High-resolution study of the $1/2^+$ analog state in ^{93}Tc†

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The ^{92}Mo(p,p) excitation function was measured with high energy resolution over the $1/2^+$ analog state at 5.3 MeV. A total of 125 s-wave resonances were resolved and analyzed. Results of the analysis of the analog-state fine structure are presented. The average level spacing and the parent-state spectroscopic factor are calculated.

[NUCLEAR REACTIONS ^{92}Mo(p,p), $E=5.13$–5.43 MeV; measured $\sigma(E,\Theta)$. ^{93}Tc deduced resonances J, π, l, Γ isobaric-analog resonance. Fine-structure distribution, resonance spacing, spectroscopic factor.]

I. INTRODUCTION

Until recently, the lack of a high-resolution proton beam of sufficient energy and intensity has limited analog-state fine-structure measurements[1] to nuclei with $A \lesssim 65$. Cross-section fluctuations in the ^{92}Mo(p,p) excitation function at 5.3 MeV were observed by Richard, Fox, Moore, and Robson,[2] and attributed by these authors to fine-structure resonances enhanced by the analog of the $\frac{1}{2}^+$ first excited state of ^{93}Mo. Only a few of the strongest resonances were actually resolved. In this paper we present results of a measurement of the fine structure of this analog. We have resolved 125 $\frac{1}{2}^+$ resonances in a 300-keV energy interval encompassing the analog state. The large number of resonances allows an unusually detailed analysis of the fine-structure distribution. In a similar experiment Meyer[3] has recently measured the ^{92}Mo(p,p) excitation function over this analog, but no analysis of this data has been presented.

II. EXPERIMENTAL RESULTS AND ANALYSIS

The high energy resolution (500 eV at 5.3 MeV) necessary to resolve individual resonances was achieved using the neutral-beam technique on the Triangle Universities Nuclear Laboratory FN tandem Van de Graaff accelerator. The method has been described elsewhere,[4] and will not be discussed here. Targets were prepared by evaporating MoO_3 (>98% ^{92}Mo) onto thin (~10–μg/cm^2) carbon foils. The thickness of ^{92}Mo was ~2 μg/cm^2. Scattered protons were detected in surface-barrier detectors mounted at laboratory angles of 165, 125, and 90°. The inelastic scattering yield was negligibly small relative to the elastic scattering yield. A portion of the data of this experiment is shown in Fig. 1. These data, taken in one continuous run, demonstrate the feasibility of measuring extended excitation functions with the neutral-beam technique.

Resonance energies, spins, parities, and widths were extracted by an R-matrix analysis of the elastic scattering cross sections. Since all neutron channels are closed and inelastic scattering is negligible, only the elastic channel was considered in the analysis. The R matrix was constructed from up to 75 levels, always including all strong levels and all resonances near a given energy. A total of 125 resonances were fitted, all with $J^\pi = \frac{1}{2}^+$. Very weak resonances ($\Gamma_p \lesssim 10$ eV) could not be observed because of finite-energy resolution. The solid line in Fig. 1 represents the fit to these data. A Gaussian resolution function with 500 eV full width at half maximum was used in obtaining this fit. Reduced widths were calculated using Coulomb penetrabilities evaluated at the matching radius $1.25(A^{1/3}+1)$ fm. The resonance parameters are listed in Table I.

III. FINE-STRUCTURE DISTRIBUTION

The reduced widths are plotted in Fig. 2(a). Several features of the fine-structure pattern can be noted immediately. The over-all line shape demonstrates the asymmetric giant-resonance pattern expected for analog states,[5] with the lower energy widths much larger, on the average, than those on the high-energy side. Large fluctuations in the reduced widths are apparent throughout the distribution; weak levels are present even near the center. A detailed study of these fluctuations[6,7] is beyond the scope of this paper. Here we con-

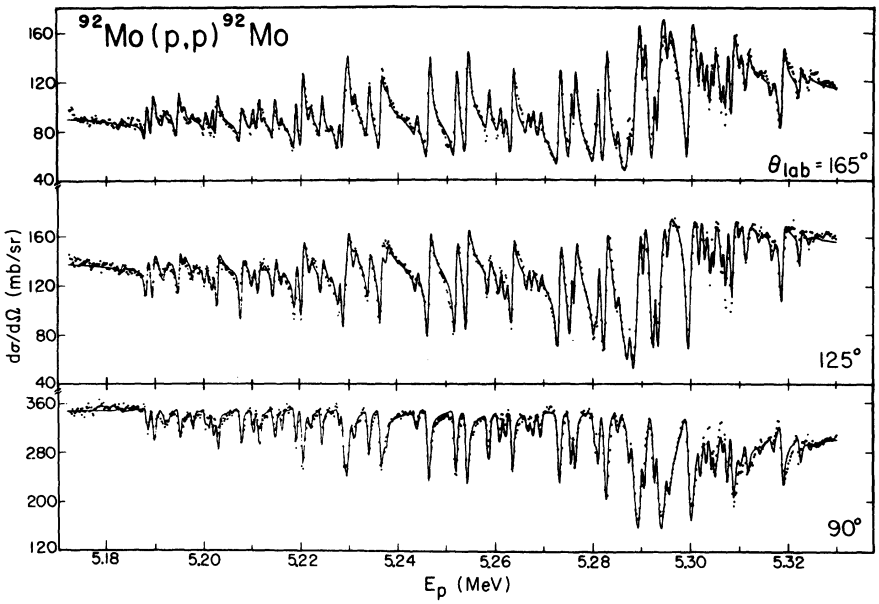

FIG. 1. Elastic scattering data in the central part of the region strongly enhanced by the analog state. The solid line is a 75-level fit to the data.

fine our attention to an analysis of the line shape.

An extensive theoretical discussion of the fine structure of analog states has been given by Lane[8]; we adopt his formalism in the following discussion. The reduced-width data were fitted to the strength function

$$s(E) = s_0 \frac{(E-E_A+\Delta)^2+\frac{1}{4}\omega^2}{(E-E_A)^2+\frac{1}{4}W_0^2}\,, \tag{1}$$

where W_0 is the spreading width, s_0 is the background strength function, and E_A is the analog-state energy. The degree of asymmetry is determined by ω and Δ through the asymmetry parameter.

$$R \equiv \frac{s(E_A+\Delta)}{s(E_A-\Delta)} = \frac{\frac{1}{4}\omega^2+4\Delta^2}{\frac{1}{4}\omega^2}\,. \tag{2}$$

The quantities of Eq. (1) are related through

$$\tfrac{1}{4}\omega^2 = \tfrac{1}{4}W_0^2 - \Delta^2 + \frac{W_0\gamma_A^2}{2\pi s_0}\,, \tag{3}$$

where γ_A^2 is the reduced width of the analog state after mixing with the background. In general s_0, Δ, ω, and γ_A^2 are channel-dependent. We omit the channel subscript here since only the elastic channel is discussed.

The strength function $s(E)$ of Eq. (1) can be interpreted as a conventional strength function $s(E) = \langle\gamma^2\rangle/D$ only in the case of strong mixing ($W_0 \gg D$, where D is the average level spacing). If the mixing is weak or intermediate ($W_0 \lesssim D$), Eq. (1) is still expected to be valid if $s(E)$ is interpreted as an ensemble strength function.[9] That is, $s(E)$ gives the expectation value of the reduced width at energy E for the ensemble of analogs defined by fixing D, W_0, Δ, γ_A^2, and s_0.

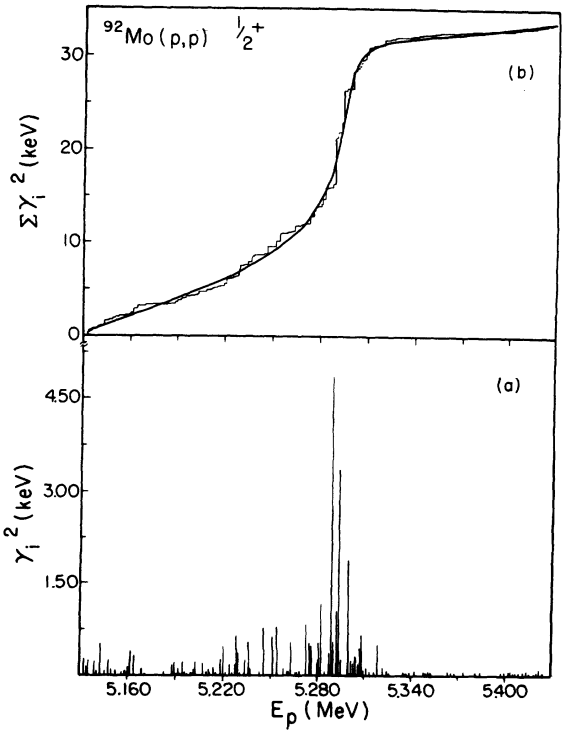

FIG. 2. (a) The fine-structure reduced widths. (b) Cumulative reduced-width plot. The smooth curve is the fit to the data—see text for the parameters used in this fit.

TABLE I. ^{93}Tc resonance parameters. All of the levels have $J^\pi = \frac{1}{2}^+$. The uncertainty in the absolute energy is 10 keV. The resonance energies quoted are the exact R-matrix energies. For most of these, the uncertainty is ~100 eV. For strongly interacting levels, a change of as little as 50 eV can produce a noticeable change in the fit.

E (MeV)	Γ_p (eV)	γ_p^2 (eV)	E (MeV)	Γ_p (eV)	γ_p^2 (eV)	E (MeV)	Γ_p (eV)	γ_p^2 (eV)	E (MeV)	Γ_p (eV)	γ_p^2 (eV)
5.132 93	110 ± 11	279	5.207 68	90 ± 10	210	5.275 41	254 ± 25	548	5.342 60	40 ± 10	80
5.134 58	60 ± 10	152	5.210 11	30 ± 10	70	5.276 18	227 ± 23	490	5.348 60	30 ± 10	60
5.135 59	100 ± 10	253	5.211 39	50 ± 10	116	5.280 32	121 ± 12	260	5.349 40	15 ± 5	30
5.139 46	100 ± 10	252	5.212 23	20 ± 5	46	5.280 92	250 ± 25	536	5.350 50	19 ± 5	38
5.140 92	30 ± 10	75	5.214 60	60 ± 10	139	5.282 53	550 ± 55	1178	5.352 20	30 ± 10	60
5.143 29	210 ± 21	527	5.216 10	20 ± 5	46	5.284 98	31 ± 10	66	5.353 68	30 ± 10	60
5.146 04	30 ± 10	75	5.219 00	115 ± 12	264	5.287 47	170 ± 17	362	5.364 30	20 ± 5	39
5.148 46	103 ± 10	257	5.220 42	210 ± 21	482	5.288 85	2300 ± 230	4892	5.366 03	20 ± 5	39
5.150 12	43 ± 10	107	5.221 99	24 ± 10	55	5.290 30	265 ± 27	563	5.367 98	20 ± 5	39
5.152 94	43 ± 10	107	5.224 35	80 ± 10	183	5.292 63	502 ± 50	1063	5.374 10	19 ± 5	37
5.154 98	5 ± 3	12	5.226 19	15 ± 5	34	5.293 57	1600 ± 160	3386	5.381 80	20 ± 5	39
5.156 67	20 ± 5	49	5.228 00	78 ± 10	178	5.295 34	124 ± 12	262	5.384 89	25 ± 10	48
5.159 22	32 ± 10	79	5.228 97	287 ± 29	652	5.299 87	900 ± 90	1891	5.398 19	30 ± 10	57
5.160 25	63 ± 10	155	5.229 48	163 ± 16	370	5.301 89	120 ± 12	252	5.400 13	30 ± 10	57
5.161 88	165 ± 17	405	5.230 98	25 ± 10	57	5.303 09	100 ± 20	209	5.403 13	30 ± 10	56
5.164 41	145 ± 15	355	5.233 56	10 ± 5	23	5.304 16	150 ± 20	314	5.403 97	31 ± 10	58
5.169 37	51 ± 10	124	5.233 97	114 ± 11	258	5.304 89	150 ± 20	314	5.405 67	19 ± 5	36
5.171 47	10 ± 5	24	5.236 52	242 ± 24	546	5.306 59	95 ± 15	198	5.407 03	12 ± 5	23
5.183 10	20 ± 5	48	5.237 10	50 ± 10	113	5.307 39	220 ± 22	459	5.409 62	15 ± 5	28
5.188 29	70 ± 10	167	5.237 59	50 ± 10	113	5.308 66	310 ± 31	645	5.412 49	25 ± 10	47
5.189 61	90 ± 10	214	5.243 74	31 ± 10	69	5.310 11	40 ± 10	83	5.414 19	40 ± 10	74
5.189 90	10 ± 5	24	5.246 28	352 ± 35	785	5.311 63	65 ± 10	135	5.416 39	20 ± 5	37
5.191 40	10 ± 5	24	5.251 81	292 ± 29	647	5.314 19	10 ± 5	21	5.418 85	20 ± 5	37
5.191 89	40 ± 10	95	5.254 12	363 ± 36	802	5.316 89	25 ± 10	52	5.420 59	35 ± 10	65
5.192 49	30 ± 10	71	5.258 51	65 ± 10	143	5.318 85	251 ± 25	517	5.422 29	34 ± 10	63
5.193 27	10 ± 5	24	5.260 81	35 ± 10	77	5.322 45	72 ± 10	148	5.424 15	29 ± 10	53
5.194 90	96 ± 10	227	5.262 01	30 ± 10	66	5.324 43	10 ± 5	20	5.429 79	30 ± 10	55
5.195 84	20 ± 5	47	5.263 41	256 ± 26	560	5.325 20	40 ± 10	82			
5.197 70	10 ± 5	24	5.266 54	32 ± 10	70	5.326 10	30 ± 10	61			
5.200 35	25 ± 10	59	5.267 59	29 ± 10	63	5.335 52	30 ± 10	61			
5.200 99	10 ± 5	23	5.269 18	32 ± 10	70	5.336 54	17 ± 5	34			
5.201 90	38 ± 10	89	5.273 03	392 ± 39	848	5.337 91	20 ± 5	40			
5.202 74	100 ± 10	234				5.339 66	30 ± 10	60			

The parameters s_0, E_A, Δ, W_0, and γ_A^2 were determined by fitting the partial integral of the strength function $\int_{E_0}^{E} s(E')dE'$ to the corresponding partial sum of reduced widths. The method is described by Lynn.[6] From Eqs. (1) and (3) the strength function can be written as

$$s(E) = s_0 + \frac{2s_0\Delta(E-E_A)}{(E-E_A)^2+\frac{1}{4}W_0^2} + \frac{1}{2\pi}\,\frac{W_0\gamma_A^2}{(E-E_A)^2+\frac{1}{4}W_0^2}\,. \tag{4}$$

From this expression it is clear that the wings of the distribution are relatively more sensitive to the values of s_0 and Δ. After a preliminary fit allowing all parameters to vary, the values of s_0 and Δ were determined by restricting the fits to data in the wings of the distribution. The remaining parameters (E_A, W_0, γ_A^2, and an integration constant) were then evaluated by fitting the entire distribution while holding s_0 and Δ fixed. The value of ω was calculated using Eq. (3).

The data and the best fit to the data are shown in Fig. 2(b). Details of the fitting procedure will be given in a future publication.[10]

The best-fit parameters with estimated uncertainties are: $s_0 = 0.04 \pm 0.01$, $\Delta = -30 \pm 5$ keV, $W_0 = 16.5 \pm 3$ keV, $\gamma_A^2 = 21.5 \pm 3$ keV, $E_A = 5.2954$ MeV, and $\omega = 48 \pm 5$ keV. The proton partial width of the analog state is $\Gamma_p = 2P\gamma_A^2 \simeq 10$ keV.

IV. DISCUSSION

Several checks on the validity of the fit are possible. The background strength function, the spreading width, and the analog-state reduced

width can be estimated directly from the data. Such estimates give values in agreement with those determined from the fitting procedure. For fixed Δ Eq. (2) provides an independent estimate of ω. Assuming $\Delta = -30$ keV, one obtains $50 \gtrsim \omega \gtrsim 40$, consistent with the best-fit value. The values of W_0, Δ, and Γ_p are in reasonable agreement with those found by Richard *et al.*[2]

The asymmetry in the fine-structure enhancement is an interference effect arising from a correlation between the reduced-width amplitudes of the background resonances and the interaction matrix element between these states and the analog.[11] This correlation is measured by[8]

$$r = \frac{2\pi s_0 \Delta^2}{W_0 \gamma_A^2}. \tag{5}$$

Using the best-fit values for these parameters yields $r = 0.64$, a strong correlation which reflects the obvious asymmetry in the data. Such pronounced asymmetry may not be common to all analogs in this mass region. Although Richard *et al.*[2] find a high degree of asymmetry for an analog in ^{91}Nb, poor-resolution polarized-beam experiments[12] on several different analogs (all with open neutron channels) have indicated that the distributions are in some cases nearly symmetric.

The present results allow a determination of the average s-wave level spacing D. The precision of this determination is limited by the uncertainty in the number of missed levels. The observed local level density is relatively constant from the low-energy side of the analog up through the center of the analog, but decreases on the high-energy side, indicating that a larger fraction of the resonances in this region are too weak to be observed. Based on the lower-energy data, the observed level spacing is $D \cong 2$ keV. Assuming that we see between 67 and 100% of the levels actually present in the 300-keV interval, $D = 2.0 \pm 0.4$ keV.

In principle, the parent-state spectroscopic factor should be well determined by the present measurement of the proton partial width of the analog state $\Gamma_p = 10$ keV. We have evaluated the spectroscopic factor using single-particle widths calculated by methods discussed by Harney and Weidenmüller.[13] Optical-model parameters were taken from Moorhead and Moyer.[14] We find for the spectroscopic factors $S^{\mathrm{TAR}} = 0.44$, and $S^{\mathrm{MM}} = S^{\mathrm{ZDH}} = 0.30$. (The superscripts refer to models used to calculate the single-particle widths.[13]) Both values are somewhat smaller than the (d,p) value,[14] $S_{dp} = 0.64$.

The reduced normalization, a quantity which is related to the spectroscopic factor but less dependent on optical-model parameters,[15] has recently been evaluated for a number of analog pairs in the mass-90 region.[16] The parent states were populated by sub-Coulomb stripping. For this $\frac{1}{2}^+$ analog in ^{93}Tc the (d,p) reduced normalization is approximately twice the analog-state value. Such a discrepancy could result from experimental error, but the possibility of this is slight. The proton partial width used in Ref. 16 was $\Gamma_p = 12$ keV. Our value ($\Gamma_p = 10$ keV) increases the discrepancy, and in fact even if the proton width of all of the resonances seen in this experiment (~ 15 keV) is attributed to the analog, the disagreement remains. The sub-Coulomb (d,p) result could be in error if the reaction populates a doublet, e.g., an $l = 4$ state along with the $l = 0$ state of interest.[17] Only the $l = 0$ state was found in Ref. 14, however, and it is unlikely that a doublet would have escaped detection with their energy resolution (4–7 keV). The disagreement between the spectroscopic factors and between the reduced normalizations as measured by analog-state experiments and by stripping experiments seems to indicate that these quantities are not reliably calculated with current theories.

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