

Elastic scattering of 7.9-MeV photons from  $^{181}\text{Ta}$ 

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Differential cross sections for elastic scattering of 7.9-MeV photons from  $^{181}\text{Ta}$  have been measured at angles ranging from 25 to 140°. These results and previously measured differential cross sections from  $^{238}\text{U}$  and  $^{232}\text{Th}$  are compared with theoretical predictions taking into account Rayleigh scattering, nuclear Thomson scattering, nuclear resonance scattering, and Delbruck scattering. It is shown that at this energy the forward elastic scattering at angles  $\leq 75^\circ$  is due almost entirely to Delbruck scattering. The experimental results were found to deviate systematically from predicted values throughout most of the angular range. A striking agreement with theory was obtained only after excluding the contribution of the real part of the Delbruck scattering amplitude.

[NUCLEAR REACTIONS  $^{181}\text{Ta}(\gamma, \gamma)$ ,  $E = 7.9$  MeV; measured Delbruck scattering,  $\sigma(\Theta)$ ,  $\Theta = 25\text{--}140^\circ$ .]

## I. INTRODUCTION

The main interest of recent elastic scattering studies of photons was directed to Delbruck scattering<sup>1-15</sup> or the elastic scattering of a photon by a nucleus, considered to be a static Coulomb field. In the intermediate state of this scattering process, an electron-positron pair is formed which subsequently annihilates producing an elastically scattered photon. The Delbruck amplitude is complex. The real (dispersive) part corresponds to virtual pair production which is related to vacuum polarization. The imaginary (absorptive) part corresponds to real pair production in the intermediate state.

In a recent study<sup>11</sup> a short report was published concerning elastic scattering of 7.9-MeV photons from  $^{238}\text{U}$  and  $^{232}\text{Th}$  and a good agreement between theory and experiment was obtained only after excluding the contribution of the calculated<sup>2</sup> real Delbruck amplitudes.

The main objective of the present work is to find whether similar results are obtained in the elastic scattering of 7.9-MeV photons from  $^{181}\text{Ta}$ . The elastic scattering of photons from  $^{181}\text{Ta}$  is composed of a coherent and an incoherent part.<sup>16</sup> The coherent part is a sum of four coherent processes: (1) Thomson scattering from the nucleus, (2) Rayleigh scattering from bound electrons, (3) nuclear resonance scattering, and (4) Delbruck scattering. The incoherent part is related to the tensorial part of the scattering amplitude or the transfer of two units of angular momentum to the nucleus in the elastic scattering process. Because of the contribution of four coherent processes to the elastic scattering, the isolation of the contribution of

Delbruck scattering is rendered very difficult. However, by properly choosing the photon energy and the scattering angle, the contribution of the other scattering processes may be minimized and the contribution of Delbruck scattering and especially the effect of the real amplitudes may be made very large and hence easy to detect experimentally. It turns out that photons of energies in the region  $E_\gamma = 7$  to 9 MeV are suitable for the present study on  $^{181}\text{Ta}$  because the elastic scattering cross section in the angular range 15–75° is due almost entirely to Delbruck scattering and the relative contribution of the real Delbruck amplitude is larger than 60% (see below). This may be seen by noting that at this energy the Rayleigh scattering at angles higher than 20° is negligible. Further, the combined contribution at  $E_\gamma = 7$  to 9 MeV of nuclear Thomson scattering and nuclear resonance scattering amplitudes, being of opposite phase, is near its minimum value. This is illustrated in Fig. 1 which shows the calculated elastic scattering cross section versus energy of nuclear Thomson and nuclear resonance scattering only, for  $^{181}\text{Ta}$ , after excluding the Delbruck and Rayleigh contributions. Figure 1 is in fact the normal doubled-peaked giant-dipole-resonance (GDR) curve<sup>16</sup> extrapolated down to low energies. It is displayed on a semilogarithmic scale to illustrate the interference dip<sup>17</sup> between nuclear Thomson and nuclear resonance scattering amplitudes. The dip may be seen to be at 8.6 MeV which is near  $E_\gamma = 7.9$  MeV used in the present work.

It is very important to note that in selecting this photon energy another condition was fulfilled namely, that the excitation energy in the nucleus is in the region of the continuum where the effect

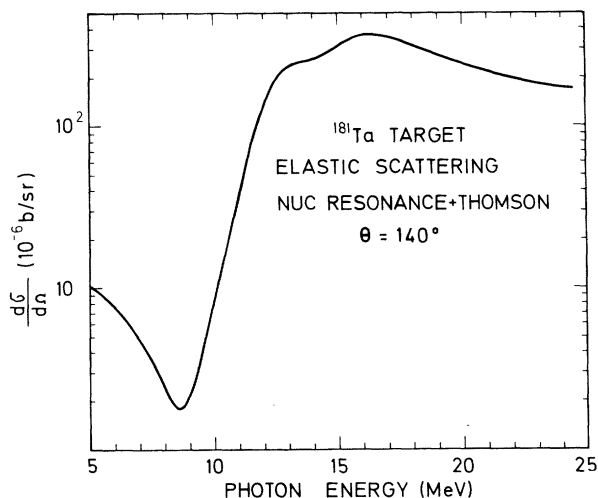


FIG. 1. Calculated elastic scattering cross section from  $^{181}\text{Ta}$  including nuclear resonance scattering and nuclear Thomson scattering only. The Rayleigh and Delbruck scattering amplitudes are excluded. The dip at 8.6 MeV is due to the interference between Thomson and nuclear resonance scatterings. The nuclear resonance amplitudes were calculated using the GDR parameters of Ref. 23.

of isolated resonances and Ericson fluctuations are negligible. The average level spacing<sup>18</sup> in  $^{181}\text{Ta}$  at 7.9 MeV excitation is less than 1 eV, and the total level width is much smaller than the energy spread ( $\Delta E = 24$  eV) of the incident 7.9 MeV  $\gamma$  line. The value of  $\Delta E$  is due to thermal Doppler broadening at a source temperature of 400 C. It should be noted that if some effects of isolated resonances do exist, then it is not permissible to calculate the nuclear resonance amplitudes by extrapolating the giant dipole resonance, nor is it possible to use the usual phase relationship between the resonance scattering amplitude and the other scattering amplitudes.

Several measurements of Delbruck scattering have been carried out employing high-energy  $\gamma$  rays. Stierlin, Scholz, and Povh<sup>7</sup> used 17-MeV  $\gamma$  rays from the  $^7\text{Li}(p, \gamma)$  reaction; Moffat and Stringfellow<sup>6</sup> used 87-MeV  $\gamma$  rays from an electron synchrotron; Bosch *et al.*<sup>8</sup> and Moreh, Salzmann, and Ben-David<sup>10</sup> used 9-MeV  $\gamma$  rays from the  $\text{Ni}(n, \gamma)$  reaction, and Jackson and Wetzel<sup>9</sup> used 10.83-MeV  $\gamma$  rays from the  $^{14}\text{N}(n, \gamma)$  reaction. In all these experiments the contribution of the imaginary Delbruck amplitude has been established. However, nothing could be said with certainty regarding the contribution of the real Delbruck amplitude.

Some experiments were also carried out at  $\sim 1$  MeV.<sup>12-15</sup> In all these experiments, a large discrepancy was found between the measured and calculated cross sections at angles higher than  $90^\circ$ .

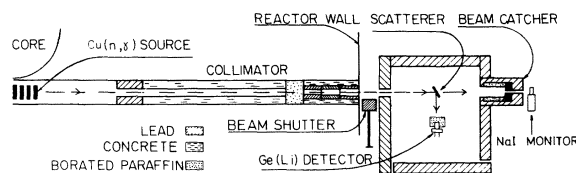


FIG. 2. Schematic diagram of the experimental system.

However, the situation here suffers from uncertainties in making an accurate calculation of the Rayleigh scattering amplitude and hence no final conclusion regarding the contribution of the real Delbruck amplitudes could be reached.

## II. EXPERIMENTAL PROCEDURE

The experimental arrangement is shown schematically in Fig. 2. The  $\gamma$  source was obtained from the  $(n, \gamma)$  reaction on five separated disks of copper, each 8 cm diam and 1.5 cm thick. The discs were placed along a tangential beam tube near the core of the IRR-2 reactor at a thermal neutron flux of  $2 \times 10^{13}$   $n/\text{cm}^2$  sec, yielding typical  $\gamma$  intensities of  $\sim 10^8$  monoenergetic photons/ $\text{cm}^2$  sec on the target position. The 7.915-MeV  $\gamma$  line, the highest energy and the most intense line in the  $\text{Cu}(n, \gamma)$  spectrum results from the ground-state transition in the reaction  $^{63}\text{Cu}(n, \gamma)^{64}\text{Cu}$ . The target was in the shape of a disk 3.5 cm diam and weighing 60 g. The scattered radiation was measured using a 20-cm<sup>3</sup> Ge(Li) detector; the detector-scatterer distance was varied between 40 cm at forward angles to 15 cm at backward angles where the scattering cross section was very small. The energy resolution of the 20-cm<sup>3</sup> Ge(Li) detector at 8 MeV was 12 keV. Time normalization for angular distributions was effected by measuring the neutron flux near the  $\gamma$  source position and by monitoring the incident  $\gamma$  beam using a NaI detector placed behind the beam catcher. The Ge(Li) detector was shielded from the effect of fast neutrons, emitted by the  $(\gamma, n)$  reaction on the target and on the surrounding lead collimators and shielding, by covering it with 4-cm-thick borated paraffin having an inner and outer mantel of borated plastic. Other details of the experimental system were published elsewhere.<sup>19</sup>

## III. THEORETICAL REMARKS

The differential elastic scattering cross section in the case of  $^{181}\text{Ta}$  may be written as<sup>16</sup>:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_c + \left(\frac{d\sigma}{d\Omega}\right)_{ic}, \quad (1)$$

where the subscripts c and ic refer to the coherent

and incoherent contributions to the elastic scattering cross section. For zero-spin even-even nuclei, such as  $^{238}\text{U}$  and  $^{232}\text{Th}$ , the incoherent term reduces to zero (see below).

In terms of the circularly polarized waves, the differential coherent scattering cross section may be written in the form<sup>12</sup>:

$$\left(\frac{d\sigma}{d\Omega}\right)_c = r_0^2(|A|^2 + |A'|^2), \quad (2)$$

where  $r_0 = e^2/mc^2 = 2.818 \times 10^{-13}$  cm is the classical electron radius and  $A$  and  $A'$  are the no-spin-flip and the spin-flip scattering amplitudes, respectively. Each of  $A$  and  $A'$  is a coherent sum of four scattering processes namely:

$$A = A_T + A_R + A_N + A_D, \quad (3)$$

written in the form:

$$\left(\frac{d\sigma}{d\Omega}\right)_c = r_0^2[(A_{T1} + A_{R1} + A_{N1} + A_{D1})^2 + (A'_{T1} + A'_{R1} + A'_{N1} + A'_{D1})^2 + (A_{N2} + A_{D2})^2 + (A'_{N2} + A'_{D2})^2], \quad (5)$$

where the subscripts 1 and 2 refer to the real and imaginary parts of the scattering amplitudes, respectively. We hereby deal with each of the scattering amplitudes separately:

#### A. Nuclear Thomson scattering amplitudes

These amplitudes are given by<sup>12</sup>:

$$\begin{aligned} A_{T1} &= -Z^2(m/M)(\cos\theta + 1)/2, \\ A'_{T1} &= -Z^2(m/M)(\cos\theta - 1)/2, \\ A_{T2} &= A'_{T2} = 0, \end{aligned} \quad (6)$$

where  $Z$  is the charge number of the scattering nucleus and  $m/M$  the ratio of the electron and nuclear mass. The minus sign of  $A_{T1}$  and  $A'_{T1}$  is due to the phase convention mentioned above.

#### B. Rayleigh scattering amplitudes

These amplitudes are very small at  $E_\gamma = 7.9$  MeV and angles higher than  $20^\circ$ . This may be seen by extending the procedure suggested by Brown and Mayers,<sup>20</sup> and used by Schumacher, Smend, and Borchert<sup>15</sup> at  $E_\gamma \approx 1$  MeV, and applying it to  $E_\gamma = 7.9$  MeV. The real parts of the Rayleigh amplitudes may thus be written as the coherent sum of the contribution of  $K$ ,  $L$ ,  $M$ , and  $N$  electrons.

$$\begin{aligned} A_{R1} &= -(A_K + A_L + A_M + A_N), \\ A'_{R1} &= -(A'_K + A'_L + A'_M + A'_N), \\ A_{R2} &= 0, \\ A'_{R2} &= 0. \end{aligned} \quad (7)$$

$$A' = A'_T + A'_R + A'_N + A'_D, \quad (4)$$

where the subscripts  $T$ ,  $R$ ,  $N$ , and  $D$  denote Thomson, Rayleigh, nuclear resonance, and Delbruck components, respectively. The relative phases of the Thomson and Rayleigh amplitudes were taken to be the same. Further, the relative phases of the nuclear resonance amplitudes and the Delbruck amplitudes were taken to be the same. The phases of the real parts of the later amplitudes were taken to be opposite to those of Thomson and Rayleigh amplitudes.<sup>2, 12, 13</sup>

In general, each of the amplitudes in Eqs. (3) and (4) are complex. However  $A_T$  and  $A'_T$  are purely real amplitudes while  $A_R$  and  $A'_R$  can also be considered as real amplitudes because the imaginary parts of  $A_R$  and  $A'_R$  can be neglected.<sup>20</sup> The coherent scattering cross section can therefore be

The minus sign of  $A_{R1}$  and  $A'_{R1}$  is due to the phase convention mentioned above.

The  $K$ -shell Rayleigh amplitudes may be calculated using the form-factor approximation<sup>21</sup> and the "corrected" form-factor approximation,<sup>20</sup> i.e.,

$$\begin{aligned} A'_K &\approx f_K(\cos\theta - 1)/2, \\ A_K &\approx g_K(\cos\theta + 1)/2. \end{aligned} \quad (8)$$

The form factor  $f_k$  and the "corrected" form factor  $g_k$  are given by:

$$f_K = \int |\psi_K|^2 e^{i\vec{k}\cdot\vec{r}} d^3r, \quad (9)$$

$$g_K = \int |\psi_K|^2 e^{i\vec{k}\cdot\vec{r}} [mc^2/(E+V)] d^3r, \quad (10)$$

where  $k = (E_\gamma/c)[2(1 - \cos\theta)]^{1/2}$  is the momentum transfer,  $E$  is the total energy of the  $K$  electron,  $V$  the potential energy, and  $\psi_k$  is the wave function of the  $k$  electron. Following Schumacher, Smend, and Borchert,<sup>15</sup>  $f_k$  and  $g_k$  were calculated using the relativistic Hartree-Fock-Slater wave functions calculated with the aid of the computer program of Liberman, Cromer, and Waber.<sup>22</sup>

The  $L$ -shell amplitudes may be written in precisely the same manner as Eqs. (8), (9), and (10) with  $f_L$ ,  $g_L$ , and  $\psi_L$  inserted instead of  $f_k$ ,  $g_k$ , and  $\psi_k$ , respectively.

For the  $M$ - and  $N$ -shell amplitudes, the fol-

lowing relations were used:

$$A'_{M,N} = f_{M,N}(\cos\theta - 1)/2, \quad (11)$$

$$A_{M,N} = f_{M,N}(\cos\theta + 1)/2$$

with  $f_M$  and  $f_N$  given by similar relations to Eq. (9).

It turned out that the effect of  $A_{R1}$  and  $A'_{R1}$  was to increase the elastic scattering cross section by about 1% at  $25^\circ$  while at higher angles the cross section changed by less than 5%. This is because the Delbruck amplitudes at forward angles ( $25^\circ$ ) are very large and the effect of  $A_{R1}$  and  $A'_{R1}$  is relatively small. At large angles, the Delbruck amplitudes decrease steeply and the relative contribution of  $A_{R1}$  and  $A'_{R1}$  become larger.

#### C. Nuclear resonance scattering amplitudes

Here it is assumed that at 7.9 MeV excitation in  $^{181}\text{Ta}$  the effect of isolated nuclear levels may be neglected and that the nuclear resonance scattering amplitudes may be obtained from an extrapolation of the giant dipole resonance. The  $^{181}\text{Ta}$  nucleus is deformed and hence the GDR is double peaked and is described by a two Lorentzian curve; the corresponding parameters of the GDR may be taken from the literature<sup>23</sup> and were obtained from photoneutron cross-section measurements. The nuclear resonance scattering amplitudes may be written in the form:

$$\begin{aligned} A_{N1} &= (\alpha_1 + \alpha_2)(\cos\theta + 1)/2, \\ A'_{N1} &= (\alpha_1 + \alpha_2)(\cos\theta - 1)/2, \\ A_{N2} &= (\beta_1 + \beta_2)(\cos\theta + 1)/2, \\ A'_{N2} &= (\beta_1 + \beta_2)(\cos\theta - 1)/2. \end{aligned} \quad (12)$$

$\alpha_j$  and  $\beta_j$  (with  $j = 1, 2$ ) are related to the complex scattering amplitudes by:

$$\alpha_j + i\beta_j = \frac{E^2 \sigma_j \Gamma_j}{2r_0 c h} \frac{(E_j^2 - E^2) + i\Gamma_j E}{(E_j^2 - E^2)^2 + \Gamma_j^2 E^2}, \quad (13)$$

where  $E_1$ ,  $\Gamma_1$ , and  $\sigma_1$  denote the peak energy, the width, and the peak cross section of the low-energy resonance, while  $E_2$ ,  $\Gamma_2$ , and  $\sigma_2$  are related to the high-energy resonance of the GDR. Here we have expressed the nuclear resonance amplitudes in terms of the peak cross sections  $\sigma_1$  and  $\sigma_2$  of the GDR and hence the result differs from that given in Ref. 16. For the actual calculation of these amplitudes, the following GDR parameters taken from the work of the Livermore group<sup>23</sup> were used:

$$\begin{aligned} E_1 &= 12.75 \text{ MeV}, & \Gamma_1 &= 3.0 \text{ MeV}, & \sigma_1 &= 198 \text{ mb}, \\ E_2 &= 15.50 \text{ MeV}, & \Gamma_2 &= 5.0 \text{ MeV}, & \sigma_2 &= 224 \text{ mb}. \end{aligned}$$

#### D. Delbruck scattering amplitudes

These amplitudes were calculated numerically by Ehlutzky and Sheppey<sup>2</sup> for several energies between  $0$  and  $120^\circ$  with a claimed accuracy of at best 5 to 10%. The amplitudes were calculated for polarizations perpendicular and parallel to the scattering plane.

The relation between the tabulated<sup>2</sup> amplitudes  $a_1^{\parallel}$  and  $a_1^{\perp}$  and those of the circular polarization representation used in the present work is given by:

$$\begin{aligned} A_{D1} &= (\alpha Z)^2 (a_1^{\parallel} + a_1^{\perp})/2, \\ A'_{D1} &= (\alpha Z)^2 (a_1^{\parallel} - a_1^{\perp})/2, \\ A_{D2} &= (\alpha Z)^2 (a_2^{\parallel} + a_2^{\perp})/2, \\ A'_{D2} &= (\alpha Z)^2 (a_2^{\parallel} - a_2^{\perp})/2, \end{aligned} \quad (14)$$

where  $\alpha = 2\pi e^2/hc$  is the fine-structure constant.

The Delbruck amplitudes at 7.9 MeV and at the angles measured in the present work were obtained from the tabulated amplitudes<sup>2</sup> by applying smooth graphical interpolations. The amplitudes at  $140^\circ$  were obtained by smooth graphical extrapolations. The contribution of the Delbruck amplitudes to the elastic cross section is dominant at forward angles ( $\leq 75^\circ$ ).

#### E. Incoherent elastic scattering

The incoherent contribution<sup>16</sup> to the elastic scattering cross section arises from one scattering process only, namely, from nuclear scattering from the GDR. This is related to the tensorial term of elastic scattering and corresponds to the transfer of two units of angular momentum in the scattering process. This contribution is given by:

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_{ic} &= r_0^2 (I_0 K_0 20 | I_0 2 I_0 K_0)^2 [(2\alpha_1 - \alpha_2)^2 \\ &+ (2\beta_1 - \beta_2)^2] \frac{(13 + \cos^2\theta)}{40}, \end{aligned} \quad (15)$$

where  $I_0$  is the nuclear spin and  $K_0$  is the projection of  $I_0$  on the symmetry axis of the nucleus;  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$ , and  $\beta_2$  were defined in Eq. (13). The Clebsch-Gordan coefficient expresses the weight of the transition to the ground state. Equation (15) was obtained by substituting the amplitudes given in Eq. (13) in the expression for the tensorial contribution to the elastic cross section derived in Ref. 16. The tensor scattering term vanishes unless  $I_0 \geq 1$ . Hence for  $^{238}\text{U}$  and  $^{232}\text{Th}$  where  $I_0 = 0$ ,  $(d\sigma/d\Omega)_{ic} = 0$ . The contribution of the incoherent term to the elastic scattering cross section is relatively small in  $^{181}\text{Ta}$ . It is 2% at  $60^\circ$  and

reaches 12% at  $140^\circ$  when the real part of the Delbruck amplitude is neglected.

#### IV. RESULTS

In order to measure the differential scattering cross section, it was necessary to measure the intensity of the incident  $\gamma$  beam and the scattered photons under similar geometrical conditions.<sup>19</sup> The measurements of the intensity of the incident  $\gamma$  beam was difficult because it required the reduction of the  $\gamma$  beam intensity by a factor of  $\sim 10^5$  to a level tolerated by the Ge(Li) detector using five pieces of lead each 4 cm thick. The attenuation of each piece of Pb and hence the attenuation coefficient  $\mu$  was measured using two different methods. The first method is highly accurate and is based on the resonance scattering<sup>24</sup> of 7.915-MeV photons by  $^{144}\text{Nd}$  using a natural Nd target.<sup>25</sup> In the other method, the value of  $\mu$  was directly determined by measuring the resulting intensity of the beam as a function of absorber thickness. Both experiments yielded almost the same value of  $\mu$  for Pb, namely:  $\mu(7.915 \text{ MeV}) = 0.04669 \pm 0.00009 \text{ cm}^2/\text{g}$  which is higher by 2% than the values available in the literature.<sup>26</sup> It should be noted that a 2% deviation in  $\mu$  can introduce a 25% error in the absolute cross section and hence an accurate determination of  $\mu$  was very important for the present results.

Figure 3 shows a portion of the elastically scattered spectrum from Ta measured at  $35^\circ$ . The steeply rising background towards lower energies is due to inelastic processes. The figure also shows the corresponding energy spectrum of the

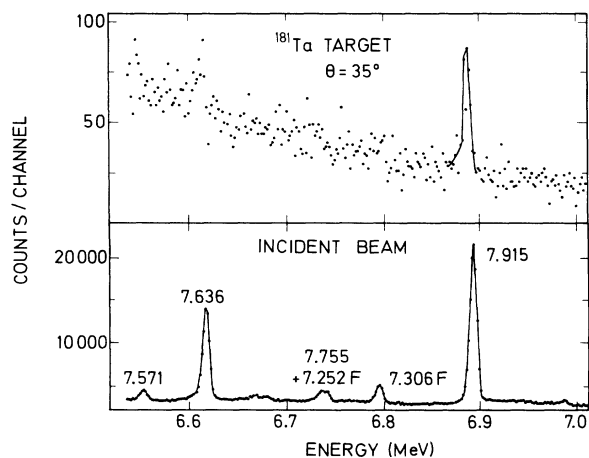


FIG. 3. High-energy part of the incident  $\text{Cu}(n, \gamma)$  spectrum and the scattered spectrum from a Ta target at  $35^\circ$  as measured by a  $20\text{-cm}^3$  Ge(Li) detector. Lines indicated by F denote single-escape peaks while other lines indicate double-escape peaks.

incident  $\gamma$  beam. The experimental data for  $^{181}\text{Ta}$  together with the calculated cross sections are given in graphical form in Fig. 4 and the numerical values are listed in Table I. The running time at each angle is of the order of 3 days depending on the cross section. The experimental points of Fig. 4 are the averages of two independent measurements. Similar results for  $^{238}\text{U}$  and  $^{232}\text{Th}$ , published earlier,<sup>11</sup> are given in graphical form in Fig. 5.

#### V. DISCUSSION

The solid curve of Fig. 4 includes contributions from Thomson, Rayleigh, nuclear resonance, and Delbruck amplitudes. In the dashed curve, the contribution of the calculated real Delbruck amplitudes is excluded. It is evident that the dashed curve is in much better agreement with experiment than the solid curve.

It should be remarked in passing that the agreement in Fig. 4 at backward angles should be treated with some reservation because at this angular range the elastic scattering cross section is very sensitive to small changes in the value of the scattering amplitudes. For example when the

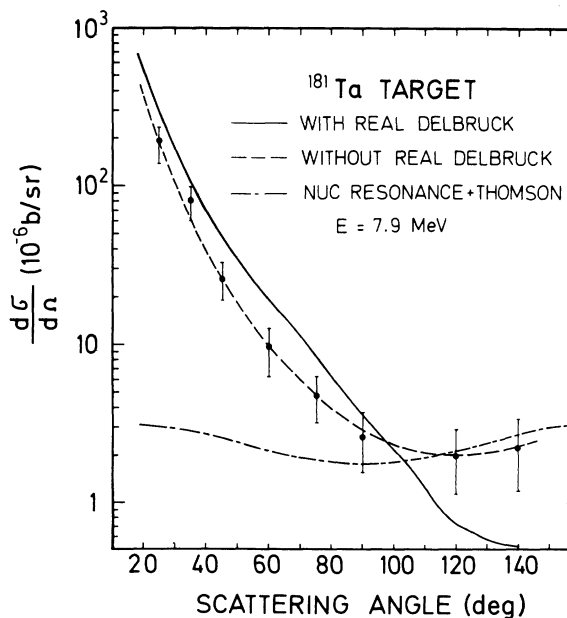


FIG. 4. Differential elastic scattering cross sections for 7.9-MeV photons from  $^{181}\text{Ta}$ . The solid curve represents the calculated values obtained by including the amplitudes of Thomson, Rayleigh, nuclear resonance, and Delbruck scatterings. The dashed curve represents the result obtained after excluding the real Delbruck amplitudes while the dash-dot curve represents the result obtained by including nuclear resonance and Thomson scattering only. The incoherent contribution is included in all calculations.

TABLE I. Measured differential cross sections ( $\mu\text{b}/\text{sr}$ ) for elastic scattering from a  $^{181}\text{Ta}$  target at various angles. The calculated cross sections include the contributions of all scattering processes with and without the real part of the Delbruck scattering amplitude.

Angle (deg)	Experiment	Calculated	
		Without real	With real
25	195 $\pm$ 50	190	290
35	82 $\pm$ 20	62	110
45	26 $\pm$ 7	25	48
60	9.7 $\pm$ 3.5	9.8	19
75	4.9 $\pm$ 1.5	4.8	8.5
90	2.6 $\pm$ 1.1	2.9	3.8
120	2.0 $\pm$ 0.9	2.1	0.76
140	2.2 $\pm$ 1.1	2.2	0.51

set of GDR parameters obtained by the Saclay group<sup>27</sup> is used instead of the Livermore set<sup>23</sup> for calculating the elastic cross section, a large deviation from experiment was obtained; however, the values at forward angles  $\leq 75^\circ$  remained essentially the same.

The results for  $^{232}\text{Th}$  and  $^{238}\text{U}$  shown in Fig. 5 also display a good agreement with theory only after throwing off the real Delbruck amplitudes and in this sense are strikingly similar to those of  $^{181}\text{Ta}$ . This discrepancy between theory and experiment can only be due to inaccuracies in the Delbruck scattering amplitudes because at forward angles ( $\leq 60^\circ$ ) the contribution of the nuclear resonance scattering, Thomson, and Rayleigh scat-

tering is very small, less than 10%, while the discrepancies between theory and experiment here are higher than 60%.

Further supporting evidence in favor of neglecting the real Delbruck amplitudes may be obtained by examining the work of Moreh, Salzmann, and Ben-David<sup>10</sup> carried out at angles 7–20° using  $E_\gamma = 9$  MeV. Here, a renewed analysis of the results yielded also a better agreement with calculated values after excluding the real Delbruck amplitudes. The results of the work of Jackson and Wetzel<sup>9</sup> using  $E_\gamma = 10.83$  MeV, may also be viewed as yielding evidence to the same effect because in their work, a good agreement between theory and experiment was also obtained after excluding the real Delbruck amplitudes. However, at 10.83 MeV, the contribution of these amplitudes to the elastic cross section is small  $\leq 20\%$  at most scattering angles. This is smaller than the experimental uncertainties of their work. These experimental data might suggest that the calculated values of the real Delbruck amplitudes as given by Ehlitzky and Sheppey<sup>2</sup> are much higher than their actual values.

It should be emphasized that the real Delbruck amplitudes must exist and we are only questioning here the actual magnitude of these amplitudes. The real amplitudes are related to vacuum polarization which is predicted by quantum electrodynamics and confirmed by the agreement between the measured and calculated value of the Lamb shift. Moreover, the real and imaginary amplitudes are connected by a dispersion relationship, hence the experimental evidence for the

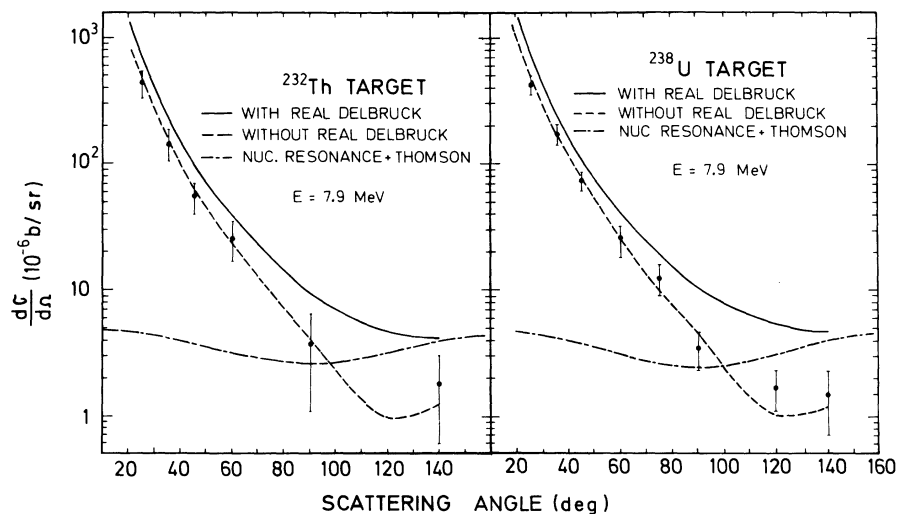


FIG. 5. Differential elastic scattering cross sections for 7.9-MeV photons from  $^{232}\text{Th}$  and  $^{238}\text{U}$ . The solid curve represents the calculated values obtained by including the amplitudes of Thomson, Rayleigh, nuclear resonance, and Delbruck scatterings. The dashed curve represents the result obtained after excluding the real Delbruck amplitudes while the dash-dot curve represents the result obtained by including nuclear resonance and Thomson scattering only.

existence of the latter provides indirect evidence for the existence of the former. A reconciliation between theory and experiment may be obtained by reducing the real Delbruck reported in Ref. 2 by at least a factor of 2.5. Alternatively, if one assumes that *both* the real and imaginary amplitudes are overestimated,<sup>2</sup> one may attempt to bring theory in agreement with experiment by reducing both amplitudes, while keeping the amplitudes of other scattering processes the same.

Such attempts, for <sup>181</sup>Ta, were only partially successful. It was necessary to reduce both Delbruck amplitudes by 25% to achieve agreement with experiment in the angular range 25–75° while at higher angles a large deviation remained. A similar situation was found to hold for <sup>232</sup>Th and <sup>238</sup>U where a reduction of about 20% was necessary to achieve agreement in the same angular range. Such reduction is higher than the 10% claimed accuracy of the calculated Delbruck amplitudes.<sup>2</sup>

It is important to note that since first-order Born approximation was employed in calculating the Delbruck amplitudes,<sup>2</sup> one might expect that by including the higher-order terms, more accurate values of the scattering amplitudes may be obtained. The effect of higher-order terms, namely, multiphoton exchange with the nucleus was evaluated at very high energies (~1 BeV) by Cheng and Wu<sup>3</sup> and resulted in a reduction of as much as a factor of 4 in the imaginary Delbruck amplitudes

for <sup>181</sup>Ta and for momentum transfers  $k \gg mc$  encountered in the present work. The effect on the real amplitudes was not calculated<sup>3</sup> because these amplitudes are negligible at such high energies. It is not clear whether this reduction in the amplitudes persists down to energies as low as 8 MeV and whether it is also true for the real amplitudes.

It should be remarked that in an earlier work using ~1-MeV photons, Basavaraju and Kane<sup>14</sup> have also questioned the correctness of the calculated values of the real Delbruck amplitudes. They assumed that the Rayleigh amplitudes at ~1 MeV are accurately known and that these amplitudes interfere destructively with the real Delbruck amplitudes. Thus they were able to deduce an empirical value of the real Delbruck amplitude at 124.5° and found it to be 4 times higher than the calculated value,<sup>2</sup> while at 90° they found it to be unexplicably smaller. Besides the self-inconsistency of this conclusion, it should be noted that the results of the present work imply that the calculated Delbruck amplitudes<sup>2</sup> are overestimated and not underestimated.

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