$^{208}Pb(\alpha,\alpha)^{208}Pb$ scattering near the Coulomb barrier*

G. M. Hudson and R. H. Davis
Physics Department, The Florida State University, Tallahassee, Florida 32306

(Received 16 November 1973)

Differential cross sections for the elastic scattering of α particles by ^{208}Pb have been measured over the laboratory energy range 16.0 to 25.5 MeV in 0.5-MeV steps and over the laboratory angle range of 30 to 170° in 5° steps. A unique real potential of 35±3 MeV is found in an optical-model analysis with $r_0=1.22$ fm in $R=r_0(A_1^{1/3}+A_2^{1/3})$ and $a=0.57$ fm. Uniqueness is expected for scattering in the vicinity of the Coulomb barrier once size parameters are fixed. No discrete ambiguities are found over the energy range investigated consistent with a square-well R-matrix model, but they may exist at higher energies.

[NUCLEAR REACTIONS $^{208}Pb(\alpha,\alpha)$, $E=16$ to 25.5 MeV, $\Delta E=0.5$ MeV; measured $\sigma(\theta)$, $\theta=30$ to 170°; extracted optical model U and W.]

I. INTRODUCTION

Eck, LaSalle, and Robson[1] proposed scattering near the Coulomb barrier as a means of resolving discrete ambiguities in optical-model potential well depths and applied the technique to the scattering of ^{16}O. Mo and Davis[2] successfully used the procedure to choose one of several discrete ambiguities in the scattering of protons by ^{208}Pb. However, the technique proved to be inapplicable in the study of α particles by medium-weight nuclei by Watson *et al.*[3] simply because the discrete ambiguities did not exist. An R-matrix model with a square well explained the absence of the discrete ambiguities as a result of mixing of single α-particle states due to a large absorption.

The scattering of α particles by ^{208}Pb has been studied to first determine whether the various optical-model parameter sets[4-6] reported for the energy range here are the result of discrete or continuous ambiguities. Second, the measurement of the energy dependencies of the optical-model parameters, insofar as the trends may be extrapolated, indicate whether or not discrete ambiguities should be found outside the energy range of the measurements. Finally, the analysis provides a set of parameters for the geometry given, and the data is available for calculations with other geometries without resort to use of empirical continuous ambiguity relations.

Several properties of the target nucleus ^{208}Pb favor the existence of discrete ambiguities not far above the barrier. It is a $4n$, doubly magic nuclide with a relatively high first excited state (3^- state at 2.615 MeV). Further, the (α,n) reaction threshold is not too different from the Coulomb barrier. Thus the reaction cross section (the absorption) should be small enough to augur minimal mixing of α-particle states and yet large enough to reduce or eliminate the complication of compound elastic scattering.

II. EXPERIMENTAL METHOD

α-particle beams were produced by the Super FN tandem Van de Graaff accelerator using a duoplasmatron ion source. Cross sections were measured in a 46-cm reaction chamber[7] fitted with a ring carrying 16 surface barrier detectors spaced at 10° intervals in two opposing quadrants.

30 point angular distributions from 30 to 170° were measured at energies separated by 0.5 MeV from 14 to 25.5 MeV. Absolute cross sections were obtained by normalization to Rutherford scattering. The over-all energy resolution estimate is 300–500 keV full width at half maximum (FWHM). Errors in the c.m. differential cross section varied continuously from 5% at forward angles and low energies to 9% at the back angles and high energies.

III. ANALYSIS AND RESULTS

As shown in Figs. 1 and 2, the cross sections vary smoothly with energy and angle except for possible structure at large angles at the highest energies which is not statistically significant within the precision of this experiment. The optical-model code JIB-3[8] was used to analyze the data. Four parameters adequately described the data. The imaginary geometrical parameters were set equal to the real geometrical parameters $r_0=1.22$ fm and $a=0.57$ fm which are the same for the Coulomb part of the potential in agreement with electron scattering from a uniformly charged sphere.[9] The nuclear and Coulomb radii were defined: $R_N=R_C=r_0(A_t^{1/2}+A_p^{1/3})$.

The nuclear optical-model potential has the usual form

$$V_n(r) = Uf(r) + iWg(r)\,. \tag{1}$$

It was sufficient in this work to set $g(r) = f(r)$ where $f(r)$ is written

$$f(r) = \left[1 - \exp\left(\frac{r - R_N}{a}\right)\right]^{-1}, \tag{2}$$

the Woods-Saxon function. With the choice of $g(r)$, a volume absorption is selected. Many other variations of the optical model were tried such as a six-parameter model, the inclusion of compound elastic scattering cross sections, different form factors for the imaginary part of the potential, and different real geometrical parameters without producing a significant improvement in the fit. Nor did these extensions of the optical model yield a change in the parameters which might provide a better understanding of the mechanism.

The first step in the optical-model parametrization was the calculation of χ^2 contours over the U, W surface with U the real-well depth and W the imaginary-well depth in Eq. (1). The χ^2 function is given by the equation

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{\sigma_e(\theta_i) - \sigma_c(\theta_i)}{\Delta\sigma_e(\theta_i)}\right]^2, \tag{3}$$

where $\sigma_e(\theta_i)$ and $\sigma_c(\theta_i)$ are the measured and calculated cross sections at θ_i and $\Delta\sigma_e(\theta_i)$ is the error in the measurement. The well depth U was varied in steps of 5 MeV from 10 to 225 MeV and W was varied in steps of 2 MeV from 1 to 21 MeV. The values of U and W corresponding to the χ^2 minimum were used as starting values for searches on U and W. The resultant values of U and W are plotted in Fig. 3. The rise in W and the decrease in U below 21 MeV incident α-particle energy is not statistically significant as indicated by the error bars.

The error bars are determined by the $(\chi^2_{min} + 1)$ contours. Over most of the energy range, χ^2_{min} is one or less for the parameters of Fig. 3. It does

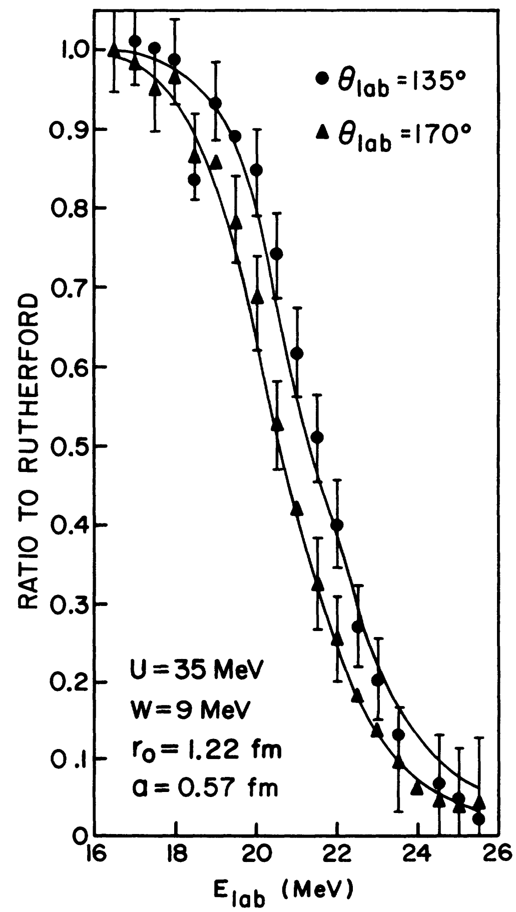

FIG. 1. Excitation curves for $^{208}Pb(\alpha, \alpha)^{208}Pb$ scattering at laboratory angles 135 and 170°. The solid curves are optical-model fits for $U = 35$ and $W = 9.0$ MeV.

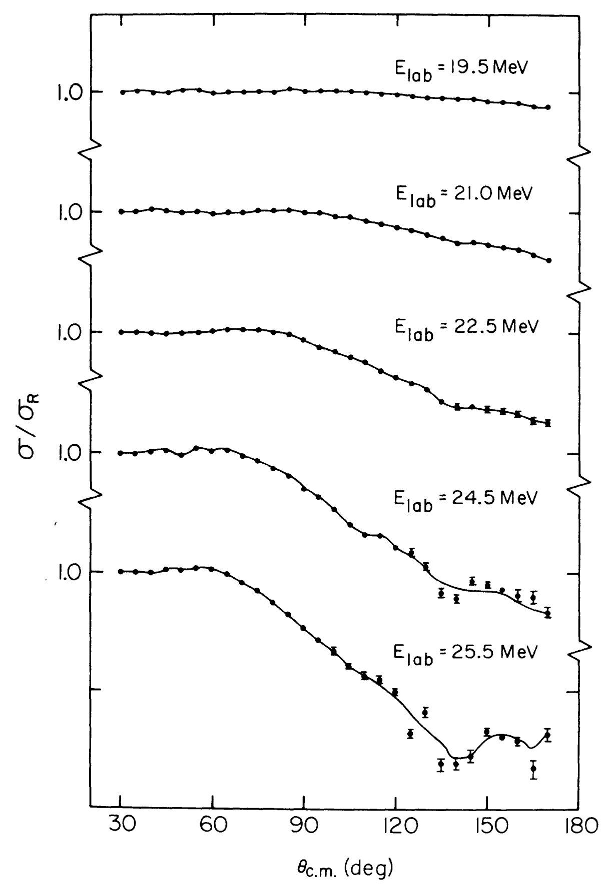

FIG. 2. Angular distributions at various laboratory energies. Lines connecting the points are optical-model fits to the data.

rise monotonically for the highest few energies and reaches a value of ~8 at 25.5 MeV.

Even though W, the critical parameter in the discreteness condition of the R-function model,[3] decreases as the energy increases, the computed total reaction cross section increases with bombarding energy. At 18.5 MeV, the computed total reaction cross section is ~50 mb and it smoothly increases to ~700 mb at 25.5 MeV.[10]

The averages of the U and W values extracted at each energy are $U = 35 \pm 3$ MeV and $W = 6 \pm 3$ MeV. As seen in Fig. 4, the values of χ^2 increase monotonically from the minimum in the U, W plane except for local minima at low W.

No discrete ambiguity is found in the energy range studied here, i.e., only a set of U, W values fits the data for the geometry parameters given. The unanswered question is whether the absence of discrete ambiguities is due to the near-barrier scattering technique or due to the properties of the colliding nuclei. An insight is provided by the R-function application to a square well by Watson *et al.*[3] Except at the highest energies, the conditions for the existence of discrete ambiguities are not satisfied by the extracted optical-model parameters, principally because W is too large. If the downward trend of W continues (see Fig. 3) for energies above the range investigated here, a discrete ambiguity should occur in an energy region above the barrier.

Both Satchler, Brock, and Yntema[11] at 43 MeV and Goldberg *et al.*[12] at 139 MeV have reported discrete ambiguities. However, since their angular distributions extended to only 60 and 90° (c.m.), respectively, spurious discrete ambiguities may be present in the results.

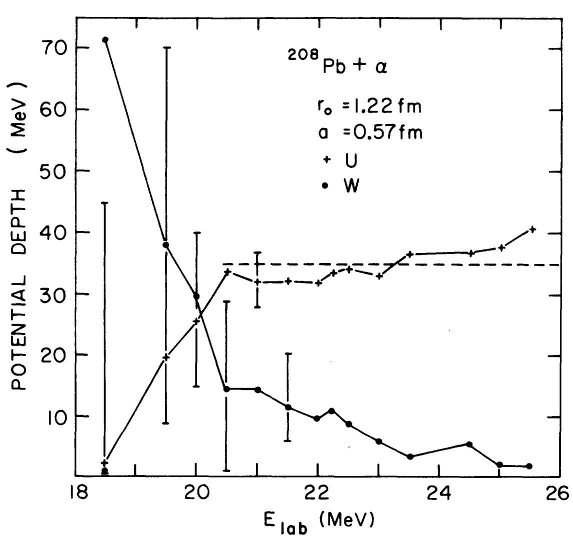

FIG. 3. Best-fit values of U and W as functions of the bombarding energy. The error bars represent the span of the $(\chi^2_{min} + 1)$ contour.

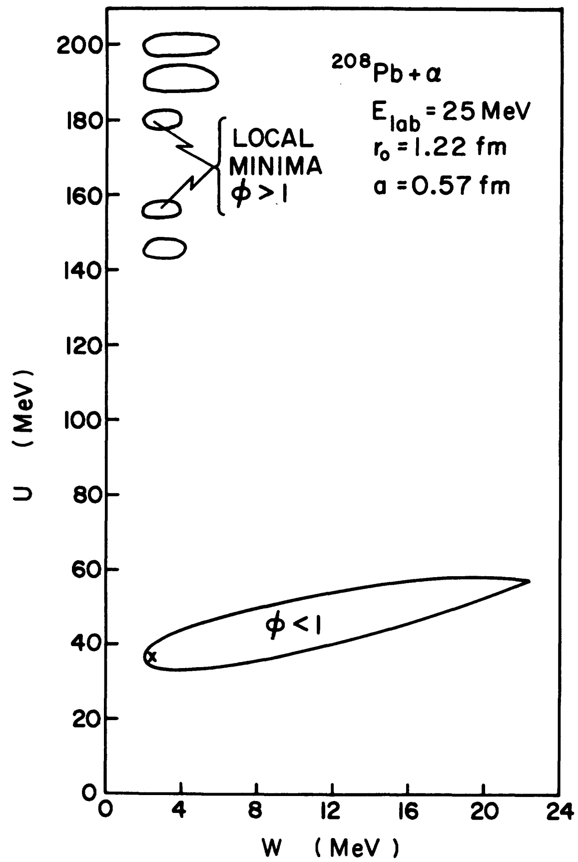

FIG. 4. Contour plot of $\phi = \log_{10}\chi^2$ on the U, W plane at bombarding energy 25 MeV. The minimum is marked by x.

The continuous relationship between U and r_0 was studied. It was found the expressions

$$UR^n = \text{const} \quad (4)$$

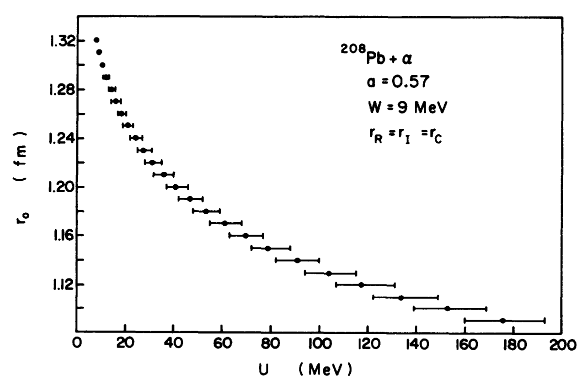

FIG. 5. The continuous ambiguity between U and R (or r_0). The data are fitted values for U with W and a held constant. Error bars show the span of I for $(\chi^2_{min} + 1)$.

and

$$U \exp(R/a) = \text{const}, \qquad (5)$$

where Eq. (5) is the continuous ambiguity form proposed by Igo,[13] both fit the experimentally derived curve (see Fig. 5) equally well. The value of n in Eq. (4) is 16 ± 1 which suggests that the volume integral used, for example, by Cage, Cole, and Pyle[14] for comparison of similar work is not applicable here.

IV. DISCUSSION

Only one real-well depth $U = 35$ MeV yields a good fit to the data for $r_0 = 1.22$ fm and $a = 0.57$ fm over the energy range 16 to 25.5 MeV (lab). Furthermore, a strong dependence on R is found for the continuous ambiguity relating U and R (see Fig. 5). Consequently the wide spread in the values of V (values range from 30 to 180 MeV) which have been reported in previous studies in this energy range is understood as a consequence of one or more continuous ambiguities.

While discrete ambiguities are not expected from the R-function model[3] over most of the energy range, the condition for discrete ambiguities is marginally satisfied by the optical-model parameters at the highest energy. If the decrease of W with energy continues above the range studied here, discrete ambiguities should be observed. The onset has not been observed, but there is some evidence for discrete ambiguities at considerably higher energies.[11,12]

---

*Work supported in part by the National Science Foundation, Grants Nos. NSF-GU-2612, NSF-GP-25974, and NSF-GJ-367.

[1] J. S. Eck, R. A. LaSalle, and D. Robson, Phys. Lett. 27B, 420 (1968).

[2] T. Mo and R. H. Davis, Phys. Rev. C 6, 231 (1972).

[3] B. D. Watson, D. Robson, D. D. Tolbert, and R. H. Davis, Phys. Rev. C 4, 2240 (1971).

[4] G. Igo, R. M. Thaler, Phys. Rev. 106, 126 (1957).

[5] A. R. Barnett, J. S. Lilley, and F. Chwieroth, Williams Nuclear Physics Laboratory Annual Report, University of Minnesota, 1969; A. R. Barnett and J. S. Lilley, *ibid.*, 1970.

[6] G. Goldring, M. Samuel, B. D. Watson, M. C. Bertin, and S. L. Tabor, Phys. Lett. 32B, 465 (1970).

[7] E. J. Feldl, P. B. Weiss, and R. H. Davis, Nucl. Instrum. Methods 28, 309 (1964).

[8] F. G. Perey, private communication.

[9] R. Hofstader and H. R. Collard, *Nuclear Radii* (Springer-Verlag, New York, 1967), pp. 21–52.

[10] G. M. Hudson, Ph.D. thesis, Florida State University, 1973 (unpublished).

[11] G. R. Satchler, H. W. Brock, and J. L. Yntema, Phys. Lett. 16, 52 (1965).

[12] D. A. Goldberg, S. M. Smith, H. G. Pugh, P. G. Roos, and N. S. Wall, Phys. Rev. C 7, 1938 (1973).

[13] G. Igo, Phys. Rev. 115, 1665 (1959).

[14] M. E. Cage, A. J. Cole, and G. J. Pyle, Nucl. Phys. A201, 418 (1973).