Comparison of sub-Coulomb (d, p) stripping to analog resonance results for the $N = 82$ isotones*

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Reduced normalizations have been extracted from proton elastic scattering data via isobaric-analog resonance for analogs of low-lying parent states in ^{139}Ba, ^{141}Ce, ^{143}Nd, and ^{145}Sm, by the use of three different analog resonance theories. These reduced normalizations were compared to those obtained from sub-Coulomb (d,p) stripping to the low-lying parent states of the above nuclei. This comparison shows that the R-matrix theory gives the best agreement to the (d,p) results.

[NUCLEAR REACTIONS ^{138}Ba, ^{140}Ce, ^{142}Nd, ^{144}Sm (d,p) comparison to (p,p_0) results. Calculated Λ for (p,p_0) IAR results.]

I. INTRODUCTION

In the study of sub-Coulomb (d, p) stripping reactions a quantity, known as the reduced normalization Λ_{lj}, has been shown to be insensitive to the optical-model parameters used in the distorted-wave Born-approximation (DWBA) analysis.[1,2] This same quantity can be extracted from (p, p_0) isobaric analog resonance (IAR) data by the use of three different IAR theories.[3] By comparison of sub-Coulomb (d, p) stripping and the (p, p_0) IAR results one may be able to choose among the three IAR theories. In the past this comparison has been attempted by comparing spectroscopic factors S_{lj} derived from (p, p_0) scattering via IAR with those found from (d, p) stripping to the low-lying parent states.

In the DWBA analysis of the (d, p) data, S_{lj} is strongly dependent on the optical-model parameters used. In many cases, the analysis of the (d, d) elastic scattering data leads to several equally good families of parameters, which when applied to the (d, p) reaction yield spectroscopic factors which may differ by as much as 50% (e.g. Ref. 4). The dependence of S_{lj} on the deuteron and proton potential parameters can be strongly reduced by performing the (d, p) experiments at energies in which both entrance and exit channels are below the Coulomb barrier. However, the dependence on the bound-state neutron potential parameters does not decrease appreciably below the Coulomb barrier. Thus, S_{lj} still cannot be determined uniquely. However, the reduced normalization, which is closely related to S_{lj}, can be determined uniquely for both the sub-Coulomb (d, p) stripping reactions and the (p, p_0) IAR experiments.

Λ_{lj} is essentially the square of the ratio of the transferred neutron's asymptotic wave function to a spherical Hankel function, and is related to S_{lj} by

$$k^3\Lambda_{lj} = N_{lj}^2 S_{lj},$$

with

$$k = (2\mu|B_n|)^{1/2}/\hbar ,$$

where N_{lj} is the ratio of the DWBA neutron bound-state wave function to a spherical Hankel function evaluated outside the nuclear radius, μ is the reduced mass in the exit channel, and B_n is the binding energy of the last neutron. Rapaport and Kerman[1] have shown that Λ_{lj} is nearly independent of the geometrical parameters used to describe the neutron bound-state potential for sub-Coulomb (d, p) stripping. This has also been shown by Kent, Morgan, and Seyler[5] and Norton *et al.*[2]

Clarkson, Von Brentano, and Harney[3] have extracted reduced normalizations from the (p, p_0) reactions in the context of three IAR theories: the R-matrix approach of Thompson, Adams, and Robson[6] (TAR) and two shell-model methods, that of Mekjian and McDonald[7] (MM) and that of Zaidi and Darmodjo[8] and Harney[9] (ZDH). These theories have been used to yield the single-particle proton width Γ_p^{sp} of the analog state. Clarkson *et al.* have defined a term called the reduced single-particle proton width G_p, which is related to Γ_p^{sp} by

$$\Gamma_p^{sp} = \frac{N_{lj}^2}{k^3} G_p .$$

For (p, p_0) reactions, the spectroscopic factor is given by

$$S_{lj} = \Gamma_p/\Gamma_p^{sp} .$$

Then, the experimentally measured proton partial width Γ_p is related to the reduced normalization by $\Gamma_p = \Lambda_{lj} G_p$. Thus, to determine the value of Λ_{lj} from (p, p_0) IAR experiments, G_p is calculated from one of the three theories and compared to the experimental value of Γ_p. However, the value of G_p is strongly dependent on the IAR theory used, and therefore the value of the reduced normalization, which can be determined uniquely from sub-Coulomb (d, p) to the parent state, is dependent on the analog resonance theory used in its extraction from (p, p_0) scattering data.

Harney and Weidenmüller[10] (HW) have compared these theories on a theoretical level and have shown that there are fundamental differences which can produce spectroscopic factors that may differ by as much as 50%. It is now possible to compare these theories on an experimental level by use of the reduced normalization.

In this paper reduced normalizations are calculated for each of the three theories mentioned above, using analog resonance parameters found in the available literature for the $N=82$ isotones; ^{138}Ba, ^{140}Ce, ^{142}Nd, and ^{144}Sm. These reduced normalizations are compared to those reported by Rapaport and Kerman[1] for sub-Coulomb (d, p) stripping to parent states in ^{139}Ba, and by Norton *et al.*[2] for sub-Coulomb stripping to parent states in ^{141}Ce, ^{143}Nd, and ^{145}Sm. This comparison is a continuation of that started by Morgan, Seyler, and Kent[11] near the $N=50$ region. The optical-model parameter dependence of Λ_{lj} from the various IAR theories is also discussed. The preliminary results of the comparisons, for both the $N=50$ and the $N=82$ region, have been previously reported.[12]

II. OPTICAL-MODEL PARAMETERS FOR (p,p_0) IAR IAR ANALYSIS

Reduced normalizations were calculated from each of the three theories by use of code BETTINA[13] with the optical-model parameters taken from Wiedner *et al.*[14] The proton partial widths were obtained from Williams *et al.*[15] for ^{138}Ba, Marquardt *et al.*[16] for ^{140}Ce, Grosse *et al.*[17] for ^{142}Nd, and Fiarman *et al.*[18] for ^{144}Sm, and are shown in Table I.

The optical-model potential used in code BETTINA is

$$U(r) = V(1+e^{x})^{-1} + V_c(r, r_{0c}) + 4iW\frac{e^{x'}}{(1+e^{x'})^2} + \alpha(l)V_{so}\left[\frac{-1.998}{r}\frac{d}{dr}(1+e^{x})^{-1}\right],$$

where

$$x = \frac{r - r_0 A^{1/3}}{a}, \quad x' = \frac{r - r_0' A^{1/3}}{a}$$

and

$$\alpha(l) = \begin{cases} l \text{ if } j = l + \frac{1}{2}, \\ -l-1 \text{ if } j = l - \frac{1}{2}. \end{cases}$$

TABLE I. Final results.

Parent	E_x	l_j	Γ_p (keV)	(p,p_0) IAR Λ_{MM}	Λ_{TAR}	Λ_{ZDH}	Λ_{dp} [a]
^{139}Ba	0.00	$f_{7/2}$	17.2	26	21	15	19.6
	0.63	$p_{3/2}$	26.0	78	88	42	107
	1.08	$p_{1/2}$	22.5	58	58	28	66
	1.42	$f_{5/2}$	9.5	1.41	1.20	0.75	···
^{141}Ce	0.00	$f_{7/2}$	12.3	50	39	26	33
	0.67	$p_{3/2}$	21.8	149	145	86	168
	1.14	$p_{1/2}$	20.4	124	103	51	104
	1.50	$f_{5/2}$	7.3	3.4	2.6	1.6	2.6
	1.78	$f_{5/2}$	(7.2)	3.2	2.3	1.5	2.1
	2.41	$(p_{1/2})$	···	···	···	···	36
		$(p_{3/2})$	···	···	···	···	19
^{143}Nd	0.00	$f_{7/2}$	10.5	111	76	52	49
	0.74	$p_{3/2}$	23.5	344	272	139	250
	1.31	$p_{1/2}$	22.7	365	210	115	165
	1.56	$f_{5/2}$	6.0	6.26	4.32	2.63	5.08
	1.92	$f_{5/2}$	···	···	···	···	1.68
^{145}Sm	0.00	$f_{7/2}$	8	212	123	84	77
	0.89	$p_{3/2}$	27	944	517	282	442
	1.61	$p_{1/2}$	30	789	350	196	367

[a] Taken from Norton *et al.* (Ref. 2) and from Rapaport and Kerman (Ref. 1) for ^{139}Ba.

The factor of 1.998 is the square of the Compton wavelength of the π meson in femtometers and V_c is the Coulomb potential of a uniformly charged sphere of radius $r_{0c}A^{1/3}$.

In order to calculate Γ_p^{sp}, code BETTINA assumes that the last neutron in the parent nucleus is in a single-particle state. Therefore, since the low-lying states of the $N=83$ isotones are not pure single-particle states, the binding energy required to determine V_n is not the binding energy of the last neutron, but a single-particle binding energy, E_{Bn}, given by

$$E_{Bn} = Q_{g.s.}(d,p) - \epsilon_{sp} + 2.224 \text{ MeV}$$

and

$$\epsilon_{sp} = \frac{\sum S_{lj} E_{lj}}{\sum S_{lj}},$$

where the sum is over states of the same spin and parity, and E_{lj} is the excitation energy of the state whose spectroscopic factor is S_{lj}. Estimates of ϵ_{sp} are available in the literature[19–22] from (d,p) work done above the Coulomb barrier.

As mentioned earlier, the reduced normalizations obtained from sub-Coulomb (d,p) stripping reactions are nearly independent of the optical-model parameters used to describe the neutron bound-state well, as opposed to the strong dependence exhibited by the spectroscopic factors obtained from the sub-Coulomb (d,p) reactions. For the proton elastic scattering data, Λ_{lj} is again more insensitive to the neutron parameter than is S_{lj}, as shown in Fig. 1. Here, as in parameter variations to follow, only one parameter was varied; in this case the neutron bound-state well radius, while all others were held constant.

Of the three theories, the R-matrix approach of TAR is the least dependent on the neutron radius for S_{lj}. For this theory, S_{lj} decreases by 80%, while a 90% decrease is noted in the MM and ZDH theories. However, for Λ_{lj} the TAR theory exhibits the greatest neutron radius dependence, since Λ_{lj} increases almost 50%, while the MM value increases by 20% and Λ_{lj} from ZDH decreases by 13%.

In the proton channel the potential parameters were varied individually to see how the various theories differed as to parameter dependence. Figure 2 shows the dependence of S_{lj} and Λ_{lj} on

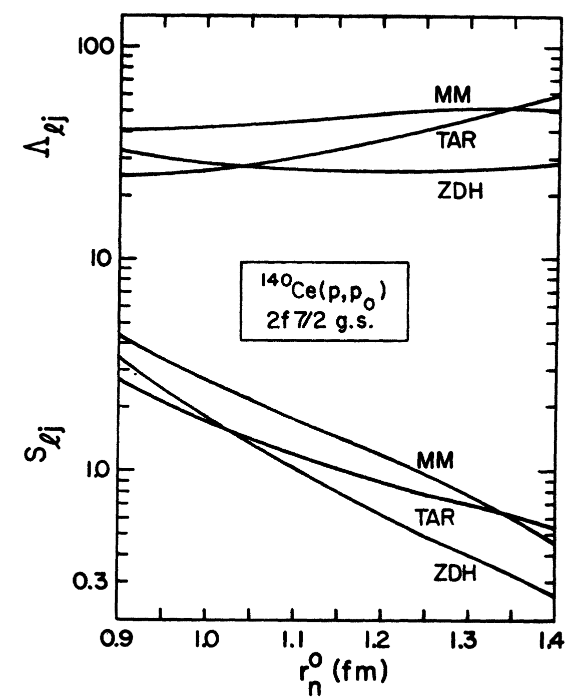

FIG. 1. Results of the variation of the neutron well radius for the $^{140}Ce(p,p_0)$ reaction. r_n^0 was varied from 0.9 to 1.4 fm while all other parameters were held constant. Although each theory exhibits a different r_n^0 dependence, in every case, Λ_{lj} is less dependent on r_n^0 than is S_{lj}.

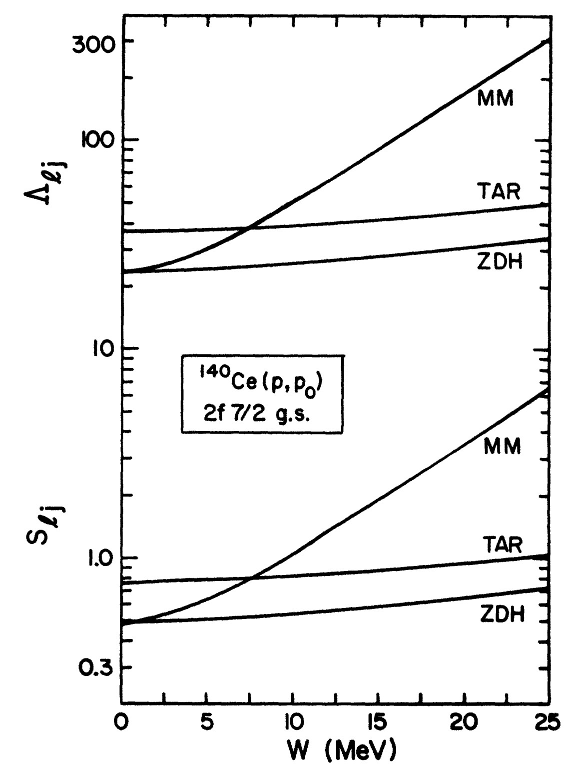

FIG. 2. Results of the variation of the absorption potential depth for the $^{140}Ce(p,p_0)$ reaction. W was varied from 0 to 25 MeV in code BETTINA, while all other parameters were held constant.

the surface absorption potential W which was varied from 0 to 25 MeV. At $W=0$, MM and ZDH give identical values for S_{lj} and Λ_{lj} with TAR and ZDH theories showing identical W dependence, which is due to the fact that TAR and ZDH have nearly the same expressions for the resonance mixing phase ϕ_c, as noted in Harney and Weidenmüller.[10] In the variation of the real proton potential (Fig. 3), strong peaking was noted in the values of S_{lj} and Λ_{lj} at $V_p \sim 50$ MeV for the MM theory only. An identical curve is obtained when the real potential radius r_{0p} is varied, with a peak occurring at $r_{0p} = 1.15$ fm for the MM theory. Also the spin-orbit potential depth V_{so} was varied from 0 to 10 MeV, which produced changes of 5 to 10% in S_{lj} and Λ_{lj} for all three theories, with the MM theory showing the greatest dependence.

These parameter variations illustrate some of the substantive differences that exist among the three theories. These differences are explored in detail by Harney and Weidenmüller.[10]

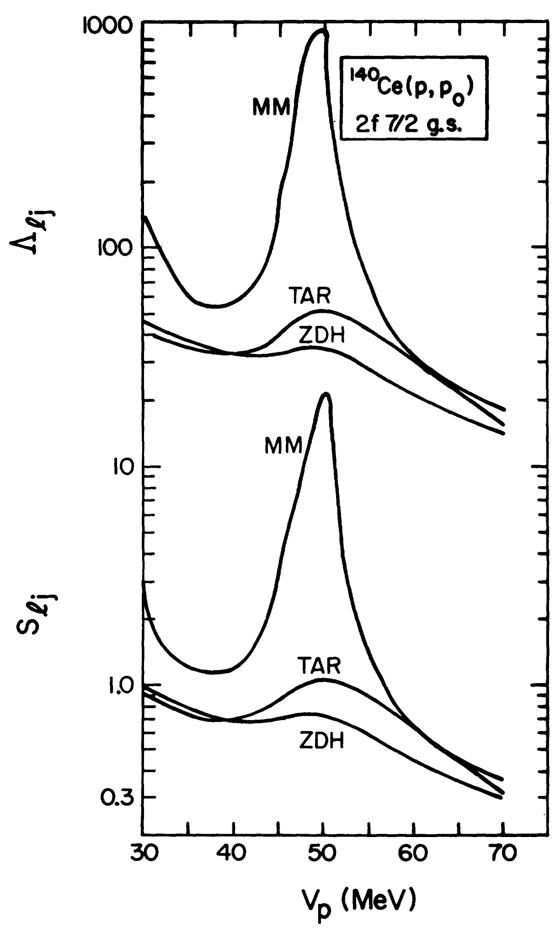

FIG. 3. Results of the variation of the real proton potential depth for the ^{140}Ce(p,p_0) reaction. V_p was varied from 30 to 70 MeV in code BETTINA, while all other parameters were held constant.

III. COMPARISON OF Λ_{lj} FROM IAR AND (d,p) RESULTS

In obtaining Λ_{lj} from the analog resonance data, the largest uncertainties are from the proton partial widths and the center-of-gravity excitation energy ϵ_{sp}. Assuming that ϵ_{sp} is known to within 150 keV, an average uncertainty of ±10% can be assigned to Λ_{lj} due to this parameter. This uncertainty, combined with the 6 to 12% experimental errors associated with Γ_p, leads to an estimate of ±20% error in Λ_{lj}. The values of the reduced normalizations are listed in Table I, along with those obtained from sub-Coulomb (d,p) stripping reactions reported by Norton *et al.*[2] and Rapaport and Kerman.[1] These reduced normalizations also have uncertainties of ±20%.

The reduced normalizations with their associated uncertainties versus excitation energy are shown in Fig. 4. The cross-hatched area represents Λ_{lj} obtained from the (d,p) analysis. Note that in each case the error bars of at least one of the analog resonance theories overlap with the (d,p) results. From this figure and Table I it can be seen that the R-matrix approach of TAR comes closest to the (d,p) reduced normalization a total of 11 times, with MM and TAR producing the same values of Λ_{lj} for the $p_{1/2}$ state of ^{139}Ba.

In order to determine which of the analog resonance theories has the best agreement to the (d,p) reduced normalizations, a "goodness-of-fit" parameter[11] I is defined similarly to χ^2. This parameter is given by

$$I=\sum_{\text{states}} \frac{(\Lambda_{dp}-\Lambda_{pp})^2}{(\Delta\Lambda_{dp})^2+(\Delta\Lambda_{pp})^2},$$

where the sum is over states of the same spin and parity and Δ is equal to 0.2, representing the 20% uncertainty in Λ_{dp} and Λ_{pp}. Note that if I is equal to 1 for a given state the difference between Λ_{dp} and Λ_{pp} is 1.4Δ, since there are two equal terms

TABLE II. Goodness-of-fit parameter for the $N=82$ isotones.

State	No. of states	I MM	I TAR	I ZDH	Isotones included
$2f_{7/2}$	4	18.6	5.3	1.7	Ba, Ce, Nd, Sm
$3p_{3/2}$	4	8.4	1.1	18.8	Ba, Ce, Nd, Sm
$3p_{1/2}$	4	12.7	0.9	18.2	Ba, Ce, Nd, Sm
$2f_{5/2}$	3	3.5	0.4	8.7	Ce, Nd
Total	15	43.1	7.8	47.5	
Total/States		2.9	0.5	3.2	

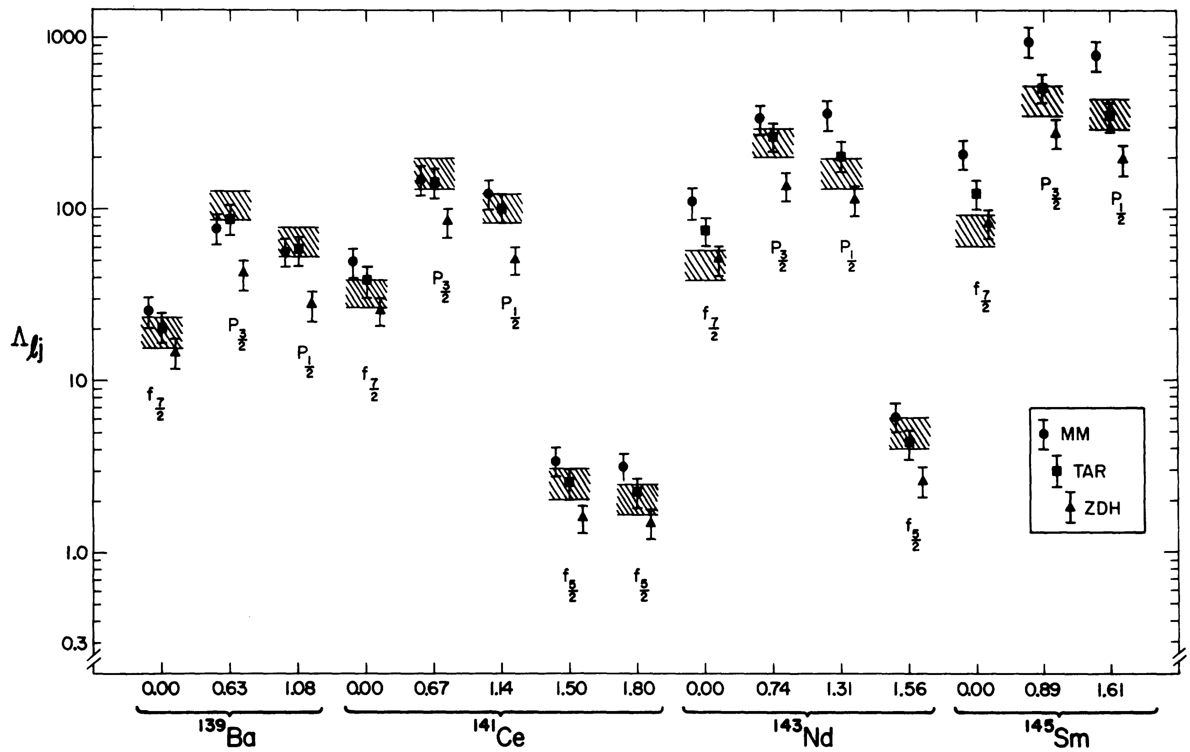

FIG. 4. The reduced normalization versus excitation energy for states in the $N=83$ isotones. The cross-hatch marks denote the value of the reduced normalizations obtained from the sub-Coulomb (d,p) stripping reactions.

in the error denominator. In the usual case of only one error term a χ^2 of 1 implies that the quantities agree to within one Δ. In this case, however, I must be less than 0.707 for the Λ's to agree within one Δ.

Table II shows the comparison of the three theories to the experimental (d, p) results. For each state except the $2f_{7/2}$ state, TAR does give the best agreement for the ^{139}Ba and ^{141}Ce isotones; only in ^{143}Nd and ^{145}Sm does the ZDH approach come closer to Λ_{dp}. It should be noted that the $2f_{7/2}$ ground states of ^{143}Nd and ^{145}Sm are the states closest to the top of the Coulomb barrier, within 8% and 5% respectively, for the proton channel, thereby causing the reduced normalizations for these states to have a greater dependence on the outgoing channel's potential parameters. Also, the TAR approach yields the lowest total value of I, and is the only theory that produces an average value for I per state that is less than 0.707.

IV. CONCLUSION

The sub-Coulomb (d, p) stripping comparisons to the proton elastic analog resonance reactions for the $N=82$ isotones indicate rather strongly that the theory of Thompson, Adams, and Robson gives the best agreement to the experimental results in this mass region. This supports the work done by Morgan, Seyler, and Kent *et al.*[11] near the $N=50$ region. The reason why the TAR R-matrix approach yields the best over-all agreement to the sub-Coulomb (d, p) stripping results is not clear. In their extensive comparison of these three theories, HW pointed out that true substantive differences exist among the theories. In particular both of the shell-model approaches utilize statistical assumptions in the construction of the analog states. These assumptions ignore second-order effects in the imaginary optical potential W, an assumption which HW show is violated even for very small values of W. Also, HW show that application of an R-matrix theory to analog resonances appears to violate the R-matrix assumptions of no internal mixing and no external polarizing potential. It may be that the better results obtained with the TAR method is an indication of which of these violations has a stronger effect on the calculation. It should be pointed out that the R-matrix approach does have an adjustable parameter, the channel radius, outside of which there exists no nuclear potential for the channel in question. In every case this parameter was set at the first maximum in the single-particle proton width outside the nuclear radius. Since this was done automatically by the code BETTINA for every state in this comparison, it is not clear that this extra parameter significantly influenced the results.

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