Possible measurement of the vacuum polarization in heavy-ion scattering*

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A measurement of the vacuum polarization potential in nonrelativistic elastic heavy-ion scattering is suggested. We find that in a typical sub-Coulomb barrier event enough momentum is transferred from the projectile to the target nucleus to make the high-q^2 behavior of the photon propagator observable. The elastic cross section is increased in a typical case by 2.5% above the Rutherford value. Only relative measurements are needed.

[NUCLEAR REACTIONS ^{208}Pb(^{16}O,^{16}O). Calculated vacuum polarization effect, sub-Coulomb, DWBA.]

The only high-precision experiment in which vacuum polarization effects give a dominant correction term to the lowest-order effect is the measurement of certain transition energies in muonic atoms, such as $5g_{7/2}$-$4f_{5/2}$ in muonic lead.[1,2] In all other precision tests of quantum electrodynamics, such as Lamb shift, g factor, etc., vacuum polarization is a small, albeit well-established, part of the effect observed. Furthermore, unlike the case of heavy muonic atoms, essentially only low-momentum-transfer contributions are tested ($q^2/m_e^2 \approx 1$). In high-energy scattering of e^+, e^-, etc. (which is of course a high-momentum-transfer test and therefore essential for detecting possible new interactions or deviations at short distances[3]) one tests in practice only Born diagrams at high energies. The effect of the vacuum polarization has also been taken into account to obtain an improved value for the scattering length[4-6] in low-energy proton-proton scattering. This experiment of course does not measure the vacuum polarization effect by itself, since strong interactions give the main unknown contribution to the calculations. Here one has to believe the theoretical predictions for vacuum polarization at $q^2 \gtrsim m_e^2$.

The purpose of this note is to point out that in nonrelativistic heavy-ion scattering we can achieve a rather direct test of the structure of the spectral function for the photon propagator at large momentum transfer $|\vec{q}|^2$, though the relative precision presently available is in no sense comparable to that of muonic atoms. For illustration, we choose the scattering of ^{16}O on ^{208}Pb. To bring both nuclei to a separation of 12 fm (at which point the nuclear densities begin to overlap) we need at least a kinetic energy E of 76 MeV for a ^{16}O ion. We choose an $E_p = 60$-MeV ^{16}O beam (which most tandem laboratories can achieve). For the relative momentum we find $p^2 \sim 1.9$ GeV$^2/c^2$. The momentum transfer is $|\vec{q}|^2 = |\vec{p}|^2\, 4\sin^2\theta/2$ where θ is the scattering angle. We see that with the chosen conditions we easily reach a momentum transfer $|\vec{q}| \cong 1$GeV$/c$ at a sub-Coulomb energy. With ^{208}Pb as the target the available energy resolution of better than 1% allows discrimination against Coulomb excitations of the target.

As we calculate below, we find an increase of the elastic scattering cross section due to vacuum polarization effects by 1% at $|\vec{q}| = 30$ MeV (corresponding to 5.7° in the above example), at $|\vec{q}| = 100$ MeV by 1.4%(19°), at $|\vec{q}| = 1$ GeV by 2.1% (30°), and at 3 GeV by 2.4% (104°). No absolute measurements of the cross section are necessary, since we may view $|\vec{q}|$ as function of the scattering angle. Also, no measurement beyond angles greater than 110° is necessary, because the deviations do not increase much more beyond this point. As the precision of the relative measurements now can be as high as 0.5%,[7] we believe that the necessary experiments can be carried out in the near future.

We will justify below that it is sufficient to consider first-order Born approximation to calculate the effects outlined above. In the nonrelativistic approximation the elastic scattering of two charged point-like particles is described completely by the photon propagator $D_{\mu\nu}(|\vec{q}|^2)(q_0=0)$. In first-order Born approximation we have

$$\frac{d\sigma}{d\Omega} \propto D_{00}(|\vec{q}|^2)^2 \,. \tag{1}$$

We also know, that $D_{00}(|\vec{q}|^2)$ may be represented using the spectral function $\sigma(t)$[8]:

$$D_{00}(\vec{q}^{\,2}) = |\vec{q}|^{-2} + \int_1^\infty \sigma(t)(|\vec{q}|^2 + 4m_e^2 t^2)^{-1}\, dt \,. \tag{2}$$

m_e is the electron mass. In first order in α the spectral function is given by[9]:

$$\sigma(t)=\frac{2\alpha}{3\pi}\left(1+\frac{1}{2t^2}\right)(t^2-1)^{1/2}t^{-2}. \tag{3}$$

Combining Eqs. (1) and (2) we obtain for the ratio of the observed to the Rutherford scattering:

$$\left(\frac{d\sigma}{d\Omega}\right)\Big/\left(\frac{d\sigma}{d\Omega}\right)_R=1+2|\vec{q}|^2 \times\int_1^\infty \sigma(t)(|\vec{q}|^2+4m_e^2t^2)^{-1}\,dt. \tag{4}$$

As all quantities on the right-hand side of Eq. (4) are positive definite, we see that the actual cross section should be larger than the Rutherford cross section. The right-hand side is easily evaluated for different values of $|\vec{q}|^2$. We find that already for $|\vec{q}|^2\sim 5(2m_e)^2$ the spectral function reaches the asymptotic form (see Uehling[9])

$$\int_1^\infty \sigma(t)(|\vec{q}|^2+4m_e^2t^2)^{-1}\,dt =\alpha/3\pi|\vec{q}|^{-2}\{\ln[|\vec{q}|^2(2m_e)^{-2}]-\tfrac{5}{3}\}. \tag{5}$$

Noting that $|\vec{q}|^2=8\sin^2(\theta/2)E_p\mu_p$, where $E_p=p^2/2\mu_p$ is the relative kinetic energy and μ_p the reduced mass, we obtain

$$\left(\frac{d\sigma}{d\Omega}\right)\Big/\left(\frac{d\sigma}{d\Omega}\right)_R=1+2\alpha/3\pi\{\ln[|\vec{q}|^2(2m_e)^{-2}]-\tfrac{5}{3}\} =1+2\alpha/3\pi \times\{\ln[2\sin^2(\theta/2)E_p\,\mu_p/m_e^2]-\tfrac{5}{3}\}. \tag{6}$$

This expression yields the numerical results quoted above.

The above consideration in momentum space can be easily understood in configuration space. As it is well known, the electron vacuum polarization potential has the same sign as the inducing potential and has a range of about $\lambda\!\!\!^{-}_e/2\sim 190$ fm where $\lambda\!\!\!^{-}_e$ is the Compton wavelength of the electron. Due to this interaction we have, in addition to the Coulomb force, an additional "long-range" force (when comparing with the nuclear sizes). Thus the probability for scattering must increase.

We will now justify the use of first-order Born approximation to derive Eq. (4). As the coupling constant is $\alpha'=Z_1Z_2\alpha>1$, we cannot use the standard argument $\alpha'<1$. The characteristic parameter for Coulomb scattering is $\eta=Z_1Z_2\alpha c/v_\infty$ where v_∞ is the asymptotic velocity of the projectile. For the case considered we have $\eta>10$. This indicates that no expansion in η is permissible. However, there is a very small parameter $\nu=(2m_ec^2)^2/p^2$. We will now consider the scattering amplitude in all orders in η and approximate the result using $\eta\nu^{1/2}$ as a small parameter. In fact, $\eta\nu^{1/2}=Z_1Z_2\alpha m_e c^2/E_p$. Setting E_p proportional to the Coulomb energy at the turning point, we obtain (Z dependence cancels out) $\eta\nu^{1/2}=R_{ca}/\lambda_e$ where R_{ca} is the distance of closest approach. We must calculate the amplitude for Coulomb scattering to all orders, but include the vacuum polarization only to first order. That means we perform a distorted-wave Born-approximation calculation.

We rely heavily on the work done in proton-proton scattering.[5] Using the expression given by Durand[5] for the vacuum polarization scattering amplitude $M_{vp}(q^2)$ calculated with Schrödinger Coulomb functions we obtain

$$M(|\vec{q}|^2)=M_c(|\vec{q}|^2)+M_{vp}(|\vec{q}|^2)\propto(\sin^2\theta/2)^{-i\eta} \times\Big(|\vec{q}|^{-2}+\int^\infty\sigma(t)(|\vec{q}|^2+4m_e^2t^2)^{-1}[(1+\nu t^2/2\sin^2\theta/2)/(1+\nu t^2/2)]^{-i\eta} \times\exp\{-2\eta\tan^{-1}[(\nu/2)^{1/2}t]\}|\Gamma(1+i\eta)|^2 \times{}_2F_1(-i\eta,1+i\eta,1;(1+t^2\nu/2\sin^2\theta/2)^{-1})\Big). \tag{7}$$

Note that the ${}_2F_1$ function is singular for $\nu\to 0$. Therefore, we consider

$$|\Gamma(1+i\eta)|^2{}_2F_1(-i\eta,1+i\eta,1;(1+\epsilon)^{-1})=-i\eta\ln[(1+\epsilon)/\epsilon]{}_2F_1(-i\eta,1+i\eta,1;\epsilon/(1+\epsilon)) -i\eta\Gamma(-i\eta)^{-1}\Gamma(1+i\eta)^{-1}\sum_{n=0}^\infty\Gamma(n-i\eta)\Gamma(n+1+i\eta)/(n!)^2 \times[\epsilon/(1+\epsilon)]^n\Big\{(n-i\eta)^{-1}\sum_{m=n+1}^\infty 2\eta^2[m(m^2+\eta^2)]^{-1}\Big\}. \tag{8}$$

The sum over n may be truncated after $n=0$ since the expansion parameter is $n^2\nu$. We thus obtain

$$\text{Eq. (8)} = 1 - i\eta \ln[(1+\epsilon)/\epsilon]\,{}_2F_1(-i\eta, 1+i\eta, \epsilon/(1+\epsilon)) + i\eta 2\,\mathrm{Re}[\psi(1+i\eta)+\gamma]\ , \qquad (9)$$

where $\psi(z) = d[\ln\Gamma(z)]/dz$ and $\gamma = 0.5772$. Even for $\eta \sim 10$ the asymptotic expansion $\mathrm{Re}[\psi(1+i\eta)]_{\eta\to\infty} = \ln(1+\eta)$ is very good. Inserting Eq. (8) into Eq. (7) we see that, in the domain of integration over t in which $\nu t^2 < 1$, we can neglect all additional factors in Eq. (7) but

$$1 - i\eta\{\ln[1+(\nu t^2/2\sin^2\theta/2)^{-1}] + 2\ln(1+\eta) + 2\gamma\}.$$

For $\nu t^2 > 1$, however, the integrand in Eq. (7) does not contribute significantly. Thus

$$M(|\vec{q}|^2) \propto |\vec{q}|^{-2} + \int \sigma(t)(|\vec{q}|^2 + 4m_e^2t^2)^{-1}\,dt + i\eta \int \{\ln(1+\eta) - \ln[1+(\nu t^2/2\sin^2\theta/2)^{-1}]\} \times \sigma(t)(|\vec{q}|^2 + 4m_e^2t^2)^{-1}\,dt. \qquad (10)$$

We emphasize that this result is valid for any value of η. Because the additional term in Eq. (10) is purely imaginary, no destructive interference between Coulomb and vacuum polarization parts in the amplitude occurs. Proceeding as before we obtain

$$\left(\frac{d\sigma}{d\Omega}\right)\Big/\left(\frac{d\sigma}{d\Omega}\right)_R = 1 + 2|\vec{q}|^2 \int_1^\infty \sigma(t)(|\vec{q}|^2 + 4m_e^2t^2)^{-1} \times dt - \eta^2(A|\vec{q}|^2)^2\ , \qquad (11)$$

where

$$A|\vec{q}|^2 = |\vec{q}|^2 \int [2\ln(1+\eta) + 2\gamma] - \ln[1+(\nu t^2/2\,\sin^2\theta/2)^{-1}] \times\sigma(t)(|\vec{q}|^2 + 4m_e^2t^2)^{-1}dt.$$

As long as $\eta\alpha < 1$ the contribution $(A|\vec{q}|^2)^2$ can be neglected. When considering possible future Pb-Pb experiments this term should be carefully evaluated, however; then other terms of the order $(Z_1\alpha)(Z_2\alpha)$ should also be included.

Let us finally note a number of additional effects which may possibly enter into the discussion of the Coulomb elastic cross section. The most obvious one is the electron screening of the target nucleus (we assume that the light projectile is completely stripped). The s electrons have essentially constant charge distribution in the nucleus; this will cause no effect as a function of the scattering angle or energy and the effects due to the variation of electronic charge in the target atom can be estimated using a model charge distribution and are found to be of the order of 0.1% and less of the total cross section in the region of interest.

Since the collision is slow compared with the electron motion, molecular orbitals[10] must be used to describe the electron states during the collision. This will give rise to effective molecular potentials similar to those found in chemical binding as has been outlined in Ref. 11. The precise calculation of those effects which are very important in very heavy-ion-atom collisions are in progress. However, if we choose light projectiles with $Z < 20$ and heavy target with $Z \sim 80$–90, this effect is also negligible.[12] All radiative effects like self-energy, vertex corrections, and bremsstrahlung are negligible due to the large mass of the projectile and its nonrelativistic motion. In the unlikely event that we use projectiles or targets with nonvanishing spin, we must also consider the spin-spin and spin-orbital coupling. In the nonrelativistic case these are also negligible.

Finally, let us make a few remarks about the possible presence of dispersive effects; these are contributions to the scattering amplitude in which intermediate excited nuclear states are populated. This effect corresponds to a very short-range interaction when compared with vacuum polarization potential. Therefore, it may become competitive with the vacuum polarization effects only in backward scattering.

In this note we have chosen the stiffest possible nuclei to measure the effect of the vacuum polarization; however, using soft nuclei and looking at backward scattering one may probably see the dispersive effects. This is similar to multiple Coulomb excitation with the final state being again the ground state of the nucleus. As the dispersive amplitude does not decrease as drastically as the Coulomb amplitude does with increasing scattering angle[13] and may be as large as the Coulomb amplitude only for head-on collisions (in the example presented in this note) we believe that in the present case the influence of dispersive effects on the elastic cross section will lie below present experimental precision for scattering between 0 and say 120°. These effects should be evaluated more carefully when higher-precision scattering data at high energies becomes available.

We conclude this discussion by observing that in scattering of very heavy ions, such as lead on lead at sub-Coulomb barrier energy, we can reach $|\vec{p}| = 134$ GeV^2 which may lead to $|\vec{q}|^2 \geq 500$ GeV^2 (center of mass) under favorable conditions. Such experiments will be possible as soon as heavy-ion accelerators in Darmstadt and Berkeley (super Hilac) are working with Pb and heavier beams. With these accelerators we will have unique tools to test vacuum polarization at large momentum transfer.

The idea that vacuum polarization may play an important role in heavy-ion scattering arose in discussions with Berndt Muller and Walter Greiner in Frankfurt. The work did not proceed until a discussion with Chris Davies, who brought this problem again to our attention.

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[1]The experiments are partially contradictory with respect to the sign of a discrepancy, which may be as large as 3.5 standard deviations compared to the best available theory, which also includes higher-order effects of vacuum polarization. See G. Backenstoss *et al.*, Phys. Lett. 31B, 233 (1970); 43B, 539 (1973); M. S. Dixit, *et al.*, Phys. Rev. Lett. 27, 878 (1971); and H. K. Walter *et al.*, Phys. Lett. 40B, 197 (1972).

[2]For theoretical calculation see M. K. Sundaresan and P. J. S. Watson, Phys. Rev. Lett. 29, 15, 1122 (1972); J. Blomquist, Nucl. Phys. B48, 95 (1972); T. L. Bell, Phys. Rev. A 7, 1480 (1973); P. Vogel, Phys. Rev. A 7, 63 (1973). For a full summary of the present status see J. Rafelski, B. Müller, G. Soff, and W. Greiner, Critical Discussion of the Vacuum Polarization Measurements with Muonic Atoms, Frankfurt, September, 1973 (to be published). In a recent report G. A. Rinker, Jr., and L. Wilets, Phys. Rev. Lett. 31, 1559 (1973), claim additional contribution to higher-order vacuum polarization calculations. This contribution enlarges the discrepancy in lead by one standard deviation.

[3]S. J. Brodsky and S. D. Drell, Annu. Rev. Nucl. Sci. 20, 147 (1970).

[4]For review see E. M. Henley, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969).

[5]L. Durand, III, Phys. Rev. 108, 1597 (1957).

[6]L. Heller, Phys. Rev. 120, 627 (1960).

[7]T. Fortune, private communication. We thank Dr. Fortune for supplying us with the experimental information needed.

[8]We refer here to any standard book on field theory, such as J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[9]J. Schwinger, Phys. Rev. 75, 651 (1949); see also E. A. Uehling, Phys. Rev. 48, 55 (1935).

[10]B. Müller, J. Rafelski, and W. Greiner, Phys. Lett. (to be published); B. Müller, Ph.D. thesis, Frankfurt, 1973 (unpublished).

[11]J. Rafelski, B. Müller, and W. Greiner, Nuovo Cimento Lett. 4, 469 (1972).

[12]In this case the molecular K orbitals are already created when both nuclei are separated by approximately 150 fm. The energy of the K shell does not change considerably when the ions further approach each other; thus no appreciable effect on the scattering cross section is found in the range of $|\vec{q}|^2$ considered.

[13]We draw this conclusion from the available calculations on electron dispersive scattering. For references see J. Eisenberg and W. Greiner, *Excitation Mechanisms of the Nucleus* (North-Holland, Amsterdam, 1970), p. 186.