Interpretation of quasifree scattering in the ^{6}Li(p, pd) and (p, p^3He) reactions*

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The large difference between the cross sections for the quasifree scattering in the ^{6}Li(p,pd) and (p,p^3He) (p,pt) reactions are accounted for on the basis of separation energies.

[NUCLEAR REACTIONS ^{6}Li(p,pd), (p,p^3He), (p,pt), $E=156$, 590 MeV; calculated $\sigma(q)$.]

Recently, Dollhopf *et al.*[1] have observed the very small cross section for the quasifree scattering (QFS) in the ^{6}Li(p,pt) reaction at the incident energy of 590 MeV, compared with that measured by Alder *et al.*[2] for the ^{6}Li(p,pd) reaction at the same incident energy. Using 50-MeV α particles, Lambert *et al.*[3] have measured the cross section for the QFS in the ^{6}Li(α, α^3He) reaction which is much smaller than that for the ^{6}Li(α, αd) and (α, 2α) reactions. The same fact has been also observed by Bachelier *et al.*[4,5] for incident protons of 156 MeV.

These data were usually accounted for in terms of clustering probabilities or effective numbers of clustering particles and the appropriate cross sections for elementary scattering occurring in the QFS processes. With known cross sections for the corresponding free scattering at lower incident projectile energies, one had to assume that the d-α configuration dominates over other ones in the ^{6}Li ground state, to explain the large difference between the cross sections of the ^{6}Li breakup reactions into a deuteron and an α particle and into a triton and a ^{3}He particle. On the other hand, at higher energies the cross section for free scattering, for example, the proton-triton cross section at 590 MeV, is not now known. To get clustering probabilities consistent with those found by analyzing data for the radiative capture of ^{3}He by tritons[6] and for the ^{6}Li(p,^{3}He) reaction,[7] one had to suppose a cross section at 90° for the p-t scattering smaller than that at 90° for the p-d scattering by two to three orders of magnitude.

We show in the present note that the large differences between cross sections for the QFS in the ^{6}Li (p,pd) and (p,p^3He) (p,pt) reactions are simply explained by the magnitude of the separation energy of the ^{6}Li nucleus into a deuteron and an α particle compared with that into a triton and a ^{3}He nucleus, and hence that the differences are not due to clustering probabilities.

The correlation cross section for the QFS in the (p,pa) reaction in the spectator model is given by

$$\frac{d^3\sigma}{dEd\Omega^2}=K\left(\frac{d\sigma}{d\Omega}\right)_{pa}|\phi_{ab}(q)|^2, \tag{1}$$

where K is a kinematical factor and $(d\sigma/d\Omega)_{pa}$ is an appropriate proton-nucleus a scattering cross section. In the plane-wave approximation the overlap integral $\phi_{ab}(q)$ is reduced to

$$\phi_{ab}(q)=\int d^3re^{i\vec{q}\cdot\vec{r}}\tilde{\psi}_{ab}(r), \tag{2}$$

where $\tilde{\psi}_{ab}(r)$ is the antisymmetrized wave function which describes the c.m. motion of clustering particles a with respect to the c.m. of the remaining particles b in the target nucleus, q being the momentum transfer which is equal to the recoil momentum of the residual nucleus.

The wave function $\tilde{\psi}_{ab}(r)$ has an oscillatory structure for the d-α and t-^{3}He configurations in the ^{6}Li ground state. The oscillatory structure of $\tilde{\psi}_{ab}(r)$ almost cancels the contribution from its interior part to the overlap integral, and hence its asymptotic part dominates the overlap integral. However, the asymptotic form of $\tilde{\psi}_{ab}(r)$ is essentially determined by the separation energy.

For convenience a harmonic-oscillator wave function is generally used to describe the motion of particles inside the nucleus. However, this function has a very poor asymptotic shape. To get an improved asymptotic form, noting that the interior part of the function has less importance for the overlap integral, we use the square-well function with its radius R,

$$\psi_{ab}(r) = A h_l^{(1)}(i\beta r) Y_{lm}(\hat{r}), \quad r > R \tag{3}$$

and

$$= B j_l(\alpha r) Y_{lm}(\hat{r}), \quad r \leq R, \tag{4}$$

where

$$\beta = (2\mu|\epsilon|)^{1/2}\hbar . \tag{5}$$

Here ϵ is the separation energy of the particle a from the nucleus and μ is the reduced mass of the particle a with respect to the others. The constants A, B, and α are determined by requiring the continuity condition for the logarithmic derivative of the function at $r = R$ and the normalization condition. The value of α is fixed so that the function $j_l(\alpha r)$ has nodes inside R corresponding to those of $\tilde{\psi}_{ab}(r)$.

The asymptotic part of $\psi_{ab}(r)$ does not differ from that of $\tilde{\psi}_{ab}(r)$, since the exchange effect is important only in the interior part. Although the difference between the functions is appreciable in the interior region due to antisymmetrization effects, the contribution to the overlap integral from the functions in this region is strongly suppressed. The antisymmetrization effects are not important if the function $\psi_{ab}(r)$ is taken to have nodes corresponding to those of $\tilde{\psi}_{ab}(r)$, that is $\psi_{ab}(r)$ with its correct radial quantum number is used. This justifies the use of the square-well function in the calculation for the cross section for the QFS in the breakup reactions.

The square-well function is normalized with the factor

$$\theta_{ab}^{\,2} = \int d^3r\, |\tilde{\psi}_{ab}(r)|^2 \tag{6}$$

calculated with the antisymmetrized wave function $\tilde{\psi}_{ab}(r)$ on the basis of harmonic-oscillator functions. The values used in the present calculation are $\theta_{d\alpha}^{\,2} = 1.07$[8] and $\theta_{t\,^3\mathrm{He}}^{\,2} = 0.6$. These values are the most probable[9] for the harmonic-oscillator cluster-model wave functions.

The radius R is chosen as follows: The antisymmetrized wave functions $r\tilde{\psi}_{ab}(r)$ based on harmonic-oscillator functions have slightly more than three quarters of a wavelength inside R for the small separation energy of the ^{6}Li nucleus into a deuteron and an α particle, and slightly less than one wavelength inside R for the large separation energy into a triton and a ^{3}He nucleus. This condition is satisfied by $R = 3.5$ fm.

The present simple calculation is entirely parameter-free except for the choice of R. It is worthwhile to see how the calculated results depend on this choice of R. A variation from the value of $R = 3.5$ fm by ± 0.5 fm changes the results by only a small amount, mostly within the experimental error bars.

Nuclear absorption effects for the incident and outgoing particles by the target and residual nuclei may diminish the overlap integral for the interior region. However, it has been shown[10] that for a wave function with nodes the overlap integral does not strongly depend on whether or not the nucleus is completely black inside R. This means that the attenuation effects for the incident and outgoing particles due to nuclear absorptions are small because of the cancellation occurring in the overlap integral for the interior region where absorption effects are expected to be important. Therefore, the correlation cross section divided by $K(d\sigma/d\Omega)_{pa}$, that is $|\phi_{ab}(q)|^2$, is rather independent of nuclear absorption effects for moderately large projectile energies when appropriate

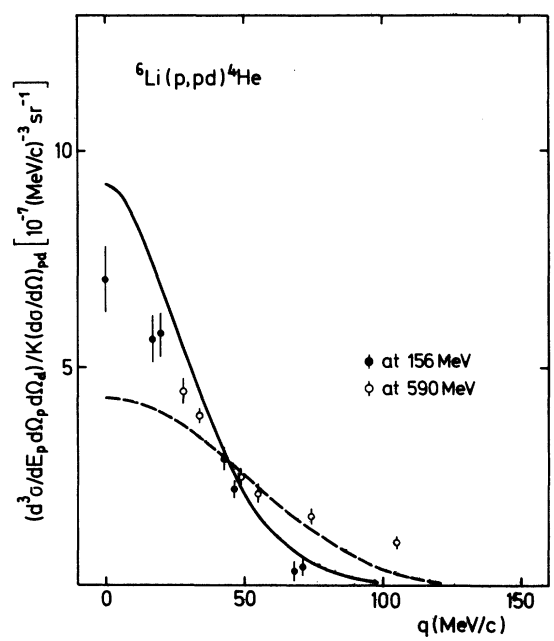

FIG. 1. The recoil momentum distributions for the QFS in the ^{6}Li(p,pd)^{4}He reaction at 156 and 590 MeV. The solid and dashed curves are calculated, respectively, with the $2S$-state square-well function $\psi_{d\alpha}(r)$ and the cluster model wave function $\tilde{\psi}_{d\alpha}(r)$ antisymmetrized on the basis of harmonic-oscillator functions. Both functions are normalized to $\theta_{d\alpha}^{\,2} = 1.07$. The experimental data are taken from Refs. 2 and 4.

kinematical conditions corresponding to QFS make possible final-state interactions less effective. This is especially the case for wave functions with small separation energies.

Figure 1 shows the calculated results compared with data for the QFS in the ^{6}Li(p,pd) reaction at incident energies of 156[4] and 590 MeV.[2] The values of $(d\sigma/d\Omega)_{pd}$ are taken from the free p-d scattering cross sections at 155 [11] and 582 MeV.[12]

The largely canceled overlap integral for the interior region gives almost the same width for the momentum distribution as that predicted by using the cutoff approximation. Hence the width is essentially determined by the separation energy. Therefore, before concluding that the momentum distribution width for the reaction at 590 MeV is twice as large as that at 156 MeV, and in agreement with the results calculated by the harmonic-oscillator cluster model function, additional data for small q values must be accumulated at 590 MeV. The curves in Fig. 1 demonstrate this.

Even if the separation energies are not small, the function $|\phi_{ab}(q)|^2$ is expected to be rather independent of projectile energy for appropriate kinematical conditions of QFS. If this is the case

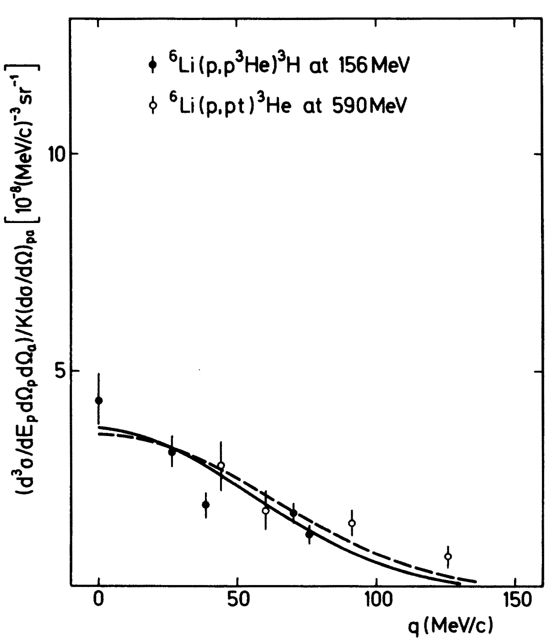

FIG. 2. The recoil momentum distributions for the QFS in the ^{6}Li(p,p^3He)^{3}H reaction at 156 MeV and ^{6}Li(p,pt)^{3}He reaction at 590 MeV. The solid and dashed curves show, respectively, the results calculated with the 2S-state square-well function $\psi_{t\,^3\mathrm{He}}(r)$ and the cluster-model wave function $\tilde{\psi}_{t\,^3\mathrm{He}}(r)$ antisymmetrized on the basis of harmonic-oscillator functions. Both functions are normalized to $\theta_{t\,^3\mathrm{He}}{}^2=0.6$. The experimental data are taken from Refs. 1, 4, and 5.

the calculated results for $|\phi_{t\,^3\mathrm{He}}(q)|^2$ can be compared with data for the ^{6}Li(p,p^3He) reaction at 156 MeV[4,5] and the ^{6}Li(p,pt) reaction at 590 MeV.[1] The shapes of the momentum distributions extracted for both the reactions are very similar as seen in Fig. 2. The p-^{3}He scattering cross section at 155 MeV [13] is used for $(d\sigma/d\Omega)_{p\,^3\mathrm{He}}$. The p-t scattering cross section is not now known. The p-t QFS cross sections at 590 MeV divided by $K(d\sigma/d\Omega)_{pt}^{90°\,\mathrm{c.m.}}$ are shown in Fig. 2, where $(d\sigma/d\Omega)_{pt}^{90°\,\mathrm{c.m.}}$ is taken to be 3.2 μb/sr which is reasonable based on data for the p-d, p-^{3}He, and p-^{4}He scattering at 590 MeV.[12]

In the above calculation off-the-energy shell effects on $(d\sigma/d\Omega)_{pa}$ are neglected when free scattering cross sections are used. Off-the-energy shell cross section takes its value between the on-the-energy shell cross sections at the relative energies E_i and E_f of the proton-nucleus a system in the initial and final states.[14] It is assumed that there is no resonance state of that system in the energy range between the E_i and E_f. The energy difference $E_i - E_f$ is small for the QFS kinematical region with small q values. The difference of on-the-energy shell cross sections at E_i and E_f is also small for high-energy projectiles. Therefore, the cross section $(d\sigma/d\Omega)_{pa}$ is well approximated by on-the-energy shell one, although off-the-energy shell effects may become appreciable for the QFS cross sections involving large q values.

For projectiles of lower energies, off-the-energy shell effects can be large and final-state interactions produce effects on the correlation cross sections even in the QFS kinematical region. Moreover, Eq. (1) becomes less valid for lower incident energies.

Compared with QFS, the situation is completely different for both elastic and inelastic scattering. The overlap integral for QFS is essentially the Fourier transform of the wave function as given by Eq. (2). On the other hand, the form factor for the elastic and inelastic scattering is the Fourier transform of the product of two bound-state wave functions, and hence this product falls off more rapidly with increasing radius. Moreover, the cancellation hardly occurs for these cases. Therefore, the correct asymptotic behavior of the wave function is much more important in the description of QFS. This is the reason why the harmonic-oscillator function fails to reproduce QFS data, although it explains well the elastic and inelastic form factors. This is particularly significant when the separation energy is small.

The overlap integral calculated with the wave function having a correct asymptotic form has little ambiguity because of the cancellation occurring in it for the interior region, and hence correlation cross sections can be predicted in agreement with data for the QFS in the ^{6}Li breakup reactions. The large difference between the cross sections for the ^{6}Li(p,pd) and ^{6}Li(p,p^3He), (p,pt) reactions with small q values reflects simply the magnitudes of separation energies of the ^{6}Li nucleus into a deuteron and an α particle and into a triton and a ^{3}He nucleus.

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