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Charge-exchange reactions on light nuclei in a multiple-scattering formalism*

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We have calculated single and double charge exchange of pions on ^{9}Be, ^{13}C, ^{11}B, and ^{18}O. The results are in rough agreement with the currently available experimental results.

[NUCLEAR REACTIONS $^{18}O(\pi^+,\pi^0)$, $E=180$ MeV; calculated $\sigma(E)$; $^{18}O(\pi^+,\pi^-)$, $E=180$ MeV; calculated $\sigma(E)$; $^{13}C(\pi^+,\pi^0)$, $E=30-200$ MeV; calculated $\sigma(E)$; $^{13}C(\pi^+,\pi^0)$, $E=180$ MeV; calculated $\sigma(\theta)$; $^{9}Be(\pi^+,\pi^0)$, $E=30-250$ MeV; calculated $\sigma(E)$; $^{9}Be(\pi^+,\pi^-)$, $E=30-250$ MeV; calculated $\sigma(E)$ to $T=\frac{3}{2}$ final states; $^{9}Be(\pi^+,\pi^0)$, $E=175$ MeV, calculated $\sigma(\theta)$ to ground and sum of final states; $^{9}B(\pi^+,\pi^-)$, $E=175$ MeV, calculated $\sigma(\theta)$ to $T=\frac{3}{2}$ final states; $^{11}B(\pi^+,\pi^0)$, $E=20-250$ MeV, calculated $\sigma(E)$ to ground and sum of final states.]

I. INTRODUCTION

In this article we present a calculation of coherent pion-nuclear single-charge-exchange (SCX) and double-charge-exchange (DCX) reactions on light nuclei in the region around 200-MeV incident pion energy.[1] The model uses as input the free pion-nucleon scattering amplitudes and makes use of multiple-scattering corrections to all orders.

Early calculations of coherent SCX and DCX generally ignored or employed only low-order-multiple-scattering corrections.[2] For those experiments in which stringent upper bounds in cross sections have been set, the results are usually high, sometimes by orders of magnitude.

Physically, the intermediate elastic pion-nucleon scattering has the effect of exciting or breaking up the nucleus. For coherent processes the nucleus remains in its initial (or analog) state; hence, the coherent cross section is lowered by the multiple scattering. The effect is particularly dramatic near the N^* resonance, where the elastic pion-nucleon cross section is large. This does not mean that DCX itself is suppressed; it simply means that DCX is accompanied by a great deal of excitation of the target. If all final states are taken into account, DCX is found experimentally to be in the mb range[3] while DCX to a *single* final state is in the μb region.[4]

Multiple scattering also has a strong effect on the A dependence of the DCX cross section. Although the total DCX cross section is usually larger for big nuclei, the likelihood of multiple scattering also grows. This leads to increased nuclear excitation. As a result, the coherent DCX may even *decrease* with A.

Two techniques have recently been used to include the effect of multiple scattering in charge-exchange (CX) processes.

The first[4,5] utilizes Glauber's formulation of multiple-scattering theory, and the second[6-9] uses the optical model. Of the Glauber models, the work of Bjørnenak *et al.*[5] is the most elaborate in this energy regime, including (for ^{13}C) the effects due to the antisymmetrization of the nuclear wave functions, spin-flip, and Fermi motion of the nucleons. Because of the large likelihood of backscattering in the energy region near the N^* resonance, the small-angle approximations used in the Glauber model limit its validity, although the approximations become increasingly better at high energies. Nonetheless, it is a very attractive model inasmuch as it leads to a simple explicit form for the scattering amplitude in terms of almost[10] on-energy-shell pion-nucleon amplitudes, and it can be used for both coherent and incoherent scattering.

The second technique uses the optical model with coupled π^+, π^0, and π^- channels. Particularly extensive calculations have been made by Miller[8] and we look forward to fuller development along these lines. In Sec. IV we will compare some of our calculations with Miller's. This type of calculation has been most extensively applied at low energies, thus complementing the Glauber calculations.

A recently proposed formulation[11] of Watson's multiple-scattering equations also forms a convenient starting point for the calculation of nuclear CX processes. In contrast to the Glauber series, the equations contain no kinematical small-angle approximations. Off-shell effects were eliminated by assuming a very simple analytic structure of

the pion-nucleus scattering amplitude (the "pole approximation"). The nucleons were assumed fixed during the scattering process, but the fundamental pion-nucleon amplitudes were averaged over the momentum distribution of the nucleons (Fermi averaging). The basic equations, given in Sec. II, are readily solved on a computer, provided the nucleus is small ($A \leq 20$). The solution reduces to Glauber's series at high energies and small angles. The method has been applied with reasonable success to π-^{12}C elastic scattering in the 100–300-MeV region.[12]

In this article we extend the system of equations to take into account charge exchange of the scattered pion. The extension is straightforward and the solution presents no new difficultes.

We would like to emphasize that these calculations are of an exploratory nature. Different calculations of cross sections for DCX have in the past, given results varying over several orders of magnitude. We believe that our present calculation, although rough, is close to the correct order of magnitude. We have included what we believe to be the dominant processes, i.e., multiple scattering to all orders interspersed between single pion-nucleon charge exchanges.

II. BASIC EQUATIONS AND METHOD OF SOLUTION

According to Ref. 11, the basic equations for the elastic scattering amplitude of π mesons on a set of fixed nucleons at $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A$ are in the "pole approximation"

$$G_i(\vec{k},\vec{k}') = f_i(\vec{k},\vec{k}')e^{i\vec{r}_i\cdot\vec{k}} + \frac{ik}{2\pi} \times \sum_{j\neq i}\int d\Omega_p f_i(\vec{p},\vec{k}')e^{i\vec{p}\cdot(\vec{r}_i-\vec{r}_j)} \times \theta[\vec{p}\cdot(\vec{r}_i-\vec{r}_j)]\, G_j(\vec{k},\vec{p}); \quad |\vec{p}| = |\vec{k}| = |\vec{k}'|, \tag{1}$$

where $f_i(\vec{k},\vec{k}')$ is the free on-shell pion-nucleon scattering amplitude and $i = 1, 2, \ldots, A$. In terms of G_i the pion-nucleus amplitude is

$$F(\vec{k},\vec{k}') = \sum_{i=1}^{A} e^{-i\vec{r}_i\cdot\vec{k}'}\, G_i(\vec{k},\vec{k}'). \tag{2}$$

The ith term in the sum corresponds to that contribution to the elastic scattering amplitude for which the *last* scatter is on the ith nucleon.

The amplitudes $F(\vec{k},\vec{k}')$ and $G_i(\vec{k},\vec{k}')$ are implicitly functions of the fixed nucleons and should be understood as $F(\vec{k},\vec{k}';\vec{r}_1,\ldots,\vec{r}_A)$. Finally $F(\vec{k},\vec{k}')$ must be integrated over the initial and final nuclear states

$$F_{fi}(\vec{k},\vec{k}') = \int\cdots\int d\vec{r}_1\cdots d\vec{r}_A\psi_f^*(\vec{r}_1,\ldots,\vec{r}_A) \times F(\vec{k},\vec{k}')\psi_i(\vec{r}_1,\ldots,\vec{r}_A). \tag{3}$$

In most of our applications the initial and final wave functions are assumed identical, i.e., the reactions are either elastic or lead, in the case of charge exchange, to analog states. It is convenient to write Eq. (1) in the shorthand form

$$G_i = f_i + f_i\sum_{j\neq i} G_j. \tag{4}$$

Physically, the first term on the right represents single scattering while the second gives multiple scattering to all orders, the last scattering being on the ith nucleon.

We now include charge exchange. Let the incident beam consist of positive pions, for example. We define G_i^+ to be the elastic, and G_i^- and G_i^0 the production amplitudes of π^- and π^0, respectively, the last scatter being off the ith nucleon. We define $f_i^{++}, f_i^{00}, f_i^{--}$ to be the free elastic pion-nucleon amplitudes, and we let $f_i^{+0}, f_i^{0-}, f_i^{0+}, f_i^{-0}$ be the free pion-nucleon–single-charge-exchange amplitudes. The coupled set of integral equations which describe the pion-nucleus scattering process assuming incident π^+ mesons is evidently

$$G_i^+ = f_i^{++} + f_i^{++}\sum_{j\neq i} G_j^+ + f_i^{+0}\sum_{j\neq i} G_j^0, \tag{5a}$$

$$G_i^0 = f_i^{0+} + f_i^{00}\sum_{j\neq i} G_j^0 + f_i^{0+}\sum_{j\neq i} G_j^+ + f_i^{0-}\sum_{j\neq i} G_j^-, \tag{5b}$$

$$G_i^- = 0 + f_i^{--}\sum_{j\neq i} G_j^- + f_i^{-0}\sum_{j\neq i} G_j^0, \tag{5c}$$

with similar equations for incident π^- mesons. For example, a term of the form $f_i^{00}\sum_{j\neq i} G_j^0$ describes a process in which a π^0 emerges from the nucleus. The last interaction (f_i^{00}) undergone by the pion was an elastic scatter on the ith nucleon. The term $f_i^{-0}\sum_{j\neq i} G_j^0$ describes a final π^- meson whose last interaction (f_i^{-0}) was a charge exchange which transformed it from a π^0 to an π^-. Equations (5) are partial wave analyzed in the final pion momentum $\vec{k}'$ and then solved by matrix techniques. As is pointed out in Ref. 12, for energy regions in which l partial waves are needed for describing the fundamental pion-nucleon interaction f_i, only l waves are needed to expand G_i to comparable accuracy. This can be seen directly from Eq. (1) where the $\vec{k}'$ dependence

on the right appears only in $f_i(\vec{k}, \vec{k}')$. For incident pion energies of 200 MeV or less, s and p waves alone give a satisfactory representation of the fundamental pion-nucleon interaction.[13] Equation (5) thus becomes a system of $3A(1+3)$ simultaneous equations. These can be solved with reasonable efficiency on a large computer for nuclei of atomic number of less than six or seven.

If the last term in each of the first two Eqs. (5) is neglected, the equations can be solved in succession instead of simultaneously. Specifically, the first equation is solved for G^+ and substituted into the second, which is then solved for G^0, etc. We expect the approximation to be reasonably good because the successive terms G^+, G^0, G^- decrease rapidly, the channels being weakly coupled. Physically, the reverse-charge-exchange term ($\pi^0 \to \pi^+$) is a small correction to the elastic amplitude. Similar comments apply to the ($\pi^- \to \pi^0$) term. The reverse-charge-exchange process was ignored entirely in Ref. 5 but was included in the optical-model calculations of Refs. 6–9.

In order to check the neglect of reverse charge exchange, we have substituted the calculated "input" values of G_j^+, G_j^0, and G_j^- into the right-hand side of Eqs. (5). The resulting values, called $G_j^{+\prime}$ and $G_j^{0\prime}$, are then compared with the input values. To check G_j^-, the amplitudes $G_j^{0\prime}$ and G_j^- are used as input to Eq. (5c) and the result is compared to G_j^-. The changes in the cross sections are very modest, amounting to only a few percent. To gain more precision it may be useful to iterate the system several times, until the iteration process converges.[14] The smallness of the reverse charge exchange is also confirmed by optical-model calculations.[15]

We performed the integration over the $3A$ nuclear coordinates by Monte Carlo techniques as discussed in Ref. 11. For elastic scattering, the initial- and final-state wave functions are identical. For each integration point, nuclear coordinates are chosen randomly, weighted by the probability distribution

$$\rho(\vec{r}_1, \ldots \vec{r}_A) = |\psi_i(\vec{r}_1, \ldots, \vec{r}_A)|^2 . \tag{6}$$

Then Eqs. (5) are solved, and Eq. (2) is used to compute the full amplitude. A few hundred points generally have been sufficient for the accuracy we desire except at deep diffraction minima where cancellations in the integrand make convergence slow.

In one example to be discussed we assume that in the process of scattering, an s-shell neutron is excited to the p shell. This problem can be formally reduced to the elastic case by rewriting the integrand of Eq. (3) as

$$\psi_i^*(\vec{r}_1, \ldots, \vec{r}_A)\left\{\left[\frac{\psi_f(\vec{r}_1, \ldots, \vec{r}_A)}{\psi_i(\vec{r}_1, \ldots, \vec{r}_A)}\right]\right\}^* F(\vec{k}, \vec{k}')\psi_i(\vec{r}_i, \ldots, \vec{r}_A) \equiv |\psi_i|^2 F'(\vec{k}, \vec{k}') . \tag{7}$$

The only difference between this and Eq. (3) is the wave-function ratio which can be absorbed into F as shown. In this simple case, using factorized wave functions, the ratio is simply proportional to r_i if the ith nucleon is excited.

This technique could also be used to include antisymmetrized wave functions. The Slater determinants are multiplied and divided by factorized wave functions. The ratio of the determinant to the factored wave function is then included formally with the amplitude for purposes of integration.

As has been pointed out by the Oxford group[16] the cross section may be quite sensitive to the shell and spin structure of the nuclear wave functions for some transitions (for example $\pi^+ {}^{14}\mathrm{N} \to \pi^0 {}^{14}\mathrm{O}$). Excepting $^{13}\mathrm{C}$ (as will be noted later), we generally ignore such refinements and use simple factorizable particle densities of Woods-Saxon or harmonic-oscillator types. The Pauli principle is included to the extent that charge exchange is allowed only on those neutrons (protons) for which there is a vacancy in the corresponding proton (neutron) shell. The parameters used for the wave functions were taken from the book of Elton.[17]

III. INCLUSIVE PROCESSES AND SPIN-FLIP

In this section we discuss the closure method of calculating summed inelastic CX scattering[18] and also indicate how spin-flip on a valence neutron can be included in our equations. These techniques will be applied in the final section.

A. Summed inelastic scattering

At incident pion energies of 200 MeV, a substantial fraction of the collisions either excite or break up the nucleus. As is indicated in Sec. II, to calculate the transition amplitude between the ground state and a specific inelastic final state n, we must integrate $F(\vec{k}, \vec{k}_n'; \vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A)$ over the overlap of the initial and final nuclear wave functions. A subscript n has been added to $\vec{k}'$ to indicate the final energy of excitation transferred from the pion during the collision.

To compute the above nuclear amplitude we must have $f(\vec{k}, \vec{k}_n')$, the pion-nucleon amplitude. If the final nuclear state is so highly excited that

$|\vec{k}_n'|$ is much less than $|\vec{k}|$, phase shifts are to no avail, and an off-shell energy extrapolation becomes necessary. One could use the correct value of $\vec{k}_n'$ in the kinematical factors of the multiple-scattering equations and set $|\vec{k}_n'| = |\vec{k}_n|$ in the pion-nucleon amplitude as a rough model.

This "off-shell" complication can be avoided if the incident pion energy is much greater than typical excitation energies of the nucleus, for in this case the approximation $|\vec{k}_n'| \simeq |\vec{k}|$ holds. In the following we restrict ourselves to this energy region and, as a consequence, are able to use the on-shell amplitudes for pion-nucleon scattering.

Next let us suppose that only the direction and charge of the outgoing pion are measured, the final state of the nucleus being unknown. The differential cross section of such "inclusive" reactions could be calculated by summing over final states, e.g.,

$$\frac{d\sigma[\pi^+ \mathfrak{N}(Z) \to \pi^+ X(Z)]}{d\Omega_{\pi^+}} = \sum_{n \text{ (accessible)}} |\langle n, Z | F^+(\vec{k}, \vec{k}_n) | 0, Z \rangle|^2 , \tag{8}$$

where $n = 0$ denotes the ground state, $X(Z)$ indicates any energetically accessible nuclear state (including breakup) of A nucleons of atomic number Z, and the sum runs over all such states. We can formally allow n to include a complete set of nuclear states and since $F^+(\vec{k}, \vec{k}_n') \simeq F^+(\vec{k}, \vec{k}')$ the use of the closure relation $\sum_n |n, Z\rangle\langle n, Z| = 1$ gives

$$\frac{d\sigma[\pi^+ \mathfrak{N}(Z) \to \pi^+ X(Z)]}{d\Omega_{\pi^+}} \simeq \langle 0, Z | F^+(\vec{k}, \vec{k}') F^{+\dagger}(\vec{k}, \vec{k}') | 0, Z \rangle . \tag{9a}$$

Similarly,

$$\frac{d\sigma[\pi^+ \mathfrak{N}(Z) \to \pi^0 X(Z+1)]}{d\Omega_{\pi^0}} \simeq \langle 0, Z | F^0(\vec{k}, \vec{k}') F^{0\dagger}(\vec{k}, \vec{k}') | 0, Z \rangle , \tag{9b}$$

$$\frac{d\sigma[\pi^+ \mathfrak{N}(Z) \to \pi^- X(Z+2)]}{d\Omega_{\pi^-}} \simeq \langle 0, Z | F^-(\vec{k}, \vec{k}') F^{-\dagger}(\vec{k}, \vec{k}') | 0, Z \rangle . \tag{9c}$$

This approximation has been used with great success at high energies in Glauber's diffraction theory.[18] When the pion energy is in the range 50–200 MeV, however, we must use this type of result with great caution since Batusov[1] has shown that the pion then frequently loses a substantial fraction of its energy in DCX processes.

B. Spin-flip effects

Since the flip and nonflip components of the elementary pion-nucleon amplitude are comparable, spin-flip may provide some changes in our predictions. The effect is expected to be most important at angles near 90° because the p-wave spin-flip amplitude is proportional to $\sin\theta$. Spin- and (isospin-) flip on the core nucleons is suppressed by the Pauli exclusion principle. This is fortunate, for it is possible to include spin-slip on one nucleon without unduly increasing the complexity of our equations, whereas including the possibility of spin-flip for each nucleon would have led to a prohibitively large set of equations. In what follows we restrict ourselves to nuclei with a single valence nucleon (e.g. ^{13}C) and consider only single charge exchange.

We previously assumed that no reverse charge exchange is possible. Physically this meant that we considered only those scattering sequences for which at most, a single-charge-exchange reaction (for SCX) took place. All orders of elastic multiple scattering were allowed before and after the charge exchange. In a similar spirit we will ignore reverse spin-flip. We emphasize that these approximations are for numerical expedience only, as the reverse processes are easily included in our equations.

Assuming an incident π^+ beam, the relevant equations for elastic and single-charge-exchange processes are, in our shorthand notation,

$$G_i^1 = f_i^{11} + f_i^{11} G_j^1 , \tag{10a}$$

$$G_i^2 = f_i^{21} + f_i^{21} G_j^1 + f_i^{22} G_j^2 , \tag{10b}$$

$$G_i^3 = f_i^{31} + f_i^{31} G_j^1 + f_i^{33} G_j^3 , \tag{10c}$$

$$G_i^4 = f_i^{41} + f_i^{41} G_j^1 + f_i^{42} G_j^2 + f_i^{43} G_j^3 + f_i^{44} G_j^4 , \tag{10d}$$

where $\sum_j$ has been suppressed, and where the upper index is defined by the pion charge and the valence-nucleon spin, viz. $1 = (\pi^+$, spin up), $2 = (\pi^+$, spin down), $3 = (\pi^0$, spin up), $4 = (\pi^0$, spin down). In the above we set $f_i^{21} = f_i^{31} = f_i^{41} = f_i^{42} = f_i^{43} = 0$ unless i corresponds to the valence neutron. The method of solution is to first solve Eq. (10a) for G^1. This is substituted into Eqs. (10b) and (10c) which are independently solved. Finally, the solutions to the first three equations are then fed into (10d) which is then solved. This procedure roughly doubles the computation time over single charge exchange without spin-flip.

We now develop the necessary formalism for evaluating the spin-flip equations. The fundamental pion-nucleon amplitude has the partial wave form[19] (suppressing isospin)

$$f(W,\cos\theta)=\frac{1}{2ik}\sum_{l=0}^{\infty}[lf_{l-}(W)+(l+1)f_{l+}(W)]P_l(\cos\theta)+[f_{l+}(W)-f_{l-}(W)]\vec{\sigma}\cdot\vec{L}P_l(\cos\theta), \tag{11}$$

where $f_{l\pm}$ are the partial-wave amplitudes corresponding to $J=l\pm\frac{1}{2}$, $\vec{L}$ is the orbital angular momentum operator, and θ is the scattering angle. Denoting $3f_1^N\equiv f_{1-}+2f_{1+}$ and $3f_1^F\equiv f_{1+}-f_{1-}$, the matrix elements of f between states of pion momenta $\vec{p}$ and $\vec{k}'$ and nucleon spinors (Z axis of quantization) are

$$\begin{pmatrix}\langle\uparrow|f(\vec{p},\vec{k}')|\uparrow\rangle\\ \langle\downarrow|f(\vec{p},\vec{k}')|\uparrow\rangle\end{pmatrix}=\frac{2\pi}{ik}\left\{f_{0+}Y_0^{0*}(\hat{p})Y_0^0(\hat{k}')+f_1^N\sum_m Y_1^{m*}(\hat{p})Y_1^m(\hat{k}')\pm f_1^F[Y_1^{1*}(\hat{p})Y_1^1(\hat{k}')-Y_1^{-1*}(\hat{p})Y_1^{-1}(\hat{k}')]+\cdots\right\}, \tag{12a}$$

$$\langle\downarrow|f(\vec{p},\vec{k}')|\uparrow\rangle=\frac{2\pi}{ik}\sqrt{2}f_1^F[Y_1^{0*}(\hat{p})Y_1^1(\hat{k}')+Y_1^{-1*}(\hat{p})Y_1^0(\hat{k}')]+\cdots, \tag{12b}$$

$$\langle\uparrow|f(\vec{p},\vec{k}')|\downarrow\rangle=\frac{2\pi}{ik}\sqrt{2}f_1^F[Y_1^{1*}(\hat{p})Y_1^0(\hat{k}')+Y_1^{0*}(\hat{p})Y_1^{-1}(\hat{k}')]+\cdots, \tag{12c}$$

where we have retained up to p-wave terms. For the inhomogeneous terms we will need $\vec{p}=\hat{e}_z k$, for which $Y_1^1(\hat{e}_z)=0$ and $Y_1^0(\hat{e}_z)=\sqrt{3}/4\pi$. It is interesting that the flip amplitude f_1^N does contribute to the nonflip π-nucleus amplitude since $\vec{p}$ is integrated over all directions in the last term of Eq. (10a). The function G_i^α is next expanded in spherical harmonics

$$G_i^\alpha(\vec{k}')=\frac{2\pi}{ik}\sum_{\lambda\mu}\frac{(2\lambda+1)^{1/2}}{4\pi}g_{i\lambda}^{\alpha\mu}Y_\lambda^\mu(\hat{k}'),$$

$$\alpha=1,\ldots,4. \tag{13}$$

If Eqs. (12) and (13) are substituted into Eq. (10) and the angular integrals performed, we are left with a set of algebraic equations to solve for the $g_{i\lambda}^{\alpha\mu}$. The details are similar to those in the Appendix of Ref. 16.

To get a feeling for the magnitude of spin-flip corrections we have used a simple Gaussian nuclear density and taken the valence neutron to have its spin initially polarized along the beam axis. The flip and nonflip cross sections are then compared.

IV. RESULTS

Our choice of examples is governed by considerations of calculational limitations, available data, and theoretical interest. The basic calculational limitation is simply that for $A\gtrsim 20$ our matrices get very large and the corresponding $3A$-dimensional integration over the final-state wave function gets prohibitively time consuming.

A. ^{18}O

It is very interesting to use a nucleus which can double charge exchange via an isobaric-analog transition since then the final- and initial-state wave functions are identical except for Coulomb effects. The smallest such nucleus is ^{18}O which is about as large a nucleus as we can handle.

^{18}O has been a popular target for theorists.[2] On the other hand, experimental results are scarce for all charge-exchange reactions, and ^{18}O is no exception. The only data presently available are given by the Oxford group,[16] who used activation methods. They used π^+ mesons to bombard water enriched with ^{18}O, and they determined the final-state nuclide by its half-life. They did not determine the precise nuclear level, however, since the nucleus could be formed in an excited state and subsequently decay to the ground or analog state.

Since ^{18}O has many excited states, we expect to calculate numbers smaller, but of the same general magnitude as those measured. At $T_{\pi^+}=180$ MeV incident energy, the activation cross section for $^{18}\mathrm{O}(\pi^+,\pi^0)^{18}\mathrm{F}$ is 3.5 ± 0.7 mb,[18] and for $^{18}\mathrm{O}(\pi^+,\pi^-)^{18}\mathrm{Ne}$ is less than 0.1 mb. Our results for the integrated SCX and DCX cross sections are approximately 0.4 mb and 5 μb, respectively, where we have assumed that only the two valence neutrons can charge exhange. We also predict $d\sigma/d\Omega|_{0^\circ}\simeq 6$ μb/sr falling to approximately 0.3 μb/sr at 30°. This is to be compared with the result of Parsons, Trefil, and Drell[20] who obtain in a double scattering model 42 μb at 0° at energy corresponding to the N^* peak. The excitation included in our multiple-scattering calculation is seen to

reduce the cross section by approximately an order of magnitude although both predictions lie below the experimental upper bound. We look forward to DCX experiments on ^{18}O, such as the one to be described for ^{9}Be, which distinguish between excited states of the final nucleus. The density function used in this calculation was a Woods-Saxon with radius parameter 2.75 and diffusivity 0.42.

B. ^{13}C

The reaction $^{13}C(\pi^+, \pi^0)^{13}N$ has also been measured by the Oxford group[16] who obtained 3.3 ± 1.0 mb for incident π^+ energy of 180 MeV. A recent Tel Aviv preprint[21] reports another series of measurements of ^{13}C charge exchange indicating a fairly flat cross section of approximately 1.0 ± 0.4 mb between 30 and 90 MeV with an over-all normalization uncertainty of 25%. Since there are no particle-stable excited levels of ^{13}N the experimental numbers give the cross section to the analog state.

Our calculations are compared with these data points in Fig. 1. Our predictions are seen to fall too quickly in the region of the Tel Aviv points, much like optical-model predictions. The high-energy predictions peak about an order of magnitude below the Oxford point. To try to account for the discrepancy we have redone the calculation using a variety of forms for the valence-$(1p_{1/2})$ neutron wave function. The curves are labeled as follows:

(a) Harmonic oscillator with range parameter 1.64 fm.

(b) Woods-Saxon of radius parameter 2.3 fm and diffusivity 0.43 fm for the core but with valence-neutron wave function consisting of a p-wave polynomial inside the core matched to a p-wave Hankel function of imaginary argument. Specifically the density of the valence neutron is

$$(ar^2 + br^3)^2, \quad r < R,$$

$$[(1 + 1/\beta r)\exp(-\beta r)]^2, \quad r > R,$$

where

$$a = (4 + 4\rho + \rho^2)\exp(-\rho)/\beta R^3,$$

$$b = (-3 - 3\rho - \rho^2)\exp(-\rho)/\beta R^4,$$

and $\beta (= 0.43\ \text{fm}^{-1})$ is calculated from the separation energy of the last neutron. The matching point R was chosen to be 2.35 fm. Other matching points cause the curve to be shifted to the right or left for small energies. For larger energies the whole character of the cross section depends on R. If R

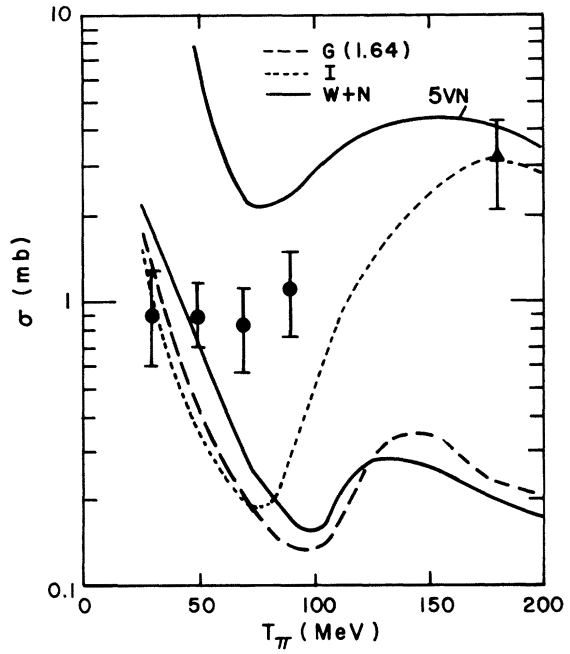

FIG. 1. ^{13}C single-charge-exchange. G stands for harmonic-oscillator density, I for impulse approximation, W+N is Woods-Saxon for core and p-wave polynomial matched to p-wave Hankel function at $R = 235_f$, and 5VN assumes that 5 (instead of 1) neutrons are free to charge exchange. The latter curve is calculated with oscillator density.

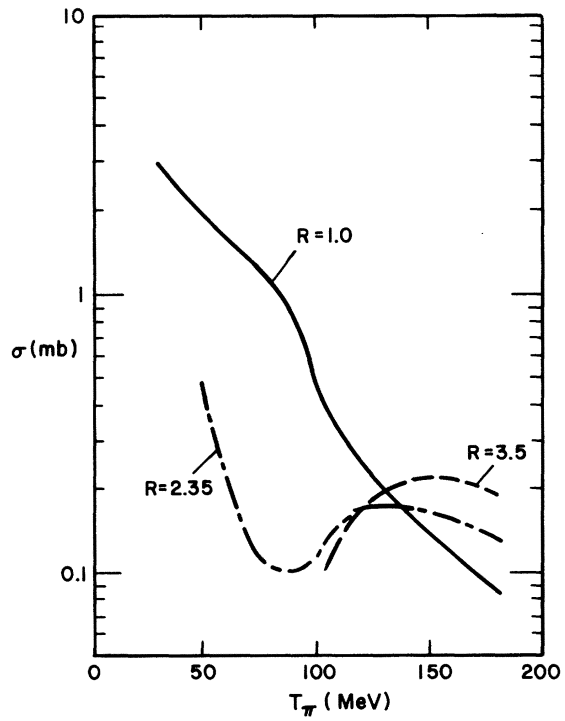

FIG. 2. ^{13}C variation of cross section with matching point of the valence-neutron wave function.

is large, and hence the neutron spends much of its time outside the core, the cross section *peaks* in the region of the resonance. If R is small, and hence the neutron spends most of its time buried in the core, the cross section *dips* in the resonance region. We attribute the dip to the fact that the incident pion must now burrow within the nucleus in order to charge exchange. It hence is more likely to cause excitation due to the short mean free path in the resonance region.[22] See Fig. 2.
(c) Plane-wave impulse approximation (PWIA). Only the first terms in Eqs. (5a) and (5b) were used.

In all cases except (c) the prediction falls below the high-energy experimental point by a factor of about 10. We consider it an interesting fluke that the PWIA matches the data as well as it does, since any multiple scattering will reduce the calculated cross section. Sakamoto[23] also considers this reaction in the PWIA using p-wave isospin $\frac{3}{2}$ phase shifts as input data. He does not find the sharp dip in cross section in the low-energy region, a fact attributable to his neglect of s waves.[24]

This reaction was the subject of an article by Bjørnenak *et al.*,[5] who argue that spin-flip of the valence neutron and antisymmetrization of the nuclear wave functions give important contributions. As the authors point out, they probably overestimate the spin-flip contribution since they take into account only a few multiple scatterings in the spin-flip part of the calculation.

We have included spin-flip as described in the preceding section. The effect (shown in Fig. 3 for 180-MeV pions) is seen to be very small, not nearly enough to account for the discrepancy. Before becoming too alarmed, we must note that the decay scheme was an extremely difficult one to untangle experimentally, "the most difficult attempted,"[16] in part due to a very large ^{12}C impurity in the target. The large value of the cross section is, however, consistent with (p,n) charge-exchange results.[16] In the Tel Aviv experiment[21] the ^{13}C sample was quite pure and the background better under control.

Figure 4 compares our result for ^{13}C SCX with the coupled channel optical-model results of Miller[8] for the same choice of nuclear densities. They agree quite well at low energies but diverge near the resonance. There is clearly more nuclear excitation taking place in the optical-model calculations. It will obviously be very interesting to know experimentally whether the charge-exchange cross sections have peaks or dips near the N^* and to study the trends as A varies. Taken together the data of the Oxford and Tel Aviv groups suggest a peak in ^{13}C SCX, but it must be remembered that neither of the predictions comes very close to explaining the data.

C. ^{9}Be

Other than the activation experiment, only three further reports have been made of coherent DCX measurements on nuclei with A less than 20.[25-27] Coherence would be assured if very little energy difference were seen between the incident π^+ and the produced π^-, but this would require a very well controlled beam energy and a precise measurement of the π^- energy. The energy difference is

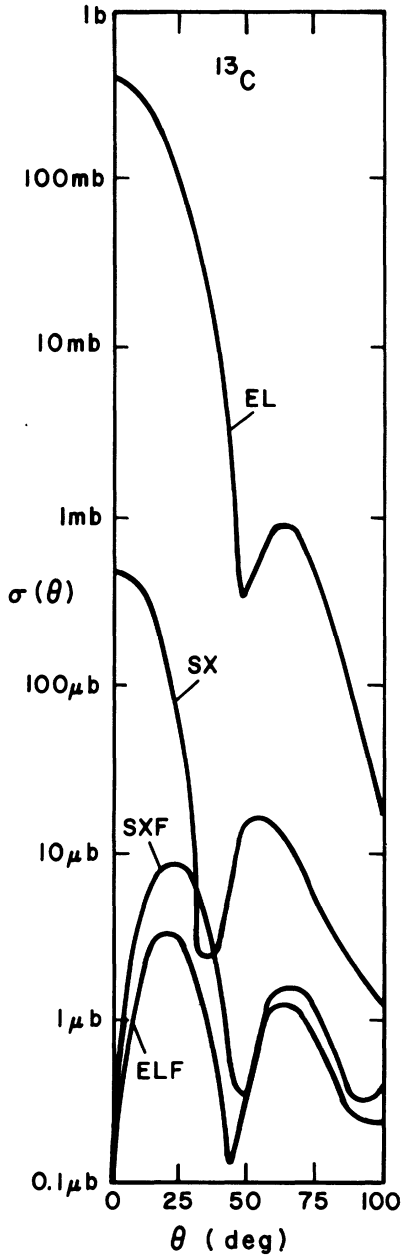

FIG. 3. ^{13}C spin-flip vs nonflip for elastic and charge exchange.

due to nuclear recoil and the difference in Coulomb energy between the initial and final nuclei. Because of these stringent requirements, in none of these experiments has the two-body nature of the final state been definitely established. Thus, as in activation experiments, it is still possible that the final nucleus can be left in an excited state. The data, then, give upper limits on SCX and DCX to the analog (or other) states only.

Prospects for a decisive experiment on ^{9}Be in the future are good, as an experimental proposal has been approved for a study of DCX on ^{9}Be and ^{56}Fe at LAMPF.[28] The experiment will use the EPICS channel and it involves a very precise energy measurement of the charge-exchanged pion. In this way it should be possible to isolate final energy levels of the nucleus.

The experiment of Gilly *et al.*[25] was also of the type which measures the energy of the produced pion. A quasielastic peak was reported, indicating a $\pi^-{}^9$C two-body final state. Similar experiments[26] failed to confirm the corresponding peak in ^{7}Li, so the experimental situation remains ambiguous. Further, the slight peak in the cross section vs the produced π^- energy is depressed some 20 MeV below that of the incident beam, a result hard to explain on the basis of Coulomb energy alone.

Assuming the existence of this excited state, Gilly *et al.* conjectured that it might be due to charge exchange on one p- and one s-shell neutron, the s-shell neutron being elevated to the $1p_{1/2}$ shell of ^{9}C and leaving the nucleus with 28 MeV of excitation energy. We will return to this point shortly.

The process ^{9}Be $(\pi^+, \pi^0)^9$B can proceed via an analog transition since the ground states of ^{9}Be and ^{9}B are members of an $T = \frac{1}{2}$ isomultiplet. The isotope ^{9}C has $T = \frac{3}{2}$, necessitating a change in wave function between the initial and final states of ^{9}Be$(\pi^+, \pi^-)^9$C. As a first attempt we assume that the initial- and final-state wave functions are identical although the overlap is actually not perfect. This leads us to overestimate the cross section somewhat. The calculation gives $d\sigma/d\Omega|_{0^\circ} \simeq 20$ μb/sr at T_{π^+} of 195 MeV falling to about half this at 20° and becoming less than 1 μb/sr at 30°. The integrated DCX cross section is about 10 μb. This is to be compared with the experimental result $d\sigma/d\Omega|_{0^\circ} \simeq 10$ μb/sr of Gilly *et al.*, where those π^- with energy within 45 MeV of the incident π^+ energy were included. To estimate the cross section if DCX is accompanied by the excitation of an s-shell neutron to a p shell as suggested by Gilly *et al.*, we

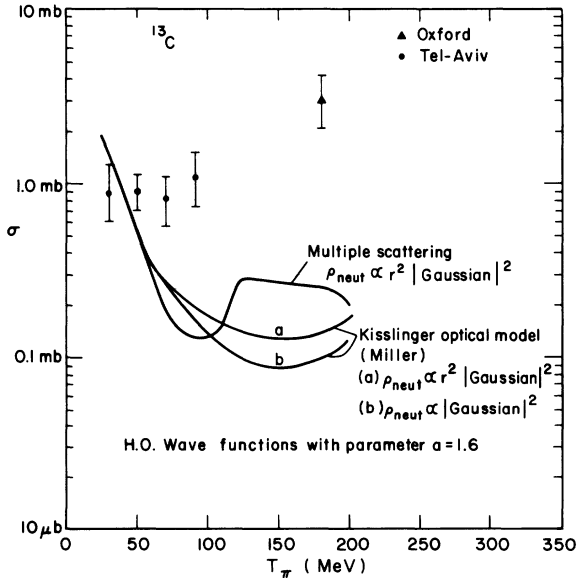

FIG. 4. ^{13}C comparison with optical model (Ref. 8).

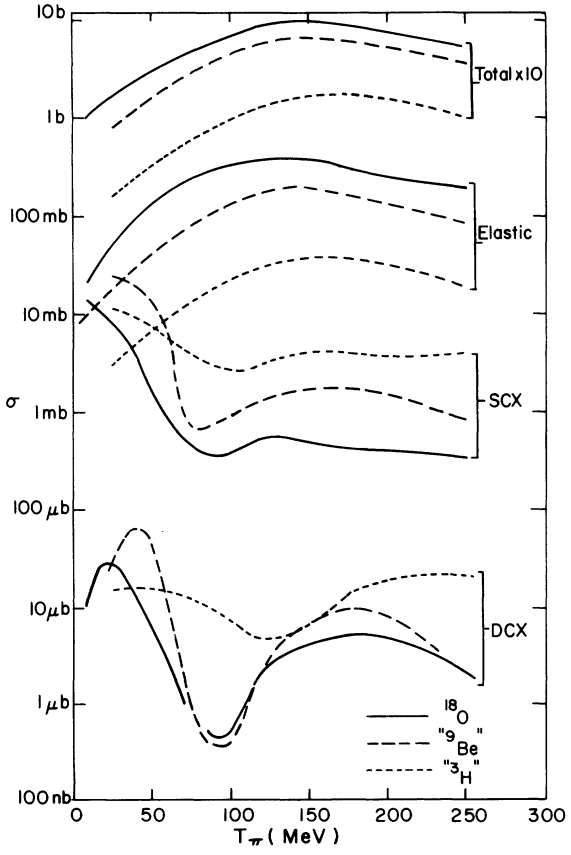

FIG. 5. Survey of cross sections for total, elastic, SCX, and DCX for ^{18}O, "^{9}Be," and "^{3}H" where " " means that two valence neutrons were assumed and a (possibly fictitious) analog transition is assumed.

have used the method described in Sec. II. Supposing the excited ^{9}C nucleus to have $T=\frac{1}{2}$ since transitions to the $T=\frac{3}{2}$ ground state are probably small, we estimate[29] the isospin overlap to be $\sqrt{\frac{3}{2}}$. The net effect of this isospin factor is to reduce the DCX process slightly in addition to the extra factor of r coming from the s to p transition. The result of the calculation is $d\sigma/d\Omega|_{0^\circ} \simeq 15$ μb falling to less than 0.6 μb/sr at 30°. The integrated DCX cross section is approximately 6 μb.

In Fig. 5, we compare the energy dependence of the ^{9}Be cross sections with a lighter (^{3}H) and heavier (^{18}O) target. In computing these curves we took each nucleus to have two valence neutrons in order to compare the trends with increasing A. (The elastic and total ^{9}Be curves are slightly higher than computed in Fig. 7 because of an approximate transformation which has been made from the π-nucleon c.m. frame to the π-nuclear c.m. frame.[13]) To put the reactions on a common footing we assume all transitions are to (possibly fictitious) analog states.

In Fig. 5: (1) The total and elastic peaks are somewhat broader and at lower energies than in π-nucleon scattering, a well known fact. (2) The elastic and total cross sections grow with A while the SCX and DCX cross sections fall, assuming the same number of valence neutrons. (3) Both SCX and DCX show broad humps in the general region of the N^* resonance. The DCX hump is somewhat flattened for the larger nuclei while the peak is shifted to higher energies for the lightest. This result may be contrasted with the standard optical-model results[7] which show pronounced dips in both SCX and DCX in the region of the N^*. (4) The SCX and DCX cross sections show dips at approximately 100 MeV. For the lightest nucleus the dip is shallow and shifted to higher energies.

Figure 6 gives the calculated angular dependence of the summed inelastic cross section for ^{9}Be assuming only the valence neutrons to charge exchange. The inelastic processes contribute little to the forward diffraction peak, but at large angles

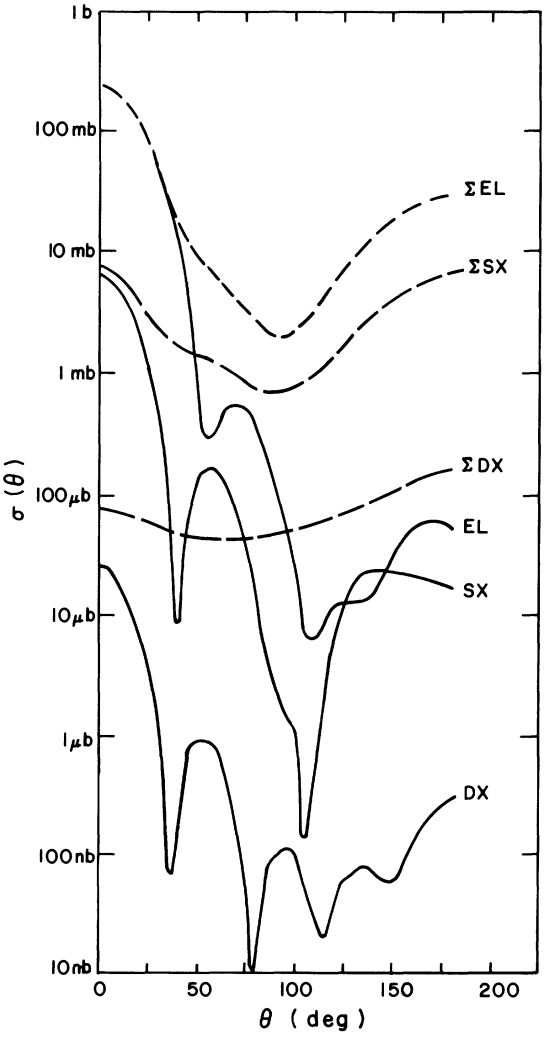

FIG. 6. ^{9}Be angular distributions of coherent and summed inelastic scattering at $T_\pi = 175$ MeV.

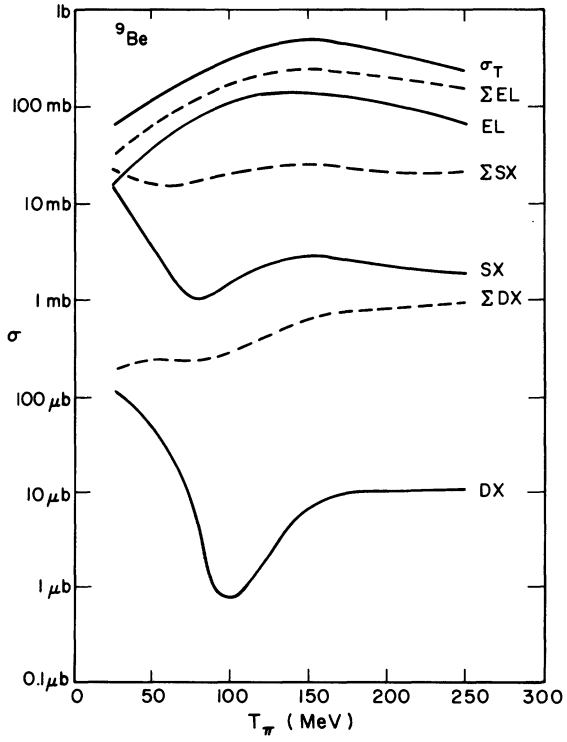

FIG. 7. ^{9}Be energy dependence of cross sections.

inelastic scattering is seen to dominate. The slight peak in the backward elastic scattering is enormously enhanced in the summed cross section. The summed DCX cross section is seen to be larger in the backward direction than the forward, a result in accord with the very different Monte Carlo calculation of Batusov.[1] Figure 7 shows the energy dependence of the summed cross sections integrated over all angles.

It is worth mentioning that the sum of the integrated summed cross sections does not equal the total cross section as calculated from the optical theorem. Pion absorption and production do not account for this since they were not included in the basic (unitary) $\pi\mathfrak{N}$ amplitudes. The real cause of the discrepancy is that the basic $\pi\mathfrak{N}$ amplitudes used include both spin- and isospin-flip. When we use these amplitudes in the *nuclear* scattering equations we have not allowed spin-flip and have allowed charge exchange on only the valence neutron. Thus our closure result sums only those inelastic events for which there is no spin-flip on individual nucleons and no charge exchange on core nucleons.

Spin and isospin may be artifically suppressed in the basic $\pi\mathfrak{N}$ amplitude by simultaneously equating $J = L \pm \frac{1}{2}$ amplitudes and taking $T = \frac{1}{2}, \frac{3}{2}$ amplitudes identical. With these basic amplitudes, the integrated closure result completely saturates the optical theorem as is expected. This has been checked both numerically for ^{9}Be and analytically for a simple two-nucleon system, at least to low orders in multiple scattering.

It may be also physically interesting to solve our scattering equations for the case in which other groups of neutrons or all neutrons are allowed to charge exchange. The latter case is most reasonable for very violent wide-angle scatters in which the recoil nucleon is likely to be knocked into an unfilled shell or into the continuum and, hence, is not restricted by the Pauli principle. We would not expect this result to be valid in the forward coherent peak where the quasielastic events predominate.

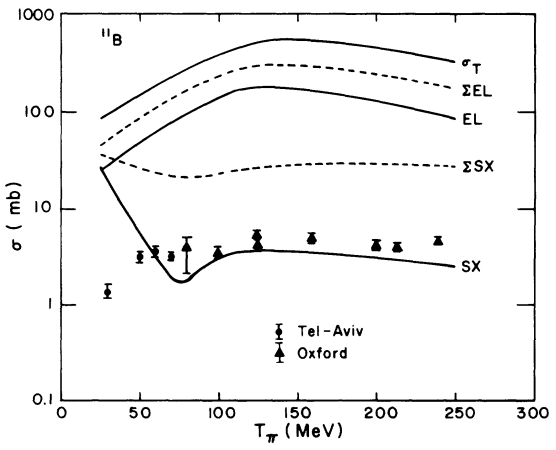

FIG. 8. ^{11}B energy dependence of cross sections.

D. $^{11}B_5$

Both the Tel Aviv and Oxford groups have studied the single-charge-exchange reaction $^{11}B_5(\pi^+, \pi^0)$-$^{11}C_6$ by activation techniques. The energy regions studied by these two groups join smoothly at 100 MeV (see Fig. 8 for an approximate rendering of the data.) Theoretical analysis of the activation cross section is complicated by the presence of nine particle-stable excited levels of ^{11}C. As is anticipated, most experimental points lie between the coherent analog curve (SX) and the summed inelastic charge-exchange curve ($\sum$ SX). As with ^{13}C, the theoretical SX curve grows quickly as E_π falls below 50 MeV. For ^{13}C the cross section was flat, while for ^{11}B the cross section falls. Of course one might seriously call into question the fixed-nucleon approximation at such low energies as 50 MeV.

IV. CONCLUSIONS

We acknowledge a number of weaknesses in the present calculation, the most important of which are the rather inadequate treatment of the transformation of the pion-nucleon amplitudes to the over-all laboratory frame and the use of rather unsophisticated nuclear wave functions. Of course Coulomb corrections will become important at very small angles. Some of these questions are now under study.

We have presented a calculation of charge-exchange processes on light nuclei using a multiple-scattering formalism which includes elastic multiple-scattering corrections to infinite order. The predictions, although certainly not in quantitative agreement (e.g. ^{13}C) with presently available data, are at least in the right general range. Until new data become available over a spectrum of energies and mass numbers we will not be able to answer even such simple questions as whether coherent DCX (and SCX) cross sections have bumps or dips in the N^* resonance region. In contrast to small-angle elastic scattering, which can be fitted with a single "size" parameter, charge-exchange data rigidly constrain[30] calculational models and will be sure to provide stimulation and embarrassment for theorists for some time to come.

ACKNOWLEDGMENTS

We would like to thank K. K. Seth and G. Miller for several enlightening discussions about charge-exchange processes. One of us (WBK) thanks R. Silbar for helpful discussions and encouragement, and Louis Rosen for the hospitality offered at LAMPF. Another of us (JCJ) thanks Leon Heller for obtaining for her a one-month appointment at Los Alamos Scientific Laboratory, which enabled her to participate in this work.

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[1]See the excellent review articles: D. S. Koltun, Adv. Nucl. Phys. 3, 71 (1969); F. Becker and Yu. A. Batusov, Riv. Nuovo Cimento 1, 309 (1971).

[2]For a short but critical summary of earlier theoretical models, see A. Reitan, *Double Charge Exchange Scattering of Pions by Nuclei* (Proceedings of the International Seminar on the π Meson Nucleus Interactions, Strasbourg, 1971). In this review Reitan emphasizes the importance of multiple-scattering corrections.

[3]Yu. A. Batusov *et al.*, Yad. Fiz. 6, 158 (1967) [transl.: Sov. J. Nucl. Phys. 6, 116 (1968)]; H. W. Bertini, Phys. Rev. C 1, 423 (1970). The Monte Carlo calculations reported in those papers are designed for incoherent processes. See also Sec. III A.

[4]M. A. Locci and P. Picchi, Nuovo Cimento 57, 803 (1968).

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[9]M. Koren, Ph.D. thesis, Massachusetts Institute of Technology, 1969 (unpublished); A. K. Kerman and R. K. Logan, Argonne National Laboratory Report No. ANL-6848, 1964 (unpublished).

[10]By "almost" we mean that the nucleon remains bound to the nucleus and so does not recoil as a free nucleon. See the review article of Koltun, Ref. 1, for example.

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[12]W. R. Gibbs, Phys. Rev. C 5, 755 (1972).

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[14]R. J. Jacob suggested this procedure to us.

[15]G. Miller has checked this for us with his optical-model codes.

[16]D. T. Chivers, E. M. Rimmer, B. W. Allardyce, R. C. Whitcomb, J. J. Domingo, and N. W. Tanner, Nucl. Phys. A126, 129 (1969).

[17]L. R. B. Elton, *Nuclear Sizes* (Oxford U. P., New York, 1961).

[18]See, for example, the article by R. J. Glauber, in *High Energy Physics and Nuclear Structure* (Plenum, New York, 1970), pp. 242–256; J. S. Trefil, Phys. Rev. 180, 1366 (1969).

[19]See, for example, S. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966), pp. 369–370.

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[21]M. Zaider, J. Alster, D. Ashery, S. Cochavi, M. A. Moinester, and A. I. Yavin, *Pion Single Charge Exchange (π^+, π^0) on ^{13}C and ^{11}B from 30 to 90 MeV* (Tel Aviv University, Ramat-Avi, Israel).

[22]It may seem puzzling that the density corresponding to smallest R gives larger cross sections at low energies. Two facts make this plausible: (a) Large R leads to shallower wave functions and hence, more weakly bound and more easily excited states. (b) The nucleus is more transparent at low energies and so the pion can more easily penetrate the inner region of the nucleus without causing excitation.

[23]Y. Sakamoto, Nucl. Phys. B10, 299 (1969).

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No. 13 (unpublished).

[29]The isospin factor was estimated (admittedly crudely) as follows: The three undisturbed s-shell nucleons have $T_1 = \frac{1}{2}$ as does the s-shell neutron to be excited ($T_2 = 1$). T_{12} vanishes since the s-shell nucleons form a closed shell. T_{12} is then coupled to the excess neutron ($T_3 = \frac{1}{2}$) to give the total nuclear isospin $T = \frac{1}{2}$. Once the neutron is excited the two excess p-shell neutrons ($T_{23} = 1$) are coupled to $T_1 (= \frac{1}{2})$ to give the total isospin $\frac{1}{2}$. The overlap of the two wave functions is $\sqrt{3}\{\begin{smallmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{smallmatrix}\} = \sqrt{\frac{3}{2}}$. See A. Bohr and B. Mottleson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. 1., pp. 72–74.

[30]Whenever data are available, the simultaneous fitting of charge-exchange and elastic data would constrain prospective models even further. The success of the plane-wave impulse models in giving approximate agreement with the ^{13}C data, for example, does not assure even approximate agreement with the elastic scattering reaction especially in the N* region where the pion mean free path is small.