# Nonlocal separable potential and $p$-$p$ scattering

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The $S$-wave $p$-$p$ scattering phase shifts for the energy range of 0–400 MeV (lab) have been calculated using a Mongan-type interaction. Coulomb effects have been included exactly.

Recently Mongan[1] gave an analysis of nucleon-nucleon scattering using separable potentials of rank two. Low-energy $p$-$p$ scattering parameters have also been calculated by Harrington,[2] Gardiner,[3] and Vo-Dai and Nogami[4] using separable potentials. More recently Pigeon *et al.*[5] have presented an analysis of low-energy $p$-$p$ scattering (0–25 MeV) by expressing the nonlocal potential in a "Coulomb representation." Considering the simplicity and success of Mongan's potential in the analysis of nucleon-nucleon data, we thought it worthwhile to calculate $p$-$p$ scattering phase shifts for bombarding energy of 0–400 MeV, using a two-body interaction of Mongan's type. For reasons mentioned in a previous paper[6] we chose to work in coordinate representation, taking into account the effect of the Coulomb potential in an exact way. Thus, we considered separable potentials of the type

$$K_l(r,r') = \sum_{i=1}^{N} \lambda_l^i g_l^i(r) g_l^i(r'), \tag{1}$$

with $N=2$. For the phase-shift calculations, use was made of the expression

$$\tan\delta_l = -\frac{1}{k}\sum_i a_l^i C_l^i \lambda_l^i , \tag{2}$$

where $a_l^i$'s satisfy the algebraic equation

$$\sum_j (\delta_{ij} - \tau_l^{ij}) a_l^j = C_l^i , \tag{3}$$

with

$$\tau_l^{ij} = \int_0^\infty g_l^i(r) Z_l^j(r)\, dr \tag{4}$$

and

$$C_l^i = \int_0^\infty g_l^i(r) F_l(r)\, dr , \tag{5}$$

the appropriate boundary conditions being

$$F_l(0)=0, \quad Z_l^i(0)=0,$$
$$Z_l^i(r) \underset{r\to\infty}{\sim} \frac{\lambda_l^i}{k} G_l(r) \int_0^\infty F_l(r) g_l^i(r)\, dr . \tag{6}$$

The functions $F_l(r)$ and $G_l(r)$ are the usual regular and irregular Coulomb wave functions, respectively. For computational purposes we find it easier to determine $Z_l^i(r)$ by numerically solving the equation

$$\left[\frac{d^2}{dr^2} + k^2 - \frac{l(l+1)}{r^2} - \frac{2\eta k}{r}\right] Z_l^i(r) = \lambda_l^i(r) g_l^i(r), \tag{7}$$

with the boundary conditions Eq. (6). The form factors $g_l^i(r)$ of Eq. (1) used for calculating the $^1S_0$ $p$-$p$ scattering phase shifts were taken to be of the form

$$g_0^i(r) = e^{-\beta_i r}, \quad i = 1, 2 . \tag{8}$$

The results of the calculations are given in Fig. 1. Curve I of this figure shows the phase shifts calculated with Mongan's parameters but with Coulomb effects properly accounted for. Curves II and III are the results of the calculations with parameters adjusted to give a good fit to the phase shifts given by MacGregor *et al.*[7]; the goodness of the fit was, however, determined by eye estimation. The values of the interaction parameters corresponding to the different curves are shown in Table I. As can be seen from this table, for curve II only the range parameters have been adjusted, whereas for curve III both the range and the depth parameters have been varied. Pigeon *et al.* adjusted only the depth parameters. The fact that the inclusion of Coulomb effect in an accurate way into Mongan's potential disturbs the good fit to the data which Mongan had ob-

TABLE I. Interaction parameters.

| | $\lambda^1$ (fm$^{-3}$) | $\lambda^2$ (fm$^{-3}$) | $\beta_1$ (fm$^{-1}$) | $\beta_2$ (fm$^{-1}$) |
|---|---|---|---|---|
| Curve I | −28.293 | 3454.79 | 1.786 | 6.157 |
| Curve II | −28.293 | 3454.79 | 1.760 | 5.800 |
| Curve III | −27.800 | 2800.00 | 1.760 | 5.800 |

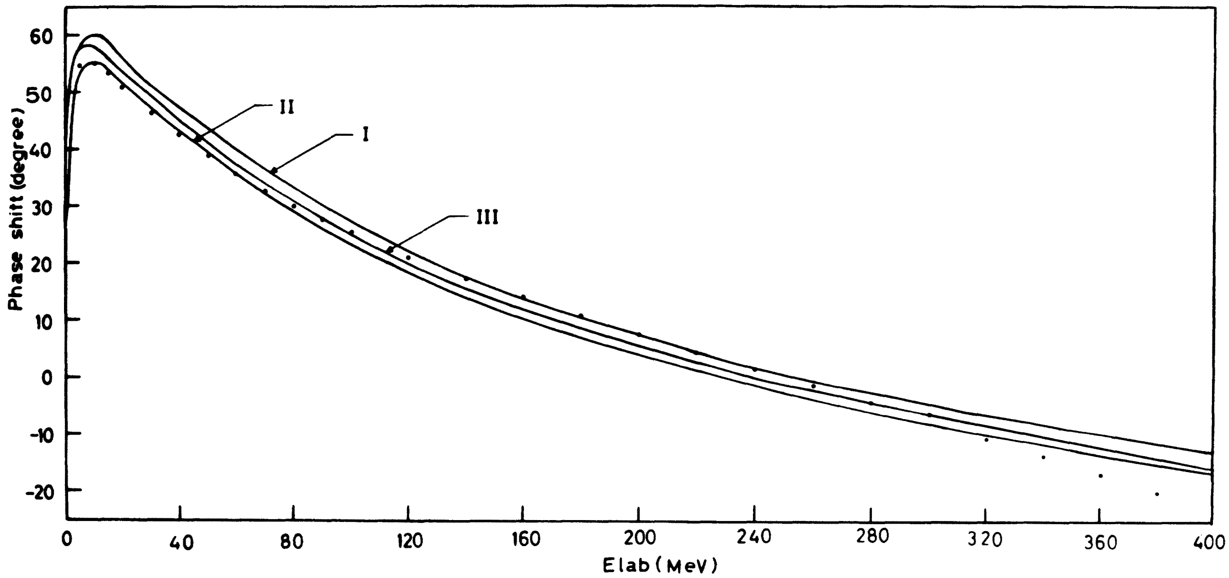


FIG. 1. $p$-$p$ phase shifts are shown as function of laboratory energy for different sets of parameters given in Table I. The dots are the experimental points.

tained at low energies, suggests that a refit of the parameters is necessary. Although curve II has been weighted more in favor of the experimental points in the low-energy region, the fit to higher energies deteriorates after 60 MeV. On the other hand, curve III makes a good compromise between low and high energies. It may seem that the fit of curve III to the high-energy data is not very satisfactory. However, note should be taken of the fact that the experimental data used here are the phase shifts obtained from the 0–400-MeV analysis of MacGregor *et al.* If, however, the data from the 0–700-MeV analysis of MacGregor *et al.*[7] are used, then the fit to the higher-energy part is found to be impressive. In fact, an extension of our calculations even up to 700 MeV showed a good fit to the data in this region. Considering all these points we conclude that Mongan's potential with properly adjusted parameters provides an over-all description of $p$-$p$ scattering for the energy range of 0–400 MeV when Coulomb effects are taken into account exactly.

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