

Energy dependence of isospin mixing in compound-statistical reactions*

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The ^{69}Ga compound nucleus was formed at excitation energies ranging from 17 to 22 MeV in the reactions $^{65}\text{Cu} + \alpha$ and $^{68}\text{Zn} + p$ and energy spectra of the emitted protons and α particles were measured between 30 and 150°. The extent of isospin mixing between the $T_>$ and $T_<$ states of the compound nucleus was determined by comparison of the quantity $\sigma(p, p')\sigma(\alpha, \alpha') / \sigma(p, \alpha)\sigma(\alpha, p)$ with theory and the fractional mixing was found to increase from 0 to 0.3 over the above energy range. The mixing width shows a corresponding increase from 0.2 to 16 keV. The independence hypothesis was verified at all energies by measurements of the α and np exit channels. The various spectra were compared with the spin-dependent statistical theory. It was not possible to fit the data for various energies with energy-independent parameters in the Fermi gas model level density expression suggesting that this expression is inadequate.

NUCLEAR REACTIONS $^{68}\text{Zn}(p, \alpha)$, (p, p') , (p, np) , $E=11-16$ MeV and $^{65}\text{Cu}(\alpha, \alpha')$, (α, p) , (α, np) , $E=13.89-19.10$ MeV differential cross sections measured for targets forming ^{69}Ga compound nucleus at excitation energies of 17-22 MeV. Deduced energy dependence of isospin mixing; tested independence hypothesis and spin-dependent statistical theory.

I. INTRODUCTION

The effect of isospin conservation on compound-statistical reactions has been the subject of recent investigation.¹⁻⁴ Isospin conservation manifests itself in the enhanced probability of proton emission in proton-induced reactions when compared to that of proton emission in α -induced reactions. This enhancement has been attributed to the fact that proton emission is virtually the only decay channel open to the $T_>$ levels of the compound nucleus. These levels are populated in $1/(2T_0 + 1)$ of the interactions induced by protons in a nucleus with isospin T_0 but are not populated in reactions induced by α particles. The decay of the $T_>$ states competes with the mixing of these states into the $T_<$ states of the compound nucleus by the Coulomb interaction and a formalism that includes fractional isospin mixing has been developed.⁵ Values of the fractional isospin mixing μ have been determined for several medium- A compound nuclei excited to approximately 20 MeV and found to be about 0.5.^{3,4} The values of μ may be combined with estimates of the decay width of the $T_>$ states to obtain the corresponding mixing widths and the latter have been found the range from 10 to 30 keV.⁴

The present study is concerned with a determination of the energy dependence of isospin mixing in compound-statistical reactions. The ^{69}Ga compound nucleus was formed at excitation energies ranging from 17 to 22 MeV in reactions of ^{65}Cu with α particles and ^{68}Zn with protons. Energy spectra and angular distributions of protons and

α particles were measured and the fractional mixing values as well as the mixing widths were deduced from the data. The results depend on the assumption that all the reactions involve the formation of a compound nucleus and this assumption was tested by determining if the independence hypothesis was obeyed by the ^{69}Ga compound system. The independence hypothesis was tested by examination of the α and np exit channels in a manner described in detail previously.⁴ The results also permit a stringent test of the validity of the spin-dependent statistical theory. Although the presently used formulation has been found to yield differential cross sections that agree very well with experiment,^{4,6,7} the comparison was generally only made at a given excitation energy of the compound nucleus. The present data require a fit over a range of energies and so permit a more severe test of the model.

II. EXPERIMENTAL

Targets of ^{68}Zn and ^{65}Cu were, respectively, bombarded by 11.0-, 12.0-, 14.0-, and 16.0-MeV protons and by 13.89-, 14.93-, 17.01-, and 19.10-MeV ^4He ions from the Purdue tandem Van de Graaff. The ^4He ion energies were chosen to form the ^{69}Ga compound nucleus at the same excitation energy as the corresponding proton energies. In determining these energies slight adjustments were made for differences in energy loss in the targets between protons and ^4He ions.⁸ The ^{68}Zn and ^{65}Cu targets consisted of self-supporting

metallic foils having an isotopic enrichment of 98.5 and 99.6%⁹ and a surface density of 0.98 and 0.83 mg/cm², respectively. Energy loss measurements performed with an ²⁴¹Am α source served for the thickness determinations.

The emitted charged particles were detected by a counter telescope consisting of two surface barrier detectors. Particle identification was based on the power-law method.¹⁰ Energy spectra were recorded at 30° intervals between 30 and 150°. Background was found to be negligibly small (< 0.3%) at all angles. The beam intensity was determined by digitizing the current collected in a Faraday cup and was also monitored by a detector located at a fixed angle to the beam. Further details of the experimental procedure have been given elsewhere.⁶

A few experiments were also performed with a ⁶⁴Zn target in order to provide correction factors for small (1.5%) amounts of ⁶⁴Zn, ⁶⁶Zn, and ⁶⁷Zn in the ⁶⁸Zn target. Since previous measurements⁷ had indicated that the ⁶⁴Zn(p, p') and ⁶⁴Zn(p, α) cross sections were about a factor of 4 larger than those of the corresponding reactions of ⁶⁸Zn with 14-MeV protons it was of some importance

to determine whether the correction factors were energy-dependent.

III. RESULTS

The data were processed with several codes designed to remove lines due to light-element impurities¹¹ and to obtain c.m. energy spectra and angular distributions.⁶ Figure 1 shows some typical proton and α -particle c.m. energy spectra. The α -particle spectra consist of a broad evaporation peak whose maximum lies at 7–8 MeV. The proton spectra have a similar shape but peak at 5–6 MeV. Below 3 MeV the proton spectra feature a second peak which is due^{4,7} to protons from the (p, np) or (α, np) reactions.

The angular distributions of protons, α particles, and second-chance protons in the energy intervals listed in Table I are shown in Fig. 2. These intervals correspond to those of the proton and α -particle evaporation peaks. The error bars are an estimate of the magnitude of the relative uncertainties in the data comprising a given angular distribution and are based on the agreement between occasional replicate determinations, mag-

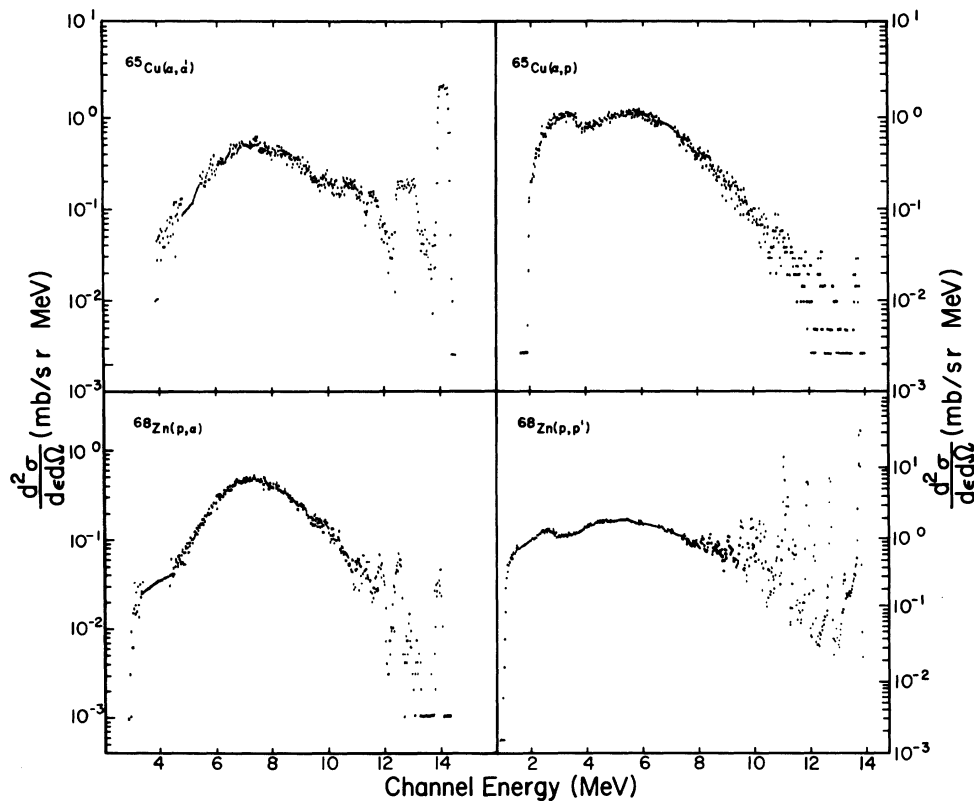


FIG. 1. Energy spectra (c.m.) of protons and α particles emitted at 150°(lab) in the decay of the ⁶⁸Ga compound nucleus at an excitation energy of 20.4 MeV.

nitude of impurity peaks, agreement between Faraday cup and monitor detector, and statistical uncertainties. The curves through the points are least-squares fits of the form $d\sigma/d\Omega = a + b \cos^2 \theta$ to the data points exhibiting symmetry about 90° . All the angular distributions are, to a greater or lesser extent, forward peaked but are consistent with symmetry at the larger angles. The b/a anisotropy coefficients range from 0 for the (p, p') to ~ 0.6 for the (α, np) reactions.

Total cross sections over the listed energy intervals were obtained by integration of the spectra consistent with a symmetric angular distribution using weighting factors obtained from the least-squares fits. The results are summarized in Table I where the quoted uncertainties include the standard deviations in the least-squares fits as well as estimated uncertainties in target thicknesses ($\sim 3\%$), current integration ($\sim 1\%$), and solid angle determination ($\sim 2\%$). The cross sections of the proton-induced reactions have been corrected by 2–7% for the contribution of the other Zn isotopes present in the ^{68}Zn target. In making a correction for ^{66}Zn and ^{67}Zn , which together constitute $\sim 0.5\%$ of the target, it was assumed that all the cross sections varied linearly with mass number between ^{64}Zn and ^{68}Zn . The cross sections of the α -induced reactions were corrected by 1–2% for the contribution of ^{63}Cu in the

^{65}Cu target on the basis of some recent¹² measurements of ^{63}Cu cross sections. The results quoted for the (p, np) and (α, np) reactions have also been corrected for the contribution from the (p, p') and (α, p) reactions to the listed energy intervals. This correction was made on the basis of the shape of the first-chance proton spectra below the threshold of the (p, np) or (α, np) reactions. The correction ranged from 3% at 18.4 MeV to 30% at 22.4 MeV. The energy intervals chosen for the (p, p') and (α, p) reactions have lower limits that in all instances are higher than the maximum second-chance proton energies.

It is well known^{6,7,13} that at low energies the (p, p') reaction, alone among the reactions of present interest, includes a contribution from precompound emission even if only those spectra consistent with a symmetric angular distribution are considered. This contribution must be subtracted from the (p, p') cross sections in order to permit a determination of the extent of isospin conservation. The precompound cross section was determined by comparison of the nuclear temperature characterizing the (p, p') spectrum with that obtained from a calculated precompound spectrum¹⁴ as well as with that expected for evaporation, as given by the mean temperature extracted from the (p, α) , (α, p) , and (α, α') spectra. This procedure has been described in

TABLE I. Experimental cross sections for specific energy intervals.

Target	Projectile	Bombarding energy (MeV)	Excitation energy (MeV)	Exit channel	c.m. energy interval (MeV)	Integrated cross section (mb)	(p, p') cross section (mb)
^{68}Zn	p	11.00	17.4	p	2.0–6.5	56 \pm 4	51 \pm 4
				α	6.0–10.5	11 \pm 1	
^{65}Cu	α	13.89		p	2.0–6.5	25 \pm 1	
				α	6.0–10.5	14 \pm 1	
^{68}Zn	p	12.00	18.4	p	2.5–7.5	71 \pm 6	64 \pm 6
				α	5.5–11.3	15 \pm 1	
				np	1.0–2.0	2.8 \pm 0.7	
^{65}Cu	α	14.93		p	2.5–7.5	33 \pm 2	
				α	5.5–11.3	17 \pm 1	
				np	1.0–2.0	0.62 \pm 0.04	
^{68}Zn	p	14.00	20.4	p	3.5–8.0	83 \pm 7	74 \pm 7
				α	5.5–12.0	19 \pm 1	
				np	1.0–3.0	20 \pm 2	
^{65}Cu	α	17.01		p	3.5–8.0	43 \pm 3	
				α	5.5–12.0	24 \pm 3	
				np	1.0–3.0	14 \pm 1	
^{68}Zn	p	16.00	22.4	p	5.5–9.5	60 \pm 5	52 \pm 5
				α	6.0–13.0	25 \pm 2	
				np	1.0–4.0	44 \pm 3	
^{65}Cu	α	19.10		p	5.5–9.5	34 \pm 2	
				α	6.0–13.0	33 \pm 3	
				np	1.0–4.0	38 \pm 2	

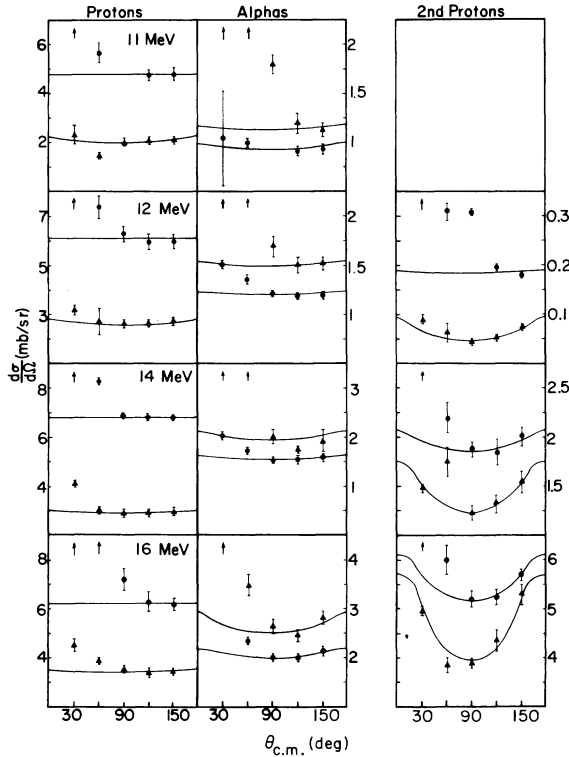


FIG. 2. Angular distributions of protons, α particles, and second-chance protons in the decay of ^{69}Ga . \bullet , proton-induced reactions; \blacktriangle , α -induced reactions. The curves are least-squares fits at backward angles. The proton bombarding energies are listed.

detail elsewhere.⁴ The precompound contribution to the $^{68}\text{Zn}(p, p')$ reaction was found to increase somewhat with bombarding energy and ranged from about 10 to 15%. The compound nuclear (p, p') cross sections are tabulated in the last column of Table I.

IV. DISCUSSION

A. Energy dependence of isospin mixing

In order to determine the extent of isospin conservation the experimentally determined compound nuclear cross sections may be recast in the form

$R_{\text{exp}} = \sigma(p, p')\sigma(\alpha, \alpha')/\sigma(p, \alpha)\sigma(\alpha, p)$. If the ^{69}Ga compound nucleus is formed with the same excitation energy and angular momentum distribution in both proton and α -particle bombardment, and moreover, if isospin is not conserved, the independence hypothesis demands that R_{exp} be unity. The extent of isospin conservation in the formation and decay of the compound nucleus is then determined by comparison of the R_{exp} with unity. If isospin is conserved to a significant extent this quantity will be larger than unity because of the enhanced value of $\sigma(p, p')$.

The situation is actually somewhat more complicated than outlined above because ^{69}Ga is not, in fact, formed with the same angular momentum distribution in both entrance channels. The mean angular momentum for the α channel thus is some $4\hbar$ units larger than that for the proton channel. This difference will in general cause R_{exp} to deviate from unity. Before the isospin effect can be ascertained the effect of angular momentum differences must first be evaluated. This can be done by means of a calculation of R_{exp} according to the spin-dependent statistical theory. Let us denote this calculated value by R_{calc} , taking note of the fact that the calculation does not take isospin conservation into account. The quantity of interest in an assessment of the extent of isospin conservation is $R_{\text{exp}}/R_{\text{calc}}$ which must be unity if isospin is not conserved and larger than unity if it is.

The quantities of interest, R_{exp} , R_{calc} , and $R_{\text{exp}}/R_{\text{calc}}$ are tabulated in columns 2–4 of Table II. The spin-dependent calculation of R_{calc} has been described in detail elsewhere^{4, 6} and is discussed in further detail in Sec. IV C. The values of R_{calc} are somewhat lower than unity reflecting the effect of angular momentum differences in the entrance channels. The 5% uncertainty in these values is a measure of the effect of reasonable variations in the various parameters in the calculation. The R_{exp} are substantially larger than unity and the ratios of R_{exp} to R_{calc} are, of course, even larger. Evidently, isospin conservation is of importance.

The largest possible value of $R_{\text{exp}}/R_{\text{calc}}$ is obtained when isospin is completely conserved. This

TABLE II. Isospin mixing in the decay of the ^{69}Ga compound nucleus.

U_{cn} (MeV)	R_{exp}	R_{calc}	$R_{\text{exp}}/R_{\text{calc}}$	$(R_{\text{exp}}/R_{\text{calc}})_{\text{max}}$	μ	$\Gamma_{\text{decay}}^>$ (keV)	$\Gamma_{\text{mixing}}^>$ (keV)
17.4	2.63 ± 0.10	0.81 ± 0.04	3.25 ± 0.20	3.32	0.03	5.7	0.18
18.4	2.28 ± 0.11	0.82 ± 0.04	2.78 ± 0.19	3.25	0.19	10.0	2.3
20.4	2.14 ± 0.23	0.82 ± 0.04	2.61 ± 0.31	3.11	0.22	22.7	6.4
22.4	2.05 ± 0.20	0.86 ± 0.04	2.39 ± 0.23	3.01	0.29	40.2	16.4

value is given by the expression³⁻⁵

$$(R_{\text{exp}}/R_{\text{calc}})_{\text{max}} = 1 + \frac{1}{2T_0} (F_{>p}/F_{<p}), \quad (1)$$

where T_0 is the isospin of the target nucleus in proton-induced reactions ($T_0 = 4$ for ^{68}Zn), and $(F_{>p}/F_{<p})$ is the ratio of the branching ratios for proton emission from the $T_>$ and $T_<$ states of the compound nucleus formed in proton bombardment. This quantity was evaluated by means of a spin- and isospin-dependent statistical model calculation described in detail elsewhere⁴ and its value ranged from 15 to 20. The resulting values of $(R_{\text{exp}}/R_{\text{calc}})_{\text{max}}$ are tabulated in column 5 of Table II. The experimental values only approach this limiting value at the lowest excitation energy of the compound nucleus. Evidently, a considerable amount of isospin mixing occurs prior to decay.

An approach to isospin conservation that allows for partial mixing of the $T_>$ and $T_<$ states has recently been formulated.⁵ The fractional mixing μ may be obtained from the expression

$$\mu = 1 - \frac{(R_{\text{exp}} - R_{\text{calc}})(2T_0 + 1)}{(R_{\text{exp}} - R_{\text{calc}}) + R_{\text{calc}}(F_{>p}/F_{<p})}. \quad (2)$$

The resulting μ values are tabulated in Table II and it is seen that the fractional mixing increases with excitation energy from 0 to 0.3.

The μ values reflect the competition between the decay of the $T_>$ states of the compound nucleus by particle emission and the mixing of the $T_>$ states into the $T_<$ states by the Coulomb interaction. It has been pointed out^{15,16} that isospin mixing should be minimal at both low and high excitation energies of the compound nucleus but substantial at intermediate energies. At low energies the compound nuclear levels are discrete and sufficiently well resolved that the Coulomb forces are unable to appreciably mix the isospins of neighboring levels. The strength of the Coulomb interaction is characterized by the Coulomb matrix element $\langle H_c \rangle$ and the condition for isospin conservation in this energy region is $\langle H_c \rangle \ll D^J$, where D^J is the spacing of levels with spin J . At higher excitation energies the compound nuclear levels overlap and this condition is no longer obeyed. Isospin mixing should then occur up to energies for which the lifetime of the $T_>$ states of the compound nucleus is sufficiently short that decay occurs prior to mixing, at which point the fractional mixing should once again become small. Since the time in which mixing occurs is of the order of $\hbar/\langle H_c \rangle$ the condition for isospin conservation is $\langle H_c \rangle \ll \Gamma_{\text{decay}}^>$, where $\Gamma_{\text{decay}}^>$ is the width for the decay of the $T_>$ states by particle (proton) emission.

The magnitude of $\langle H_c \rangle$ has been evaluated from

β -decay data¹⁷ and varies between 1 and 40 keV. The level spacing of the compound nucleus may be obtained from the Fermi gas level density formula; at the lowest excitation energy of ^{69}Ga , 17 MeV, the spacing of $J=4$ levels, which lie near the peak of the compound nucleus spin distribution, is only ~ 10 eV. The condition $\langle H_c \rangle \ll D^J$ is clearly not fulfilled.

The decay width of the $T_>$ states may be estimated by means of the statistical theory and the results are summarized in Table II. The uncertainty in these values is about a factor of 2. This estimate was arrived at on the basis of reasonable (see Sec. IV C) changes in the value of the level density parameter of the residual nucleus and their effect on the calculated decay widths. Since mixing and decay are competing processes, the values of $\Gamma_{\text{decay}}^>$ and μ may be used to obtain the mixing widths, also summarized in Table II. The energy dependence of these widths is displayed in Fig. 3. The decay widths increase from 5 to 40 keV over the energy range of present interest. The condition $\langle H_c \rangle \ll \Gamma_{\text{decay}}^>$ does not appear to be fulfilled at the lowest energy although it may begin to be approached at the highest energy. One might expect that under these conditions isospin mixing would be significant at all energies, but particu-

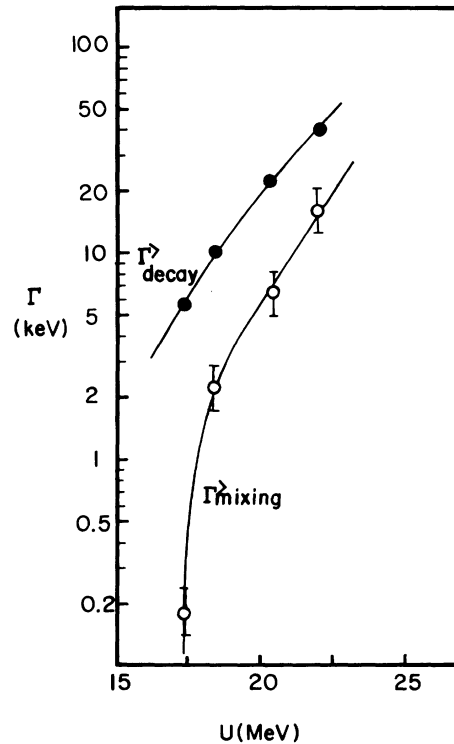


FIG. 3. Energy dependence of the mixing (open points) and decay (closed points) widths of the $T_>$ states of the compound nucleus. The lines show the trend of the data.

larly so at the lower ones, where neither one of the above conditions is obeyed. It is thus surprising to see that mixing increases with energy and is essentially zero at the energy where it might be expected to be largest.

The values of $\Gamma_{\text{mixing}}^{\Delta}$ should be closely correlated with those of $\langle H_c \rangle$ and it is seen that at the higher energies the $\Gamma_{\text{mixing}}^{\Delta}$ are comparable to the reported values of $\langle H_c \rangle$.¹⁷ On the other hand, the strong energy dependence of $\Gamma_{\text{mixing}}^{\Delta}$ is difficult to understand unless $\langle H_c \rangle$ itself is strongly energy-dependent. It is, of course, quite possible that this result merely reflects some shortcoming of the isospin mixing formalism such as the assumption that particle transmission coefficients are independent of isospin, or the neglect of higher-order terms in μ . However, if the values of μ derived from the data are indeed correct it will be necessary to revise present thinking on isospin mixing at moderate excitation energies.

B. Test of the independence hypothesis by comparison of the α and np exit channels

The analysis described in the last section is based on the assumption that the independence hypothesis is obeyed. The validity of the hypothesis has recently⁴ been demonstrated for the formation and decay of the ⁵⁶Fe, ⁶⁰Ni, ⁶³Cu, and ⁶⁴Zn compound nuclei by comparison of the α and np exit channels. As discussed in a previous report,⁴ isospin conservation does not affect a comparison of these particular exit channels because neither one is open to the T_2 states. The present results permit a test of the independence hypothesis for ⁶⁹Ga as a function of excitation energy.

Once again, the independence hypothesis demands that the quantity $R_{\text{exp}} = \sigma(p, np)\sigma(\alpha, \alpha')/\sigma(p, \alpha)\sigma(\alpha, np)$ be equal to unity provided the compound nucleus is formed with the same angular momentum distribution and excitation energy for both entrance channels. The previously noted difference in the angular momentum distributions is expected to have a more sizable effect in a comparison involving the np exit channel than in those discussed above. Low-energy neutrons and protons are not very effective at removing much angular momentum from the compound nucleus so that $\sigma(\alpha, np)$ can be expected to be low relative to $\sigma(p, np)$ whenever the angular momentum in the α entrance channel is larger than that in the p entrance channel. Another manifestation of this effect can be seen in the angular distributions, which are considerably more anisotropic for the (α, np) than the (p, np) reactions. Table III summarizes the values of R_{exp} . The above expectation is borne

out by the data, particularly at the lowest energy which is close to the reaction threshold.

The values of R_{exp} expected for the particular angular momentum distributions in the p and α entrance channels, denoted by R_{calc} , were obtained by means of the spin-dependent statistical model calculation referred to in the last section. The results are tabulated in Table III and are close to those of R_{exp} . Within the limits of error, the values of $R_{\text{exp}}/R_{\text{calc}}$ are equal to unity confirming the validity of the independence hypothesis over the energy range in question.

C. Comparison of the differential cross sections with the statistical theory

A comparison of the differential cross sections with the statistical theory is necessary for the determination of the angular momentum correction factors discussed above and, in addition, permits a rather stringent test of the model. We have made such comparisons in previous publications^{4,6,7} and found that the calculation was able to reproduce both the shapes and magnitudes of the energy spectra for reasonable values of the various parameters in the calculation. The present data permit a more severe test of the theory because they involve the decay of a given compound nucleus formed over a range of excitation energies.

Our particular formulation of the theory has been discussed in detail elsewhere.⁴ The parameters affecting the calculated spectra are those appearing in the spin-dependent level density expression as well as the optical-model parameters required for the calculation of transmission coefficients.¹⁸ The expression for the level density used in our calculation is based on the Fermi gas model

$$\Omega(U, J) = \frac{(2J+1)(U-\delta+t)^{-5/4}}{12a^{1/4}(2\sigma^2)^{3/2}} \times \exp[2a^{1/2}(U-\delta)^{1/2} - J(J+1)/2\sigma^2] \quad \text{for } J \leq J_{\text{max}}, \quad (3)$$

$$\Omega(U, J) = 0 \quad \text{for } J > J_{\text{max}}, \quad (4)$$

where a is the level density parameter, δ is the pairing energy, and σ^2 is the spin cutoff param-

TABLE III. Test of the independence hypothesis by comparison of the α and np exit channels.

U (MeV)	R_{exp}	R_{calc}	$R_{\text{exp}}/R_{\text{calc}}$
18.4	5.17 ± 1.26	4.81 ± 0.24	1.07 ± 0.27
20.4	1.87 ± 0.20	2.09 ± 0.10	0.89 ± 0.11
22.4	1.50 ± 0.14	1.60 ± 0.08	0.94 ± 0.10

eter; σ^2 is related to the nuclear moment of inertia \mathcal{I} and the temperature t by $\sigma^2 = \mathcal{I}t/\hbar^2$. The spin of the yrast level J_{\max} was assumed to obey the relation $J_{\max} = k(\mathcal{I}U)^{1/2}$, where k is an adjustable constant.

As indicated before,⁶ the values of a and δ have the biggest effects on the shapes and magnitudes of the energy spectra. A search routine was developed in which a and δ of the residual nuclei resulting from neutron, proton, or α -particle emission were systematically varied in order to give the best fit to the various spectra. It was assumed that a and δ of each nuclide were independent of both entrance channel and bombarding energy. This should be the case if the theoretical formulation, including the level density expression, is valid.

Figures 4 and 5 show a comparison of the experimental and the best-fit calculated spectra for the (α, p) and (α, α') reactions, respectively. The calculated spectra are based on $\mathcal{I} = \mathcal{I}_{\text{rigid}}$ ($r_0 = 1.2$ fm) and $J_{\max} = J_{\text{rot}}$, where J_{rot} is the value obtained on the assumption that all the excitation energy of the residual nucleus is tied up as rotational energy of a rigid sphere. It is seen that there are significant systematic differences between experiment and calculation. The calculated cross sections for proton emission are larger than the experimental values at the lowest energy but

smaller than the latter at the highest energy. On the other hand, the calculated cross sections for α -particle emission are too small at the lowest energy and too large at the highest. The over-all shapes of the calculated spectra are in reasonably good agreement with experiment. The apparent difference in the shape of the (α, p) spectra at 19.1 MeV is, at least in part, due to the experimental energy broadening, which causes the p and np groups to merge.

The comparison of the (p, α) and (p, p') spectra shows precisely the same kind of discrepancies as those noted for the (α, α') and (α, p) reactions, respectively.

In order to determine whether these discrepancies could be eliminated by changes in the other adjustable parameters a number of additional calculations were performed. The results are summarized in Table IV, where the experimental peak differential cross sections obtained for the four reactions of interest at the lowest and highest energies are compared with the corresponding quantities obtained from several calculations. The results of the calculation described above are listed in column 4. It should be mentioned that all the calculations for the (p, p') reaction were performed separately for the T_- and T_+ states and the experimentally determined μ values were used to determine the contributions from these two states. As a measure of the quantitative agree-

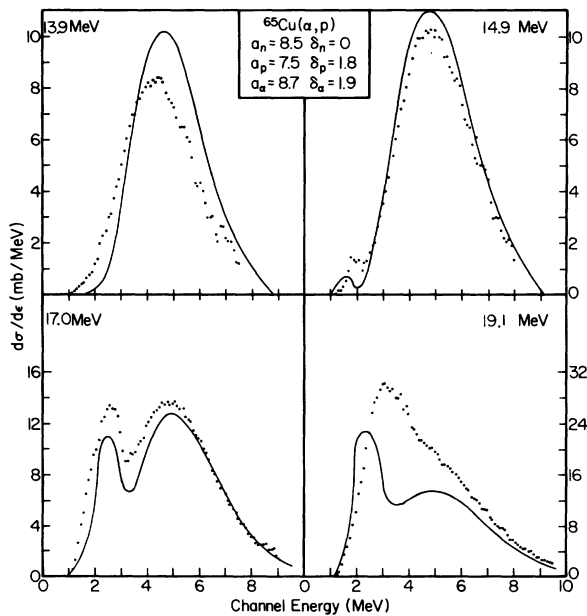


FIG. 4. Comparison of the calculated (curves) and experimental (points) differential cross sections of the $^{65}\text{Cu}(\alpha, p)$ reaction at the indicated bombarding energies. The tabulated parameters were used in the level density expression at all energies.

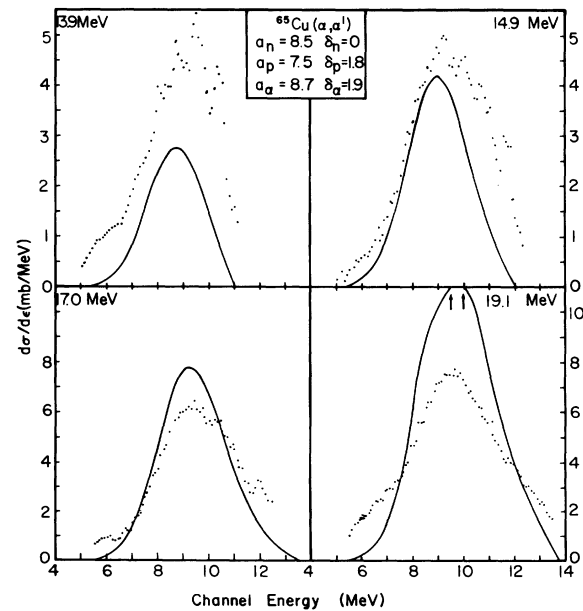


FIG. 5. Comparison of calculated and experimental differential cross sections of the $^{65}\text{Cu}(\alpha, \alpha')$ reaction. See Fig. 4 for details.

TABLE IV. Comparison of peak differential cross sections with various parametrizations of the spin-dependent statistical theory.

Reaction	U_{en} (MeV)	Experimental (mb/MeV)	Standard calculation (mb/MeV)	$\mathcal{J} = 0.4\mathcal{J}_{rigid}$ (mb/MeV)	$J_{max} = 0.5J_{rot}$ (mb/MeV)	Optical parameters changed (exit channel) (mb/MeV)	Optical parameters changed (entrance channel) (mb/MeV)	$U = at^2$ $n = 1.25$ (mb/MeV)	$U = at^2 - t$ $n = 2$ (mb/MeV)
(α, p)	17.4	8.3	10.2	10.8	10.8	11.0	10.3	10.2	11.4
(α, α')	22.4	18.0	13.3	13.0	12.2	14.0	13.6	13.4	13.9
	17.4	4.2	2.7	4.2	4.0	4.8	2.7	2.7	5.0
(p, p')	22.4	7.5	12.0	24.3	17.6	18.5	12.4	12.2	16.0
	17.4	18.8	24.9	26.4	24.9	26.5	24.7	25.9	27.1
	22.4	26.8	20.3	21.4	20.3	21.5	20.1	20.4	21.2
(p, α)	17.4	4.7	2.7	2.5	2.6	4.6	2.7	2.7	5.2
	22.4	6.1	9.9	9.2	9.7	14.6	9.7	9.8	12.8
χ^2			11.7	46.5	22.9	34.0	11.8	12.5	24.1

ment with experiment the values of χ^2 are tabulated in the last row of each column.

The results of a calculation in which \mathcal{J} was reduced to $0.4\mathcal{J}_{rigid}$ are summarized in column 5 of Table IV. The cross sections of all but the (α, α') reaction are affected by less than 10% by this change. On the other hand, the (α, α') cross section increases by as much as a factor of 2. A further decrease in \mathcal{J} leads to an additional increase in the (α, α') cross section but has little effect on the other reactions. This effect can be ascribed to the fact that a reduction in \mathcal{J} leads to a lower density of high-spin states in the residual nucleus. These states are normally more heavily populated in α -induced reactions because of the higher angular momentum of the compound nucleus formed in this manner. A reduction in the density of these states favors the emission of particles that are most effective in removing angular momentum from the compound nucleus, i.e., α particles. Subsidiary calculations indicate that the observed effect arises in nearly equal measure from the effect of \mathcal{J} on σ^2 and on J_{max} . Although a lower value of \mathcal{J} improves the fit to the (α, α') cross section at the lowest energy, it increases the discrepancy at higher energies and leads to a poorer over-all fit to the data.

Calculations of yrast levels based on the shell model¹⁹ indicate that for many nuclides J_{max} is significantly smaller than J_{rot} at low energies. Column 6 of Table IV lists the peak cross sections calculated for $J_{max} = 0.5J_{rot}$. Since a reduction in J_{max} eliminates the highest spin levels of the residual nuclei the effect of this change is qualitatively similar to the reduction of \mathcal{J} : a large increase of the (α, α') cross section and small changes in the other values. The over-all fit to the data is once again distinctly poorer. In addition to the effect on the magnitude of the cross sections, the reduction of J_{max} also has a striking effect on the shape of the calculated spectra, particularly for values below $0.5J_{rot}$. It was found that the evaporation peaks were split into two distinct peaks, an effect that had been previously predicted²⁰ for evaporation processes in which the yrast levels play a limiting role. The absence of such an effect in the experimental spectra indicates that the yrast spins of the nuclides of interest cannot be much lower than $0.5J_{rot}$.

In order to determine whether changes in the optical-model parameters could lead to improved agreement with experiment the values of the real and imaginary potentials for protons²¹ and α particles²² were systematically varied and the resulting transmission coefficients generated by the ABACUS-2 code¹⁸ used as input for evaporation calculations. The results of a 5% change in the

parameters for the exit channels are tabulated in column 7. The other parameters were identical to those of the standard calculation. It is seen that the differential cross sections for proton emission increase by 5–10% and those for α emission by 50–75%. The changes are comparable at both bombarding energies which is not surprising in view of the fact that the evaporation spectra extend over essentially the same energy interval for both cases. Thus, although the α -particle cross sections are quite sensitive to the optical parameters, it does not appear possible to make an adjustment that will yield the proper energy dependence.

The results of a calculation in which the optical parameters of the entrance channel were changed by 5% are tabulated in column 8. The effects on the various cross sections are negligibly small. Consequently a change in the energy dependence of the optical parameters cannot lead to a substantial improvement in the agreement with experiment.

A number of variants of the Fermi gas model level density differing in the preexponential term of the expression have been proposed²³ and their effect on the differential cross sections was determined. Column 9 summarizes the results of a calculation in which the preexponential energy term in the level density was $(U - \delta)^{-5/4}$, corresponding to the equation of state $U = at^2$. This form of the level density leads to virtually the same cross sections as the standard calculation. Column 10 lists the results obtained with a level density whose preexponential energy term was $(U - \delta + t)^{-2}$. This form leads to a sizable increase in the α cross sections and a slight increase in the proton values. However, the agreement with experiment is not improved.

It should be pointed out that the agreement with experiment of some of the calculations could be improved by appropriate adjustments in the a and δ values. However, all the calculations predict a larger increase with energy than is observed for the emission of α particles and a smaller increase than observed for that of protons.

In order to obtain good agreement with the data at all energies it is necessary to relax the condition that a or δ be independent of bombarding energy. Figure 6 shows the energy dependence of the parameters for which experiment and calculation are in good agreement at all energies. The agreement can be obtained by letting either just the a 's or the δ 's vary with energy. It is thus necessary to let a_p increase and a_α decrease with increasing energy or, alternatively, to let δ_p decrease and δ_α increase. The fact that the level density parameters appear to depend on bom-

barding energy, and hence on the excitation energy of the residual nuclei, suggests that the Fermi gas level density expression may give an incorrect energy dependence. More realistic calculations of the level density based on the shell model²⁴ indicate that the Fermi gas formula does indeed have various shortcomings and that these vary from one nucleus to another depending on the particular features of the nuclear shell structure. These shortcomings manifest themselves in rigorous spin-dependent statistical-model calculations only when comparisons are made with spectra measured at several bombarding energies and so were previously undetected.

It should be mentioned that the failure of the spin-dependent calculation to reproduce the data with energy-independent parameters has only a minor effect on the R_{calc} factors used in Secs. IV A and IV B. These ratios were calculated both with the energy-independent parameters and the best-fit values shown in Fig. 6 and were found to be nearly the same in all cases. In practice, the R_{calc} values tabulated in Tables II and III represent an average of the two calculations.

V. CONCLUSIONS

We have presented in this paper the first measurements of the energy dependence of the isospin mixing fraction in compound-statistical reactions. It was found that the fractional mixing of the T_1

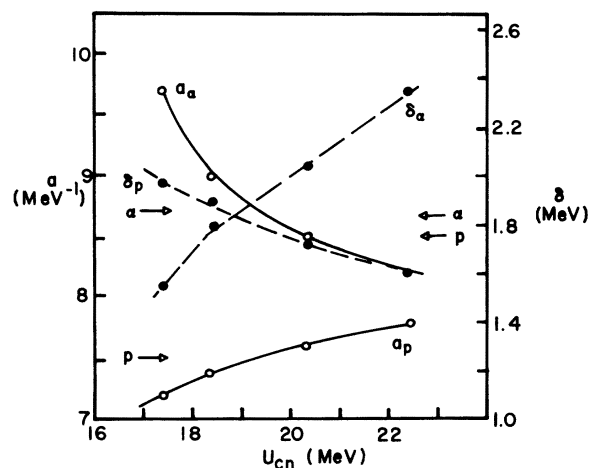


FIG. 6. Energy dependence of level density parameters a and pairing energies δ for which calculated and experimental spectra agree at all energies. \circ , a values (left ordinate); \bullet , δ values (right ordinate). The arrows point to the a (\rightarrow) and δ (\leftarrow) values used in the constant parameter fits shown in Figs. 4 and 5. The best fits are obtained by use of the energy-dependent a values and the constant δ or the energy-dependent δ and the constant a values.

and T_{\leq} states of the ^{69}Ga compound nucleus increased from 0 to 0.3 between 17- and 22-MeV excitation energy. The mixing width showed a concomitant increase from 0.2 to 16 keV.

These results are at marked variance with theoretical expectations which lead one to believe that the mixing fraction should be sizable at all energies of present interest but particularly so at the lower end of the interval. These results may merely indicate that the isospin mixing formalism is inadequate to the present task. On the other hand, if this is not the case and if similar results are obtained for other compound nuclei, it would require a revision of current ideas on isospin mixing in compound nuclear reactions.

The independence hypothesis of compound nuclear reactions was tested by a comparison of the α and $n\text{p}$ exit channels. The hypothesis was found to be valid at all excitation energies of present interest.

The various differential cross sections were compared with the spin-dependent statistical theory. The present data lend themselves to a par-

ticularly meaningful comparison with theory because the residual nuclei were formed both in two distinct ways and for a number of bombarding energies. A large number of spectra thus have to be fitted with a given set of a and δ values. It was found that while all the data at a given energy could be fitted with a given set of parameters, it was impossible to obtain an adequate fit with parameters that were independent of energy. An exhaustive search for a better fit by various changes in other parameters, such as the moment of inertia, yrast spin, optical parameters, as well as by the use of several forms of the level density failed to improve the fit. This shortcoming of the calculation suggests that the Fermi model level density formula does not give an adequate representation of the energy dependence of the level density of real nuclei.

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¹M. J. Fluss, J. M. Miller, J. M. D'Auria, N. Dudgey, B. M. Foreman, L. Kowalski, and R. C. Reedy, *Phys. Rev.* **187**, 1449 (1969).

²C. C. Lu, J. R. Huizenga, C. J. Stephan, and A. J. Gorski, *Nucl. Phys.* **A164**, 225 (1971).

³L. C. Vaz, C. C. Lu, and J. R. Huizenga, *Phys. Rev. C* **5**, 463 (1972).

⁴J. Wiley, J. C. Pacer, C. R. Lux, and N. T. Porile, *Nucl. Phys.* **A212**, 1 (1973).

⁵S. M. Grimes, J. D. Anderson, A. K. Kerman, and C. Wong, *Phys. Rev. C* **5**, 85 (1972).

⁶A. J. Kennedy, J. C. Pacer, A. Sprinzak, J. Wiley, and N. T. Porile, *Phys. Rev. C* **5**, 500 (1972).

⁷A. Sprinzak, A. J. Kennedy, J. C. Pacer, J. Wiley, and N. T. Porile, *Nucl. Phys.* **A203**, 280 (1973).

⁸C. F. Williamson, J. P. Boujot, and J. Picard, *Centre d'Etudes Nucléaires Saclay Report No. CEA-R3042*, 1966 (unpublished).

⁹Targets were obtained from Oak Ridge National Laboratory.

¹⁰F. S. Goulding, D. A. Landis, J. Cerny, and R. H. Pehl, *Nucl. Instrum. Methods* **31**, 1 (1964).

¹¹A. J. Kennedy, J. C. Pacer, A. Sprinzak, J. Wiley,

and N. T. Porile, *Nucl. Instrum. Methods* **101**, 471 (1972).

¹²J. C. Pacer, J. Wiley, C. R. Lux, and N. T. Porile, *Nucl. Phys.* (to be published).

¹³C. C. Lu, L. C. Vaz, and J. R. Huizenga, *Nucl. Phys.* **A190**, 229 (1972).

¹⁴M. Blann and F. M. Lanzafame, *Nucl. Phys.* **A142**, 559 (1970).

¹⁵D. H. Wilkinson, *Phil. Mag.* **1**, 379 (1956).

¹⁶A. M. Lane and R. G. Thomas, *Rev. Mod. Phys.* **30**, 257 (1958).

¹⁷S. D. Bloom, in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic, New York, 1966), p. 123.

¹⁸E. H. Auerbach, Brookhaven National Laboratory Report No. BNL-652, 1964 (unpublished).

¹⁹J. R. Grover, *Phys. Rev.* **157**, 832 (1967).

²⁰J. R. Grover and J. Gilat, *Phys. Rev.* **157**, 823 (1967).

²¹F. G. Perey, *Phys. Rev.* **131**, 745 (1963).

²²J. R. Huizenga and G. Igo, *Nucl. Phys.* **29**, 462 (1962).

²³A. Gilbert and A. G. W. Cameron, *Can. J. Phys.* **43**, 1446 (1965).

²⁴M. Hillman and J. R. Grover, *Phys. Rev.* **185**, 1303 (1969).