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Off-shell effects in elastic pion-nucleus scattering*

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For elastic scattering of pions from nuclei in the energy region of the (3, 3) resonance, the effects of the binding of the target nucleons are shown to be important in the calculation of the pion-nucleus optical potential. The size of these binding effects is estimated for the case in which the free pion-nucleon scattering amplitude is represented by a separable form.

Recently there has been a great deal of interest in the scattering of pions from nuclei. Various calculational approaches are available, including those which are based on Glauber theory,[1] semi-phenomenological optical models,[2] the Watson[3] multiple scattering theory, the impulse approximation,[4,5] or the extension of Chew-Low theory[6] to the case of bound nucleons. In order to construct a microscopic description of this process, one must take account of a number of interrelated effects.

These effects include the off-shell aspects of the pion-nucleon T matrix, dispersive effects due to multiple scatterings, the quenching of the pion-nucleon interaction due to the Pauli principle, other effects of the Pauli principle, pion interactions with nucleon pairs, etc. Because of the resonant nature of the free pion-nucleon T matrix, one expects that calculations in this energy region will be particularly sensitive to the proper treatment of off-shell effects. This situation is in contrast to the case of nucleon-nucleus scattering at intermediate energies, where the relevant T matrices are slowly varying functions of the energy.

In this paper, therefore, we shall consider the role of off-shell effects in pion-nucleus scattering which arise from the fact that the target nucleon is bound in some orbit $|\phi_b\rangle$, with energy $\epsilon=-|\epsilon_b|$. The leading term in a systematic hole-line expansion for the optical potential has been derived in Ref. 7. In Ref. 7 the interaction of a nucleon with a correlated nucleus was considered. For the pion, we must omit those effects which arise from the identity of the incident particle and the target particles. Furthermore, in order to simplify the discussion we will neglect correlations among the target particles. In that case, we may write the leading term in the pion-nucleus optical potential as

$$\langle \vec{k}_\pi' | V^{\text{opt}} | \vec{k}_\pi\rangle = \sum_b \int d^3k_N' d^3k_N \langle \phi_b | \vec{k}_N'\rangle \langle \vec{k}_\pi' \vec{k}_N' | \hat{t}_b(\epsilon_{\vec{k}_\pi}) | \vec{k}_\pi \vec{k}_N\rangle \langle \vec{k}_N | \phi_b\rangle \,. \tag{1}$$

A derivation of this formula for the case where the incident particle is not identical to the target particles and in which the target particles are assumed to be uncorrelated has also been given in Ref. 8. The T matrix, $\hat{t}_b(\epsilon_{\vec{k}_\pi})$, was shown in these references to be a modified Bethe-Goldstone reaction matrix. If we neglect the Pauli effects, which restrict the recoil of the target nucleon, and the distortion effects, which describe the fact that the pion and nucleon are scattering in the presence of other nuclei, the T matrix in Eq. (1) becomes the *free pion-nucleon scattering amplitude* $t_{\pi N}$ with the energy parameter shifted, viz.,

$$\langle \vec{k}_\pi' \vec{k}_N' | \hat{t}_b(\epsilon_{\vec{k}_\pi}) | \vec{k}_\pi \vec{k}_N\rangle \sim \langle \vec{k}_\pi' \vec{k}_N' | t_{\pi N}(\epsilon_{\vec{k}_\pi} - |\epsilon_b|) | \vec{k}_\pi \vec{k}_N\rangle \,, \tag{2}$$

where $\epsilon_{\vec{k}_\pi} = (k_\pi^2 + m_\pi^2)^{1/2} - m_\pi$ is the kinetic energy of the incident pion.

It is not uncommon to use the approximation of Eq. (2) with the energy parameter of the T matrix chosen as something other than $\epsilon_{\vec{k}_\pi} - |\epsilon_b|$. We are therefore led to consider the approximation

$$\langle \vec{k}_\pi' | V^{\text{opt}} | \vec{k}_\pi\rangle \simeq \sum_b \int d^3k_N\, d^3k_N' \langle \phi_b | \vec{k}_N'\rangle \langle \vec{k}_\pi' \vec{k}_N' | t_{\pi N}(\epsilon_{\vec{k}_\pi} - |\epsilon_b| + \Delta_b) | \vec{k}_\pi \vec{k}_N\rangle \langle \vec{k}_N | \phi_N\rangle \,. \tag{3}$$

We shall examine several choices of the parameter Δ_b which correspond to some customary choices for the T matrix in Eq. (2). A measure of the sensitivity of the T matrix to the choice of the energy parameter can be obtained by considering the ratio of the "incorrect" to the "correct" off-shell T matrix,

$$R \equiv \frac{\langle \vec{k}_\pi'' \vec{k}_N'' | t_{\pi N}(\epsilon_{\vec{k}_\pi} - |\epsilon_b| + \Delta_b) | \vec{k}_\pi' \vec{k}_N' \rangle}{\langle \vec{k}_\pi'' \vec{k}_N'' | t_{\pi N}(\epsilon_{\vec{k}_\pi} - |\epsilon_b|) | \vec{k}_\pi' \vec{k}_N' \rangle} . \tag{4}$$

Of course, we have $R=1$ for $\Delta_b = 0$.

We should like to transform the T matrices in Eq. (4) to the center-of-mass frame for the pion-nucleon system. Such a transformation is known not to be unique[5] for a fully off-shell relativistic T matrix. We shall here make the simple assumption, however, that the total momentum and total energy in the laboratory constitute a four-vector, which we will transform into the pion-nucleon center-of-mass system. With this assumption the energy which occurs in the denominator of Eq. (4), $\epsilon_{\vec{k}_\pi} - |\epsilon_b| + \Delta_b$, becomes

$$\omega' \equiv [(\epsilon_{\vec{k}_\pi} + m_\pi + m_N - |\epsilon_b| + \Delta_b)^2 - (\vec{k}_\pi + \vec{k}_N)^2]^{1/2} - (m_\pi + m_N), \tag{5}$$

where m_N is the rest mass of the nucleon (units with $c=1$ are used).

As Δ_b will be small in comparison with the total mass of the pion and nucleon, we may expand Eq. (5) as

$$\omega' \simeq \omega + \Delta_b \frac{[\epsilon_{\vec{k}_\pi} - |\epsilon_b| + (m_\pi + m_N)]}{\omega + (m_\pi + m_N)} \simeq \omega + \Delta_b , \tag{6}$$

where ω is defined as

$$\omega \equiv \{[\epsilon_{\vec{k}_\pi} + (m_\pi + m_N) - |\epsilon_b|]^2 - [k_\pi + k_N]^2\}^{1/2} - (m_\pi + m_N). \tag{7}$$

With these relations the ratio R, defined in Eq. (4), becomes in the center-of-mass frame of the pion-nucleon system

$$R = \frac{\langle \vec{\kappa}'' | t_{\pi N}(\omega + \Delta_b) | \vec{\kappa}' \rangle}{\langle \vec{\kappa}'' | t_{\pi N}(\omega) | \vec{\kappa}' \rangle} . \tag{8}$$

The momenta $\vec{\kappa}''$ and $\vec{\kappa}'$ are the relative momenta of the pion and the nucleon in the pion-nucleon center-of-mass system, which correspond to the laboratory momenta $\vec{k}_\pi''$, $\vec{k}_N''$ and $\vec{k}_\pi'$, $\vec{k}_N'$, respectively.

The ratio R may be further simplified if we assume that the pion-nucleon T matrix $t_{\pi N}$ is given by a separable form in the vicinity of the (3, 3) resonance, viz.

$$\langle \vec{\kappa}'' | t_{\pi N}(\omega) | \vec{\kappa}' \rangle = \frac{g_{3,3}(\vec{\kappa}'') g_{3,3}(\vec{\kappa}')}{D_{3,3}(\omega)} + \cdots , \tag{9}$$

where the subscript (3, 3) represents the dominant channel (total angular momentum equal to $\frac{3}{2}$, isospin equal to $\frac{3}{2}$, orbital angular momentum equal to 1). If we thus keep only this term in Eq. (9), we have the simple relation

$$R(\omega, \Delta_b) = D_{3,3}(\omega)/D_{3,3}(\omega, \Delta_b) . \tag{10}$$

It is straightforward to generate a separable form for $t_{\pi N}$ which is exact on the energy shell by employing the scattering theory for the inverse problem[9] to generate a separable potential. This potential may then be used in a Lippmann-Schwinger equation with relativistic kinematics to give the separable T matrix of Eq. (9). We have used the separable potential of Walker and Piepho[10] to calculate $R(\omega, \Delta_b)$ for two values of Δ_b.

It is quite common to use the on-shell two-body T matrix in the impulse approximation. This on-shell approximation requires that we take Δ_b to be

$$\Delta_b = |\epsilon_b| + \epsilon_{\vec{k}_N} , \tag{11}$$

where $\epsilon_{\vec{k}_N}$ is the kinetic energy of the nucleon. An average value for Δ_b in a typical nucleus would be approximately 40 MeV. The magnitude and the phase of $R(\omega, \Delta_b)$ for $\Delta_b = 40$ MeV are plotted in Figs. 1 and 2. One sees immediately that the shift of 40 MeV in the energy parameter of the T matrix to be used in the impulse approximation, Eq. (1), alters this T matrix quite significantly.

One should note that the phase of the ratio R is given quite simply in terms of the phase shifts in our separable approximation. This is because $D(\omega)$ is the Jost function, whose phase is just

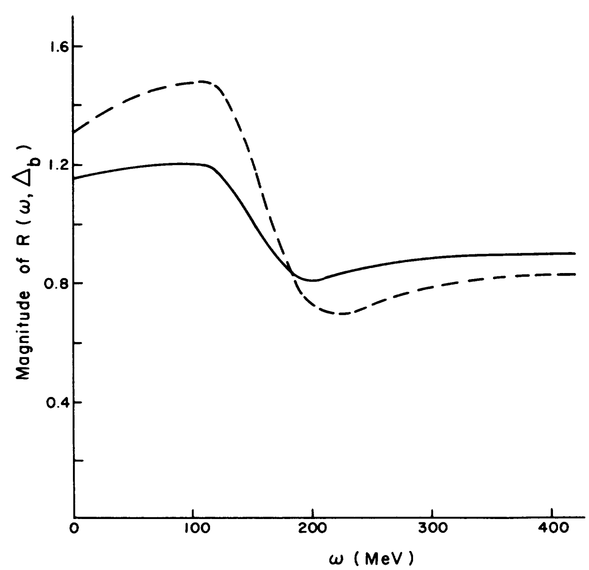

FIG. 1. The magnitude of $R(\omega, \Delta_b)$ versus ω for $\Delta_b = 20$ MeV (solid line) and $\Delta_b = 40$ MeV (dashed line).

equal to the phase shift $\delta(\omega)$. Thus the phase of $R(\omega, \Delta_b)$ is just $\delta(\omega + \Delta_b) - \delta(\omega)$.

We cannot make a trivial quantitative estimate of the shift in the position of the resonance due to the proper choice of Δ_b. Qualitatively however, one might use a Breit-Wigner form for $D(\omega)$, viz.

$$D(\omega) \approx \omega - E_R + \tfrac{1}{2} i \Gamma . \quad (12)$$

Such a parametrization of the pion-nucleon (3, 3) resonance has been found to be quantitatively deficient. Qualitatively however, it suggests the intuitive estimate that the effect of the binding of the nucleon produces a resonance in the laboratory which is ~40 MeV higher in energy than the energy of the resonance in the *pion-nucleon* center-of-mass system.

Another choice of Δ_b that has been used[5] is

$$\Delta_b = |\epsilon_b| . \quad (13)$$

An average binding energy for a nucleon in a nucleus would be approximately 20 MeV. We have plotted the magnitude and the phase of $R(\omega, \Delta_b)$ for Δ_b equal to 20 MeV in Figs. 1 and 2. It is thus apparent that there is a significant correction caused by this shift in energy.

In summary, we have examined the sensitivity of the two-body T matrix which occurs in the impulse approximation to the choice of the energy parameter. The choice of the energy parameter according to a systematic hole-line expansion takes the T matrix in the impulse approximation off shell by about 40 MeV, which results in a significant correction in the region of the pion-nucleon (3, 3) resonance. This correction, together with other effects,[1-6] must be included if one is to make a quantitative comparison with experiment.

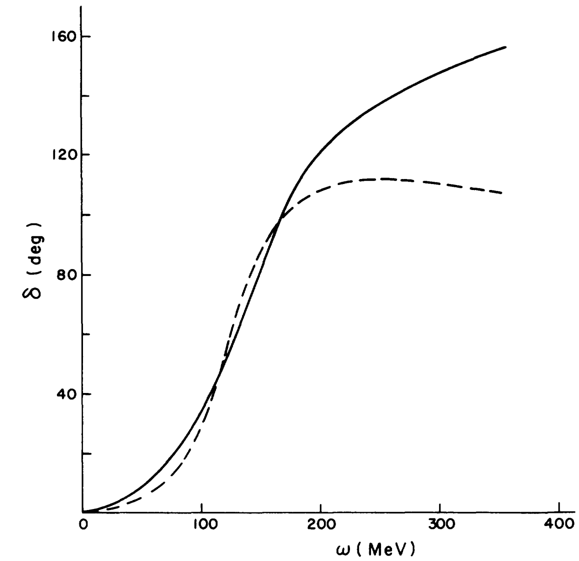

FIG. 3. The phase shifts for the separable potential given in the Appendix (dashed line) and the experimental phase shifts (solid line) versus ω.

APPENDIX

The two-body potential generated as a solution to the inverse scattering problem[9] is not Hermitian below the inelastic scattering threshold.[11] Such a potential can still satisfy the optical theo-

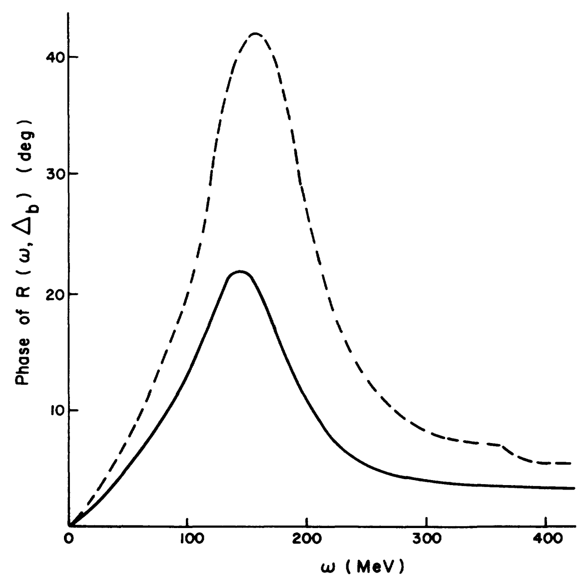

FIG. 2. The phase of $R(\omega, \Delta_b)$ versus ω for $\Delta_b = 20$ MeV (solid line) and $\Delta_b = 40$ MeV (dashed line).

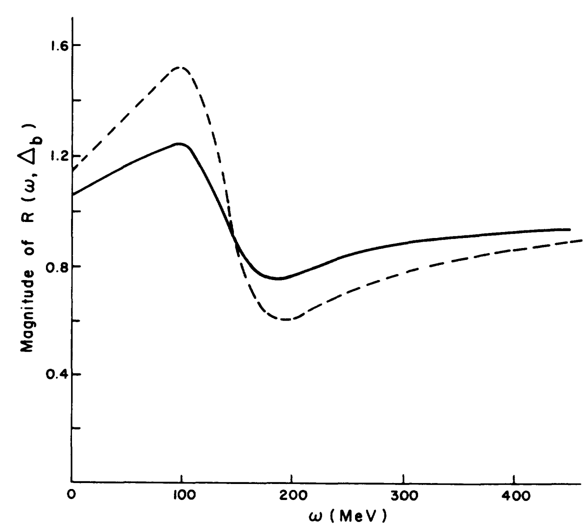

FIG. 4. The magnitude of $R(\omega, \Delta_b)$ versus ω for the separable potential given in Eq. (A3). The solid line is for $\Delta_b = 20$ MeV; the dashed line, $\Delta_b = 40$ MeV.

rem in the two-body problem, however. This can be seen from the "generalized unitarity" relation[8] for the T matrix.

$$T(E)-T^{\dagger}(E)=-2\pi i\,T^{\dagger}(E)\delta(E-H_0)T(E)+\Omega^{(+)\dagger}(E)(V-V^{\dagger})\Omega^{(+)}(E), \quad (A1)$$

where the symbols above are defined in the usual manner. If we examine the matrix element of $T(E)$ above, corresponding to elastic scattering in the forward direction, we see that the requirement that there be no absorption below the inelastic threshold, ϵ_{inel}, is

$$(V-V^{\dagger})\Omega^{(+)}(E)|\vec{k}_E\rangle=(V-V^{\dagger})|\psi^{(+)}_{\vec{k}_E}\rangle=0 \qquad (A2)$$

for $\epsilon_{\vec{k}_E}<\epsilon_{\text{inel}}$. That this property holds in the two-body problem and thus assures that the optical theorem is satisfied in the two-body problem does not imply that one will not encounter difficulties with unitarity in the many-body problem. For the potential used in this paper,[10] the anti-Hermitian part of the potential is quite small below the inelastic threshold. One would thus expect the violation of unitarity to be quite small.

We have examined this problem further by fitting the phase shifts in the (3, 3) channel below the inelastic threshold with a Hermitian separable potential of the simple form

$$\langle r'|V|r\rangle=V_0(\alpha r')e^{-(\alpha r')^2}(\alpha r)e^{-(\alpha r)^2}. \qquad (A3)$$

Such a simple form does not fit the phase shifts exactly; the best fit is given in Fig. 3. The magnitude[12] of $R(\omega,\Delta_b)$ is plotted in Fig. 4 for Δ_b equal to 20 and 40 MeV. The results are quite similar to those presented in Fig. 2. This suggests that the lack of Hermiticity of the potential below the inelastic threshold may not be a critical problem.

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