Structure of ^{113}In studied via the reactions ^{115}In(p, t)^{113}In, ^{112}Cd(^{3}He, d)^{113}In, and ^{112}Cd(^{4}He, t)^{113}In

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^{115}In(p,t)^{113}In, ^{112}Cd(^{3}He,d)^{113}In, and ^{112}Cd(α,t)^{113}In experiments have been done and distorted-wave Born approximation analysis has been used to obtain L transfers and spectroscopic factors. Several new levels are reported and J^π assignments are proposed. Two sets of states observed via the two types of reactions are classified as hole-core levels formed by coupling a proton hole to the states of ^{114}Sn, and particle-core levels formed by coupling a proton to the states of ^{112}Cd. Results of particle-core and hole-core model calculations are consistent with the observed spectroscopic factors and with the proposed level scheme.

[NUCLEAR REACTIONS ^{115}In(p,t), E=17,19 MeV; ^{112}Cd(^{3}He,d), (α,t), E=27 MeV; measured $\sigma(\theta)$; ^{113}In levels deduced J^π, S. Enriched targets, spectrograph, DWBA analysis.]

I. INTRODUCTION

When this work was begun relatively little was known about the level structure of ^{113}In, but the level structures of ^{115}In and ^{117}In had already been shown to be complicated. Each of these nuclei has an even number of neutrons and a single hole in the $1g_{9/2}$ proton shell, so a similar level structure was anticipated. Similarities in the ^{115}In and ^{117}In level structures had already been seen. In particular the rotational band structure previously identified experimentally in ^{115}In and ^{117}In was also expected to appear in ^{113}In, along with the $g_{9/2}^{-1}\times 2^+$ (^{114}Sn) states observed via Coulomb excitation. Thus the purpose of this study was to find new levels, to assign spins and parities wherever possible, and to compare the results with predictions of nuclear models, especially the particle-core (p.c.) or equivalently, the hole-core (h.c.) model.

We used two different types of direct reaction in order to look for states predominantly of two different types. The two-neutron transfer reaction ^{115}In(p, t)^{113}In was the probe for h.c. states consisting of a $g_{9/2}$ proton hole coupled to a ^{114}Sn nuclear core; the one-proton transfer reaction ^{112}Cd(^{3}He, d)-^{113}In was the probe for proton 1p-2h states. The supplementary single-proton transfer reaction ^{112}Cd(^{4}He, t)^{113}In was added because it tends to favor the formation of states having large spin.

We have made h.c. and p.c. calculations of the level structure for comparison with the experimental results.

II. EXPERIMENTAL METHOD

The bombarding particles came from the Rochester Van de Graaff accelerator. Reaction products were analyzed and recorded with a magnetic spectrograph having a spark counter at its focal plane. For the (p, t) work both 17- and 19-MeV protons were used. For the (^{3}He, d) and (^{4}He, t) work 27-MeV particles were used. In the (p, t) experiments, observations were made from 5 to 50°, in 5° steps. In the (^{3}He, d) experiments the range was 5 to 40°, at 12 angles, and in the (^{4}He, t) experiments it was 10 to 80°, at 10 angles.

Both natural In and enriched ^{115}In targets were used, 50 to 150 μg/cm^2 thick, on carbon backings. Natural Cd and enriched ^{112}Cd were used in the other experiments, on carbon backings. All targets were made by vacuum evaporation. The Cd targets proved difficult to make and unstable under bombardment. A counter monitoring the elastic scattering (at 45°) was therefore essential to normalize the results. Small-angle elastic scattering was used to establish absolute cross sections.

III. EXPERIMENTAL RESULTS

A. ^{115}In(p,t)^{113}In work

A spectrum from this reaction is shown in Fig. 1. The energy resolution is about 12 keV, full width at half maximum (FWHM). The background is very small. Eight states of ^{113}In are clearly seen in the 1.0- to 1.8-MeV region of excitation. A cluster of

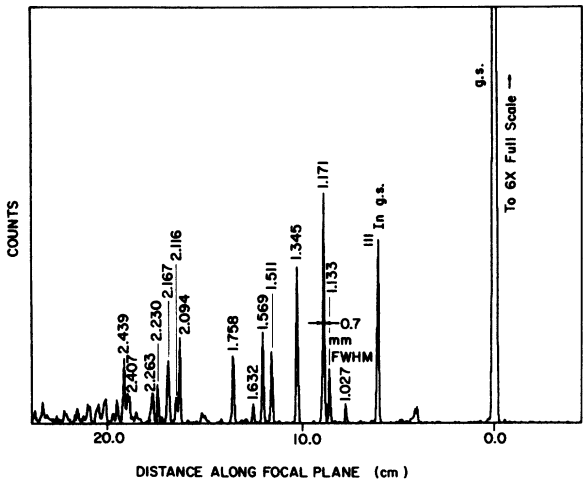

FIG. 1. Triton spectrum at 25° for 19-MeV protons on natural In.

at least seven more states is seen at 2.0 to 2.5 MeV.

The ^{111}In ground-state group (seen because of the presence of ^{113}In in the natural In target) has been used to calculate the difference in Q values between the two ground-state reactions. The result is

$$Q = Q_{115-113} - Q_{113-111} = 810 \pm 10 \text{ keV},$$

in disagreement with the value 717 ± 40 keV tabulated by Gove and Wapstra.[1]

Angular distributions found are shown in Fig. 2. They will be discussed below.

B. ^{112}Cd(^{3}He,d)^{113}In and ^{112}Cd(^{4}He,t)^{113}In

Spectra from these reactions are shown in Fig. 3. The energy resolution is about 33 keV in the (^{3}He, d) case and about 15 keV in the (^{4}He, t) case. Little evidence of target contamination is seen. The (^{4}He, t) spectrum is essentially free of background. The levels at 1.066, 1.194, 1.634, 1.700, and 1.831 MeV as well as several lying higher, apparently have not been reported before.

Angular distributions are shown in Figs. 4 and 5. Unambiguous identification of the l values for each transition can readily be made from the (^{3}He, d) angular distributions.

IV. DISTORTED-WAVE BORN-APPROXIMATION ANALYSIS

A. Distorted-wave Born-approximation analysis of ^{115}In(p,t)^{113}In results

After careful consideration we concluded that an ordinary one-step zero-range distorted-wave Born-approximation (DWBA) analysis would be adequate here. It appears unnecessary to introduce a two-step component, partly because of experimental evidence from the work of Fleming *et al.*[2] on a series of Sn nuclei studied via the (p, t) reaction: Using a consistent microscopic

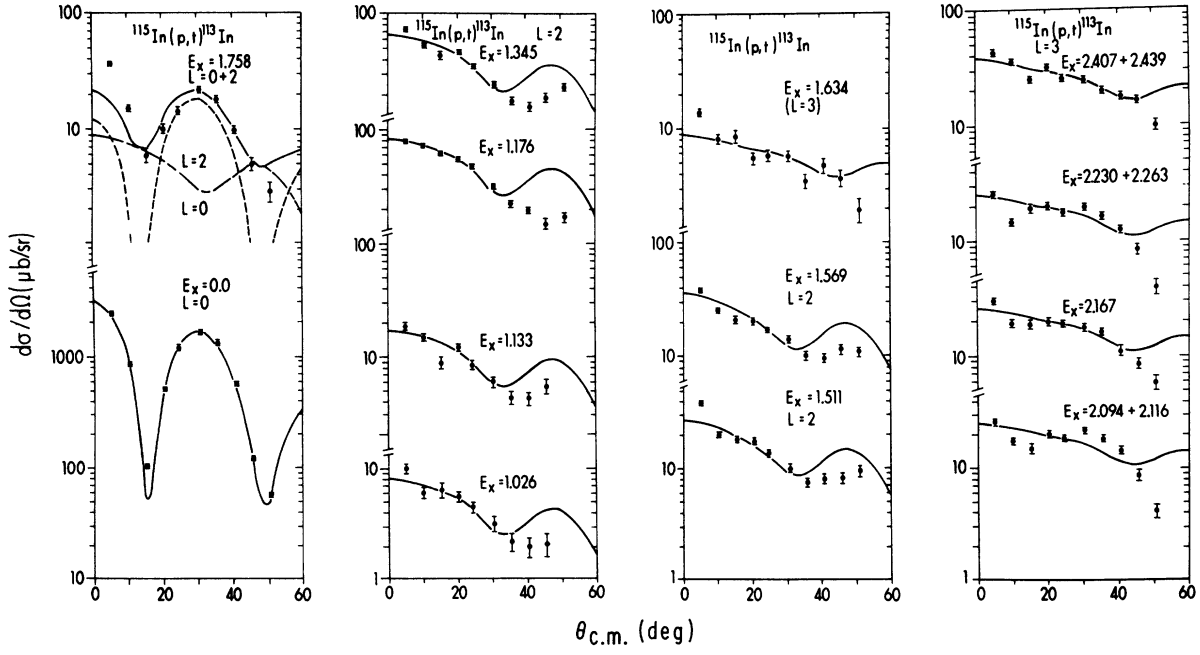

FIG. 2. Angular distributions for ^{115}In(p,t)^{113}In with 19-MeV protons. The curves are DWBA predictions arbitrarily normalized to fit the data.

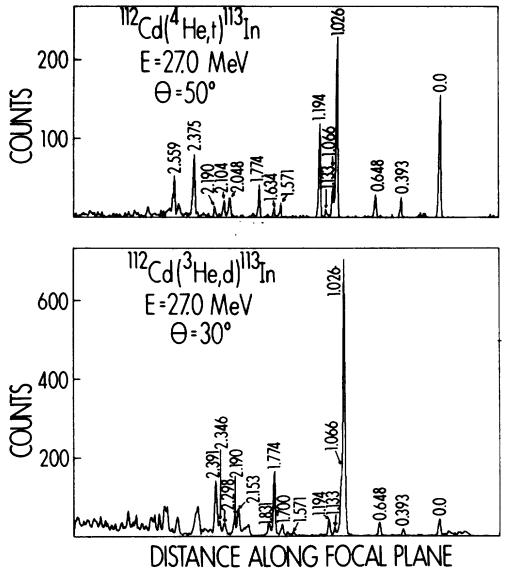

FIG. 3. ^{112}Cd(^{3}He,d) and (α,t) spectra at 30 and 50° for 27-MeV incident particles.

description in the analysis, they showed that the cross-section ratios $\sigma(2_1^+)/\sigma(0_1^+)$ and to some extent $\sigma(0_2^+)/\sigma(0_1^+)$ and $\sigma(4_1^+)/\sigma(0_1^+)$ were predicted correctly by ordinary DWBA calculations. Also, two-step processes should not affect the relative selection of $g_{9/2}{}^{-1}\times 2^+_{\rm core}$ states over 1p-2h$\times 0^+_{\rm core}$ states, which is our immediate concern.

The choice of form factor and optical potentials is not critical, except for predicting absolute magnitudes of cross sections. We tried pure j^2 form factors calculated via Bayman-Kallio techniques, with a Woods-Saxon well for the single nucleons, and also tried the multiconfiguration harmonic-oscillator functions of Ref. 2. Several combinations of optical potentials were tested. After no great sensitivity of results was found we finally chose the form factors and potentials (Set DX) of Ref. 2.

Excellent agreement with the data was obtained for the $L=0$ ground-state transition. (See Fig. 2.) Fair agreement was found for the $L=2$ transitions to the levels from 1.0 to 1.6 MeV. Factors conceivably contributing to the relatively poor $L=2$ fits are $L=4$ admixtures (possible for all states having J between $\frac{1}{2}$ and $\frac{17}{2}$), momentum mismatch and Q-value effects, and multistep processes. The pattern of good agreement with angular distributions from $L=0$ transitions and worse agreement with higher-L transitions seen by Fleming *et al.*[2] is consistent with our results. The 1.758-MeV state seems to be formed by a mixture of $L=0$ and $L=2$ pickup, implying $J^\pi=\frac{9}{2}^+$. The 2.0- to 2.4-MeV states appear to be formed via $L=3$ transitions, but this is not certain. The weak transition to the 1.633-MeV state may also have $L=3$ character; this is even less certain.

The simple h.c. model predicts a set of seven

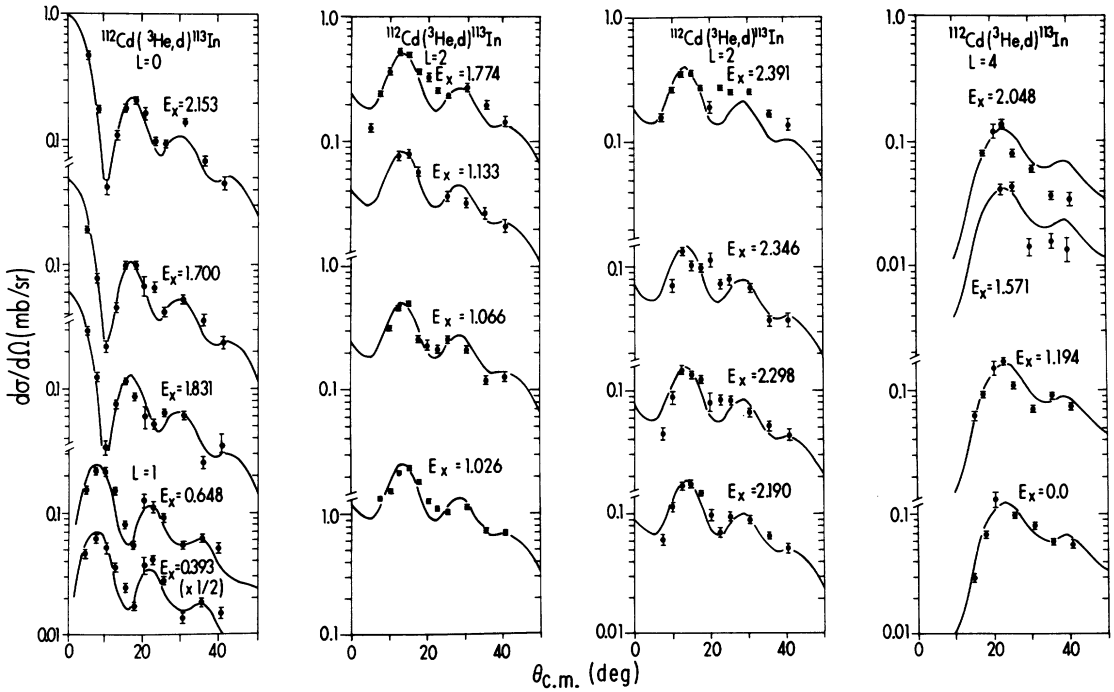

FIG. 4. Angular distributions observed in the ^{112}Cd(^{3}He,d) experiment. The curves are DWBA predictions.

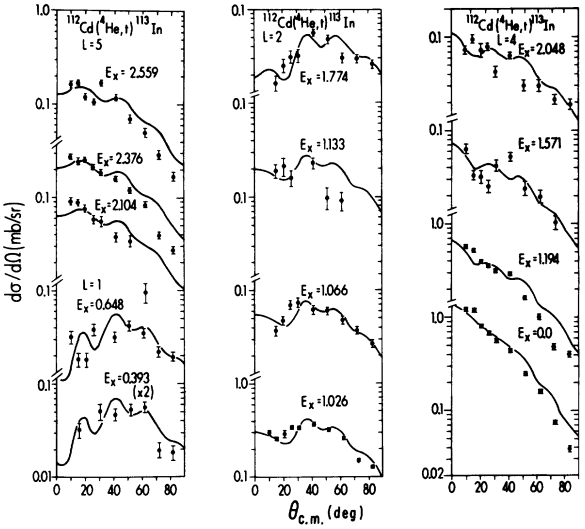

FIG. 5. Angular distributions observed in the ^{112}Cd (α,t) experiment. The curves are DWBA predictions.

states having J^π from $\frac{3}{2}^-$ to $\frac{15}{2}^-$, formed by coupling a $g_{9/2}$ hole to the 3_1^- octupole-vibrational state at 2.28 MeV in the core. The seven states seen in the range 2.0 to 2.5 MeV are conceivably members of that set. Their angular distributions resemble $L=3$ DWBA results. However, there is too much $L=3$ strength (see Table I). Possibly the experimental data contain unresolved contributions from other states.

According to our simple model the ^{114}Sn core, and not the odd-proton hole is involved in the (p,t) reaction. Therefore, it is interesting to compare the (p,t) cross sections seen here with those seen in the ^{114}Sn reaction (Ref. 2). We have calculated the ratio of the total $L=2$ cross section for all states to the $L=0$ cross section for the ground state, for comparison with corresponding figures for the first 2^+ and ground-state transitions in ^{114}Sn. The results, displayed in Table I, are in good agreement, tending to support our model of the reaction.

B. ^{112}Cd(^{3}He,d)^{113}In and ^{112}Cd(^{4}He,t)^{113}In

The standard Woods-Saxon bound-state well parameters $r_0=1.25$ and $a=0.65$ fm were used with the sets of scattering parameters given in Table II. Good scattering-potential parameters exist for the ^{3}He, d and t, but not for ^{4}He. There are few scattering data available for α particles in the appropriate energy range. The parameters seen in the literature all include large, equal radii for the real and imaginary potentials, in violation of the rule that the radius of the real potential well should equal the radius of the single-nucleon potential. In view of these uncertainties we arbitrarily chose a set of ^{4}He scattering-potential parameters that gave fairly satisfactory fits.

The DWBA fits to the (^{3}He, d) data are excellent, allowing unambiguous identification of $l=0$, 1, 2, and 4 transfers. No case with l transfer greater than 4 is seen in these results.

The fits to the (^{4}He, t) data are not as good. A dependence of shape on excitation energy is seen. Here it is harder to distinguish l values. No $l=0$ transition is seen. In general, transitions with higher l values are stronger than in the (^{3}He, d) case. The normalization factor $N=46$ assumed initially in the DWBA analysis led to disagreements with experimental data implying that $N=250$ would be about right here.

Table III gives C^2S values normalized to the sum-rule limit for the lowest three levels, assumed to be pure $g_{9/2}{}^{-1}$, $p_{1/2}{}^{-1}$, and $p_{3/2}{}^{-1}$ single-hole states. Table IV gives ratios of the C^2S values for individual states in both reactions. Although for a given l transfer the relative values agree, for different l transfers they disagree; the particular choice of optical-model potentials for the α particle may be responsible. In later discussion the (^{3}He, d) C^2S values will be used, normalized as just described. This normalization is 17% larger than the accepted factor of 4.42.

Figure 6 shows an energy level diagram for ^{113}In. Some of the energies, spins, and parities included

TABLE I. Sn(p,t) cross sections (see Ref. 2).

A target	112	114	116 [a]	118	120	122	124
$\sigma(2^+_1)/\sigma(0^+$ g.s.)	0.061	0.103	0.128	0.159	0.266	0.266	0.350
$\sigma(3^-_1)/\sigma(0^+$ g.s.)	···	0.039	0.044	0.059	0.138	0.122	0.162

[a] The ^{115}In(p,t)^{113}In results yield:

$$\frac{\sum_i \sigma_i(L=2)}{\sigma_{g.s.}(L=0)}=0.133 \quad \text{and} \quad \frac{\sum_i \sigma_i(L=3)}{\sigma_{g.s.}}=0.080.$$

TABLE II. Optical-model parameters.

	V	r_r	a_r	W_d	W	r_I	a_I	V_{so}	r_c	Ref.
^{3}He	146	1.24	0.652	···	32	1.19	0.927	···	1.25	3
d	101	1.15	0.810	66.5	···	1.34	0.680	6	1.15	4
^{4}He	60.1	1.46	0.712	···	18.7	1.46	0.712	···	1.25	5
^{3}H	170.1	1.16	0.759	···	18.8	1.489	0.839	···	1.25	6

were deduced from the results of the three reactions studied here. Evidently the single-nucleon and two-nucleon transfer reactions do indeed tend to pick out two essentially different sets of states. Some states not seen in the (p,t) reaction yield large C^2S values in the $(^3\text{He},d)$ reaction; they could be interpreted as single-particle states. But others yield small C^2S values; they must be more complex. Furthermore, there are too many states to be accounted for by the simple single-particle model. It must therefore be concluded that configuration mixing is present, probably involving excited states of the ^{112}Cd core.

TABLE III. Spectroscopic factors.

E_x [a] (MeV)	lj assumed	C^2S [b] (α,t)	C^2S [b] $(^3\text{He},d)$	C^2S theory
0.0	$g_{9/2}$	0.15	0.17	···
0.393	$p_{1/2}$	0.083	0.059	···
0.648	$p_{3/2}$	0.078	0.048	···
1.026	$d_{5/2}$	0.30	0.52	0.41
1.066	$d_{3/2}$	0.090	0.15	0.26
1.133	$d_{5/2}$	0.021	0.02	···
1.194	$g_{7/2}$	0.19	0.21	0.35
1.571	$(g_{9/2})$	(0.022)	(0.03)	···
	$(g_{7/2})$	(0.027)	(0.04)	···
1.634	$(l=3,4)$	···	···	···
1.700	$s_{1/2}$	···	0.024	···
1.774	$d_{3/2}$	0.099	0.14	···
1.831	$s_{1/2}$	···	0.029	···
2.048	$g_{7/2}$	0.063	0.097	···
2.104	$h_{11/2}$	0.028	···	···
2.153	$s_{1/2}$	···	0.048	···
2.190	$(d_{5/2})$	···	0.034	···
	$(d_{3/2})$		0.045	
2.298	$(d_{5/2})$	···	0.024	···
	$(d_{3/2})$		0.037	
2.346	$(d_{5/2})$	···	0.022	···
	$(d_{3/2})$		0.033	
2.376	$h_{11/2}$	0.11	···	···
2.391	$(d_{5/2})$	···	0.064	···
	$(d_{3/2})$		0.097	
2.559	$h_{11/2}$	0.080	···	···

[a] Estimated uncertainties are ±5 keV below about 2 MeV and increase to about ±10 keV at 2.5 MeV.

[b] Normalized by $\sum_j(2j+1)\,C^2S = 2.0$ where $j^\pi = \frac{9}{2}^+$, $\frac{1}{2}^-$, $\frac{3}{2}^-$.

V. STRUCTURE OF ^{113}In

A. Introduction

In discussing the results of the (p,t) reaction it is convenient to use the h.c. model, in which ^{113}In is seen as a $1g_{9/2}$ proton hole coupled to a ^{114}Sn core, and ^{115}In is similar in structure, but with two more neutrons which are picked up in the reaction. Accordingly, the states formed in this reaction should be the ground state and excited states based on coupling the $1g_{9/2}$ proton hole to excited states of the core. Thus five low-lying states should be seen corresponding to the 2^+ first excited state of ^{114}Sn, with J^π ranging from $\frac{5}{2}^+$ to $\frac{13}{2}^+$ etc.

On the other hand, in the case of the ^{112}Cd$(^3\text{He},d)$-^{113}Sn reaction it is convenient to use the p.c. model, with ^{113}In seen as a proton coupled to a ^{112}Cd core. Accordingly, in this reaction a proton is stripped into a shell-model orbit, essentially without disturbance of the core structure, so one expects to see the ground state plus others based on low-lying single-particle states with $J^\pi = \frac{7}{2}^+$, $\frac{5}{2}^+$, $\frac{3}{2}^+$, etc. According to the h.c. model, the $J^\pi = \frac{1}{2}^-$ and $\frac{3}{2}^-$ first and second excited states should not be seen in the ^{115}In$(p,t)^{113}$In reaction, but according to the p.c. model they can be excited in the $(^3\text{He},d)$ and $(^4\text{He},t)$ reactions if the ground-state wave function of ^{112}Cd has significant $|p_{1/2}{}^{-2}\rangle$ and $|p_{3/2}{}^{-2}\rangle$ components, as turns out to be the case.

TABLE IV. Comparison of (α,t) and $(^3\text{He},d)$ spectroscopic factors.

E_x (MeV)	lj assumed	$\dfrac{C^2S(\alpha,t)}{C^2S(^3\text{He},d)}$
0.0	$g_{9/2}$	0.88
0.393	$p_{1/2}$	1.4
0.648	$p_{3/2}$	1.6
1.026	$d_{5/2}$	0.58
1.066	$d_{3/2}$	0.60
1.133	$d_{5/2}$	1.3
1.194	$g_{7/2}$	0.90
1.571	$g_{9/2}$	0.67
1.774	$d_{3/2}$	0.71
2.048	$g_{7/2}$	0.65

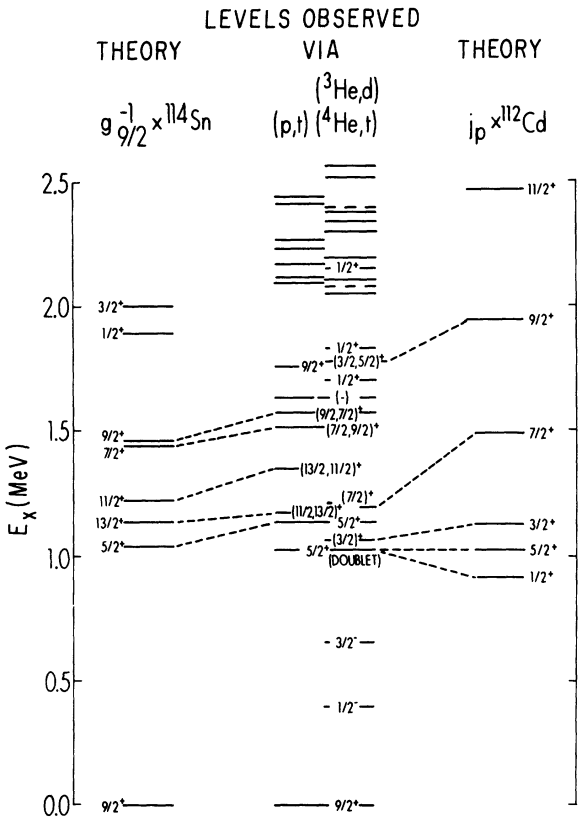

FIG. 6. Experimental and theoretical level schemes. On the left and right are the levels predicted by the hole-core and particle-core models. In the center are separately the levels observed via the two types of reactions. Tentative assignments are shown.

B. Hole- (particle-) core model

Many attempts have been made to calculate the spectrum of low-lying positive-parity states of the odd In isotopes in terms of a $g_{9/2}$ hole coupled to the states of the even-even Sn core,[7,8] usually treated as vibrational. The most recent of these have been able to reproduce a number of energy level positions, $B(E2)$ values, and pickup spectroscopic factors. However, in all cases more states are seen than are predicted by the model; furthermore the "extra" states are found to share some of the expected $E2$ strength.

In his calculations for ^{115}In and ^{117}In, Sen[8] included a $K=\frac{1}{2}^+$ rotational band proposed earlier by Backlin[9] and Pandharipande.[7] Because the proposed rotational band is based on proton 1p-2h excitations presumably not present in the h.c. basis this procedure does not count states twice.[10] Sen included a further refinement, an arbitrary interaction between the rotational states and the h.c. states. His scheme appears to work very well for ^{115}In and ^{117}In in that it succeeds in reproducing proton pickup spectroscopic factors and $B(E2)$ values. However it does not directly predict proton stripping spectroscopic factors and it predicts high-spin ($J^\pi=\frac{9}{2}^+$, $\frac{11}{2}^+$, etc.) states at low-excitation energy which have not been located. One might hope for a more unified model which treats both types of excitations equally.

A more satisfactory treatment of the problem might be a calculation in the space of proton 0p-1h and 1p-2h configurations coupled to the states of the even-even Sn core, including all sources of interaction between various configurations while retaining a manageable basis. The cruder and simpler technique adopted here is to treat the 1p-2h excitations separately, using a p.c. model in which a proton in an orbit outside the $1g_{9/2}$ shell closure is coupled to the states of ^{112}Cd, with all interaction between the 1p-2h and 1h states neglected. In the approximation that ^{112}Cd is a pure proton 2h nucleus and that ^{114}Sn has a closed proton shell for all low-lying states, there is no double-counting of states. Besides having obvious simplicity and symmetry, this model yields proton stripping spectroscopic factors directly.

Essentially two calculations have been carried out. In both cases the proton (particle or hole) was taken to interact with the core via a pure quadrupole-quadrupole force

$$H_{\rm int}=-\zeta r_p^{\,2}Y_2(p)Q2 \quad \text{(core)}.$$

In each case the core $Q2$ matrix elements were evaluated with the relation

$$\langle J'\,\|\,Q2\,\|\,J\rangle = e^{-1}\langle J'\,\|\,E2\,\|\,J\rangle\,,$$

where empirical $E2$ core matrix elements were used wherever possible. This procedure is particularly valuable for nuclei which are neither vibrational nor rotational in character. In the p.c. case, the first five states of the ^{112}Cd core were used, the only ones for which the required experimental $Q2$ matrix elements were known (Table V), and the $d_{5/2}$, $d_{3/2}$, $g_{7/2}$, and $s_{1/2}$ proton orbits were included, with single-particle energies taken from

TABLE V. Reduced core $Q2$ matrix elements for ^{112}Cd.

E_x (MeV)	J^π	$Q2$ [a] (fm^2)				
0	0^+	0	72	0	10.2	0
0.617	2^+	72	−34.3 [b]	38	59	131
1.221	0^+	0	38	0	0	0
1.312	2^+	10.2	59	0	0	0
1.413	4^+	0	131	0	0	0

[a] Reference 12, unless otherwise designated.
[b] Reference 13, $Q(2^+)=-26\,e\,\text{fm}^2$.

TABLE VI. Reduced core $Q2$ matrix elements for ^{114}Sn.

E_x (MeV)	J^π	$Q2$ (fm^2)						
0	0^+	0	47[a]	0	0	0	0	0
1.300	2^+	47[a]	20[b]	29.7[c]	66.5[c]	89.2[c]	0	0
2.200[d]	0^+	0	29.7[c]	0	0	0	0	0
2.200[d]	2^+	0	66.5[c]	0	0	0	0	0
2.200[d]	4^+	0	89.2[c]	0	0	.0	109[c]	131[c]
3.300[d,e]	4^+	0	0	0	0	109[c]	0	0
3.300[d,e]	6^+	0	0	0	0	131[c]	0	0

[a] Reference 14.

[b] Arbitrary value corresponding to $Q(2^+) = 15\ e\,fm^2$ (Ref. 13).

[c] Vibrator estimates.

[d] Assumed level positions; not overly critical.

[e] Used only in the $J^\pi = \frac{1}{2}^+$ and $\frac{5}{2}^+$ calculations.

Ref. 11. In the h.c. case seven states of the ^{114}Sn core were used, up to 3.3 MeV in excitation; unfortunately little experimental $Q2$ information was available, so typical vibrational values were assumed (Table VI). The value of ζ was 0.004 MeV/fm^4.

C. Results: hole-core calculations

The calculated h.c. level spectrum is shown in Fig. 6 together with the experimental spectrum (and the calculated p.c. spectrum). Because the model used is crude and because the number of parameters is large, no attempt has been made to force a detailed fit. However, in this connection two observations were made. First, the value assigned to $\langle 2_1^+ \| Q2(\text{core}) \| 2_1^+ \rangle$ in the Hamiltonian dramatically affects the calculated spectrum. The value used here is $Q(2^+) = +15\ e\,fm^2$. If, instead, the value zero is used, as in the hole-vibrator model, the $J^\pi = \frac{11}{2}^+$ and $\frac{13}{2}^+$ levels change order, and if the sign is changed a more dramatic rearrangement of levels occurs. We obviously cannot make assignments simply on the basis of these calculations. (However, given enough information about the odd nucleus and a sufficiently refined model we may hope one day to gain information about the even-even core nucleus.) The second observation is the prediction of $J^\pi = \frac{1}{2}^+$ and $\frac{3}{2}^+$ states at about 2.0 MeV: They lie this low only when $J^\pi = 4^+$ and 6^+ three-phonon core states are included in the calculation. Otherwise, space truncation effects are small for the low-lying states.

D. Results of the particle-core calculations

The p.c. case is more complicated than the h.c. case because the core states are much more closely spaced and because four single-particle orbitals are included. Both conditions enhance the complexity of the wave functions and the sensitivity of the level positions to the parameters of the calculation. The calculated level positions are shown in Fig. 6. The p.c. states do not include the ground state so it was necessary to establish their positions relative to the h.c. levels. This was done by identifying the $J^\pi = \frac{5}{2}^+$ state at 1.026 MeV as the lowest $J^\pi = \frac{5}{2}^+$ p.c. state.

Variations of the coupling strength ζ and the $\langle 2_1^+ \| Q2 \| 2_1^+ \rangle$ matrix element show that, although the exact level positions vary a great deal relative to the first $J^\pi = \frac{5}{2}^+$ state one always finds at least one $J^\pi = \frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$, $\frac{7}{2}^+$, and $\frac{9}{2}^+$ state below 2 MeV of excitation. The first $J^\pi = \frac{11}{2}^+$ always appears at about 2.5 MeV. If the $\langle 2_1^+ \| Q2 \| 2_1^+ \rangle$ matrix element is taken to have the opposite sign, a second $J^\pi = \frac{7}{2}^+$ state appears below 2 MeV and the first $J^\pi = \frac{1}{2}^+$ and $\frac{3}{2}^+$ states move upward by as much as 0.5 MeV. The predicted spectroscopic factors for the low-lying $J^\pi = \frac{3}{2}^+$, $\frac{5}{2}^+$, and $\frac{7}{2}^+$ states are listed in Table III. The predicted spectroscopic factor for the first $J^\pi = \frac{1}{2}^+$ presumably at 1.030 MeV, as will be discussed later, is 0.33; no experimental value is available.

E. Discussion of results

We now discuss our experimental data in terms of the results of the model calculations, making use of other available data, especially on the level structure of ^{115}In and ^{117}In.

The ground state was seen both in pickup and stripping, as expected.

The first and second excited states, with $J^\pi = \frac{1}{2}^-$ and $\frac{3}{2}^-$, are not seen in the (p,t) reaction. If these are essentially pure $p_{1/2}{}^{-1} \times (0_1^+)$ and $p_{3/2}{}^{-1} \times (0_1^+)$ states the former should not be seen, according to the simple h.c. model, but the latter might be seen if it has a substantial $g_{9/2}{}^{-1} \times (3^-)$ component. Both states are seen in the stripping reactions, where they can appear if the ground-state wave function of ^{112}Cd contains $p_{3/2}{}^{-2}$ and $p_{3/2}{}^{-2}$ (proton) components. According to the strengths deduced from the $(^3\text{He},d)$ results (Table III) the ground-state wave function of ^{112}Cd is 85% $g_{9/2}{}^{-2}$, 6% $p_{1/2}{}^{-2}$, and 9% $p_{3/2}{}^{-2}$, assuming this degree of simplicity. In a similar analysis of the results from their ^{116}Cd$(^3\text{He},d)^{117}$In experiment, Harar and Horoshko[15] found implied 14% $p_{1/2}{}^{-2}$ and 18% $p_{3/2}{}^{-2}$ components, respectively, for the ^{116}Cd ground state.

In the (p,t) reaction the five states expected from coupling the $g_{9/2}$ hole to the 2_1^+ state of the core should be populated mostly by $L = 2$ pickup, but they can be excited via $L = 4$ pickup if they contain admixtures of the form $g_{9/2}{}^{-1} \times 4^+$. Our h.c. calculations and those of Sen suggest that 10% ad-

TABLE VII. Cross sections for ^{115}In$(p,t)^{113}$In.

Ex	L	J^π	$\sigma(p,t)$ [a]
0.0	0	$\frac{9}{2}^+$	8.93
1.758	0	$\frac{9}{2}^+$	0.14
1.026	2	$\frac{5}{2}^+$	0.042
1.133	2	$\frac{5}{2}^+$	0.083
1.171	2	$(\frac{11}{2}, \frac{13}{2})^+$	0.42
1.345	2	$(\frac{13}{2}, \frac{11}{2})^+$	0.33
1.511	2	$(\frac{7}{2}, \frac{9}{2})^+$	0.14
1.569	2	$(\frac{9}{2}, \frac{7}{2})^+$	0.17

[a] $\sum (d\sigma/d\Omega)(\theta i)$ (mb/sr), $\theta = 5$–$45°$.

mixtures may well be present.

Seven states formed in such transitions are found in the 1.0- to 2.0-MeV region. See Table VII. The spins and parities of these states are not well known; in fact, only the 1.026- ($\frac{5}{2}^+$) and 1.133-MeV ($\frac{5}{2}^+$) assignments were reasonably certain beforehand.

In the (p,t) reaction the 1.758-MeV state appears to be formed via mixed $L=0$ and $L=2$ capture, hence it is assigned $J^\pi = \frac{9}{2}^+$. Its cross section is 1.6% that of the ground-state group. It is not seen in the (^{3}He,d) reaction, so we doubt that it is the $\frac{9}{2}^+$ member of the h.c. group. The (p,t) yield can be accounted for by assuring that the ground state of ^{115}In (and presumably also ^{113}In) contains some admixture of higher p.c. states. [The p.c. basis includes no $\frac{9}{2}^+$ orbit and the (p,t) reaction is coherent in nature making it sensitive to small components.] The true $\frac{9}{2}^+$ member of the h.c. multiplet should probably show a larger $L=2$ component.

Of the two $J^\pi = \frac{5}{2}^+$ states the one at 1.133 MeV is populated about twice as strongly as the one at 1.026 MeV in the (p,t) reaction, hence contains the larger $(g_{9/2}^{-1} \times 2_1^+)^{5/2+}$ component in its wave function. The 1.026-MeV state is populated about 25 times as strongly in the (^{3}He,d) reaction, hence it is identified as a p.c. state mixed with the h.c. state. The spectroscopic factor is in good agreement with the predicted value.

The 1.171- and 1.345-MeV states are thought to be the $J^\pi = \frac{11}{2}^+$ and $\frac{13}{2}^+$ members of the h.c. multiplet because of their large cross sections in the (p,t) reaction and in Coulomb excitation. Ideally we expect the cross sections to be proportional to $2J+1$, but mixing with other h.c. states can alter this relationship significantly. Coulomb excitation work[16] showed a marked similarity between the 1.171- and 1.345-MeV states of ^{113}In and the 1.131- and 1.289-MeV states of ^{115}In. The latter have been assigned $J^\pi = \frac{11}{2}^+$ and $\frac{13}{2}^+$, respectively. Thus we feel that the pair in ^{113}In also have the same character, but we are unsure about the order.

The 1.511- and 1.569-MeV states may both be members of the h.c. multiplet; then one of them would have character $J^\pi = \frac{7}{2}^+$, the other $\frac{9}{2}^+$, and the $\frac{9}{2}^+$ state at 1.758 MeV would not be a member. If this were the case, the 1.569-MeV state would have $J^\pi = \frac{9}{2}^+$ since it is seen in the stripping experiments and this would not be the case for a pure h.c. $J^\pi = \frac{7}{2}^+$ state. One might hope that the $\frac{9}{2}^+$ stripping strengths would be proportional to the $\frac{9}{2}^+$ pickup strength predicted by the h.c. model. If this were so the $J^\pi = \frac{9}{2}^+$ h.c. excited state would have a stripping spectroscopic factor 0.04 (0.23 times the ground-state value) compared with the measured value of 0.03 for the 1.569-MeV state. (However, we cannot exclude the possibility that both states have $J^\pi = \frac{7}{2}^+$, sharing the h.c. parentage. Either possibility would be consistent with the magnitudes of the cross sections and with the $L=2$ character of the transitions.) We have ruled out $J^\pi = \frac{5}{2}^+$, $\frac{11}{2}^+$, and $\frac{13}{2}^+$ by analogy with ^{115}In and ^{117}In, where extra $\frac{11}{2}^+$ and $\frac{13}{2}^+$ states are unknown and no more than two $\frac{5}{2}^+$ states have been identified in either spectrum.

Both ^{115}In and ^{117}In have $\frac{3}{2}^+$ and $\frac{1}{2}^+$ levels just above the $\frac{3}{2}^-$ level, furthermore our p.c. calculations predict such a pair similarly placed in ^{113}In. The level we see at 1.066 MeV in the stripping reactions is a good candidate for assignment as the $\frac{3}{2}^+$ level, since it is formed by $l=2$ capture. The predicted spectroscopic factor is 60% larger than the measured value with the remaining measured strength found in the 1.774-MeV level which is not identified with a predicted p.c. state.

There is evidence for $J^\pi = \frac{1}{2}^+$ state very near the well-known $J^\pi = \frac{5}{2}^+$ state at 1.026 MeV. On the basis of the rate of β decay of ^{113}Sn to a level at 1.030 MeV, Delucci and Meyer[17] have concluded that it must have $J^\pi = \frac{1}{2}^+$ or $\frac{3}{2}^+$. It decays 100% via a cascade through the first two excited states, but from Coulomb-excitation studies the state at 1.026 MeV is known to decay entirely to the ground state. Unfortunately, limitations in energy resolution and the presence of a background contamination problem at small angles prevented us from confirming the presence of the $\frac{1}{2}^+$ level.

The state seen by $l=4$ at 1.194 MeV in the stripping experiments is assumed to have $J^\pi = \frac{7}{2}^+$. A $J^\pi = \frac{9}{2}^+$ assignment is rejected because of the measured large spectroscopic factor, and because a $J^\pi = \frac{7}{2}^+$ state is also seen at a similar excitation energy in ^{115}In. The predicted spectroscopic factor is again found to be 67% larger than the measured one.

Our h.c. calculations predict $\frac{1}{2}^+$ and $\frac{3}{2}^+$ levels in

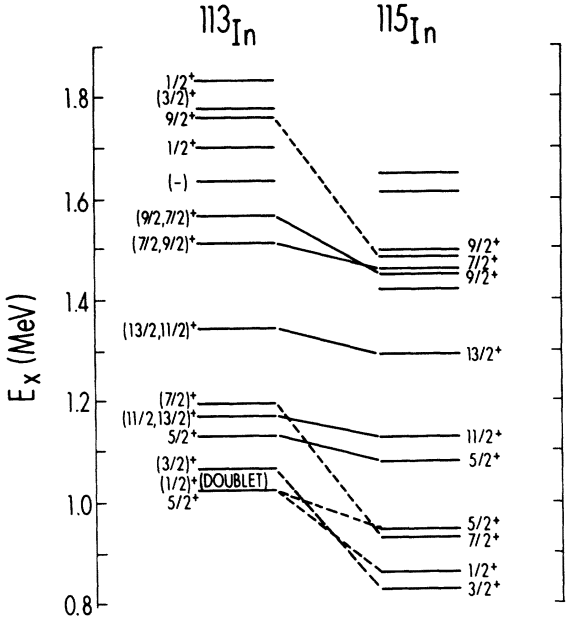

FIG. 7. Comparison of the ^{113}In and ^{115}In level schemes.

the neighborhood of 2.0 MeV (Sec. V C). Several states seen at about the right energy could be candidates for these assignments, but none was seen in the (p,t) experiment and all were seen with at least moderate strength in the stripping experiments. Such levels are neither predicted by Sen's calculations nor yet identified in the corresponding parts of the ^{115}In and ^{117}In spectra.

Rotational bands have tentatively been identified[8,15] in ^{117}In and ^{115}In. In both cases use of similar inertial and decoupling parameters led to fairly satisfactory predictions of positions for the low-lying $J^\pi = \frac{1}{2}^+ - \frac{9}{2}^+$ members. In ^{113}In, however, the levels which one might associate with such a band have a very different ordering (see Fig. 7) for which only the positions of the $J^\pi = \frac{1}{2}^+ - \frac{7}{2}^+$ members can be reproduced and then only by a very different set of parameters. This change in level structure and known changes in single-particle level positions suggest that one may be seeing a crossing of two $K = \frac{1}{2}$ bands; one being the [431] band built on the $g_{7/2}$ orbit and the other, the [420] band built on the $d_{5/2}$ orbit.

No $J^\pi = \frac{11}{2}^+$ member has been found in any of the isotopes although the rotational model predicts it should lie at about 1.5 MeV. In contrast, the p.c. model predicts the first $\frac{11}{2}^+$ state at about 2.5 MeV and reasonable positions for the other levels in question.

Since the energy level predictions of the rotational and p.c. models differ so little (except for the first $\frac{11}{2}^+$ state), it is interesting to compare their other predictions. Electromagnetic $E2$ matrix elements were calculated in both pictures. In the p.c. calculations core $E2$ matrix elements were taken to be the same as those used to evaluate the p.c. Hamiltonian (Table V) and the proton was assumed to have unit charge.

If one assumes that each p.c. state is a member of a $K = \frac{1}{2}$ rotational band one may calculate the implied intrinsic quadrupole moment Q_0. The values of Q_0, listed in Table VIII, are very nearly equal, in agreement with the concept of a common intrinsic state. In Table IX we display the $B(E2)$ values between these p.c. states and compare them with the simple rotational-model prediction

$$B(E2: J_i K \to J_f K) = \frac{5e^2}{16\pi} Q_0^2 (2J_f + 1) \begin{pmatrix} J_i & 2 & J_f \\ K & 0 & -K \end{pmatrix}^2,$$

where the value of Q_0 was taken from the average of the values listed in Table VIII. Again we observe a striking correspondence between the p.c. and rotational-model predictions suggesting a strong similarity in the two types of excitations.

TABLE VIII. Implied intrinsic quadrupole moments of the lowest p.c. states.

J	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$	$\frac{11}{2}$
$-Q_0$ (e fm^2)	228	257	246	227	235

VI. SUMMARY

Two-nucleon pickup and single-proton stripping experiments leading to ^{113}In have been made. Two $L=0$, six $L=2$, and seven $L=3$ angular distributions were observed in the (p,t) reaction experi-

TABLE IX. Comparison of p.c. and rotational $B(E2)$ predictions.

J_i	J_f	$B(E2)$ p.c. (e^2 fm^4)	$B(E2)$ rel. p.c./rotor
$\frac{11}{2}$	$\frac{9}{2}$	32	0.485
$\frac{11}{2}$	$\frac{7}{2}$	1523	0.884
$\frac{9}{2}$	$\frac{7}{2}$	31	0.315
$\frac{9}{2}$	$\frac{5}{2}$	1605	0.988
$\frac{7}{2}$	$\frac{5}{2}$	186	1.144
$\frac{7}{2}$	$\frac{3}{2}$	1093	1.121
$\frac{5}{2}$	$\frac{3}{2}$	265	0.816
$\frac{5}{2}$	$\frac{1}{2}$	1043	0.917
$\frac{3}{2}$	$\frac{1}{2}$	1147	1.001

ment. Stripping angular distributions for $l=0$, 1, 2, 4, and possibly 5 were found in the (^{3}He, d) and (α, t) experiments. In general the two types of reactions tended to populate different states. Each type revealed several new states.

DWBA calculations were carried out for all three reactions. Good fits were found for the $L=0$ (p, t) angular distributions. Poorer fits were found for $L=2$ and 3. Good fits were obtained for all (^{3}He, d) and (α, t) angular distributions, although the spectroscopic factors obtained in these two reactions disagreed somewhat. In the analysis reliance was placed on the (^{3}He, d) results.

Spins and parities of a number of states were assigned on the basis of the stripping and pickup information, with additional clues provided by the known level structures of ^{117}In and ^{115}In and from the results of the model calculations. Both h.c. and p.c. model calculations were carried out. Results were found to be consistent with the observed distribution of states. Predicted stripping spectroscopic factors were in qualitative agreement with experiment.

It was concluded that two distinct sets of states exist, one set arising predominantly from coupling a proton hole to the low-lying states of ^{114}Sn, the other arising from coupling a proton to the low-lying states of ^{112}Cd. Within each set, especially the latter, there appears to be considerable configuration mixing. Between the sets there seems to be little mixing.

Note added in proof: Evidence from ^{113}Cd(p, n) and ($p, n\gamma$) experiments has recently appeared, supporting our belief that a $J^\pi = \frac{1}{2}^+$ level lies just above the 1.026-MeV $J^\pi = \frac{5}{2}^+$ level; also, the presence of a level at 1.064 MeV is confirmed.[18]

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