Spin and isospin effects in π-^{4}He scattering*

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We derive the amplitude for π-^{4}He elastic collisions, including all multiple scattering, spin-flip, and charge-exchange effects, in the Glauber approximation. A totally antisymmetric ^{4}He ground-state wave function, and πN elastic scattering amplitudes obtained from phase-shift analyses (through G waves when necessary) are used as input. The results are applied to π-^{4}He total cross sections below 1.2 GeV and compared with measurements. The real part of the π-^{4}He forward elastic scattering amplitude is calculated and compared with dispersion-relation results. The differential cross section for π-^{4}He elastic scattering is calculated for pion energies between 50 and 1120 MeV and compared with data.

[NUCLEAR REACTIONS ^{4}He($\pi^\pm$, $\pi^\pm$), ($\pi^\pm$, x), E = 50–1200 MeV; calculated $\sigma(\theta)$ and σ_{tot}.]

The Glauber approximation for scattering of hadrons by nuclei has had extensive application in the past decade. Calculations have generally employed oversimplified basic hadron-nucleon scattering amplitudes, or have neglected the effects of spin and or isospin, or have treated spin and isospin degrees of freedom rather approximately. At energies below ~1 GeV, however, the spin and isospin parts of the hadron-nucleon amplitudes are often quite significant. Since marked improvements in measurements below 1 GeV will soon be forthcoming, it is important that analyses at such energies include the effects of spin and isospin as accurately as possible. A step in this direction has recently been made.[1,2] In those analyses, additional approximations were introduced regarding the possible ways in which charge exchange and spin flip may occur in nuclei.

In the present analysis we consider the effects of charge exchange and spin flip in collisions of pions with ^{4}He. Within the context of the Glauber approximation, which considers single, double, triple, and quadruple scattering for collisions with ^{4}He targets, we make no additional approximations regarding the ways in which charge exchange and spin flip occur. All possible combinations of charge exchange, spin flip, and elastic collisions through fourth-order (i.e., quadruple) scattering are considered.

The scattering amplitude for elastic collisions between pions and nuclei with mass number A is given by[3]

$$F_{ii}(q)=(ik_{\pi A}/2\pi)\int e^{i\vec{q}\cdot\vec{b}}\Psi_i^\dagger(\vec{r}_1,\ldots,\vec{r}_A,\vec{\sigma}_1,\ldots,\vec{\sigma}_A,\vec{\tau}_1,\ldots,\vec{\tau}_A)\delta^{(3)}(A^{-1}\sum_j\vec{r}_j)$$
$$\times\left\{1-\prod_j\left[1-(2\pi ik_{\pi N})^{-1}\int e^{-i\vec{q}'\cdot(\vec{b}-\vec{s}_j)}f(\vec{q}',\vec{\sigma}_j,\vec{\tau}_j,\vec{T})d^2q'\right]\right\}$$
$$\times\Psi_i(\vec{r}_1,\ldots,\vec{r}_A,\vec{\sigma}_1,\ldots,\vec{\sigma}_A,\vec{\tau}_1,\ldots,\vec{\tau}_A)d\vec{r}_1\cdots d\vec{r}_A\,d^2b\,, \qquad (1)$$

where f is the pion-nucleon (πN) amplitude for elastic scattering, Ψ_i is the ^{4}He ground-state wave function, $\vec{T}$ is the isospin operator for the pion, and $\vec{\sigma}_j$ and $\vec{\tau}_j$ are the spin and isospin operators, respectively, for the jth nucleon.

The wave function Ψ_i is totally antisymmetric. We take the ground state to be a pure S state, and write the wave function as

$$\Psi_i=\psi(\vec{r}_1,\ldots,\vec{r}_A)\phi(\vec{\sigma}_1,\ldots,\vec{\sigma}_A,\vec{\tau}_1,\ldots,\vec{\tau}_A)\,. \qquad (2)$$

The spatial part ψ is taken to be completely symmetric and the combined spin-isospin part ϕ is taken to be completely antisymmetric. There are two $S=0$ spin wave functions:

$$\chi=\tfrac{1}{2}(\alpha_1\beta_2-\beta_1\alpha_2)(\alpha_3\beta_4-\beta_3\alpha_4)\,, \qquad (3)$$

and

$$\overline{\chi}=3^{-1/2}\vec{\sigma}_1\cdot\vec{\sigma}_3\chi\,, \qquad (4)$$

where α, β are the spin wave functions for each nucleon. Similarly there are two corresponding $I=0$ isotopic-spin wave functions which we denote by η and $\bar{\eta}$. The completely antisymmetric spin-isospin wave function is given by

$$\phi = 2^{-1/2}(\chi\bar{\eta} - \bar{\chi}\eta). \tag{5}$$

The general form for the πN elastic scattering amplitude operator is

$$f(\vec{q}, \vec{\sigma}_i, \vec{\tau}_i, \vec{T}) = f_1(q) + if_2(q)\vec{\sigma}_i\cdot\hat{n} + [g_1(q) + ig_2(q)\vec{\sigma}_i\cdot\hat{n}]\vec{\tau}_i\cdot\vec{T} \tag{6}$$

$$= f_1(q) + \Delta(\vec{q}, \vec{\sigma}_i, \vec{\tau}_i, \vec{T}), \tag{7}$$

where Eqs. (6) and (7) define $\Delta(\vec{q}, \vec{\sigma}_i, \vec{\tau}_i, \vec{T})$.

Combining Eqs. (1), (2), and (7) we obtain

$$F_{ii}(q) = (ik_{\pi A}/2\pi)\int e^{i\vec{q}\cdot\vec{b}}\psi^*\delta^{(3)}\left(\frac{1}{4}\sum_j \vec{r}_j\right) \times \left\langle \phi \left| \left\{ 1 - \prod_j \left[1 - (2\pi i k_{\pi N})^{-1}\int e^{-i\vec{q}\,'\cdot(\vec{b}-\vec{s}_j)}(f_1+\Delta)d^2q' \right] \right\} \right| \phi \right\rangle \psi d\vec{r}_1\cdots d\vec{r}_4 d^2b, \tag{8}$$

where ϕ is given explicitly by Eqs. (3)–(5), and the spin-isospin dependence of Δ is given explicitly by Eqs. (6) and (7). The matrix elements in spin-isospin space can now be carried out in detail using this general result.[4] (We might add that the same method may be used to calculate p-^{4}He amplitudes.)

To illustrate the result Eq. (8), let us consider ψ to be a product of single-particle harmonic-oscillator wave functions and use the Gartenhaus-Schwartz transformation[5] to eliminate the δ function. This enables us to replace $\psi^*\psi\delta^{(3)}$ by $(\alpha^2/\pi)^6\exp[-\alpha^2(r_1^2+r_2^2+r_3^2+r_4^2)]$ provided we multiply the resulting expression for F_{ii} by $\exp(q^2/16\alpha^2)$. The scattering amplitude then becomes

$$F_{ii}(q) = (ik_{\pi A}/2\pi)\,e^{q^2/16\alpha^2}\int e^{i\vec{q}\cdot\vec{b}}\left\{1 - \left\langle \phi \left| \prod_{j=1}^{4}[\gamma(b) + \delta(\vec{b}, \vec{\sigma}_j, \vec{\tau}_j, \vec{T})] \right| \phi \right\rangle\right\} d^2b, \tag{9}$$

where

$$\gamma(b) = 1 + ik_{\pi N}^{-1}\int_0^\infty S(q)J_0(qb)f_1(q)q\,dq, \tag{10}$$

$$\delta(\vec{b}, \vec{\sigma}_j, \vec{\tau}_j, \vec{T}) = -(2\pi i k_{\pi N})^{-1} \times \int S(q)e^{i\vec{q}\cdot\vec{b}}\Delta(\vec{q}, \vec{\sigma}_j, \vec{\tau}_j, \vec{T})d^2q, \tag{11}$$

and where $S(q) = \exp(-q^2/4\alpha^2)$.

If we expand the product in Eq. (9), the expectation value may be written as

$$\left\langle \phi \left| \prod(\gamma+\delta) \right| \phi \right\rangle = \gamma^4 + 4\gamma^3\langle\phi|\delta|\phi\rangle + 6\gamma^2\left\langle \phi \left| \prod_{j=1}^{2}\delta \right| \phi \right\rangle + 4\gamma\left\langle \phi \left| \prod_{j=1}^{3}\delta \right| \phi \right\rangle + \left\langle \phi \left| \prod_{j=1}^{4}\delta \right| \phi \right\rangle. \tag{12}$$

In order that the final state be the ^{4}He ground state, πN charge-exchange collisions and spin-flip collisions must occur an even number of times. Consequently $\langle\phi|\delta|\phi\rangle = 0$ since δ corresponds to a single spin flip and or charge exchange. The remaining matrix elements in Eq. (12) may be expressed in terms of three integrals:

$$A(b) \equiv k_{\pi N}^{-1}\int_0^\infty S(q)f_2(q)J_1(qb)q\,dq, \tag{13}$$

$$B(b) \equiv k_{\pi N}^{-1}\int_0^\infty S(q)g_1(q)J_0(qb)q\,dq, \tag{14}$$

$$C(b) \equiv k_{\pi N}^{-1}\int_0^\infty S(q)g_2(q)J_1(qb)q\,dq. \tag{15}$$

The results we obtain are

$$\left\langle \phi \left| \prod_{i=1}^{2}\delta \right| \phi \right\rangle = \tfrac{1}{3}A^2 + \tfrac{2}{3}B^2 + \tfrac{2}{3}C^2, \tag{16}$$

$$\left\langle \phi \left| \prod_{i=1}^{3}\delta \right| \phi \right\rangle = -4iABC, \tag{17}$$

$$\left\langle \phi \left| \prod_{i=1}^{4}\delta \right| \phi \right\rangle = A^4 + 10(B^4 + C^4)/3 - 4(A^2B^2 + B^2C^2 + C^2A^2). \tag{18}$$

The final form taken by F_{ii} is obtained from Eqs.

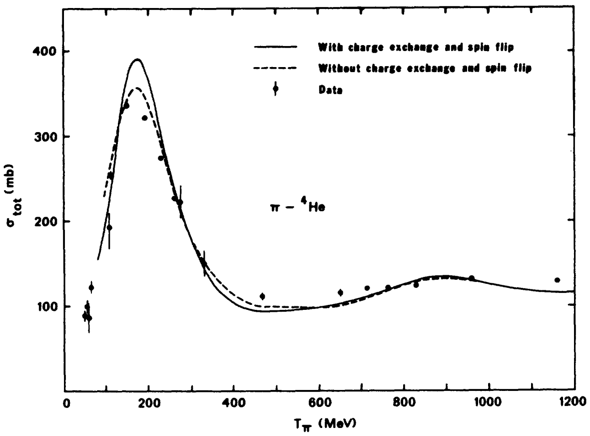

FIG. 1. Calculated π-^{4}He total cross sections compared with data of Refs. 8 and 9.

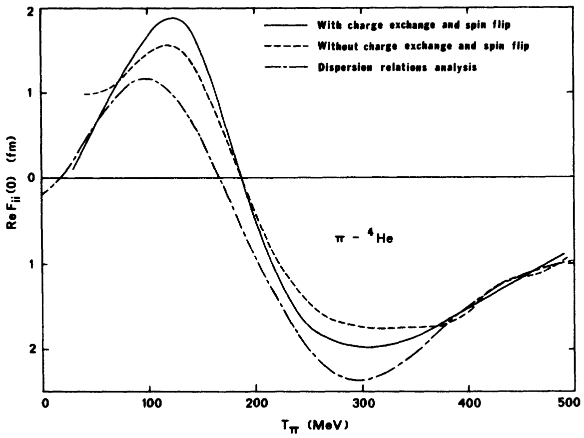

FIG. 2. Real part of π-^{4}He forward elastic scattering amplitude. The dispersion relation analysis is from Ref. 10.

(9), (12), and (16)–(18), and is given by

$$F_{ii}(q)=ik_{\pi A}e^{q^2/16\alpha^2}\int_0^\infty J_0(qb)\{1-\gamma^4-2\gamma^2[A^2+2(B^2+C^2)]+16i\gamma ABC-A^4-\tfrac{10}{3}(B^4+C^4)+4(A^2B^2+B^2C^2+C^2A^2)\}\,b\,db\,, \tag{19}$$

with γ, A, B, and C given by Eqs. (10), (13)–(15).

We have applied this result to $\pi-{}^4$He total cross sections and elastic scattering at energies below 1.2 GeV. For the πN amplitudes f_1, f_2, g_1, and g_2, we have used the CERN "theoretical" phase shifts.[6] The parameter α^2 was taken to be 0.512 fm^2 to fit the ^{4}He rms radius of 1.71 fm.[7]

In Fig. 1 we compare the calculations for total cross sections, obtained from the optical theorem, with data[8,9] below 1.2 GeV. The effects of charge exchange and spin flip are not large above 250 MeV. The disagreement between theory and measurement near 175 MeV diminishes if the effects of Fermi motion are considered.

In Fig. 2, we compare the calculations for Re F_{ii} (0) with recent dispersion-relation calculations.[10] The two methods yield reasonable agreement with each other above ~200 MeV. The effects of charge exchange and spin flip are seen to be significant.

In Fig. 3, we compare calculations of the differential cross section for elastic scattering at 1.12 GeV with the data.[11] The theory agrees rather well with the measurements, and the effects of charge exchange and spin flip are rather small. However, we point out that we have performed similar calculations for the lower energies of 50–260 MeV, where data also exist, and find quite different results. Below 100 MeV the effects of spin flip and charge exchange are *very* large and significantly improve the comparison of theory with data. Despite this improvement, however, the theory is in poor quantitative agreement with the data. (Qualitatively the comparison is good, as the minima that appear in the data also appear in the calculations at approximately the correct scattering angle. However, the calculated magnitudes are high by a factor of ~4–10.)

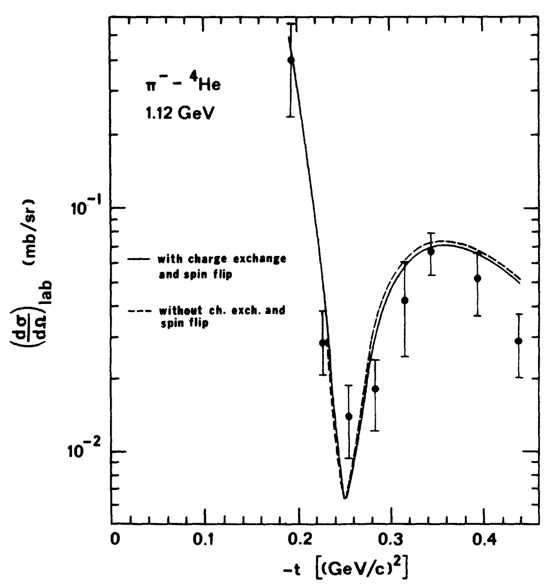

FIG. 3. π^--^{4}He elastic scattering intensity at 1.12 GeV. The data is from Ref. 11.

Between 100 and 260 MeV spin-flip and charge-exchange effects are significant. However, except for angles less than those corresponding to the first diffraction minimum (i.e., $\theta_{c.m.} \lesssim 65°$), the agreement of theory with data is rather poor. The disagreement between 50 and 260 MeV is not totally surprising since $1 \lesssim kR \lesssim 3$, which is not very large, and the data extend to very large angles.

The analysis we have presented can be extended in an approximate way to other selected nuclei such as ^{40}Ca and ^{12}C (for which kR would be larger than for ^{4}He) by considering them to be α clusters and restricting the ways in which multiple spin flip and charge exchange may occur in nuclei.[1]

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[1]V. Franco, invited talk, in Lectures from the LAMPF Summer School on the Theory of π-Nucleus Scattering, edited by W. R. Gibbs and B. F. Gibson, 1973 (unpublished).

[2]A. T. Hess and J. M. Eisenberg, contributed talk, in Lectures from the LAMPF Summer School on the Theory of π-Nucleus Scattering (see Ref. 1).

[3]R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. Brittin *et al.* (Interscience, New York, 1959), Vol. I; V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966).

[4]See C. Wilkin, CERN Report No. 71-14, 1971 (unpublished) for an alternative and presumably equivalent method for obtaining a formal expression for F_{ii} in the Glauber approximation.

[5]S. Gartenhaus and C. Schwartz, Phys. Rev. 108, 482 (1957).

[6]A. Donnachie *et al.*, CERN Report No. TH. 838 Addendum, 1967 (unpublished).

[7]R. F. Frosch *et al.*, Phys. Rev. 160, 874 (1967).

[8]M. M. Block *et al.*, Phys. Rev. 169, 1074 (1968); Ph. Chavanon *et al.*, Nuovo Cimento 40A, 935 (1965).

[9]C. Wilkin *et al.*, Nucl. Phys. B62, 61 (1973).

[10]C. J. Batty *et al.*, Rutherford Laboratory Report No. RL-73-118, 1973 (unpublished).

[11]J. Combe *et al.*, Nuovo Cimento 3A, 663 (1971).