

Coriolis perturbation effects for $K^\pi = \frac{3}{2}^-$ bands in odd-neutron rare-earth nuclei

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The energy spacings of rotational levels in $K^\pi = \frac{3}{2}^-$ bands in odd-neutron rare-earth nuclei are seen to exhibit an alternating behavior with the level spin. This behavior is analyzed empirically, phenomenologically, and theoretically. It is shown that the alternating term in phenomenological rotational energy expression is even more significant than the usual $I^2(I+1)^2$ correction term in most of the nuclei. Theoretical calculations employing Nilsson-model single-particle orbitals and including the contributions of admixtures from various $N=5$ oscillator shell $\frac{1}{2}^-$ single-particle states to perturb the rotational levels based on $\frac{3}{2}^-$ [521] states show that this behavior may reasonably be described as resulting from the third-order perturbation effects of the Coriolis interaction term in the Hamiltonian.

[NUCLEAR STRUCTURE Odd- N even- Z rare-earth nuclei, calculated Coriolis perturbation effects to $K = \frac{3}{2}$ rotational band levels.]

1. INTRODUCTION

In the case of deformed nuclei the application of a rigid rotor concept implies that the low-lying energy levels will group into rotational bands with the characteristic $I(I+1)$ dependence for the energies of the respective band levels. Nonadiabatic corrections to the rotational energies are taken into account by writing^{1,2} the energy expression as a power series in $I(I+1)$:

$$E(I) = AI(I+1) + BI^2(I+1)^2 + CI^3(I+1)^3 + \dots \quad (1)$$

For $K \neq 0$ bands, the Coriolis coupling term in the collective-model Hamiltonian gives rise to an additional power series whose successive individual terms are alternating in sign with the spin of the level. The energy expression in these cases is written as^{1,2}

$$E_K(I) = E_K^0 + AI(I+1) + BI^2(I+1)^2 + \dots + (-1)^{I+K} \prod_{i=1-K}^K (I+i) [A_{2K} + B_{2K}I(I+1) + \dots] \quad (2)$$

Contribution of the leading term in the alternating series is of the order I^{2K} . For $K = \frac{1}{2}$ bands this term contributes in the lowest order (the diagonal matrix elements of the Coriolis interaction are nonzero in this case) and has been long in evidence; for this case the coefficient (A_{2K}/A^{2K}) is the well-known decoupling parameter a which can be evaluated from the corresponding single-particle Nilsson wave functions.

If the first two terms in Eq. (1) are sufficient to describe the rotational energies, then a plot of

$$A_I = [E(I+1) - E(I)]/2(I+1) \quad (3)$$

versus $[2(I+1)]^2$ gives a straight line. The relative importance of the alternating term is exhibited by the deviation from linearity of such a plot. This effect was demonstrated³ about ten years ago in the $\frac{3}{2}^+$ [411] ground-state band levels for the odd- Z nucleus ¹⁵⁹Tb and its interpretation in terms of Nilsson model was discussed. More recently the problem has been discussed⁴ in somewhat more detail for the levels in the $\frac{3}{2}^-$ [521] ground-state bands for ¹⁵⁵Gd and ¹⁵⁷Gd nuclei and a fair agreement between the theory and experiment was reported. Aside from these two studies, the problem has not received much attention, although several such bands with quite a few members have been established during the recent years. We have carried out^{5,6} a systematic detailed investigation of the problem and report here our results for the odd- N nuclei in the rare-earth region. The odd- Z nuclei, in a preliminary analysis,⁵ are found to give some perplexing results and will be discussed separately. The lowest-order contribution from the alternating series, being of the order of I^{2K} , is not expected to show up significantly for $K \geq \frac{5}{2}$ bands and even then, can be studied simultaneously only with the inclusion of $I^3(I+1)^3$ and higher-order terms in the first series which would mean additional parameters and hence, the necessity of many more experimentally observed levels to determine all such parameters. In view of these considerations our study is presently limited to a discussion of the levels in $K = \frac{3}{2}$ bands.

In Sec. 2 we look at the experimental information on the availability of such bands in odd- N rare-earth nuclei and present the same in terms of the plots referred to above. Such an analysis clearly depicts the trends and, while specifying the cases

where sufficient experimental information is presently available to admit theoretical analysis, also points out the nuclei where more experimental information on higher-spin levels will be desirable and sets out qualitative guidelines for their identification. Section 3 includes the results of fitting the experimental spectra using phenomenological formulas to find the relative contributions from the individual terms and to obtain the "experimental" measure of the coefficients. The coefficients are theoretically evaluated in Sec. 4 using Nilsson-model wave functions to calculate the Coriolis admixture of all possible $K = \frac{1}{2}$ bands into the $K = \frac{3}{2}$ band levels. A discussion of our results and conclusions are given in the last section.

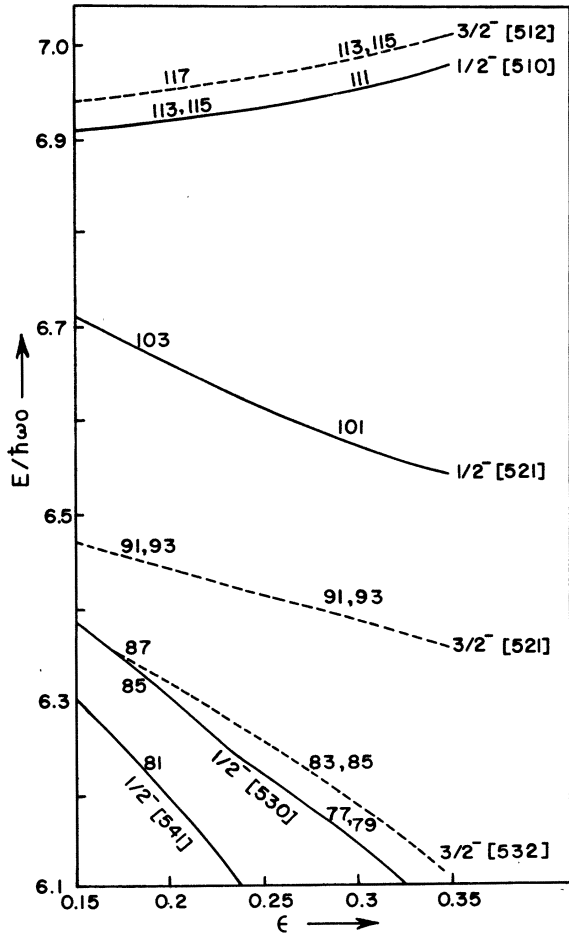


FIG. 1. Part of the Nilsson diagram showing expected $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states in the rare-earth region. The dashed line corresponds to $\frac{3}{2}^-$ states. The plot taken from Ref. 7 uses ϵ as the deformation parameter. All other plots use δ as the deformation parameter from Ref. 10 as noted in the text.

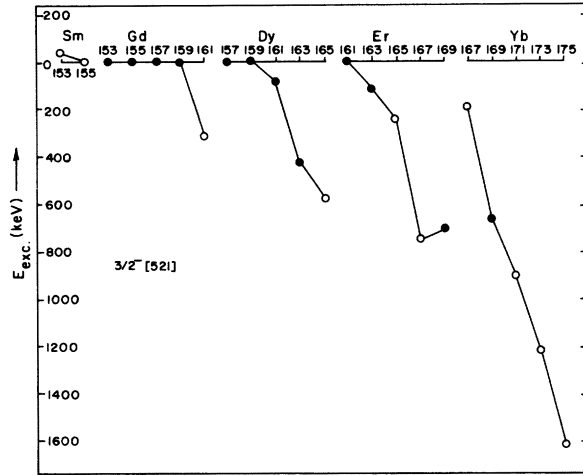


FIG. 2. Excitation energies of presently available $\frac{3}{2}^- [521]$ states are plotted for each set of isotopes as a function of neutron number. The filled circles denote the nuclei for which a rotational band having more than three members is available.

2. EXPERIMENTAL DATA AND ITS EMPIRICAL ANALYSIS

With a view to identify the single-particle neutron levels of interest in the rare-earth region we show in Fig. 1 the relevant portion of the Nilsson diagram⁷ giving the expected positions of the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ intrinsic states for $91 \leq N \leq 113$. Three $\frac{3}{2}^-$ states are seen in the diagram. The $\frac{3}{2}^- [532]$ state is already filled before the onset of deformation around $N \approx 90$ and can at best be expected to appear as a hole state. Moreover, confirmatory identification of this state is not as yet available.⁸ Thus, one can only look for the $\frac{3}{2}^- [521]$ state to occur in

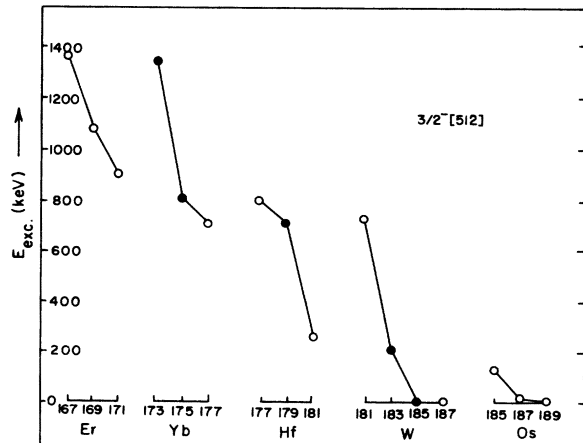


FIG. 3. Excitation energies of presently available $\frac{3}{2}^- [512]$ states. See Fig. 2.

the lighter and $\frac{3}{2}^- [512]$ state to occur in the somewhat heavier rare-earth nuclei.

The presently available experimental information on the location of these two single-particle levels is summarized in Figs. 2 and 3, respectively. It is seen that $\frac{3}{2}^- [521]$ state occurs as a particle state only in ^{153}Sm while in all other nuclei it occurs either as the ground state or as a hole state. Further, as shown in Fig. 2, although the state has been identified in about 22 nuclei, only about half of them have a sufficient number (4 or more) of rotational levels identified in each band to admit an analysis in terms of the formalism outlined in the following sections. The situation for the $\frac{3}{2}^- [512]$ state is much less promising, as shown in Fig. 3. It occurs at quite high excitation energies (700 keV or more) for $A < 181$ nuclei and only in heavier nuclei it occurs with low (100–300 keV) excitation energies or as a ground state.

The relative significance of the alternating energy term can be viewed in Fig. 4 which gives the plots of A_I [defined in Eq. (3)] versus $[2(I+1)]^2$. As mentioned earlier, deviation of such plots from

linearity provides a measure of the contribution from the alternating series. Figure 4 clearly shows the inadequacy of the presently available data (except for ^{183}W and ^{185}W) for a meaningful analysis of bands built on the $\frac{3}{2}^- [512]$ state. As such we discuss here only the results of our theoretical calculations for the rotational levels built on the $\frac{3}{2}^- [521]$ state. The available experimental data for this state are also presented in Fig. 4. We now proceed to a detailed investigation of these bands in the following.

3. PHENOMENOLOGICAL ANALYSIS

For comparison of the experiment with theory, we first evaluate the coefficients appearing in the phenomenological expression Eq. (2) for the energy. We restrict ourselves to the inclusion of the first two terms in the first series and only the first term in the second series thus giving us the energy expression for the $K = \frac{3}{2}$ case.

$$E_{3/2}(I) = E_{3/2}^0 + AI(I+1) + BI^2(I+1)^2 + (-1)^{I+3/2} A_3(I + \frac{3}{2})(I - \frac{1}{2}). \quad (4)$$

Here we have four undetermined parameters $-E_{3/2}^0$, A , B , and A_3 —requiring the availability of at least four levels in a band for their evaluation. The nuclei for which this condition is satisfied for the bands built on the $\frac{3}{2}^- [521]$ state are listed in Table I.

First we assess the relative importance of the two correction terms in Eq. (4). This is done by making three-parameter fits to the experimental energies by only including either the B term or the A_3 term in Eq. (4) one at a time and comparing the rms deviations of the calculated energies in each case with the observed ones. These are listed in the columns labeled AB and AA_3 , respectively, in Table I. It is found that in 8 out of 12 nuclei available for discussion AA_3 predictions are decidedly in better agreement with the experiment than AB predictions. We conclude that any attempt to fit the experimental energies only by Eq. (1) with the exclusion of alternating terms is not justifiable.

Accordingly we have made a least-squares fit to the available energy levels using Eq. (4) and the parameters A , B , and A_3 so obtained are listed in Table I. For comparison with the theory we have also listed the parameter

$$d = \frac{A_3 K}{A^{2K}} = \frac{A_3}{A^3} \quad (5)$$

which is similar to the decoupling parameter in the case of $K = \frac{1}{2}$ bands.

Interestingly enough we find that the coefficients

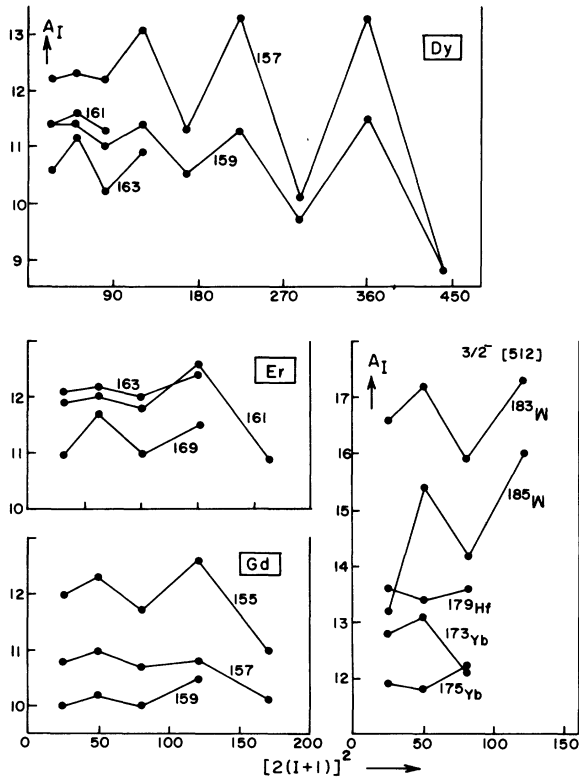


FIG. 4. Weighted transition energy $A_I = [E_{I+1} - E_I] / 2(I+1)$ is plotted as a function of $[2(I+1)]^2$ for $\frac{3}{2}^- [521]$ based bands (left-hand side). Each set of isotopes is plotted in a separate block. The right-hand side figure gives the corresponding plot for the $\frac{3}{2}^- [512]$ state bands.

B and A_3 are of the same order of magnitude in all cases; however there exists no correlation between their signs. It is also seen that, in contrast with the case of even nuclei wherein B is always negative, the sign of B varies at random for odd-mass nuclei; in the cases under discussion it is as often positive as negative. The B term is considered to arise from the rotation-vibration interaction and, as such, is always expected to bring in a negative contribution. However, as shown by Hamamoto and Udagawa,⁹ randomly varying sign of B in odd-mass nuclei can be said to originate from the consideration of Coriolis and Coriolis antipairing effects. In this study we concern ourselves only with the alternating energy term and now proceed to its theoretical evaluation.

4. THEORETICAL ANALYSIS

Treating the Coriolis interaction

$$H_C = -\frac{\hbar^2}{2\mathcal{I}}(I_+j_- + I_-j_+) \quad (6)$$

by a perturbation expansion it is evident that, except for $K = \frac{1}{2}$ bands in which case the diagonal matrix elements of H_C are nonzero, H_C produces a coupling between bands with $\Delta K = \pm 1$ to the leading order. For $K > \frac{1}{2}$, the diagonal matrix elements of H_C are zero and hence, there is no contribution in the first order. The effect on the energy in the second order has the characteristic $I(I+1)$ dependence and thus corresponds to a renormalization of the moment of inertia. To the third order in H_C , we have² a contribution δE to the energy of $K = \frac{3}{2}$

TABLE I. The results obtained by phenomenological fits to the levels in rotational bands built on the $\frac{3}{2}^-$ [521] odd-neutron state in the listed nuclei. The various columns list, respectively, the excitation energy of the $\frac{3}{2}^-$ state, total number of rotational levels available in the band, the rms deviations obtained by making least-squares fits for AB , AA_3 , and ABA_3 as explained in the text, the parameters A , B , and A_3 obtained by making a least-squares fit to experimental levels using Eq. (4), and the coefficient d , defined in Eq. (5), in specified units. The reference from which the experimental data are taken is listed in the third column.

Nucleus	Band head (keV)	Ref.	No. of levels	% rms Deviation			A (keV)	B (eV)	A_3 (eV)	d	
				AB	AA_3	ABA_3				(MeV) ⁻²	($\hbar\omega_0$) ⁻²
¹⁵³ Gd	0	a	4	2.15	1.12	0	8.4	-19.0	41.3	69.9	4130.7
¹⁵⁵ Gd	0	b	6	0.90	0.27	0.09	12.1	-2.6	-20.4	-11.4	-659.2
¹⁵⁷ Gd	0	b	7	0.53	0.88	0.24	11.0	-5.8	-9.0	-6.8	-392.6
¹⁵⁹ Gd	0	c	5	0.31	0.64	0.29	9.8	7.8	-2.0	-2.1	-117.6
¹⁵⁷ Dy	0	d	10	1.53	1.34	0.71	12.5	-4.2	-22.3	-11.4	-658.5
¹⁵⁹ Dy	0	e	10	0.67	2.05	0.08	11.5	-7.2	-9.8	-6.4	-369.7
¹⁶¹ Dy	74.9	f	4	0.51	0.51	0	11.4	1.7	-10.1	-6.8	-380.5
¹⁶³ Dy	421.0	g	5	0.24	0.18	0.08	11.0	-10.7	-18.7	-13.9	-781.9
¹⁶¹ Er	0	h	6	0.85	0.36	0.36	12.0	-0.3	-17.4	-10.1	-574.1
¹⁶³ Er	103.8	h	5	0.10	0.01	0.00	12.1	1.7	-4.4	-2.5	-139.3
¹⁶⁹ Er	714.5	i	5	0.14	0.10	0.07	11.6	-7.3	-13.6	-8.8	-481.9
¹⁶⁹ Yb	659.8	j	4	0.19	0.05	0	12.1	11.8	40.2	23.0	+1264.7

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bands:

$$\delta E = \sum_{K', K''} \frac{\langle K = \frac{3}{2} I | H_C | K' = \frac{1}{2} I \rangle \langle K' = \frac{1}{2} I | H_C | K'' = \frac{1}{2} I \rangle \langle K'' = \frac{1}{2} I | H_C | K = \frac{3}{2} I \rangle}{(E_{K'} - E_K)(E_{K''} - E_K)}$$

$$= A_3 (-1)^{I+3/2} (I - \frac{1}{2})(I + \frac{1}{2})(I + \frac{3}{2}), \quad (7)$$

where A_3 is given by the relation

$$d = \frac{A_3}{A^3} = - \sum_{K', K''} \frac{\langle K = \frac{3}{2} | j_+ | K' = \frac{1}{2} \rangle \langle K' = \frac{1}{2} | j_+ | \bar{K}'' = \frac{1}{2} \rangle \langle \bar{K}'' = \frac{1}{2} | j_+ | \bar{K} = \frac{3}{2} \rangle}{(E_{K'} - E_K)(E_{K''} - E_K)}. \quad (8)$$

The summation extends over all bands with K' , $K'' = \frac{1}{2}$; the barred state vectors are the corresponding time-reversed parts of the single-particle wave functions and E_K 's represent the intrinsic excitation energies.

In the fourth order we obtain² energy terms proportional to $I^2(I+1)^2$ in addition to $I(I+1)$ dependent and constant terms. Higher-order corrections similarly appear as higher terms in the two power series in Eq. (2). Thus Eq. (4), adopted in our present calculations, is correct up to fourth order.

In the particular case in which the two intermediate states K' and K'' represent the same $K = \frac{1}{2}$ band, the parameter d becomes

$$d = -a \frac{[\langle K = \frac{3}{2} | j_+ | K' = \frac{1}{2} \rangle]^2}{(E_K - E_{K'})^2}, \quad (9)$$

where a is the decoupling parameter of the associated $K = \frac{1}{2}$ band.

In our calculations we have adopted the modified deformed-oscillator potential⁷ for calculation of Nilsson-model single-particle wave functions and energies. The variation of all quantities, including energies, matrix elements etc., has been studied as a function of the deformation for which we have adopted δ of Nilsson.¹⁰ The relationship between various deformation parameters adopted by different investigators has been recently discussed by Löbner, Vetter, and Honig.¹¹ We report the results of our calculations for the range $\delta = 0.20-0.36$ which covers the range of δ deduced for rare-earth nuclei from experimental data.

We are interested in studying the perturbation of the levels built on the $\frac{3}{2}^-$ [521] due to Coriolis interaction. As discussed above this will arise from coupling with the same spin levels in all other K' , $K'' = \frac{1}{2}$ bands. Since parity is a good quantum number, admixtures can arise only with the $K^\pi = \frac{1}{2}^-$ bands which, in this case, are taken as the ones belonging to the $N=5$ oscillator shell only.

As seen from Eq. (9), the relative contribution from a particular $K = \frac{1}{2}$ state depends on its energy separation from the $\frac{3}{2}^-$ state, its decoupling parameter, and the matrix elements $\langle \frac{3}{2} | j_+ | \frac{1}{2} \rangle$. First we discuss the role of energy separations. Two K^π

$= \frac{1}{2}^-$ states i.e., [501] and [550] lie outside the energy range of Fig. 1 and their separations from $\frac{3}{2}^-$ [521] are very large. Since the energy separation appears in the denominator in Eq. (8), these two states are expected to bring in very small admixture; also neither of them has been experimentally observed in any of the nuclei under discussion. The other four states can reasonably be expected to contribute. Excepting [541] the others have been observed in one or the other of these nuclei, though the experimental situation as discussed later is far from satisfactory. For the present we restrict ourselves to the use of the predicted positions of energy levels only. We find from Fig. 1 that these separation energies are not drastically changing with deformation. Thus the energy denominators by themselves do not cause any appreciable variation of d with δ .

The numerator in Eq. (8) is however found to be very much deformation-dependent for some of the cases. The contribution of the numerator is shown in Fig. 5. The dominating contribution of the [530] state is quite evident and is a rapidly varying function of δ . This variation with δ is caused by the variation of its decoupling parameter. The other neighbor which from Fig. 1 is the [521] state, is found to bring in negligible contribution as a direct term but enters significantly through the cross terms.

The calculated values of d (in $\hbar\omega_0$ units) from Eq. (8) are shown in Fig. 6 which gives, on a semi-log plot, the contributions of the significant direct and cross terms. It is seen that at small deformations the value of d is effectively determined by admixture from the [530] state alone. As the deformation increases beyond 0.25, the contributions from other direct and cross terms start playing a significant role and the over-all effect is to change the sign of d at a smaller deformation than that obtained with [530] alone.

For comparison with the experiment we have also shown by horizontal lines the value of d (in $\hbar\omega_0$ units) from Table I. It is seen that agreement with the experiment can be obtained for the ten cases shown in the figure for deformation values in the

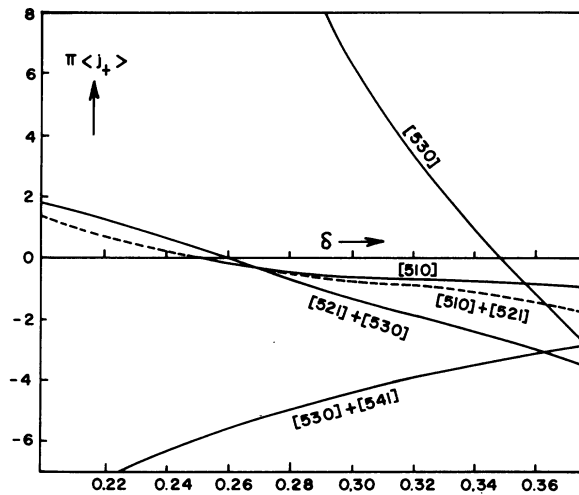


FIG. 5. Plot of the numerator in Eq. (8) for the K' and K'' as labeled on individual curves. A single-state label on the curve shows $K' = K''$. Contributions from other K', K'' combinations, not shown in this figure, are all negligible on this scale.

range 0.24–0.30. Our calculations fail to reproduce the experimental results for the two nuclei ^{153}Gd and ^{169}Yb for any reasonable value of δ . Of course, as seen in Fig. 6, for large δ the calculated value of d shows a rather abrupt change of sign accompanied by a steep rise in its magnitude so that for sufficient large values of δ one can obtain the abnormally large positive values of d quoted for these two nuclei in Table I. However, this procedure does not appear to be satisfactory, particularly since one cannot reasonably expect such a large deformation for a nearly spherical nucleus like ^{153}Gd which, anyway, should not be more deformed than the heavier Gd isotopes. The explanation for discrepancy in these two cases may lie possibly in assignment of levels to the $\frac{3}{2}^-$ rotational bands other than presently suggested. Alternatively one may look for the existence of other physical phenomena¹² for these two nuclei.

5. CALCULATIONS WITH OBSERVED LEVEL POSITIONS

So far we have performed all our calculations employing the energy separations theoretically predicted by the adopted deformed potential.⁷ Since this potential uses an average set of parameters, it is not expected to reproduce the observed excitation energies of the various single-particle levels. Consequently, the separation energies ΔE need not agree with the actual separation energies for individual nuclei. Our calculations should, as far as feasible, use the experimentally observed separa-

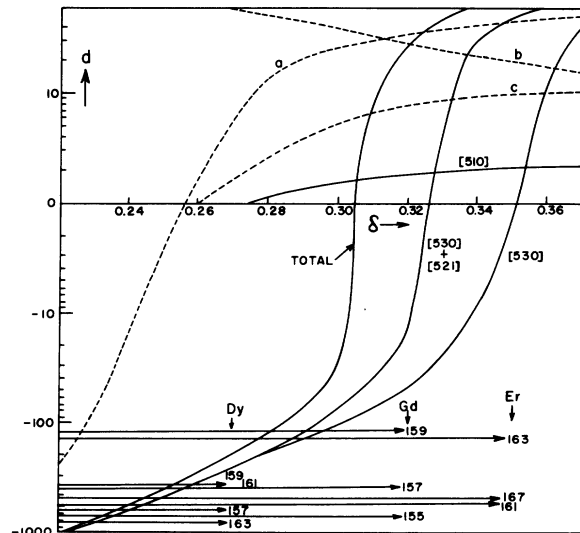


FIG. 6. The calculated (curves) and experimental (horizontal lines) values of the coefficient $d = A_3/A^3$ in $\hbar\omega_0$ units plotted on a semilog scale. The dashed curves show the cross-term contributions only for the combinations (a) [530] and [521], (b) [530] and [541], and (c) [521] and [510]. Individual contributions of the [530] and [510] are also shown along with the total of the direct- and cross-term contributions when both [530] and [521] states are simultaneously considered. The curve labeled "total" shows the calculated results when contributions from direct and cross terms from all the $\frac{1}{2}^-$ states from the $N = 5$ oscillator shell are included.

tion energies in individual cases. However, unfortunately, the present situation in this regard is quite unsatisfactory.

The $\frac{1}{2}^-$ [521] state is identified⁸ in all the nuclei under consideration (except for ^{157}Dy and ^{161}Er). We have concluded above that this state contributes negligibly in the evaluation of d because of its very small matrix element. However, when this state and the $\frac{1}{2}^-$ [530] couple simultaneously, the cross term in Eq. (8) gives significant contribution. The dominant contribution is found to come from the $\frac{1}{2}^-$ [530] state, which has so far been observed only in ^{155}Gd . The rotational band built over this state has also been identified¹³ in ^{157}Gd and ^{159}Gd , but the band head of these bands is yet to be identified. $\frac{1}{2}^-$ [510] has also been identified⁸ in ^{163}Dy , ^{163}Er , ^{169}Er , and ^{169}Yb nuclei. However, as discussed above, this state brings in negligible direct- as well as cross-term contributions. No other $K = \frac{1}{2}$ state has been observed in any of the nuclei under consideration. Thus we find that the calculations with experimental energy position is possible only in some of the Gd isotopes.

In order to assess the changes, if any, brought about in our conclusions in the previous section, we have evaluated d for ^{155}Gd , ^{157}Gd , and ^{159}Gd

employing the experimental positions and empirical values of decoupling parameters of the $\frac{1}{2}^-$ [530] and $\frac{1}{2}^-$ [521] states. The band heads in ^{157}Gd and ^{159}Gd for the $\frac{1}{2}^-$ [530] state were deduced by using the reported¹³ decoupling and inertial parameters of these bands. It is found that the agreement between experiment and theory needs $\delta = 0.25$ for ^{155}Gd and ^{159}Gd and $\delta = 0.20$ for ^{157}Gd . The experimentally observed¹¹ value of deformation for these nuclei is ~ 0.27 .

We conclude that on the whole the explanation of the alternating behavior (depicted in Fig. 4) of the experimental levels in rotational bands built on the $\frac{3}{2}^-$ [521] neutron state in odd-mass rare-earth nu-

clei can be reasonably interpreted as arising from the third-order Coriolis perturbation from the various $K^\pi = \frac{1}{2}^-$ bands. For this purpose the main contribution appears to come from the $\frac{1}{2}^-$ [530] state which still awaits experimental identification in most of these nuclei.

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