mass regions. To confirm such a conclusion much more data are needed than are presently available. However, such an effect is indicated, and is physically plausible as well as consistent with the observed characteristics.

Thus one may say that a linear A dependence of the real-well depths of the proton optical potentials as depicted in Fig. 1 and given in Table I may be

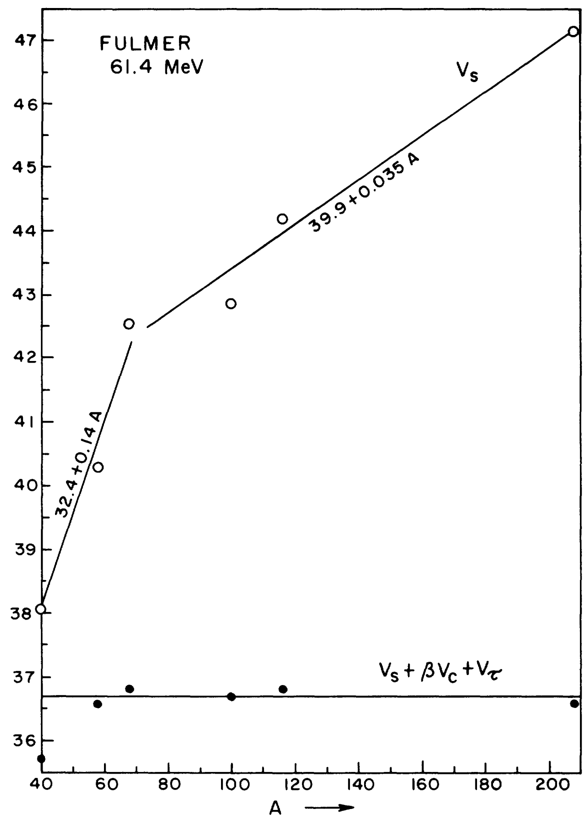

FIG. 3. Results obtained by Fulmer *et al.* (Ref. 17) from the analysis of 61.4-MeV proton scattering data. Open circles represent the real-well depth V_s obtained from the search procedure while the dots represent the well depths obtained after the Coulomb correction term ($=0.4\,Z/A^{1/3}$) and the symmetry term [$=24(N-Z)/A$] have been subtracted.

simulated because of the isospin-dependent potential, the Coulomb correction term, and possibly the shell effects.

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