PHYSICAL REVIEW C VOLUME 9, NUMBER 4 APRIL 1974

Sensitivity of the small-angle charge-exchange polarization-transfer reaction to spin-flip forces

V. A. Madsen*
Oregon State University, Corvallis, Oregon 97331

J. D. Anderson† and V. R. Brown†
Lawrence Livermore Laboratory, Livermore, California 94550
(Received 15 October 1973)

We present a derivation of the zero-angle polarization transfer $K_y^y(0°)$ in charge exchange and inelastic scattering using direct-reaction theory with central and tensor spin-dependent forces. The energy dependence of the calculated polarization transfer is sensitive to the relative strengths of the spin-independent, central spin-spin, and tensor forces. Adjustment of the parameters of a purely central force cannot reproduce the calculated results for central-plus-tensor forces, giving rise to the hope that measurements of data of this type will allow one to distinguish between central spin-spin and tensor forces. Numerical results are presented for ^{15}N, ^{11}B, and $^{3}H(p,n)$ reactions.

[NUCLEAR REACTIONS DWBA derivation of 0° P transfer; calculation of $^{15}N(p,n)$, $^{11}B(p,n)$ analogs $E=16–28$ MeV; $^{3}H(p,n)$ comparison with experiment.]

I. INTRODUCTION

It is well known that the spin-dependent part of the two-body effective interaction derived from microscopic model calculations includes tensor and spin-spin as well as spin-orbit contributions. The increasing availability of polarized beams permits one to learn more about the details of such spin-dependent interactions. We wish to point out that measurements of small-angle polarization-transfer reactions are sensitive to the *relative* contributions of the individual components of the spin-dependent force. In this note numerical results are presented for the polarization of forward-emitted neutrons in (p,n) transitions due to a polarized beam of protons.

Calculations are made in distorted-wave Born approximation (DWBA), using a central-plus-tensor two-body interaction, of the polarization transfer[1] in the forward direction $K_y^y(0°)$. The quantity $K_y^y(\theta)$ is the y component of the polarization of the scattered or emitted particle per unit y polarization of the incident projectile. In Sec. II we present a theoretical DWBA derivation of the forward-scattering polarization transfer, showing how it depends on direct-reaction amplitudes. In Sec. III we present calculated energy dependence of $K_y^y(0°)$ for the analog (p,n) reaction on the light nuclei ^{15}N and ^{11}B using Watson's systematic optical potentials.[2] We also give some results for the $^{3}H(p,n)^{3}He$ reaction, for which there are experimental $K_y^y(\theta)$ data.[1]

II. THEORY

For an unpolarized target the final density matrix averaged and summed over target spins can be written

$$\underline{\rho}=\sum_{IN}\underline{\mathfrak{A}}(IN)\underline{\rho}^0\underline{\mathfrak{A}}^\dagger(IN)\,, \tag{1}$$

where I and N are total angular momentum transfer and z projection, $\underline{\rho}^0$ is the initial density matrix for the projectile, and the elements of the matrix $\underline{\mathfrak{A}}$ are defined by the relation

$$A_{M_f'M_f,\,M_i'M_i}=\sum_{IN}(J_iJ_fM_i-M_f|I-N)\times(-1)^{J_i-M_i}\mathfrak{A}_{M_f'M_i'}(IN)\,, \tag{2}$$

where $A_{M_f'M_f,\,M_i'M_i}$ is the transition amplitude from initial projectile and target-spin projections $M_i'M_i$ to final projections $M_f'M_f$. The polarization generally is given by

$$P=\frac{\mathrm{tr}(\underline{\rho}\,\underline{\sigma})}{\mathrm{tr}\underline{\rho}}. \tag{3}$$

We now use direct-reaction theory with no spin-orbit distortions to derive a simple formula for the polarization transfer at zero degrees. The amplitude matrix elements of Eq. (1) are

$$\mathfrak{A}_{M_f'M_i'}(IN)=\sum_{\substack{LI'\\MN'}}(-1)^{I-I'-L}\hat{I}^{-1}(LI'MN'|IN)(-1)^N(J'J'M_i'-M_f'|I'-N')(-1)^{J'-M_i'}f_{II'LM}(\hat{k}_f)\,, \tag{4}$$

where

$$f_{II'LM} = \langle \chi_f^{(-)} | G_{II'L}(R) Y_L^M(\hat{R}) | \chi_i^{(+)} \rangle , \tag{5}$$

and $G_{II'L}(R)$ is the direct-reaction radial form factor[3] for transfer I of total angular momentum, I' of spin, and L of orbital angular momentum. We take the beam direction to be along the z axis and initial polarization P_0 in the y direction, so we have $\rho^0 = (1+\sigma_y)$. Combining Eqs. (1), (3), and (4), we obtain after considerable calculation

$$K_y^y(0°) = \frac{P_y(0°)}{P_0} = \frac{\sum_L \frac{|f_{L0L0}|^2}{2L+1} - \sum_I \frac{1}{2I+1} \left| \sum_L (L100|I0) f_{I1L0} \right|^2}{\sum_{II'L} \frac{|f_{II'L0}|^2}{2L+1}} . \tag{6}$$

Aside from constant numerical factors the denominator is the differential cross section. The numerator consists of positive terms due to spin-independent forces ($I'=0$) and negative terms due to the spin-dependent forces ($I'=1$). In the latter term there is interference between amplitudes for different orbital transfer L.

Although in this paper we use Eq. (6) for analog transitions, it is by no means so restricted; it applies to any direct inelastic scattering or charge-exchange transition. For transitions that proceed entirely by spin-flip, such as $^{14}C(p,n)^{14}N_{g.s.}$, $K_y^y(0°)$ should be negative according to Eq. (6). For those in which both spin and spin-independent forces contribute, $K_y^y(0°)$ can be either positive or negative. This fact may yield information on the nature of the effective two-body interaction in nucleon-nucleus scattering or nuclear spectroscopic information. For example, the $^{14}N(p,p')$ 3.95-MeV 1^+ and the $^{89}Y(p,p')$ 0.908-MeV $\frac{9}{2}^+$ excitations each proceed through both a transition with spin transfer ($L=0$ and 3, respectively) and one with no spin transfer ($L=2$ and 5, respectively). The extent of the spin-flip and non-spin-flip contributions could be better determined by measuring polarization transfer than by just comparing experiment with calculated angular distributions. (For charged-particle reactions, where zero degrees is inaccessible, measurements at a few degrees scattering angle are of interest.)

If we include only the $L=0$ orbital angular momentum transfer in Eq. (6), we get a very simple result:

$$K_y^y(0°) = \frac{\sigma_{I'=0} - \frac{1}{3}\sigma_{I'=1}}{\sigma_{I'=0} + \sigma_{I'=1}} . \tag{7}$$

For an $l=0$ target nucleon and a central force Eq. (7) can be derived just by considering the fraction of scatters in which a spin-spin force causes the z projection of the spin to reverse its direction. As we shall see in Sec. III, the fact that $L=0$ transfers dominate the total cross section has little relevance to the zero-degree polarization. The higher-L values contribute very significantly and the polarization is strongly energy-dependent in contrast to Eq. (7), which for a purely central force with a common form factor for V_τ and $V_{\sigma\tau}$ is energy-independent. Equation (7) is approached only for unrealistically long-range central interactions.

We assume a simple charge-exchange effective interaction

$$V_{\text{eff}} = \vec{\tau} \cdot \vec{\tau} \left\{ \frac{e^{-\alpha r}}{\alpha r} (V_\tau + V_{\sigma\tau} \vec{\sigma} \cdot \vec{\sigma}) + V_T \left[h_2^{(1)}(i\alpha' r) - \left(\frac{\beta}{\alpha'}\right)^3 h_2^{(1)}(i\beta r) \right] S_{12} \right\} , \tag{8}$$

where S_{12} is the tensor operator and $h_2^{(1)}$ is the spherical Hankel function. In Sec. III we present results for the zero-degree polarization using an empirical interaction with $V_\tau = 9.0$ MeV, $V_{\sigma\tau} = 6.0$ MeV, $\alpha = 0.714$ fm^{-1} taken from the work of Anderson *et al.*[4] and another central interaction with the commonly used[5] range parameter $\alpha = 1$ fm^{-1}. In both cases we use $\alpha' = 0.714$ fm^{-1}, $\beta = 4$ fm^{-1}, and $V_T = 3.9$ MeV from the work of Crawley *et al.*[6] and Ref. 3. We have not included a two-body spin-orbit force, which is considerably weaker than the central-tensor forces. It is usually important primarily in high-angular-momentum states[7] and states that are weakly connected by central and tensor forces.[8]

Recently Moss *et al.*[9] have shown convincing evidence for a Lane-type spin-orbit term in the interaction by comparison of calculations with their

TABLE I. $^{15}N(p,n)$ analog.

Energy (MeV)	K_y^y (0°) Central	Tensor	Central plus tensor
16	0.791	−0.210	0.746
22	0.457	−0.295	0.048

data on analyzing power in analog (p, n) transitions. Such a term is probably important at other angles, but cannot contribute to the transfer of polarization at 0°. The reason for this is that perpendicular components of spin transfer take place through a transfer of the perpendicular component of the orbital angular momentum, which is zero in the forward direction.

III. RESULTS

Table I presents calculated polarization transfer for the $^{15}N(p, n)^{15}O$ analog reaction at 16- and 22-MeV proton energies using the empirical-central and central-plus-tensor interactions. The polarization changes drastically between these two energies. ^{15}N is a relatively simple nucleus for understanding the rapid change of the small-angle polarization transfer. Because it has spin $\frac{1}{2}$, only $L=0$ and 2 orbital transfers are possible in Eq. (6). As has been pointed out previously,[10] the capability of wave functions to produce experimental ft values must be regarded as a prerequisite for their being used in charge-exchange calculations. The simple $p_{1/2}$ hole model for ^{15}N satisfies this requirement.[11]

We have calculated the amplitudes $f_{II'L0}$ for $L=0$ and 2 in order to understand how the rapid variation in zero-degree neutron polarization occurs. The principle reason for the rapid change with energy is that the $|f_{0000}|^2$ drops by about a factor of 6 over this energy range and at the same time the $|f_{1120}|^2$ term increases by a factor of about 2. As a result both the calculated differential cross section at zero degrees and polarization transfer change rapidly between these two energies. For the tensor force there is an additional effect from a large $|f_{1100}|^2$ term which increases considerably, giving both a lower and a more rapidly decreasing polarization than in the case of the central force. The extent of the $L=0$ and 2 contributions to $d\sigma(0°)/d\Omega$ depends on the optical potential. In the case of ^{15}N, the Watson potentials rather poorly reproduce the experimental differential cross section near zero degrees. In the data there is a minimum at 0° for 18-MeV protons, while the Watson potentials do not give the minimum until 28 MeV. For this reason the ^{15}N case must be thought of as a simple, typical case and not a prediction.

We have found that the Watson optical potentials, along with the empirical force and Cohen-Kurath p-shell wave functions, do much better at producing agreement with the ^{11}B small-angle experimental differential cross section data[12] than the ^{15}N calculation described above. The ^{11}C β decay is retarded compared to the single-particle $(p_{3/2})^{-1}$ result by a factor of 2.27.[13] The Cohen-Kurath wave functions[14] give a retardation of 2.67, 17% more than the data. For these reasons we consider the $^{11}B(p, n)$ to be a fairly good case for examining the information one can obtain from polarization transfer measurements. Results of the calculations are presented in Fig. 1, which shows a comparison of the neutron polarization results as a function of energy for five cases as calculated with A, the simple formula Eq. (7); B, the empirical force with the tensor part set to zero; C, the empirical force including the tensor force; D, an effective V_τ and $V_{\sigma\tau}$ adjusted to give the same polarization and analog cross section as the central-plus-tensor interaction at 16 MeV; and E, same as D except with $\alpha=1$ fm^{-1} in Eq. (8). Comparison of cases A and B shows the effect of higher-L transfers on $K_y^y(0°)$. Comparison of curve C with B shows the effect of including the tensor force. Comparison of curve C with curves D and E indicates that the energy dependence of the polarization including the tensor force cannot be simulated by adjusting the strengths V_τ and $V_{\sigma\tau}$ or the range α^{-1} of the central interaction. The calculations were done assuming no energy dependence of the

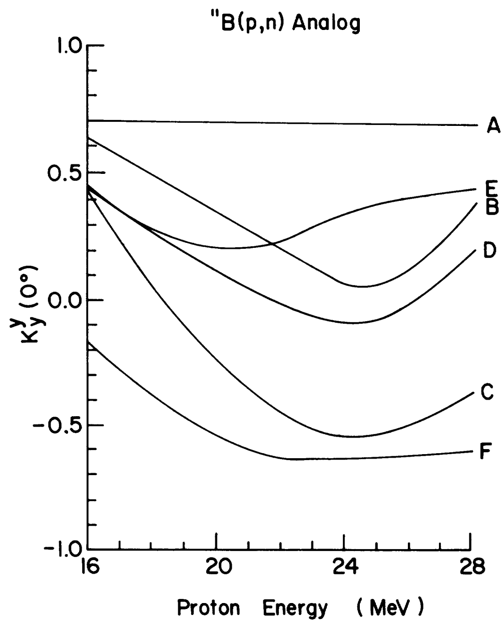

FIG. 1. The polarization transfer K_y^y (0°), Eq. (6), vs proton energy for $^{11}B(p,n)$ calculated with A, the simple $L=0$ formula, Eq. (7), using the ratio of the empirical forces; B, the empirical force with the tensor term set to zero; C, the empirical central-plus-tensor force, Eq. (8), with $\alpha=\alpha'=0.714$ fm^{-1}; D, the central force $\alpha=0.714$ fm^{-1} with V_τ(9.27 MeV) and $V_{\sigma\tau}$(8.16 MeV) adjusted to give K_y^y (0°) and total cross section at 16 MeV in agreement with C; E, a central force with $\alpha=1$ fm^{-1} in Eq. (8) and with V_τ(23.1 MeV) and $V_{\sigma\tau}$(18.4 MeV) adjusted to agree at 16 MeV with C; and F, same as C except with increased relative tensor strength $V_T=6.0$ MeV, $V_\tau=6.0$ MeV, and $V_{\sigma\tau}=4.0$ MeV.

interaction. Analog data do indicate that some energy dependence of V_T may be required, but this would affect the calculations C, D, and E in the same way and would not qualitatively change our argument. Thus, we see that the energy dependence of the neutron polarization from analog states can be used to gain empirical information about the spin-flip contribution in the charge-exchange effective interaction and further that the central-tensor mixture of this spin-flip force could not be adjusted arbitrarily because the two have a different energy dependence.

However, none of cases B through E gives an adequate description of the measured differential cross section at zero degrees. In Fig. 1 we show by curve F what an increase in the relative tensor strength does to $K_y^y(0°)$. In Fig. 2 we show the differential cross section at zero degrees for both cases C and F along with data.[12] Although the zero-degree differential cross section has been considerably improved in going to the stronger tensor force, we found that the over-all angular distribution was worsened slightly. Before we can predict $K_y^y(0°)$ reliably, considerable work needs to be done just in fitting angular distributions. In any case, however, it is fairly consistently true that the effect of the tensor force is to make $K_y^y(0°)$ more negative.[15]

The only experimental results reported for K_y^y are those for the ^{3}H(p,n) analog reaction. No systematic optical potentials are available for scattering from mass-3 nuclei. Watson optical parameters give entirely the wrong behavior both in (p,p) and (p,n). A good fit to neutron scattering from tritium has been obtained by Sherif and Podmore.[16] We have calculated ^{3}H(p,n) at 9 and 18 MeV using their parameters but correcting the real part of the optical potential for isospin effects. We used about the same value of the imaginary potential at 9 and 18 MeV trying qualitatively to account for the lower reaction threshold in p scattering than in n scattering. We used $V_0 = 63$ and 59 MeV and $W_0 = 4$ and 4.69 MeV. We obtained values of $K_y^y(0°) = 0.26$ and 0.29 at 9 and 18 MeV compared to 0.7 and 0.34 at 10 and 16 MeV in the data of Ref. 1. The calculated energy dependence between 15 and 21 MeV is consistent with an extrapolation of the experimental data, which ends at 16 MeV.

IV. SUMMARY AND DISCUSSION

As expected, direct-reaction theory shows the transfer of polarization K_y^y to be highly sensitive to spin-dependent forces. Measurements of $K_y^y(0°)$ supply spectroscopic information about a given transition since spin-independent and spin-dependent amplitudes enter with opposite sign. Central spin-spin and tensor interactions both affect the polarization strongly. Central forces alone do not appear to be able to reproduce the energy dependence of $K_y^y(0°)$ calculated using both central and tensor forces. Not only does measurement of polarization give information about the extent of spin-dependent forces, it may also allow one to distinguish between central and tensor spin forces. The use of this tool for extracting spectroscopic information and strengths of central and tensor parts of the interaction must await further measurements and analyses of spin transfer reactions.

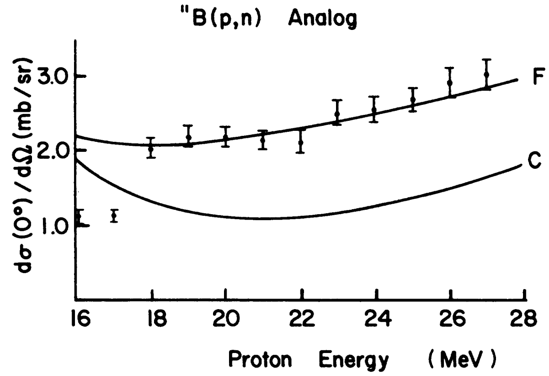

FIG. 2. The measured differential cross sections at 0° as a function of energy are compared with calculations for cases C and F of Fig. 1.

The advantage of the 0° measurements over other angles is that the effect of spin-orbit forces is minimized. A Lane spin-orbit interaction contributes nothing at 0°, and by calculation with and without a spin-orbit optical potential we find its effect to be small at 0°.

Our attempt to fit the ^{3}H(p,n) data has had mixed success. The extent of the polarization at the highest energies is in good agreement with experiment, but the energy dependence is wrong at the lower energies. These calculations should not be taken too seriously, however, because of the lack of appropriate optical potentials, which do not, in fact, reproduce the experimental[17] energy dependence of the zero-degree ^{3}H(p,n) differential cross section.

We emphasize that it is essential to the interpretation of subsequent experiments on polarization transfer to have reliable systematic optical parameters. The L terms in the cross section change fairly rapidly with energy in a way which is different for different L values and for different optical potentials. As a minimum criterion for the applicability of the optical parameters to the $K_y^y(0°)$ data, the DWBA must give correct forward-angle differential cross section, since $K_y^y(0°)$ depends so critically on the strength of various L transfers in the cross section.

*Work supported in part by the National Science Foundation.

†Work done under the auspices of the U. S. Atomic Energy Commission.

[1]R. C. Haight, J. J. Jarmer, J. E. Simmons, J. C. Martin, T. R. Donoghue, Phys. Rev. C 5, 1826 (1971).

[2]B. A. Watson, P. P. Singh, and R. E. Segel, Phys. Rev. 182, 977 (1969).

[3]C. Wong, J. D. Anderson, V. A. Madsen, F. A. Schmittroth, and M. J. Stomp, Phys. Rev. C 3, 1904 (1971).

[4]J. D. Anderson, S. D. Bloom, C. Wong, W. F. Hornyak, and V. A. Madsen, Phys. Rev. 177, 1416 (1969).

[5]F. Petrovich, H. McManus, V. A. Madsen, and J. Atkinson, Phys. Rev. Lett. 22, 895 (1969).

[6]G. M. Crawley, S. M. Austin, W. Benenson, V. A. Madsen, F. A. Schmittroth, and M. J. Stomp, Phys. Lett. 32B, 92 (1970).

[7]W. G. Love, Phys. Lett. 35B, 371 (1971).

[8]S. M. Austin, in *The Two-Body Force in Nuclei*, edited by S. M. Austin and G. M. Crawley (Plenum, New York, 1972).

[9]J. M. Moss, C. Brassard, R. Vyse, and J. Gasset, Phys. Rev. C 6, 1698 (1972).

[10]V. A. Madsen, V. R. Brown, F. Becchetti, and G. W. Greenlees, Phys. Rev. Lett. 26, 454 (1971).

[11]E. J. Konopinski, *The Theory of Beta Radioactivity* (Oxford U. P. Oxford, 1966).

[12]J. D. Anderson, C. Wong, S. M. Grimes, and J. Davis, unpublished data.

[13]C. Wong, Phys. Rev. 95, 765 (1954).

[14]S. Cohen and D. Kurath, Nucl. Phys. 73, 1 (1965).

[15]This does not necessarily mean that K_y^y (0°) will be negative for ^{11}B(p,n) with a tensor force. We have, in fact, been able to find sets of parameters which gave both reasonable angular distributions and positive K_y^y (0°). They were, however, cases in which the tensor force was relatively weak.

[16]H. S. Sherif and B. S. Podmore, in *Proceedings of the Los Angeles Conference on Few Particle Problems in the Nuclear Interaction, Los Angeles, 1972,* edited by I. Šlaus, S. A. Moszkowski, R. P. Haddock, and W. T. H. Van Oers (North-Holland, Amsterdam, London, 1972), p. 691.

[17]D. K. McDaniels, M. Drosg, J. C. Hopkins, and J. D. Seagrave, Phys. Rev. C 6, 1593 (1972).