Bound states of the multichannel ($n\alpha\alpha$) system

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The multichannel three-body model is applied to the ($n\alpha\alpha$) system to study the bound states of ^{9}Be. The α particle involved in this system is assumed to be a two-level particle to take care of its internal structure. The energies of the ground state [$\frac{3}{2}^-$] and an excited state [$\frac{1}{2}^+$] are calculated to be -1.76 and $+0.68$ MeV, respectively.

[NUCLEAR STRUCTURE ^{9}Be; calculated levels. Multichannel three-body model.]

Recently, calculations of the bound states of ^{6}Li and ^{12}C have been made on the multichannel three-body model.[1,2] In such a model, the α particle involved in the α-nucleon three-body system is assumed to be a two-level particle to take care of its internal structure approximately. The N-α and α-α two-body potentials are obtained phenomenologically by multichannel analyses of the low-energy scattering data. Thus the picture has effectively taken into account all the inelastic channel contributions except for breakup of the α particle.

The multichannel Faddeev equations are generalized to allow spin and internal-structure quantum numbers of the particles and are solved in the separable t-matrix approximation for bound-state energies. The results calculated for both the nuclei agree very well with the experimental values. It seems therefore interesting to carry out the same model calculation for ^{9}Be. Since the formulation of this problem is essentially the same as that of ^{6}Li which has been described in detail in Ref. 1, we shall only give a brief outline of the procedures in this note. The general formulas given in Ref. 1 can be readily applied to the present case and will not be reproduced here.

To treat a system of three particles that may have different spin and internal-structure states, we introduce a set of quantum numbers (r, m_r) in addition to the usual spin quantum numbers (s, m_s). These new quantum numbers can be treated in exactly the same manner as the spin quantum numbers except that they do not couple with the angular-momentum operators. Thus for an elementary particle, we have $r = m_r = 0$ and for a two-level particle such as the α particle in our model, we have $r = \frac{1}{2}, m_r = \pm\frac{1}{2}$. In a representation in which particles j and k form a subsystem with particle i left free, the internal state of the subsystem is characterized by $\vec{R}_i = \vec{r}_j + \vec{r}_k$, while $\vec{R} = \vec{R}_i + \vec{r}_i$ is the corresponding quantum number of the three-body system.

The three-body states in general involve three parts, the internal-structure part, the spin part, and the spatial part. Written in these three-body states, the Faddeev equations represent a set of coupled integral equations in two continuous variables, i.e., the magnitudes of the two relative momenta. The angular-momentum states involved in the kernel can be decomposed completely by a procedure similar to those given by Omnes[3] and by Ahmadzadeh and Tjon[4] for single-channel case. The details of the calculations are presented in a thesis by Chuu.[5] One of the variables can be explicitly integrated after making the separable t-matrix approximation as described by Ball and Wong.[6] For bound-state problems, the two-body t matrix can be expressed in terms of a complete set of eigenfunctions of the homogeneous Lippman-Schwinger equation. The eigenvalue problem can be solved for given two-body potentials and these solutions are then used to calculate all the matrix elements of the kernel as a function of the three-body energy for a state of definite spin and parity. The bound-state energy in question is given by that energy for which the eigenvalue of the kernel matrix is unity.

The N-α potential is represented by a 2×2 matrix and the α-α potential by a 3×3 matrix because of symmetry considerations. They are both l-dependent with each matrix element represented by a square well. In Refs. 1 and 2 two sets of parameters are given for each of these potentials. They fit the two-body scattering data equally well but yield slightly different results in the three-body calculations. We only take the best sets in the present work.

For the ($n\alpha\alpha$) system under consideration, we can apply the formulas in Sec. II of Ref. 1 by labeling, for instance, the neutron as particle 1 and the two α's as particle 2 and particle 3. Then we have $r_1 = 0$, $s_1 = \frac{1}{2}$, and $r_2 = r_3 = \frac{1}{2}$, $s_2 = s_3 = 0$. We have calculated the binding energies for the $\frac{3}{2}^-$ state, the ground state of ^{9}Be, and the $\frac{1}{2}^+$ state. Since the higher partial-wave contributions are insignificant in the three-body system, we have included only s and d waves for α-α interaction, and s and p waves for N-α interaction. All the angular-momentum states that are consistent with the correct J^π values are included. Thus there are nine partial three-body states in the $\frac{3}{2}^-$ state calculation and six partial three-body states in the $\frac{1}{2}^+$ state calculation.

After Coulomb-energy correction which is 1.84 MeV for ($n\alpha\alpha$) system, we obtain -1.76 MeV for the ground-state energy of ^{9}Be and $+0.68$ MeV for the excited $\frac{1}{2}^+$ state. The corresponding experimental values are -1.57 MeV and $+0.18$ MeV. The only existing work that we are aware of has been made by Grubman and Witten[7] who obtained -1.22 MeV for the ground state from a single-channel calculation.

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