Cluster model and the photodisintegration of ^{6}Li[†]

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The total cross section for the photodisintegration of ^{6}Li, leading to three-body final states is considered in a sequential decay model. Various cluster decompositions of the bound and continuum states are studied. The influence of the (^{3}He + ^{3}He) channel on the photoeffect is considered in a schematic coupled-channel model. The models investigated are able to provide a qualitative description of the data. The cluster model for ^{6}Li appears to provide a more satisfactory over-all description than the shell model. Various open questions remain concerning the importance of nonsequential amplitudes.

[NUCLEAR REACTIONS Photodisintegration of ^{6}Li; coupled channels; cluster model.]

I. INTRODUCTION

In a previous publication[1] we discussed the total cross section near threshold for the photodisintegration of ^{6}Li. Our analysis was based on a sequential decay model. In this work we extend our analysis to higher energies and discuss various modifications of the model used previously.

The previous model, when used at higher γ-ray energies ($E_\gamma \gtrsim 12$ MeV) yields a cross section that is about a factor of 2 too large (see Fig. 1). Here the ground state of ^{6}Li is taken to be the $(p_{3/2})^2$ shell-model configuration. The model is based on the assumption that the γ ray ejects a $p_{3/2}$ proton or $p_{3/2}$ neutron leaving the system in the ^{5}He or ^{5}Li ground state. The latter states, which are unstable, are then considered to decay, leading ultimately to a three-body final state ($n+p+{}^4$He). For the purposes of this work we will call the aforementioned model, the "shell model." This terminology is used to distinguish that model from the various cluster-model decompositions used in the following discussion. The *shell-model* result (Fig. 1) may be placed in better all over agreement with the experimental data by introducing a numerical factor α_1 which reduces the amplitude of the $(p_{3/2})^2$ configuration in the shell-model description of the ^{6}Li ground state. If we neglect, without any justification, the contributions of the other parts of the ^{6}Li ground state and put $\alpha_1{}^2 \approx \frac{1}{2}$ we can achieve qualitative agreement between theory and experiments over a large range of γ-ray energies. However, the previous agreement[1] in the threshold region for the magnitude of the cross section is destroyed, while the excellent agreement for the average neutron energy in that region[1] ($E_\gamma \simeq 6$ MeV to $E_\gamma \simeq 12$ MeV) is retained. The addition of contributions from the $(p_{1/2})^2$ or $(p_{1/2}p_{3/2})$ portions of the ^{6}Li ground-state shell-model wave function would tend to again produce an overestimate of the cross section over most of the range of γ-ray energies. We note that the processes involving the ejection of a $p_{1/2}$ nucleon from the $(p_{1/2})^2$ configuration or the $(p_{3/2})$ nucleon from the $(p_{1/2}p_{3/2})$ configuration are not readily encompassed by the sequential model, as in this case, the low-lying ($J=\frac{1}{2}^-$) states of the residual nuclei (^{5}Li or ^{5}He) are very broad. In this work we do not attempt to describe the photoeffect leading directly to three-body channels, but limit ourselves to the processes that may be considered sequential. Therefore, we drop the $(p_{1/2})^2$ and $(p_{1/2}p_{3/2})$ configurations from consideration.

Because of some of the difficulties encountered with the shell-model approach we have also investigated various cluster models for the ^{6}Li ground state and continuum states. We discuss the theoretical basis of the cluster model and the results of our calculations in the next section. As we will see, if we discard those parts of the electromagnetic interaction which connect different cluster decompositions, we are able to provide a reasonable fit to the data. The neglect of these terms again corresponds in part to the neglect of nonsequential processes.

II. CLUSTER MODELS–GENERAL CONSIDERATIONS

It is useful at this stage to introduce a decomposition of the ^{6}Li ground and continuum states into various cluster partitions. It is apparent that the cluster decomposition has certain advantages for the description of a light nucleus such as ^{6}Li. This description has a certain intuitive appeal in the case of the photoeffect, where the long-range portions of the wave function (that portion in which the "clusters" are well separated) is important. However, as is well known, the various cluster decompositions are not linearly independent except in the far asymptotic regions. The use of *non-orthogonal* cluster states leads to major ambiguities in the meaning of those spectroscopic factors which might be extracted from experimental investigations. However, the use of *orthogonalized* cluster decompositions has a great advantage in that standard Dirac algebra may be applied in the analysis. For example, if $\{|\Phi_\alpha\rangle\}$ denotes a set of (closed) orthogonalized channel states one may write for the ground state of ^{6}Li, $|\Psi_{^6\mathrm{Li}}\rangle$,

$$|\Psi_{^6\mathrm{Li}}\rangle = \sum_\alpha |\Phi_\alpha\rangle\langle\Phi_\alpha|\Psi_{^6\mathrm{Li}}\rangle, \tag{2.1}$$

where the summed absolute squares of the amplitudes $\langle\Phi_\alpha|\Psi_{^6\mathrm{Li}}\rangle$ are equal to unity. We can be more precise concerning the nature of the $|\Phi_\alpha\rangle$, particularly if we limit ourselves to two-body channels. For example, we may start with two wave functions, each internally antisymmetrized, for two nuclei separated by a distance r and having relative orbital angular momentum l. The intrinsic spins of the clusters (S_1 and S_2) may be coupled to l in some particular order, and the resulting state, *after* full antisymmetrization, may be denoted as $|r_c, c\rangle$. Here c sistinguishes the particular intrinsic states used, the coupling scheme, and the various angular momentum quantum numbers. In general, one would find

$$\langle r'_{c'}, c'|r_c, c\rangle = \delta_{cc'}\delta(r_c - r'_{c'}) - \langle r'_{c'}, c'|\rho|r_c, c\rangle, \tag{2.2}$$

where the second term is a measure of the nonorthogonality. We then propose the determination of an operator F such that the channel states

$$|\Phi_c(r_c)\rangle\rangle = \sum_{c'}\int |r'_{c'}, c'\rangle\langle r'_{c'}, c'|F|r_c, c\rangle dr'_{c'} \tag{2.3}$$

are orthonormal,[3] i.e.,

$$\langle\Phi_{c'}(r'_{c'})|\Phi_c(r_c)\rangle\rangle = \delta_{cc'}\delta(r_c - r'_{c'}). \tag{2.4}$$

Thus if we write

$$|\Psi_{^6\mathrm{Li}}\rangle = \sum_c \int |\Phi_c(r_c)\rangle\rangle\,\phi_c(r_c)dr_c, \tag{2.5}$$

with

$$\langle\Phi_{c'}(r_{c'})|\Psi_{^6\mathrm{Li}}\rangle = \phi_{c'}(r_{c'}), \tag{2.6}$$

we have

$$\sum_c \int |\phi_c(r_c)|^2 dr_c = 1. \tag{2.7}$$

Introducing a set of normalized states $u_{n,c}(r_c)$ such that

$$\phi_c(r_c) = \sum_n \lambda_n^c u_{n,c}(r_c) \tag{2.8}$$

we would have

$$\sum_{n,c} |\lambda_n^c|^2 = 1.$$

In general, the construction of the operators F is a difficult problem and we will not attempt to make that construction in this work.[3] We have, however, presented the foregoing discussion in order that the nature of the approximations used in the following discussion will be apparent.

Continuing in this manner we may also provide a cluster decomposition of the relevant *continuum* states of ^{6}Li, that is, those states which are reached in the dipole photodisintegration. Since the ground state has ($J=1^+$; $T=0$), the relevant continuum channels have ($J=0^-, 1^-, 2^-$; $T=1$). These continuum solutions may be written as

$$|\Psi_{c'c}^{(+)}\rangle \sum_{c'}\int |\Phi_{c'}(r_{c'})\rangle\rangle\,\psi_{c'c}^{(+)}(r_{c'})dr_{c'}, \tag{2.9}$$

where c denotes the channel in which there are incoming waves; $\psi_{c'c}^{(+)}(r_{c'})$ is a wave function having

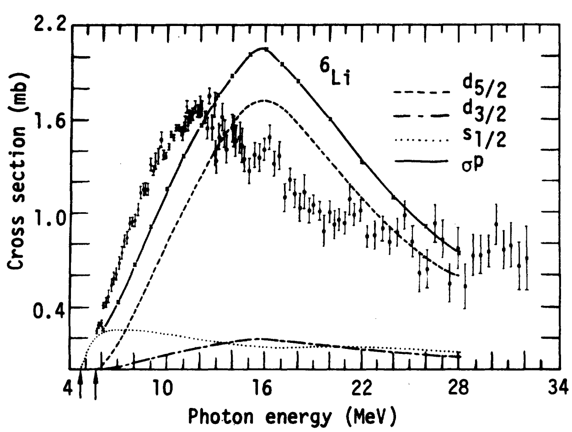

FIG. 1. Total cross section (solid line) for *proton* ejection from ^{6}Li based upon the shell-model description. The contribution from each continuum partial wave considered is also shown. The neutron curves may be obtained by displacing the proton curves 1 MeV to the right (threshold effect). The arrows indicate the proton and neutron thresholds in the sequential decay model; the ($n+p+{}^3$He) threshold is at 3.70 MeV. (a) To obtain the total cross section in the shell model one adds the neutron and proton contributions $\sigma_T = \sigma^p + \sigma^n$. (b) To obtain the total cross section in the cluster model (without channel coupling) one adds one half the proton and neutron contributions $\sigma_T = \sigma^p/2 + \sigma^n/2$. The total cross section data are from Ref. 2.

outgoing waves in channels c' (including $c' = c$). We may obtain integral equations for the wave functions $\psi_{c'c}^{(+)}(r_{c'})$ by writing the total Hamiltonian H as $H = \mathcal{K} + (H - \mathcal{K})$, where $\mathcal{K}$ is an appropriately defined channel Hamiltonian.

It is possible to obtain a symmetrical description of the particles if the channel Hamiltonian is written in terms of an appropriate set of *orthonormal* states (see the Appendix). In terms of the states

$$|\Phi_c(k_c')\rangle \equiv \int |\Phi_c(r_c)\rangle\, dr_c \langle r_c | k_c'\rangle , \tag{2.10}$$

where

$$\langle \Phi_{c'}(k_{c'}') | \Phi_c(k_c)\rangle = \delta_{cc'}\delta(k_{c'}' - k_c) \tag{2.11}$$

we may define

$$\tilde{\mathcal{K}} = \sum_c \int |\Phi_c(k_c)\rangle E_{k_c} \langle \Phi_c(k_c)|\, dk_c . \tag{2.12}$$

In Eq. (2.12) E_{k_c} is the channel energy defined by the asymptotic configurations. We may write the formal Lippmann-Schwinger equation

$$|\Psi_{c'c}^{(+)}\rangle = |\Phi_c(k_c)\rangle\, \delta_{c'c} + \frac{1}{E_{k_c} - \tilde{\mathcal{K}} + i\epsilon} (H - \tilde{\mathcal{K}}) |\Psi_{c'c}^{(+)}\rangle , \tag{2.13}$$

$$|\Psi_{c'c}^{(+)}\rangle = |\Phi_c(k_c)\rangle\, \delta_{c'c} + \sum_{c'} \int \frac{|\Phi_{c'}(k_{c'}')\rangle\, dk_{c'}' \langle \Phi_{c'}(k_{c'}')|(H - \tilde{\mathcal{K}})|\Psi_{c'c}^{(+)}\rangle}{E_{k_c} - E_{k'c'} + i\epsilon} . \tag{2.14}$$

It is now useful to introduce another channel Hamiltonian in which the effects of the parts of H diagonal in the channel indices are summed. This will lead to a two-potential formulation of the theory. The equations for the problem without channel coupling may be written

$$|\Theta_c^{(+)}(k_c)\rangle = |\Phi_c(k_c)\rangle + \int \frac{|\Phi_c(k_c')\rangle\, dk_c' \langle \Phi_c(k_c')|(H - \tilde{\mathcal{K}})|\theta_c^{(+)}(k_c)\rangle}{E_{k_c} - E_{k_c'} + i\epsilon} . \tag{2.15}$$

Using the solutions of Eq. (2.14) we may rewrite our equations as

$$|\Psi_{c'c}^{(+)}\rangle = |\Theta_c^{(+)}(k_c)\rangle\, \delta_{cc'} + \sum_{c' \neq c} \int \frac{|\Theta_{c'}^{(+)}(k_{c'}')\rangle\, dk_{c'}' \langle \Theta_{c'}^{(+)}(k_{c'}')|(H - \mathcal{K})|\Psi_{c'c}^{(+)}\rangle}{E_{k_c} - E_{k'c'} + i\epsilon} \tag{2.16}$$

with

$$\mathcal{K} \equiv \sum_c \int |\Theta_c^{(+)}(k_c)\rangle E_{k_c} \langle \Theta_c^{(+)}(k_c)|\, dk_c . \tag{2.17}$$

Here $\mathcal{K}$ is a new channel Hamiltonian which incorporates the effects of those portions of H which are diagonal in the channel labels.

In order to reduce Eq. (2.15) to an equation involving one-body amplitudes we write the $|\Theta_c^{(+)}(k_c)\rangle$ in a form analogous to Eq. (2.5), i.e.,

$$|\Theta_c^{(+)}(k_c)\rangle = \int |\Phi_c(r_c)\rangle\, \theta_c^{(+)}(r_c) dr_c , \tag{2.18}$$

where the momentum label k_c is included in the index c of the wave function $\theta_c^{(+)}(r_c)$.

The next step involves projecting Eq. (2.16) from the left with $\langle \Phi_{c'}(r_{c'})|$ to yield

$$\langle \Phi_{c'}(r_{c'})|\Psi_{c'c}^{(+)}\rangle = \langle \Phi_{c'}(r_{c'})|\Theta_c^{(+)}(k_c)\rangle + \sum_{c''} \int \langle \Phi_{c'}(r_{c'})| G_{c'}^{(+)}(H - \mathcal{K})|\Phi_{c''}(r_{c''})\rangle\, dr_{c''} \langle \Phi_{c''}(r_{c''})|\Psi_{c''c}^{(+)}\rangle \tag{2.19}$$

with

$$G_{c'}^{(+)} = \int \frac{|\Theta_{c'}^{(+)}(k_{c'}')\rangle\, dk_{c'}' \langle \Theta_{c'}^{(+)}(k_{c'}')|}{E_{k_{c'}} - E_{k_{c'}'} + i\epsilon} . \tag{2.20}$$

Making use of Eqs. (2.9), (2.18), and (2.4) we obtain

$$\Psi_{c'c}^{(+)}(r_{c'}) = \theta_c^{(+)}(r_c)\delta_{cc'} \sum_{c'' \neq c'} \int\int \mathcal{G}_{c'}^{(+)}(r_{c'}, r_{c'}') dr_{c'}' \langle r_{c'}' | V | r_{c''}\rangle\, dr_{c''} \Psi_{c''c}^{(+)}(r_{c''}) , \tag{2.21}$$

where we have defined for $c' \neq c''$

$$\langle r_{c'}' | V | r_{c''}\rangle = \langle \Phi_{c'}(r_{c'}')|(H - \mathcal{K})|\Phi_{c''}(r_{c''})\rangle \tag{2.22}$$

and

$$\mathcal{G}_{c'}^{(+)}(r_{c'}, r'_{c'}) = \int \frac{\langle \Phi_{c'}(r_{c'})|\Theta_{c'}^{(+)}(k'_{c'})\rangle dk'_{c'}\langle\Theta_{c'}^{(+)}(k'_{c'})|\Phi_{c'}(r'_{c'})\rangle}{E_{k_{c'}} - E_{k'_{c'}} + i\epsilon} = \int \frac{\theta_{c'}^{(+)}(r_{c'})\theta_{c'}^{(+)\dagger}(r'_{c'})dk'_{c'}}{E_{k_{c'}} - E_{k'_{c'}} + i\epsilon} . \tag{2.23}$$

Equation (2.21) is of the form of a set of coupled integral equations which are familiar in nuclear physics. This equation may be readily solved if we make the approximation that the channel coupling terms are separable, i.e.,

$$\langle r_{c'}| V | r_{c''}\rangle = \lambda_{c'c''}\langle r_{c'} | f_{c'}\rangle \langle f_{c''} | r_{c''}\rangle . \tag{2.24}$$

Here $\lambda_{c'c''}$ is an measure of the channel coupling and the $\langle r_c|f_c\rangle$ may be termed, "form factors." The modification of the preceding formalism necessary in the presence of redundant solutions is discussed in the Appendix.

III. APPLICATIONS

For application to the photodisintegration of ^{6}Li we first consider the distorted waves $\theta_c^{(+)}(r_c)$ in the absence of channel coupling. For the continuum channels we include $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$ waves (for both protons and neutrons). These continuum wave functions, obtained from a (real) potential of the Woods-Saxon type, are coupled to the ^{5}Li and ^{5}He ground states to form states with $J = 0^-, 1^-, 2^-$. For the ^{6}Li ground state we consider a similar cluster decomposition with a $p_{3/2}$ neutron (or proton) coupled to ^{5}Li or ^{5}He in their ground states. Thus we may write

$$|\Psi_{^6\mathrm{Li}}\rangle = \frac{1}{\sqrt{2}} \int |\Phi_{n[(ljI)J]}(r_n)\rangle \phi_{lj}(r_n) dr_n + \frac{1}{\sqrt{2}} \int |\Phi_{p[(ljI)J]}(r_p)\rangle \phi_{lj}(r_p) dr_p \tag{3.1}$$

with $l = 1$, $j = \frac{3}{2}$, $J = 1^+$, and where p and n refer to the proton and neutron terms, respectively. In Eq. (3.1), I denotes the angular momentum of ground states of ^{5}Li or ^{5}He ($I = \frac{3}{2}^-$). We have also assumed that the $\phi_{lj}(r_n)$ and $\phi_{lj}(r_p)$ are normalized to unity so that all-over normalization of the wave function requires the introduction of the factors of $1/\sqrt{2}$.

We could supplement Eq. (3.1) with terms in which $l = 1$, $j = \frac{1}{2}$, i.e., $p_{1/2}$ particles coupled to the same five-nucleon states as above. The wave function should then be renormalized. This would introduce an extra parameter into the theory and leave the results for the total cross section essentially unchanged. (Although we have used slightly different wave functions for the $d_{3/2}$ and $d_{5/2}$ waves, the spin-orbit effects on the d waves are quite small and the calculation could have been done in an L-S coupling scheme.)

When using the cluster model to calculate the photodisintegration of ^{6}Li we neglect those matrix elements of the dipole operator that change the cluster structure; that is, we assume the ($n + {^5}$Li) portion of the bound state is connected to the ($n + {^5}$Li) portion of the final continuum channels. Similarly the ($p + {^5}$He) portion of the bound state is only connected to the ($p + {^5}$He) continuum wave by the dipole operator. Therefore, in this approximation we are neglecting the photodisintegration of the ^{5}Li or ^{5}He ground-state clusters. This corresponds, in part, to neglecting disintegration *directly* into three-body channels, a processes which we do not consider in our sequential decay model. We recall that similar direct three-body processes were neglected in what we have termed the shell model. Note that also neglected is a process which *is* sequential: For example, starting with the ($n + {^5}$Li) portion of the ^{6}Li ground state, the γ ray can eject a proton from the ^{5}Li cluster leaving the system in the ^{5}He ground state (which then decays to a neutron plus an α particle). These latter processes are included in the shell-model description since there the p-shell particles are treated in a symmetrical fashion.

Now the use of the cluster decomposition of Eq. (3.1) yields a result that is reduced from the "shell-model" result for the $(p_{3/2})^2$ configuration by a factor of $\frac{1}{2}$. (This factor has its origin in the neglect of processes involving the ejection of nucleons from the ^{5}Li or ^{5}He clusters in the ^{6}Li ground state.) *With the particular assumptions made,* we may conclude that the cluster-model version of the theory is in better all-over agreement with the data then the shell model. The agreement in the threshold region is unsatisfactory and may indicate the presence of some three-body breakup amplitudes which are not described in our sequential decay model.

It is possible to extend our considerations to include a wave function more general than that of Eq. (3.1). For example, we may add amplitudes corresponding to other two-body channel ($\alpha + d$) or (^{3}He + ^{3}H). *Without carrying through the program of channel orthogonalization, the addition of extra channels becomes increasingly ambiguous.* However, we have made some investigation of the role of other channels. In particular for the discussion of the photoeffect it is convenient to add some of the (^{3}He + ^{3}H) decomposition to the ground state and simultaneously consider the coupling of this channel to the continuum channels. We neglect the ($\alpha + d$) channel and argue that except in the asymptotic region the channels ($p + {}^5$He), ($n + {}^5$Li), and (^{3}He + ^{3}H) may provide a sufficiently complete basis. More precisely, we may conjecture that as we add channels (in some definite sequence) and systematically orthogonalize and orthornormalize these channel states, the channels added later in the sequence will have small amplitudes for small cluster separation. This feature may be exhibited by studying the nature of the operator F of Eq. (2.3). In this work we have not attempted to actually carry out the orthogonalization program. Some justification of this neglect may be found in the observation that most of the contribution to the photodisintegration matrix elements comes from the region of configuration space where the clusters are well separated and where they have some degree of linear independence. Clearly, these questions deserve further study; however, with these reservations in mind we continue our discussion.

If we include the (^{3}He + ^{3}H) channel with the constituent nuclei in a relative ^{31}S state we may replace Eq. (3.1) by

$$|\Psi_{^6\mathrm{Li}}\rangle = \alpha \int |\Phi_{n[(ljI)J]}(r_n)\rangle\, \phi_{lj}(r_n) dr_n + \alpha \int |\Phi_{p[(ljI)J]}(r_p)\rangle\, \phi_{lj}(r_p) dr_p + \beta \int |\Sigma_{LST}(r_3)\rangle\, \phi_{LST}(r_3) dr_3\,, \tag{3.2}$$

where $|\Sigma_{LST}(r_3)\rangle$ is the (^{3}He + ^{3}H) state vector with separation of the clusters given by r_3.

As noted previously we have considered the $d_{3/2}$, $d_{5/2}$, and $s_{1/2}$ (uncoupled) continuum channels for both neutrons and protons. In the continuum problem it is possible to couple the [$^{33}P_1$] (^{3}He + ^{3}H) channel to all of these; however, that is a formidable problem. As the $d_{5/2}$ channel is the most important by far in the total cross section, we consider coupling the (^{3}He + ^{3}H) channel to the $d_{5/2}$ channel. As a further simplifying assumption we couple separately to the $d_{5/2}$ neutron channel and the $d_{5/2}$ proton channel. This makes the continuum coupled-channel problem a two-channel problem for the cases in which either a $d_{5/2}$ neutron or a $d_{5/2}$ proton is incident. (We have also investigated the same two-channel problem with $d_{3/2}$ waves replacing the $d_{5/2}$ waves.)

Since our coupled-channel investigation is highly schematic we have used a separable approximation for the channel coupling as in Eq. (2.24). Let us denote the two coupled channels by indices 1 and 2, the latter index for the (^{3}He + ^{3}H) channel. Also, let Δ be the threshold energy for the latter channel, $\Delta = 15.69$ MeV. Then we have equations of the form

$$|\psi_1^{(+)}\rangle = |\theta_1^{(+)}\rangle + \lambda \mathcal{G}_1^{(+)}(E)|f_1\rangle\langle f_2|\psi_2^{(+)}\rangle\,, \tag{3.3}$$

$$|\psi_2^{(+)}\rangle = \lambda \mathcal{G}_2^{(+)}(E-\Delta)|f_2\rangle\langle f_1|\psi_1^{(+)}\rangle \tag{3.4}$$

which are readily solved to give

$$|\psi_1^{(+)}\rangle = |\theta_1^{(+)}\rangle + \frac{\lambda^2 \mathcal{G}_1^{(+)}(E)|f_1\rangle\langle f_2|\mathcal{G}_2^{(+)}(E-\Delta)|f_2\rangle\langle f_1|\theta_1^{(+)}\rangle}{1-\lambda^2\langle f_1|\mathcal{G}_1^{(+)}(E)|f_1\rangle\langle f_2|\mathcal{G}_2^{(+)}(E-\Delta)|f_2\rangle} \tag{3.5}$$

and

$$|\psi_2^{(+)}\rangle = \frac{\lambda \mathcal{G}_2^{(+)}(E-\Delta)|f_2\rangle\langle f_1|\theta_1^{(+)}\rangle}{1-\lambda^2\langle f_1|\mathcal{G}_1^{(+)}(E)|f_1\rangle\langle f_2|\mathcal{G}_2^{(+)}(E-\Delta)|f_2\rangle}\,. \tag{3.6}$$

We note that for $|E| < \Delta$, the wave function in channel 2 is exponentially decaying.

The potentials introduced in the ($n + {}^5$Li) and ($p + {}^5$He) channels were discussed in Ref. 1. (These were of standard Wood-Saxon form.) The potentials determining the propagation in the (^{3}He + ^{3}H) channles were taken from Ref. 4. The potential in the ^{31}S channel was adjusted to give a bound state at 15.69 MeV, the threshold for breakup of ^{6}Li into ^{3}He plus a triton. The resulting wave function has a node since the potential used has an additional more deeply bound, bound state. The latter state is redundant (see Appendix and Ref. 4) and was not used in the calculation.

The ^{33}P potential of Ref. 4 was found to have a bound state with a binding energy of about 11 MeV. This solution was also assumed to be redundant (see the Appendix) and was projected out of the wave function in the (^{3}He + ^{3}H) channel; this projection was accomplished by requiring that the form factor $|f_2\rangle$ be orthogonal to the redundant bound state. In keeping with the schematic nature of the coupled-channel calculation, simple Gaussian functions were used for the form factors, the results being rather insensitive to modifications made in these functions.

On the basis of the bound-state model of Eq. (3.2) and the continuum model of Eqs. (3.5) and (3.6), the transition amplitude, leading to a photoneutron or photoproton of momentum $\vec{k}$, may be written as

$$\langle \vec{k}|T_\gamma^C|^6\mathrm{Li}\rangle = \alpha\langle\psi_1^{(-)}|D|\phi_{p_{3/2}}\rangle + \beta\langle\psi_2^{(-)}|D|\Sigma_{010}\rangle\,. \tag{3.7}$$

In Eq. (3.7), $\phi_{p_{3/2}}$ refers to the bound-neutron or -proton wave function in Eq. (3.2) and $|\Sigma_{010}\rangle$ is the bound wave function for the (^{3}He + ^{3}H) portion of the ^{6}Li ground state; D denotes the dipole operator.

The amplitude in Eq. (3.7) may be compared to the uncoupled amplitude (i.e., $\lambda = 0$)

$$\langle \vec{k} | T_\gamma | ^6\text{Li} \rangle = \alpha \langle \theta_1^{(-)} | D | \phi_{p_{3/2}} \rangle \quad (3.8)$$

for neutron or proton emission, the results for this amplitude having been presented in Fig. 1.

In the coupled-channel model one has several parameters to specify. Once the form factors and potentials in the various channels are chosen, the parameter λ remains to be specified. We have also introduced an addition parameter on the basis of the following argument: Since the (n + ^{5}Li) and (p + ^{5}He) channels are constructed using an unstable five-body cluster it is possible to associate a complex energy with these channels. One way to accomplish this is to make the energy parameter in the propagators $\mathcal{G}_1^{(+)}(E)$ and $\mathcal{G}_2^{(+)}(E-\Delta)$ complex; however, since these functions are not developed using their spectral representation this procedure is difficult. We have tried to simulate the effect of the instability of one of the channel clusters by increasing the imaginary part of the denominators in Eqs. (3.5) and (3.6) by a constant $W/2$. This crude approximation has the effect of making the S-matrix nonunitary even if $E < 0$, i.e., when the (^{3}He + ^{3}H) channel is closed. This additional inelasticity represents the effect of the neglect of the three-body ($n + p + ^4$He) channels in our coupled-channel calculation of the final-state wave function based on two-body dynamics.

As we now have essentially three free parameters (α, λ, and W) it is difficult to explore all possible values and we will only present a few typical results. It is useful to divide the total cross section based on the amplitude of Eq. (3.7) into three parts, the coefficients of α^2, $\alpha\beta$, and β^2. We write

$$\sigma_T = \sigma_T^n + \sigma_T^p , \quad (3.9)$$

$$\sigma_T^n = \alpha^2 \sigma_D^n + \alpha\beta\sigma_{INT}^n + \beta^2 \sigma_{IND}^n , \quad (3.10)$$

$$\sigma_T^p = \alpha^2 \sigma_D^p + \alpha\beta\sigma_{INT}^p + \beta^2 \sigma_{IND}^p . \quad (3.11)$$

In these equations n and p again refer to the photoneutron and photoproton cross sections in the sequential decay model. (For example, in the photoneutron case the final-state *proton* is emitted by the ^{5}Li system after the neutron has been ejected.) We may speak of σ_D, σ_{INT}, and σ_{IND} as the direct, interference, and indirect cross sections, for want of better names.

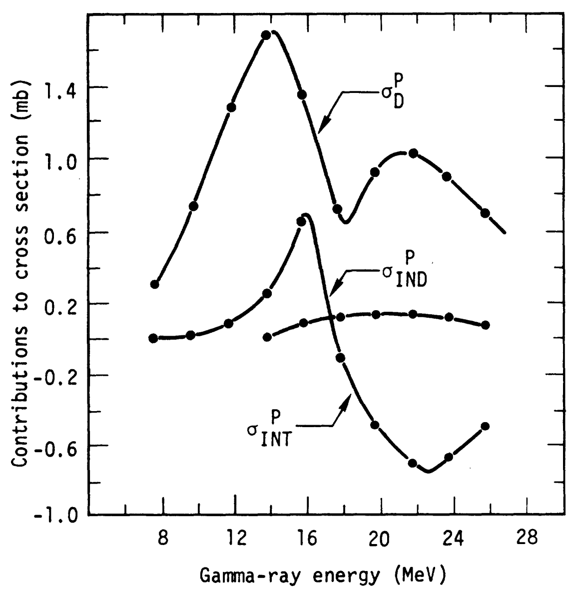

FIG. 2. The contributions from the $d_{5/2}$ proton channel to σ_D^p, σ_{INP}^p, and σ_{IND}^p as defined in Eq. (3.11). (These curves correspond to the parameter values $\lambda = 7.0$ MeV, $W = 0.5$ MeV.) For larger values of λ the oscillations in σ_D are more pronounced.

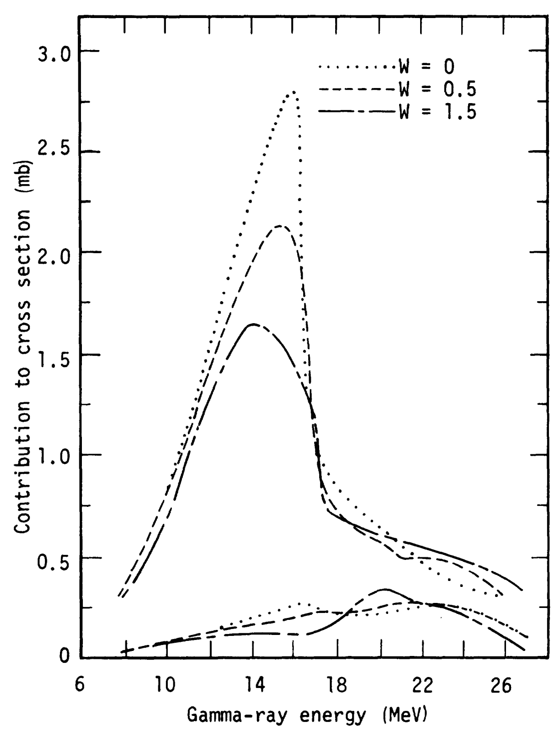

FIG. 3. The quantity ($\sigma_D^p + \sigma_{INT}^p + \sigma_{IND}^p$) for $d_{5/2}$ proton and $d_{3/2}$ proton emission in the coupled-channel model (with $\lambda = 7$ MeV) and various values of W. [See Eqs. (3.13) and (3.14)]. The $d_{5/2}$ cross sections are given in the upper group of curves and the $d_{3/2}$ contribution in the lower group.

The results of this model have the property that for the values of λ investigated ($\lambda = 5$ to $\lambda = 30$) the channel coupling induces rather large oscillations in σ_D (see Fig. 2.). These oscillations, which would make agreement with the experimental data poor, may be canceled out by the interference term σ_{INT} if one chooses $\alpha \simeq \beta$ or ($\alpha = \beta = 1/\sqrt{3}$). We have thus investigated the much simpler form of the model:

$$\sigma_T^n = \tfrac{1}{3}(\sigma_D^n + \sigma_{INT}^n + \sigma_{IND}^n), \tag{3.12}$$

$$\sigma_T^p = \tfrac{1}{3}(\sigma_D^p + \sigma_{INT}^p + \sigma_{IND}^p). \tag{3.13}$$

Since, except for some relatively small threshold effects $\sigma_T^n \simeq \sigma_T^p$ we have the approximation,

$$\sigma_T \simeq \tfrac{2}{3}(\sigma_D^p + \sigma_{INT}^p + \sigma_{IND}^p). \tag{3.14}$$

In Fig. 3 we present some results for the $d_{5/2}$ and $d_{3/2}$ continuum (coupled-) channel contributions to the quantity ($\sigma_D^p + \sigma_{INT}^p + \sigma_{IND}^p$) for various values of W and $\lambda = 7.0$ MeV. In Fig. 4 we compare the quantity given in Eq. (3.14) (including $d_{5/2}, d_{3/2}$, and $s_{1/2}$ contributions) with the experimental data of Ref. 2 for the case $W = 0.5$ MeV.

IV. CONCLUSIONS

In this work we have compared a series of approximate calculations of the photodisintegration of ^{6}Li leading to three-body channels. We find that the shell model gives a good account of the cross section in the threshold region but overestimates the cross section by a factor of about 2 elsewhere. The cluster-model result is of the correct magnitude at the higher energies but is too small near threshold. The difference between these models may be attributed to the neglect of certain processes in the case of the cluster model, as discussed previously. Whether this is a good approximation requires further investigation.

The schematic coupled-channel model also provides a result that is in reasonable agreement with the data. In the application of the cluster model we have not carried through the orthogonalization program as given in the formal theory. However, inspection of the integrands of the photodisintegration matrix elements indicates that most of the contribution arises from regions where the clusters are well separated. Therefore the channel orthogonalization may be somewhat less important for this problem. This feature also deserves further study.

Further experimental investigations of the angular distribution and energy spectra of the particles emitted in the three-body photodisintegration would be of help in understanding the nature of the basic processes. In particular, the applicability of the sequential decay model to the interpretation of the data could receive further clarification and justification.

Further, the study of the reaction ^{3}He + ^{3}H → n + p + ^{4}He would upon interpretation, give one some idea of the strength of the cross-channel coupling and some measure of the parameter λ, if the reaction is indeed sequential, i.e., ^{3}He + ^{3}H → n + ^{5}Li → n + p + ^{4}He or ^{3}He + ^{3}H → p + ^{5}He → n + p + ^{4}He.

APPENDIX

In this Appendix we discuss the modification of the theory necessary in the presence of redundant functions, that is functions $u_c^\alpha(r_c)$ which satisfy the relations

$$\sum_c \int |r_c, c\rangle u_c^\alpha(r_c)\, dr_c = 0 \tag{A1}$$

and

$$\sum_c \int |\Phi_c(r_c)\rangle u_c^\alpha(r_c)\, dr_c = 0\,. \tag{A2}$$

These are eigenfunctions of the matrix

$$\langle r_{c'}, c' | \rho | r_c, c\rangle$$

with eigenvalue unity

$$\sum_c \int \langle r'_{c'}, c' | \rho | r_c, c\rangle u_c^\alpha(r_c)\, dr_c = u_{c'}^\alpha(r'_{c'})\,. \tag{A3}$$

In the case such functions exist, Eq. (2.4) becomes

$$\langle \Phi_{c'}(r'_{c'}) | \Phi_c(r_c)\rangle = \delta_{cc'}\delta(r_c - r'_{c'}) - \sum_\alpha u_{c'}^\alpha(r'_{c'}) u_c^\alpha(r_c)\,. \tag{A4}$$

Since the $|\Phi_c(r_c)\rangle$ are not orthonormal, $\tilde{\mathcal{K}}$ of Eq.

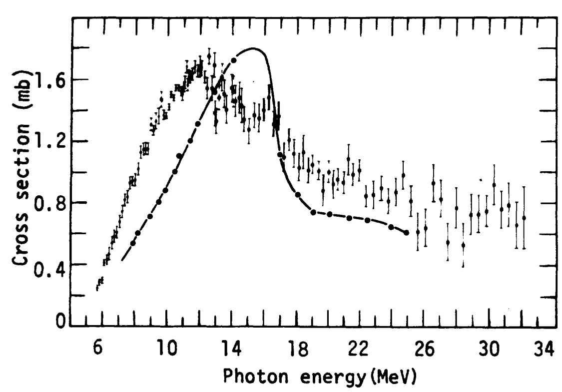

FIG. 4. The total cross section as based on Eq. (3.14) calculated for $\lambda = 7.0$ MeV and $W = 0.5$ MeV. The contributions of the $d_{3/2}$ and $d_{5/2}$ continuum channels are included using the coupled-channel approach. The $s_{1/2}$ channel is included but without the coupling to the (^{3}He + ^{3}H) channel.

(2.12) is unsatisfactory as a channel Hamiltonian. This problem can be circumvented by introducing a new set of *orthonormal* channel states

$$|A_c^{(+)}(k_c)\rangle = \sum_{c'} \int |\Phi_{c'}(r_{c'})\rangle \chi_{c'c}^{(+)}(r_{c'}) dr_{c'}, \quad \text{(A5)}$$

where the $\chi_{c'c}^{(+)}(r_{c'})$ are defined below.

It is useful to define the projection operator

$$\langle r'_{c'} c' |p| r_c, c\rangle = \delta_{cc'}\delta(r_c - r'_{c'}) - \sum_\alpha u_{c'}^\alpha(r'_{c'}) u_c^\alpha(r_c), \quad \text{(A6)}$$

or suppressing the coordinates,

$$\langle c'|p|c\rangle = \delta_{cc'} - \sum_\alpha |u_{c'}^\alpha\rangle\langle u_c^\alpha| . \quad \text{(A7)}$$

We now note that if

$$\sum_{c'} \langle c''|p|c'\rangle |\chi_{c'c}^{(+)}\rangle = |\chi_{c'',c}^{(+)}\rangle \quad \text{(A8)}$$

and if

$$\sum_{c''} \langle \chi_{c'',c'}^{(+)} | \chi_{c'',c}^{(+)}\rangle = \delta_{cc'} \quad \text{(A9)}$$

we have

$$\langle A_{c'}^{(+)}(k'_{c'}) | A_c^{(+)}(k_c)\rangle = \delta_{cc'}\delta(k_c - k'_{c'}) . \quad \text{(A10)}$$

An appropriate set of $|\chi_{c'c}^{(+)}\rangle$ may be obtained as follows. We introduce the kinetic energy operator for channel c, $h_{0,c}$, and define $|\chi_{c'c}^{(+)}\rangle$ to be the solution with incoming waves in channel c (and outgoing waves in c') of the equation

$$\left[E_{k_c} - \sum_{cc'} \langle c''|p|c\rangle h_{0,c} \langle c|p|c'\rangle\right] |\chi_{c'c}^{(+)}\rangle = 0 . \quad \text{(A11)}$$

This last equation may be written in a still more abstract form as

$$[E_{k_c} - p h_0 p] |\chi_c^{(+)}\rangle = 0, \quad \text{(A12)}$$

with the subsidiary condition $p|\chi_c^{(+)}\rangle = |\chi_c^{(+)}\rangle$. We may also write the following integral equation

$$|\chi_c^{(+)}\rangle = |k_c\rangle - \frac{1}{E_{k_c} - h_0 + i\epsilon}(1-p) h_0 |\chi_c^{(+)}\rangle, \quad \text{(A13)}$$

where all quantities are now matrices in the channel space. In Eq. (A13) $|k_c\rangle$ is a "plane-wave" state, i.e. $[E_{k_c} - h_{0,c}]|k_c\rangle = 0$.

As an example, we present the solution of Eq. (A13) in the case there is a single u_c^α. In that case we have

$$|\chi_{c'c}^{(+)}\rangle = \delta_{c'c}|k_c\rangle - \frac{G_{0,c'}^{(+)}(E_{k_c})|u_{c'}^\alpha\rangle\langle u_c^\alpha|k_c\rangle}{\sum_{c''}\langle u_{c''}^\alpha|G_{0,c''}^{(+)}(E_{k_c})|u_{c''}^\alpha\rangle}, \quad \text{(A14)}$$

where

$$G_{0,c'}^{(+)}(E_{k_c}) \equiv (E_{k_c} - h_{0,c'} + i\epsilon)^{-1} \quad \text{(A15)}$$

is diagonal in the channel indices. Note that $\sum_{c'}\langle u_{c'}^\alpha | \chi_{c'c}^{(+)}\rangle = 0$ as required.

In the presence of redundant solutions a satisfactory channel Hamiltonian is then obtained by replacing the $|\Phi_c(k_c)\rangle$ of Eq. (2.12) by the $|A_c^{(+)}(k_c)\rangle$. The modification of the remaining equations of Sec. II is fairly straightforward.

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[3]The orthonormality condition given in Eq. (2.4) cannot be achieved in general if the matrix ρ has eigenvalues equal to unity. The corresponding eigenvectors span a space of redundant functions. To deal with this problem one may construct the channel Hamiltonian from the states defined in Eq. (2.18). These latter states may be made truly orthornormal if the $\theta_c^{(+)}(r_c)$ of Eq. (2.18) are in the nonredundant space. This problem is discussed in great detail by R. R. Scheerbaum and C. M. Shakin, in a paper, Nuclear Rearrangement Scattering, II (to be published). We refer the reader to this paper for a discussion of a straightforward procedure for treating the problem of redundant solutions and the construction of the operator F. In the Appendix we present a short discussion of the modifications necessary in our theory of orthogonalized cluster channels in the presence of redundant solutions.

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