Configuration mixing of two-quasiparticle states in even-even deformed nuclei*

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Evidence of configuration mixing of higher-K bands in deformed even nuclei is surveyed. A general formulation for configuration mixing due to a two-body neutron-proton force is developed. A fit to the energy splittings of Gallagher-Moszkowski pairs in odd-odd nuclei is made to obtain an effective Gaussian central force except for the undetermined Wigner component. With this force, off-diagonal band-mixing matrix elements are calculated for various configurations in ^{176}Hf, ^{178}Hf, and ^{174}Yb. By solving BCS equations, the relevant occupation amplitudes are calculated. The effective n-p Wigner force component is fixed to give best over-all agreement to experimental band-mixing information. The resulting force is compared with the Jones, Onishi, Hess, and Sheline central force for deformed nuclei.

[NUCLEAR STRUCTURE ^{174,176,178,180}Hf, ^{174}Yb; calculated configuration mixing higher K bands, fit energy splittings, Gallagher-Moszkowski pairs, n-p force deduced.]

I. INTRODUCTION

Among certain classes of two-quasiparticle states of deformed nuclei configuration mixing has been extensively treated, while for other classes there has been almost no attention to this problem.

On the one hand, the excited bands of $K^\pi = 0^+$, 1^+, 2^+, 0^-, 1^-, 2^-, and 3^- in even-even nuclei have been extensively treated microscopically. These treatments are usually carried out with some simple separable interactions (quadrupole-quadrupole, octupole-octupole, spin-quadrupole, or surface δ interaction). Some bands in the systems treated may become "collective" and consist of a linear combination of many two-quasiparticle basis states.

On the other hand, there has been little theoretical attention to the question of configuration mixing of higher-K bands than those mentioned above, and the general question of the effective nucleon-nucleon force appropriate in this context is quite open.

II. EXPERIMENTAL EVIDENCE

In the past several years interesting measurements have been made concerning band mixing of two-quasiparticle states in even-even nuclei. The even-even nuclei in the region around ^{178}Hf are interesting because of their prolific isomerism, associated with the availability of only large-Ω Nilsson orbitals near the Fermi energy: For protons the orbitals involved are $\frac{7}{2}^+[404]$, $\frac{9}{2}^-[514]$, and $\frac{5}{2}^+[402]$, and for neutrons they are $\frac{5}{2}^-[512]$, $\frac{7}{2}^-[514]$, and $\frac{9}{2}^+[624]$. Thus, relatively low-lying $K^\pi = 6^+$, 8^-, and 7^- states can be formed either as two-quasiproton or two-quasineutron states.

Khoo *et al.*[1] have carried out impressive measurements of excited bands in ^{176}Hf. Their analysis shows that the $K^\pi = 6^+$ bands at 1333.1 and 1761.5 keV are highly mixed between two-quasi-

proton and two-quasineutron configurations with a $2p$-$2n$ mixing ratio of 38 : 62. Ejiri, Hagemann, and Hammer[2] have independently made similar measurements and come to a similar conclusion. Ejiri, Hagemann, and Hammer analyzed the 1549-keV $K^\pi = 6^+$ band in ^{174}Hf as being at least 90% two-quasiproton, whereas we determine, from the comparison of the $E2$ hindrance factors of ^{174}Yb with those of ^{176}Hf, that the $K^\pi = 6^+$ isomeric state in ^{174}Yb is nearly pure two quasineutron.

Ejiri, Hagemann, and Hammer and Khoo *et al.* differ somewhat in their analysis of mixing of the $K^\pi = 8^-$ bands in ^{176}Hf, due in part to the additional complication of Coriolis mixing. Because of the Coriolis complication, we shall exclude this case from our quantitative analyses to follow. That is, one needs to consider mixing of more than two bands, and we wish here to confine ourselves to cases of two band mixing.

In ^{178}Hf the two $K^\pi = 8^-$ states have been known from β-decay properties to be highly mixed. Studies by Helmer and Reich[3] and by Ward, Chu, and Cumming[4] indicate that the 1147-keV state (mainly two-quasineutron) and the 1480-keV state (mainly two-quasiproton) have mixing ratios of 33 : 67 and 35 : 65, respectively. The ^{178}Lu^m β-decay rates measured by Tamura[5] give mixing of 36 : 64 in good agreement.

Körner, Wagner, and Dunlap[6] measured the magnetic moment of the 1142-keV $K^\pi = 8^-$ state in ^{180}Hf as $(8.6 \pm 1.0)\mu_N$, signifying nearly pure two-quasiproton configuration.

Another case of such configuration mixing occurs in ^{174}Yb. The ^{174}Tm (5.2-min) ground state has been assigned a Nilsson configuration of $\frac{1}{2}^+[411]_p$, $\frac{7}{2}^-[514]_n$ (which is consistent with general systematics). The ^{174}Tm ground-state β ray decays to two states of ^{174}Yb at 1886 (~80%) and 2383 keV (~20%) with logft values of 4.90 and 4.65, respectively. The logft values of less than 5.0 would require a β transition involving the $\frac{7}{2}^-[514]_n \rightarrow \frac{9}{2}^-[514]_p$ orbitals. The strong γ transition between the two states in ^{174}Yb, the observed logft values, and the general energy systematics of two-quasiparticle states would suggest that the state at 1886 keV (mainly two neutron) and the state at 2383 keV (mainly two proton) are highly mixed. These two states can be assigned to the $\frac{1}{2}^-[521]_n$, $\frac{9}{2}^+[624]_n$, and $\frac{1}{2}^+[411]_p$, $\frac{9}{2}^-[514]_p$ orbitals, respectively, with $K^\pi = 5^-$. From the logft values one would deduce a 35 : 65 mixing ratio for these bands.

Bernthal, Rasmussen, and Hollander[7] measured electron capture logft values to ^{176}Hf of 7.21 and 6.85 to the $K^\pi = 1^+$ states at 1672.3 and 1862.8 keV, respectively. With the reasonable ^{176}Ta ground-state assignment by Valentin and Santoni[8] of $K^\pi = 1^-$ ($\frac{7}{2}^+[404]_p$, $\frac{5}{2}^-[512]_n$), it is easy to see that the β decay can proceed via a first-forbidden unhindered transition to the two-quasineutron component of the final $K^\pi = 1^+$ state but not to the two-quasiproton part. This conclusion is the same even if configuration mixing of

$$(\tfrac{5}{2}^+[402]_p, \tfrac{7}{2}^-[514]_n)$$

is allowed in the initial state. Thus, the ratio of ft values tells us that the lower 1^+ state is 67.3% two quasiproton and 32.7% two quasineutron, while the upper 1^+ state is the reverse. We note that the mixing of the 1^+ states is also implied by the strong 190.4-keV γ-ray transition between these levels. There might be some reservations about treating these two $K^\pi = 1^+$ states in isolation from all other $K^\pi = 1^+$ basis states. Gabrakov *et al.*[9] and Hamamoto and Birbrair[10] have made random-phase-approximation calculations of 1^+ states taking into account the spurious state problem associated with the rotational degree of freedom. We feel that our isolated treatment here may be approximately justified, since the j_+ matrix elements between our $\frac{5}{2}$ and $\frac{7}{2}$ states are very small (hence, little coupling with the spurion) and since other 1^+ basis states should be considerably higher lying in ^{176}Hf.

Stripping or pickup reactions into the mixed bands treated here would be of great interest. We note that Zaitz and Sheline[11] have studied the levels of ^{176}Hf populated in the (d, t) reaction. Their analysis assumes no band mixing. In their level scheme (Fig. 2) they do not show the levels of Khoo's upper $K^\pi = 6^+$ band, which, if pure two quasiproton, should not be populated by the (d, t) reaction. (The first two levels of this upper $K^\pi = 6^+$ band are at $E_{6^+} = 1761.5$ keV and $E_{7^+} = 1926.7$ keV.) However, to the extent that this is mixed with the $K^\pi = 6^+$ band at 1333 keV (which is mainly two quasineutron), then the levels of the upper $K^\pi = 6^+$ band should be populated relative to the lower $K^\pi = 6^+$ in proportion to their mixing ratios. In their Fig. 1, showing the experimental spectrum, there are two unassigned peaks (peaks numbers 11 and 16) with the appropriate energies and intensities to be the members of the upper $K^\pi = 6^+$ band, consistent with the mixing ratio of Khoo *et al.*[1] The $K^\pi = 1^+$ bands are evidently populated too weakly to be seen in the spectrum of Fig. 1, and their theoretical estimates give weak population of the 1^+ bands. Thus, from their work we can gain no evidence to supplement the β-decay information on the 1^+ band mixing.

III. GENERAL THEORY AND RESULTS

Can we obtain from theory at least a qualitative explanation of the phenomena so far observed?

Very generally, when considering two states which interact with one another (but do not interact with any other state), the Schrödinger equation can be written in the form:

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} \alpha\psi_1(\vec{r}) \\ \beta\psi_2(\vec{r}) \end{pmatrix} = \lambda \begin{pmatrix} \alpha\psi_1(\vec{r}) \\ \beta\psi_2(\vec{r}) \end{pmatrix},$$

$$\alpha^2 + \beta^2 = 1. \quad (1)$$

We introduce χ as the mixing ratio between the two states, letting $\chi = \beta^2/\alpha^2$. Also, let us write $m \equiv \langle\psi_1|H_{12}|\psi_2\rangle = \langle\psi_2|H_{21}|\psi_1\rangle$. If $m = 0$ (i.e., no mixing) then $\chi = 0$ or ∞. When solving Eq. (1), one easily finds:

$$m = \frac{\Delta\lambda\sqrt{\chi}}{1+\chi}, \quad (2)$$

where $\Delta\lambda = \lambda_> - \lambda_<$ is the difference of the eigenvalues of the 2×2 matrix, i.e., is the energy difference of like-spin members of the two bands of the configurations considered. In Fig. 1 we have graphed the relationship of Eq. (2). The different mixing ratios form a family of straight lines relating the proportionality of the energy separation of the bands with the absolute value of the mixing matrix element $|m|$. The lines have been labeled with mixing ratios less than unity, but those ratios χ greater than unity correspond to the line of their reciprocals $1/\chi$.

When the mixing ratio is near unity ($0.5 \lesssim \chi \lesssim 2$) the mixing matrix element $|m|$ is very insensitive to χ and dependent on $\Delta\lambda$.

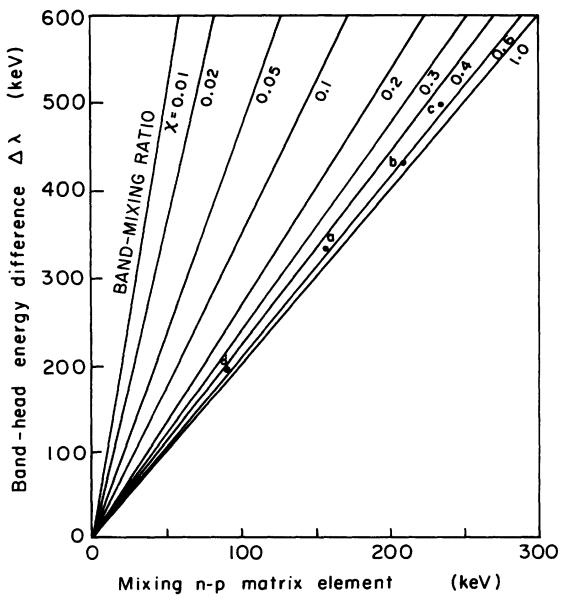

FIG. 1. Relation between $\Delta\lambda$, m, and χ. See text for definitions. The experimental points correspond to: a, $K^\pi = 8^-$ states in ^{178}Hf; b, $K^\pi = 6^+$ states in ^{176}Hf; c, $K^\pi = 5^-$ states in ^{174}Yb; and d, $K^\pi = 1^+$ states in ^{176}Hf.

We shall now consider a specific nuclear Hamiltonian for further analysis of this problem

$$H = H_{\text{Nils}} + H_{\text{pair}}^{(p)} + H_{\text{pair}}^{(n)} + V_{\text{res}}^{(n-p)}. \quad (3)$$

The first term is the single-particle Nilsson Hamiltonian. We calculated eigenvalues by the sophisticated 1969 version of the Nilsson model.[12] The Hamiltonian includes hexadecapole as well as quadrupole deformation, and diagonalization is across 11 oscillator shells, without truncation to a single shell. (We shall not be quite consistent in that the Nilsson eigenfunctions used in the calculation of the matrix elements of the residual interaction are single-shell truncated eigenfunctions without hexadecapole deformation, as in the 1955 Nilsson calculations.[13]) The second and third terms of the Hamiltonian are the proton and neutron pairing interactions, taken in the simple constant pairing approximation

$$H_{\text{pair}}^{(\tau)} = -G_\tau \sum_{ij} a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{j}} a_j. \quad (4)$$

The fourth term, the residual neutron-proton interaction, must be taken of a more sophisticated form to treat the phenomena considered here. The effective neutron-proton potential is taken to be of finite-range Gaussian central form

$$V_{np} = \exp(-r^2/r_0^2)(V_{\text{TE}} P_{\text{TE}} + V_{\text{TO}} P_{\text{TO}} + V_{\text{SE}} P_{\text{SE}} + V_{\text{SO}} P_{\text{SO}}), \quad (5)$$

where P_{TE}, P_{TO}, P_{SE}, and P_{SO} are projection operators for the spin triplet (T) or singlet (S) and even (E) or odd (O) relative orbital angular momentum.

$V_{\text{res}}^{(n-p)}$ can be written as:

$$V_{\text{res}}^{n-p} = \sum_{npn'p'} \langle np|V_{np}|n'p'\rangle a_n^\dagger a_p^\dagger a_{n'} a_{p'}. \quad (6)$$

Here a and $a^\dagger$ are the nucleon annihilation and creation operators. Indices n and n' represent all the quantum numbers which are necessary to specify the Nilsson orbitals occupied by the two neutrons.

We construct BCS solutions to the first three terms of the Hamiltonian [Eq. (3)] by iteratively solving the BCS equations for the ground state of proton and neutron systems.

Consider now what the residual neutron-proton interaction gives for the off-diagonal matrix element m, which can be expressed as

$$m = \langle\psi_{n_1 n_2}|V_{\text{res}}^{n-p}|\psi_{p_1 p_2}\rangle. \quad (7)$$

$|\psi_{n_1 n_2}\rangle$ is the ket for the two-quasineutron excited nuclear state.

If the two-quasiparticle states we are consider-

ing both have parallel angular momentum projections, that is $\Omega_{n_1}+\Omega_{n_2}$ and $\Omega_{p_1}+\Omega_{p_2}$, then $|\psi_{n_1 n_2}\rangle$ is given by

$$|\psi_{n_1 n_2}\rangle = (a^\dagger_{n_1} a^\dagger_{n_2})|\phi(n_1 n_2)\rangle\rangle\ , \tag{8}$$

where

$$|\phi(n_1 n_2)\rangle\rangle = \prod_{i\neq n_1 n_2} (u_i + v_i\, a^\dagger_i a^\dagger_{\bar{i}}) \prod_j (u_j + v_j\, a^\dagger_j a^\dagger_{\bar{j}})|0\rangle\ . \tag{9}$$

The index i goes over neutron orbitals, the index j over proton orbitals. $|0\rangle$ represents the vacuum. A bar over a subscript denotes the time-reversed orbital.

For the case of parallel angular momentum projection, the only terms of the residual interaction operator which may connect the two two-quasiparticle states are:

$$\begin{aligned}
&-\langle n_1\bar{p}_2|V_{np}|\bar{n}_2 p_1\rangle (a^\dagger_{n_1} a_{\bar{n}_2})(a^\dagger_{p_1} a_{\bar{p}_2})^\dagger \\
&-\langle n_1\bar{p}_1|V_{np}|\bar{n}_2 p_2\rangle (a^\dagger_{n_1} a_{\bar{n}_2})(a^\dagger_{p_2} a_{\bar{p}_1})^\dagger \\
&-\langle n_2\bar{p}_2|V_{np}|\bar{n}_1 p_1\rangle (a^\dagger_{n_2} a_{\bar{n}_1})(a^\dagger_{p_1} a_{\bar{p}_2})^\dagger \\
&-\langle n_2\bar{p}_1|V_{np}|\bar{n}_1 p_2\rangle (a^\dagger_{n_2} a_{\bar{n}_1})(a^\dagger_{p_2} a_{\bar{p}_1})^\dagger\ .
\end{aligned} \tag{10}$$

The matrix element of V^{n-p}_{res} may be factored, this because we are not taking into account higher-order correlations in the ground state. Let us do this for one of the terms of Eq. (10).

$$\begin{aligned}
\langle\psi_{n_1 n_2}|[\langle n_1\bar{p}_2|V_{np}|\bar{n}_2 p_1\rangle (a^\dagger_{n_1} a_{n_2})(a^\dagger_{p_1} a_{\bar{p}_2})^\dagger]|\psi_{p_1 p_2}\rangle \\
= \langle n_1\bar{p}_2|V_{np}|\bar{n}_2 p_1\rangle \langle\psi_{n_1 n_2}|a^\dagger_{n_1} a_{\bar{n}_2}|\phi\rangle \\
\times\langle\phi|(a^\dagger_{p_1} a_{\bar{p}_2})^\dagger|\psi_{p_1 p_2}\rangle\ ,
\end{aligned} \tag{11}$$

where $|\phi\rangle$, the BCS ground state of the system, is given by

$$|\phi\rangle = \prod_i (u_i + v_i\, a^\dagger_i a^\dagger_{\bar{i}}) \prod_j (u_j + v_j\, a^\dagger_j a^\dagger_{\bar{j}})|0\rangle\ . \tag{12}$$

Consider the neutron factor in Eq. (11). Using Eq. (9), we can write:

$$\begin{aligned}
|\phi\rangle = (&u_{n_1}u_{n_2} + u_{n_1}v_{n_2}a^\dagger_{n_2}a^\dagger_{\bar{n}_2} + v_{n_1}u_{n_2}a^\dagger_{n_1}a^\dagger_{\bar{n}_1} \\
&+ v_{n_1}v_{n_2}a^\dagger_{n_1}a^\dagger_{\bar{n}_1}a^\dagger_{n_2}a^\dagger_{\bar{n}_2})|\phi(n_1 n_2)\rangle\rangle
\end{aligned} \tag{13}$$

and we get, using Eqs. (8) and (13), that

$$\langle\psi_{n_1 n_2}|a^\dagger_{n_1}a_{\bar{n}_2}|\phi\rangle = -u_{n_1}v_{n_2}\ . \tag{14}$$

Similarly one gets

$$\langle\psi_{n_1 n_2}|a^\dagger_{n_2}a_{\bar{n}_1}|\phi\rangle = v_{n_1}u_{n_2}\ . \tag{15}$$

Using Eqs. (14) and (15) and similar equations for the proton part, and using the fact that $\langle np|V_{np}|n'p'\rangle = \langle\bar{n}'\bar{p}'|V_{np}|\bar{n}\bar{p}\rangle$, one gets for the case of parallel angular momentum projections $\Omega_{n_1}+\Omega_{n_2}$ and $\Omega_{p_1}+\Omega_{p_2}$

$$\begin{aligned}
|m| = |&(u_{n_1}v_{n_2}u_{p_1}v_{p_2} + v_{n_1}u_{n_2}v_{p_1}u_{p_2})\langle n_1\bar{p}_2|V_{np}|\bar{n}_2 p_1\rangle \\
&-(u_{n_1}v_{n_2}v_{p_1}u_{p_2} + v_{n_1}u_{n_2}u_{p_1}v_{p_2})\langle n_1\bar{p}_1|V_{np}|\bar{n}_2 p_2\rangle\ |\ .
\end{aligned} \tag{16}$$

Similar expressions have been given by Rowe[14] and Soloviev.[15]

In the case of antiparallel angular momentum projection $\Omega_{p_1}-\Omega_{p_2}$ and $\Omega_{n_1}-\Omega_{n_2}$, the two-quasineutron ket and the terms of residual interaction that connect the two states are given by

$$|\psi_{n_1\bar{n}_2}\rangle = a^\dagger_{n_1}a^\dagger_{\bar{n}_2}|\phi(n_1 n_2)\rangle\rangle\ , \tag{17}$$

$$\begin{aligned}
&-\langle n_1 p_2|V_{np}|n_2 p_1\rangle (a^\dagger_{n_1}a_{n_2})(a^\dagger_{p_1}a_{p_2})^\dagger \\
&-\langle n_1\bar{p}_1|V_{np}|n_2\bar{p}_2\rangle (a^\dagger_{n_1}a_{n_2})(a^\dagger_{\bar{p}_2}a_{\bar{p}_1})^\dagger \\
&-\langle\bar{n}_2 p_2|V_{np}|\bar{n}_1 p_1\rangle (a^\dagger_{n_2}a_{\bar{n}_1})(a^\dagger_{p_1}a_{p_2})^\dagger \\
&-\langle\bar{n}_2\bar{p}_1|V_{np}|\bar{n}_1\bar{p}_2\rangle (a^\dagger_{n_2}a_{\bar{n}_1})(a^\dagger_{\bar{p}_2}a_{\bar{p}_1})^\dagger\ .
\end{aligned} \tag{18}$$

Performing the algebra one obtains for m, in this case

$$\begin{aligned}
|m| = |&(u_{n_1}v_{n_2}u_{p_1}v_{p_2} + v_{n_1}u_{n_2}v_{p_1}u_{p_2})\langle n_1 p_2|V_{np}|n_2 p_1\rangle \\
&+(u_{n_1}v_{n_2}v_{p_1}u_{p_2} + v_{n_1}u_{n_2}u_{p_1}v_{p_2})\langle n_1\bar{p}_1|V_{np}|n_2\bar{p}_2\rangle\ |\ .
\end{aligned} \tag{19}$$

For the "mixed" cases, that is for cases where one of the two-quasiparticle states has parallel angular momentum projections and the other antiparallel angular momentum projections, one can derive similar equations.

In the derivations of Eqs. (16) and (19) we have assumed no blocking effects, that is, the BCS amplitudes u_i and v_i are the same for the ground and two-quasiparticle states. If blocking is taken into account, the BCS equations are solved separately for u', v' with the orbitals n_1, n_2 (p_1, p_2) removed from the system and with one pair less of nucleons. Core overlap reduction factors of the form (in the neutron case)

$$R_n = \prod_{i\neq n_1 n_2} (u_i u'_i + v_i v'_i) \tag{20}$$

modify Eqs. (16) and (19). We have not concerned ourselves with blocking corrections at this stage. They may be partly compensated if one further refines to number-projected methods such as FBCS.[16]

In the pairing factors only the combination uv occurs. Thus, the matrix elements can only be large when the two orbitals are on opposite sides of the Fermi energy. This condition is approxi-

mately fulfilled for the 72-proton configuration of Hf, since the Fermi energy should be nearly at the degenerate $\frac{7}{2}^+$ and $\frac{9}{2}^-$ orbitals with the $\frac{5}{2}^+$ orbital lying slightly higher [cf., Figs. 2(c) and 2(d) of Nilsson *et al.*[12]].

A. Pairing correction factors

The *uv* factors for the cases here considered are shown in Table I. They have been calculated by solving the BCS equations with the single-particle energies obtained with the same program used in Ref. 12, taking into account for each nucleus its quadrupole and hexadecapole deformation as given in Fig. 12(a) in the same reference. However, close to the Fermi energy we used energies obtained by interpolating the empirical single-particle energies obtained from experiment by Ogle *et al.*[17] in order to get the most realistic *uv* factors. We used the experimental gap parameters Δ_{exp} given in Figs. 4(a) and 4(b) of Ref. 12 and then adjusted the pairing strength parameter G so that Δ_{exp} was in agreement with the value of Δ obtained by solving the BCS equations.

For the specific case of $K^\pi = 8^-$ states in Hf, with n_1 the $\frac{9}{2}^+$, n_2 the $\frac{7}{2}^-$, p_1 the $\frac{9}{2}^-$, and p_2 the $\frac{7}{2}^+$ orbitals, the first two involve $K^\pi = 0^-$ couplings for the n-p matrix elements, and the second two terms involve $K^\pi = 1^+$ couplings.

More specifically, for the $K^\pi = 8^-$ states in ^{178}Hf, one has from Eq. (5) and Table I

$$|m_{8^-}(^{178}\text{Hf})| = |0.471\langle \tfrac{9}{2}_n \bar{\tfrac{9}{2}}_p | V_{np} | \bar{\tfrac{7}{2}}_n \tfrac{7}{2}_p\rangle_{0^-} - 0.501\langle \tfrac{9}{2}_n \bar{\tfrac{7}{2}}_p | V_{np} | \bar{\tfrac{7}{2}}_n \tfrac{9}{2}_p\rangle_{1^+}|. \quad (21a)$$

In the matrix elements only the Ω quantum numbers are shown. For a complete labeling of the states see Table I. All the contributing matrix elements are of large momentum change (or exchange-interaction) type, like those of the odd-even shift terms in odd-odd nuclei. Similarly, one has

$$|m_{6^+}(^{176}\text{Hf})| = |0.515\langle \tfrac{7}{2}_n \bar{\tfrac{7}{2}}_p | V_{np} | \bar{\tfrac{5}{2}}_n \tfrac{5}{2}_p\rangle_{0^-} - 0.395\langle \tfrac{7}{2}_n \bar{\tfrac{5}{2}}_p | V_{np} | \bar{\tfrac{5}{2}}_n \tfrac{7}{2}_p\rangle_{1^-}|, \quad (21b)$$

$$|m_{5^-}(^{174}\text{Yb})| = |0.254\langle \tfrac{1}{2}_n \bar{\tfrac{1}{2}}_p | V_{np} | \bar{\tfrac{9}{2}}_n \tfrac{9}{2}_p\rangle_{0^-} - 0.725\langle \tfrac{1}{2}_n \bar{\tfrac{9}{2}}_p | V_{np} | \bar{\tfrac{9}{2}}_n \tfrac{1}{2}_p\rangle_{4^+}|. \quad (21c)$$

For the $K^\pi = 1^+$ state in ^{176}Hf one has to use Eq. (19), since in this case we have antiparallel angular momentum projections. We get the following:

$$|m_{1^+}(^{176}\text{Hf})| = |0.395\langle \tfrac{7}{2}_n \tfrac{5}{2}_p | V_{np} | \tfrac{5}{2}_n \tfrac{7}{2}_p\rangle_{6^-} + 0.515\langle \bar{\tfrac{5}{2}}_n \tfrac{5}{2}_p | V_{np} | \bar{\tfrac{7}{2}}_n \tfrac{7}{2}_p\rangle_{0^-}|. \quad (21d)$$

For those cases where only one of the levels is known (in other words, $\Delta\lambda$ is not known), we have insufficient data to solve for the matrix element m. However, if the mixing is small, then we may make a theoretical estimate for $\Delta\lambda$ and still get a good value for m. This follows because, as pointed out before, m is not very sensitive to $\Delta\lambda$ if mixing is small (see Fig. 1).

TABLE I. Values of u and v amplitudes from BCS wave function.

Configuration		^{174}Yb	^{174}Hf	^{176}Hf	^{178}Hf	^{180}Hf
$\frac{1}{2}^+(411)_p$	U	0.490	0.292	0.292	0.292	0.292
	V	0.871	0.956	0.956	0.956	0.956
$\frac{7}{2}^+(404)_p$	U	0.928	0.747	0.747	0.747	0.747
	V	0.372	0.665	0.665	0.665	0.665
$\frac{9}{2}^-(514)_p$	U	0.938	0.819	0.819	0.819	0.819
	V	0.347	0.574	0.574	0.574	0.574
$\frac{5}{2}^+(402)_p$	U	0.972	0.911	0.911	0.911	0.911
	V	0.233	0.412	0.412	0.412	0.412
$\frac{1}{2}^-(521)_n$	U	0.308	0.614	0.449	0.346	0.261
	V	0.951	0.789	0.893	0.938	0.965
$\frac{5}{2}^-(512)_n$	U	0.483	0.685	0.507	0.397	0.285
	V	0.876	0.728	0.862	0.918	0.958
$\frac{7}{2}^-(514)_n$	U	0.886	0.904	0.812	0.590	0.466
	V	0.465	0.427	0.584	0.807	0.885
$\frac{9}{2}^+(624)_n$	U	0.904	0.944	0.898	0.778	0.619
	V	0.427	0.331	0.440	0.628	0.785

B. n-p force

We will use two different forces in our calculations, force I with range $r_0 = 1.5$ fm, force II with range $r_0 = 1.0$ fm. Following Jones *et al.*[18] and Ogle[19] we derive the potential strengths (up to a common additive constant) by doing a least-squares fit of energy splittings of Gallagher-Moszkowski pairs. The arbitrary additive constant of the potential strength arises because a pure Wigner force does not contribute to the splittings. Recently Ogle[19] has pointed out that there is still no satisfactory effective force for calculating the energy splitting of Gallagher-Moszkowski pairs. It may be that the off-diagonal couplings with other bands of the same K (n-p force coupling) or $K \pm 1$ (Coriolis coupling) are responsible for the inability to obtain detailed agreement in first-order splitting calculations.[20] From the general consideration of the greater occurrence of low-Ω orbitals in an oscillator shell, it is evident that the level density of two-quasiparticle bands is a rapidly decreasing function of K. Thus, we have sought an effective force by excluding from the fit all cases involving $K = 0$ or 1 bands,

TABLE II. Fit of energy splittings of Gallagher-Moszkowski pairs.

Nucleus	Configuration Proton	Neutron	K^π $\Sigma_n+\Sigma_p=0$	$\Sigma_n+\Sigma_p=1$	ΔE_{th} $r_0=1.5$ fm (keV)	ΔE_{th} $r_0=1.0$ fm (keV)	ΔE_{exp} (keV)	Ref.
^{164}Ho	$\frac{7}{2}^-(523)\uparrow$	$\frac{1}{2}^+(400)\uparrow$	3^-	4^-	118	124	102	18
		$\frac{3}{2}^+(402)\downarrow$	5^-	2^-	−123	−131	−85	18
		$\frac{3}{2}^-(521)\uparrow$	2^+	5^+	165	159	171	18
^{166}Ho	$\frac{7}{2}^-(523)\uparrow$	$\frac{1}{2}^-(521)\downarrow$	4^+	3^+	−147	−156	−171	18
^{168}Tm	$\frac{1}{2}^+(411)\downarrow$	$\frac{5}{2}^-(512)\uparrow$	3^-	2^-	−242	−231	−234	18
		$\frac{7}{2}^+(633)\uparrow$	4^+	3^+	−138	−145	−157	18
^{170}Tm	$\frac{1}{2}^+(411)\downarrow$	$\frac{5}{2}^-(512)\uparrow$	3^-	2^-	−239	−236	−232	18
^{174}Lu	$\frac{7}{2}^+(404)\downarrow$	$\frac{3}{2}^-(521)\uparrow$	5^-	2^-	−59	−58	−90	18
		$\frac{1}{2}^-(521)\downarrow$	3^-	4^-	93	69	80	18
^{176}Lu	$\frac{7}{2}^+(404)\downarrow$	$\frac{1}{2}^-(510)\uparrow$	4^-	3^-	−130	−138	−118	18
^{182}Ta	$\frac{7}{2}^+(404)\downarrow$	$\frac{1}{2}^-(510)\uparrow$	4^-	3^-	−127	−135	−174 [a]	24
		$\frac{3}{2}^-(512)\downarrow$	2^-	5^-	166	168	154 [b]	24

[a] A more recent experimental value [R. G. Helmer, R. C. Greenwood, and C. W. Reich, Nucl. Phys. A168, 449 (1971)] is −100.

[b] A more recent experimental value (see Helmer, Greenwood, and Reich) is 140.

which would be most susceptible to the higher-order corrections. This exclusion meant we did not attempt to fit the shift terms in $K=0$ bands. These shift terms have been shown by Jones *et al.*[18] to be sensitive to tensor components, another reason for not demanding that a central force fit them. Table II shows the result of the fitting procedure. The optimum force strengths at the different ranges are shown in Table V. From Table II we see a scatter in fit to experiment of about ±20 keV. The fit depends only weakly on range of force, and as Jones *et al.*[18] showed, even a δ force is rather satisfactory in fitting pair splittings.

Table III gives, for the two optimal forces at different range, the theoretical values of off-diagonal single-particle n-p matrix elements entering into the band-mixing cases for which we reviewed the experimental evidence in the first part of this paper. These are the single-particle matrix elements entering into the Eqs. (21) with appropriate weighting factors from the BCS wave functions. The matrix elements in Table III were calculated from a program due to Dr. Gordon Struble. In this program the Nilsson wave functions are calculated by truncating the set of basis states to a single oscillator shell in the isotropic harmonic-oscillator basis. The deformation parameters used were: for Table II

TABLE III. V_{np} matrix elements.

Matrix element [a]	Force I (keV)	Force II (keV)
$\langle \frac{1}{2}_n \bar{\frac{1}{2}}_p \lvert V_{np} \rvert \bar{\frac{9}{2}}_n \frac{9}{2}_p \rangle$	−152	−148
$\langle \frac{1}{2}_n \bar{\frac{9}{2}}_p \lvert V_{np} \rvert \bar{\frac{9}{2}}_n \frac{1}{2}_p \rangle$	319	292
$\langle \frac{7}{2}_n \bar{\frac{7}{2}}_p \lvert V_{np} \rvert \bar{\frac{5}{2}}_n \frac{5}{2}_p \rangle$	−151	−165
$\langle \frac{7}{2}_n \bar{\frac{5}{2}}_p \lvert V_{np} \rvert \bar{\frac{5}{2}}_n \frac{7}{2}_p \rangle$	318	288
$\langle \frac{7}{2}_n \frac{5}{2}_p \lvert V_{np} \rvert \frac{5}{2}_n \frac{7}{2}_p \rangle$	191	140
$\langle \bar{\frac{5}{2}}_n \frac{5}{2}_p \lvert V_{np} \rvert \bar{\frac{7}{2}}_n \frac{7}{2}_p \rangle$	−123	−155
$\langle \frac{9}{2}_n \bar{\frac{9}{2}}_p \lvert V_{np} \rvert \bar{\frac{7}{2}}_n \frac{7}{2}_p \rangle$	−61	−108
$\langle \frac{9}{2}_n \bar{\frac{7}{2}}_p \lvert V_{np} \rvert \bar{\frac{7}{2}}_n \frac{9}{2}_p \rangle$	209	213

[a] Only the Ω quantum numbers are shown; for a complete labeling of the states see Table I.

TABLE IV. Comparison of theoretical and experimental mixing matrix elements (keV).

Case	Exp. ref.	$\lvert m_{th} \rvert$ (Force I)	$\lvert m_{th} \rvert$ (Force II)	$\lvert m_{exp} \rvert$
$m_{8^-}(^{178}\mathrm{Hf})$	Eq. (21a)	133	158	158
$m_{6^+}(^{176}\mathrm{Hf})$	Eq. (21b)	203	199	208
$m_{5^-}(^{174}\mathrm{Yb})$	Eq. (21c)	270	249	235
$m_{1^+}(^{176}\mathrm{Hf})$	Eq. (21d)	12	25	89
$m_{8^-}(^{180}\mathrm{Hf})$		122	144	
$m_{6^+}(^{174}\mathrm{Hf})$		194	190	$<0.3\Delta\lambda$
$m_{6^+}(^{174}\mathrm{Yb})$		129	126	

TABLE V. Potential strength of forces used in this work and comparison with the one used in Ref. 18.

Force component	Force I ($r_0 = 1.5$ fm) (MeV)	Force II ($r_0 = 1.0$ fm) (MeV)	Jones *et al.* (Ref. 18) ($r_0 = 1.5$ fm) (MeV)	($r_0 = 1.0$ fm) (MeV)
V_{TE}	−117	−262	−105	−181
V_{SO}	−72	+35	−37	+48
V_{TO}	−62	−169	−120	−236
V_{SE}	−25	−51	−74	−79

approximately $\eta_n = \eta_p = 3.9$, for Table III approximately $\eta_n = 3.70$, $\eta_p = 3.75$. The Nilsson potential parameters μ_n, μ_p, κ_n, κ_p were obtained from equations on p. 14 of Ref. 12. The major shell spacings $\hbar\omega_n$ and $\hbar\omega_p$ were obtained from equations on p. 6 of Ref. 12.

With three of the four force strength parameters fixed by the Gallagher-Moszkowski pair fitting of Table II, we allowed only the freedom to adjust the fourth component (Wigner force) strength to obtain best agreement with experimental band-mixing elements from Eqs. (21) as shown in Table IV. In the last three cases there are data showing band mixing is small, but with the energy separation of the admixed band unknown, only limits can be set on mixing. The agreement is quite satisfactory, though somewhat better for the shorter range force II.

Table V summarizes the strengths of our optimal Gaussian central forces at the two different, somewhat arbitrary ranges, and compares it to the force obtained by Jones *et al.*[18] In Table III of Ref. 17 a slight error was made and the strengths of the nuclear force should read (for theory C) $V_B = -71.2$ MeV, $V_M = -13.1$ MeV, $V_H = 33.1$ MeV, $r_0 = 1.4$ fm. To make the comparison possible, we "transform" the force to the ranges of our force I and II by multiplying the potential strengths by the cube of the ratio of the ranges; that is, by $(1.4/1.5)^3$ for force I and $(1.4/1.0)^3$ for force II. Since V_W is not determined by the Gallagher-Moszkowski splittings, we use our values for it.

We are not able to make a detailed comparison with the remarkably successful spherical shell-model n-p force of Anantaraman and Schiffer[21] or of Kim and Rasmussen,[22] since our code will not handle a tensor-force component.

When an effective force of some reliability is available, one can address the problem of comprehensive prediction of the energies and band-mixing character of the multi-quasiparticle bands expected in the region. Soloviev has summarized[15] many calculations on three- and four-quasiparticle states of deformed nuclei; these calculations are for the most part done in the framework of the independent quasiparticle picture. There is also need for more data on such states. Awaiting discovery are surely other four-quasiparticle isomers like the 16^+ isomer[3] of ^{178}Hf and perhaps five-quasiparticle isomers like the $\frac{37}{2}^-$ state[23] in ^{177}Hf. Work is also being carried out by Khoo and Bertsch[23] to calculate the matrix elements connecting quasiparticle states in deformed nuclei.

IV. CONCLUSIONS

An interesting pattern emerges from analysis of band mixing of two-quasineutron and two-quasiproton basis states of deformed even nuclei. The conditions for strong mixing to occur are (1) that the zero-order energies of the $2qp$ and $2qn$ basis states be reasonably close and (2) that the two proton (neutron) orbital energies be on opposite sides of the Fermi surface. The first condition follows from the requirement of a small energy denominator in the mixing determinant, and the second from the appearance of BCS weighting factors of form $u_1 v_2$. From analysis of these mixing ratios we can deduce a value for the Wigner force strength in the effective n-p residual interaction. The Wigner component cannot be determined from the splitting of Gallagher-Moszkowski doublets in odd-odd nuclei. The off-diagonal matrix elements extracted here also may manifest themselves in configuration mixing in odd-odd nuclei. It will be valuable to have more experimental information to further constrain the effective force.

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[1]T. L. Khoo, J. C. Waddington, R. A. O'Neil, Z. Preibisz, D. G. Burke, and M. W. Johns, Phys. Rev. Lett. 28, 1717 (1972); T. L. Khoo, Ph.D. thesis, McMaster University, 1972 (unpublished).

[2]H. Ejiri, G. B. Hagemann, and T. Hammer, in Proceedings of the International Conference on Nuclear Moments and Nuclear Structure, Osaka, 1972; cf. also T. Hammer, H. Ejiri, and G. B. Hagemann, Nucl. Phys. A202, 321 (1973).

[3]R. G. Helmer and C. W. Reich, Nucl. Phys. A114, 649 (1968).

[4]T. E. Ward, Y. Y. Chu, and J. B. Cumming, private communication.

[5]T. Tamura, J. Phys. Soc. Jap. 23, 691 (1967).

[6]H. J. Körner, F. E. Wagner, and B. D. Dunlap, Phys. Rev. Lett. 27, 1593 (1971); Argonne National Laboratory Report No. ANL-7971, 1972 (unpublished), p. 153.

[7]F. M. Bernthal, J. O. Rasmussen, and J. M. Hollander, Phys. Rev. C 3, 1294 (1971).

[8]J. Valentin and A. Santoni, Nucl. Phys. 47, 303 (1963).

[9]S. I. Gabrakov, A. A. Kuliev, N. I. Pyatov, D. I. Salamov, and H. Schultz, Nucl. Phys. A182, 625 (1972).

[10]I. Hamamoto, Nucl. Phys. A177, 484 (1971); cf. also B. L. Birbrair, Nucl. Phys. A108, 449 (1968).

[11]J. I. Zaitz and R. K. Sheline, Phys. Rev. C 6, 506(1972).

[12]S. G. Nilsson, C. F. Tsang, A. Sobiczewski, Z. Szymanski, S. Wycech, C. Gustafson, I. Lamm, P. Moller, and B. Nilsson, Nucl. Phys. A131, 1 (1969).

[13]S. G. Nilsson, K. Dan. Vid. Selsk. Mat. Fys. Medd. 29, No. 16 (1955).

[14]D. J. Rowe, *Nuclear Collective Motion* (Metheun, London, 1970).

[15]V. G. Soloviev, *Theory of Complex Nuclei* (Science Press, USSR 1971) (English translation: Pergamon Press, to be published), cf. Tables 4.11, 4.12, and 4.14.

[16]H. J. Mang, J. K. Poggenburg, and J. O. Rasmussen, Nucl. Phys. 64, 353 (1965).

[17]W. Ogle, S. Wahlborn, P. Piepenbring, S. Fredriksson, Rev. Mod. Phys. 43, 424 (1971).

[18]H. D. Jones, N. Onishi, T. Hess, and R. K. Sheline, Phys. Rev. C 3, 529 (1971).

[19]W. Ogle, Research Institute of Physics Annual Report, Stockholm, Sweden, 1972 (unpublished), p. 154.

[20]Cf. G. L. Struble and J. O. Rasmussen, Phys. Lett. 17, 283 (1965).

[21]N. Anantaraman and J. P. Schiffer, Phys. Lett. 37B, 229 (1971).

[22]Y. E. Kim and J. O. Rasmussen, Nucl. Phys. 47, 184 (1963).

[23]T. L. Khoo and G. F. Bertsch, Michigan State University Cyclotron Laboratory Annual Report, 1972-1973 (unpublished).

[24]P. Nunberg and D. Prosperi, Nuovo Cimento, XLB, No. 1, 318 (1965).