Angle transformation for the π-nucleus optical potential*

E. Kujawski and G. A. Miller
Department of Physics, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 9 November 1973)

The transformation of the π-nucleon T matrix from the π-nucleon to the π-nucleus center-of-mass systems is investigated. The correct angle transformation is presented, and it is shown to yield straightforward corrections to the usual Kisslinger and local Laplacian potentials consisting of terms proportional to $\nabla^2\rho$ and $\nabla^4\rho$ where ρ is the target density. Numerical results are presented for π^--^{12}C elastic scattering at 120- and 180-MeV lab kinetic energy.

[NUCLEAR REACTIONS π^--^{12}C elastic scattering at 120 and 180 MeV, theoretical optical potential.]

Recently, there have been many attempts[1] at fitting the elastic π-nucleus scattering data of Binon *et al.*[2] Most of these have made use of the first-order optical potential obtained in the "static or factored" impulse approximation[3]:

$$\langle \vec{k}'|V|\vec{k}\rangle = A\rho(q)\langle \vec{k}', \vec{P}_0-\vec{q}\,|(N/A)t_{\pi n}(\omega_k)+(Z/A)t_{\pi p}(\omega_k)|\,\vec{k}, \vec{P}_0\rangle , \tag{1}$$

where N and Z are respectively the number of target neutrons and protons; $A=N+Z$; ρ is the target form factor; $\vec{q}=\vec{k}'-\vec{k}$; $\vec{P}_0$ is some average value of the target nucleon momenta by which one hopes to approximately account for the internal nuclear motion and the target recoil. It is important to realize that the π-nucleon T-matrix elements in Eq. (1) refer to the π-nucleus center-of-mass system (acm) while the π-nucleon scattering amplitude is conventionally determined in the two-body π-nucleon center-of-mass system (2cm).[4] As recently stressed by Landau, Phatak, and Tabakin[3] and Kisslinger and Tabakin[5] the transformation from the 2cm to the acm requires transforming both the momenta and the scattering angle. These authors introduced an angular transformation the effect of which was that a given partial wave of the π-nucleon T matrix in the 2cm contributes to the lower partial waves of the π-nucleon T matrix in the acm. Their results suggested that this angular transformation was very important in fitting the data. However, objections to the specific angular transformation employed by these authors have been raised by Fäldt.[6] Some of these objections have been answered by Kisslinger and Tabakin,[5] but the problem still remains. In this note we reexamine the angle transformation and present what we believe to be the proper Lorentz transformation for the π-nucleon T matrix from the 2cm to the acm. We also show that this transformation yields corrections to the usual Kisslinger[7] or local Laplacian potentials[6] consisting of terms proportional to $\nabla^2\rho$ and $\nabla^4\rho$. The resulting potentials are as convenient to use as the previous ones, the explicit expressions being given in terms of the coefficients tabulated by Sternheim and Auerbach.[8] These corrections influence mainly the large-angle scattering and consistently improve the agreement with the data.

We now consider the transformation of the π-nucleon T matrix from the 2cm to the acm. Let k, k', θ_k refer to the magnitudes of the momenta and the scattering angle in the acm, and p, p', θ_p to the corresponding variables in the 2cm specified by the initial conditions. The T-matrix elements in the two frames are related by[9]

$$t(\epsilon_k; k', k, \cos\theta_k) = \gamma t(\epsilon_p; p', p, \cos\theta_p), \tag{2}$$

where ϵ_k and ϵ_p are the corresponding collision energies, and γ is the usual factor arising from the Lorentz transformation. In the 2cm, from 0- to ~300-MeV kinetic energy only the S and P waves are important for π-nucleon scattering. Furthermore, as shown by Landau, Phatak, and Tabakin, it is reasonable to consider γ in Eq. (2) to be a function of the scattering energy only.

The magnitudes of the relative momenta p and p' are related to k and k' using the standard on-shell relationship.[8] The π-nucleon T matrix in the acm is then given by

$$t_{\text{acm}}(\epsilon_k; k', k, \cos\theta_k) = ak_\epsilon{}^2 + bkk'\cos\theta_p , \tag{3}$$

where k_ϵ denotes the on-shell momentum, and

$$a=-\frac{1}{2(2\pi)^3\epsilon_k}b_0(\epsilon_p), \quad b=-\frac{1}{2(2\pi)^3\epsilon_k}b_1(\epsilon_p), \tag{4}$$

with $b_0(\epsilon_p)$ and $b_1(\epsilon_p)$ being the parameters as defined by Auerbach, Fleming, and Sternheim.[8] At this point we wish to emphasize that the T-matrix element specified by Eq. (3) is still expressed in terms of the scattering angle in the 2cm rather than the scattering angle in the acm. However, it is a simple matter to relate the scattering angles in the 2cm and acm.[10] Denoting the velocity of the π in the acm by v_k and the relative velocity of the 2cm and acm by V one readily obtains

$$\cos\theta_p = \frac{v_k \cos\theta_k - V}{[\bar{v}_k^2 - 2v_k V\cos\theta_k + (V^2/c^2)v_k^2\cos^2\theta_k]^{1/2}}, \tag{5a}$$

where

$$\bar{v}_k = [v_k^2 - (V^2/c^2)v_k^2 + V^2]^{1/2}, \tag{5b}$$

and c is the speed of light. Clearly, both $\cos\theta_k$ and $\cos\theta_p$ range from -1 to $+1$. It is worth observing that the specific relativistic effects arise only from the terms involving c, and the angular transformation would be present even in the nonrelativistic limit.

Because of the small ratio of the masses of the π and the nucleon ($m_\pi/m \simeq \frac{1}{7}$) in the region of interest (π kinetic energy <300 MeV) $V/v_k \ll 1$. We then proceed by expanding the denominator in Eq. (5a) as a power series in $\cos\theta_k$. Proceeding as indicated we obtain

$$\cos\theta_p = \alpha_0 + \alpha_1\cos\theta_k + \alpha_2\cos^2\theta_k + \cdots, \tag{6a}$$

where

$$\alpha_0 = -V/\bar{v}_k, \tag{6b}$$

$$\alpha_1 = \frac{v_k}{\bar{v}_k} - \frac{V^2}{\bar{v}_k^2}\frac{v_k}{\bar{v}_k}, \tag{6c}$$

$$\alpha_2 = \frac{V}{\bar{v}_k}\frac{v_k^2}{\bar{v}_k^2} - \frac{3}{2}\frac{V^3}{\bar{v}_k^3}\frac{v_k^2}{\bar{v}_k^2} + \frac{V^3 v_k^2}{2c^2\bar{v}_k^3}. \tag{6d}$$

The order of magnitude of the above terms should be apparent to the reader: $\bar{v}_k \simeq v_k$, $\alpha_0 \sim V/v_k$, $\alpha_1 \sim 1$, $\alpha_2 \sim V/v_k$. It may readily be seen that the higher order terms in the series (6a) are at least of order $(V/v_k)^2$ and we neglect them. To substantiate the above remarks α_0, α_1, α_2 are tabulated in Table I for several energies of interest.

We now proceed to determine the π-nucleus optical potential in coordinate space. As is well known, the optical potential depends on the off-shell extrapolation of the π-nucleon T matrix. In order to obtain a simple potential in coordinate space as well as to make contact with previous works we express $\cos^2\theta_k$ as

$$\cos^2\theta_k = (1 - q^2/2k_\epsilon^2)^2, \tag{7}$$

where $\vec{q}$ is the momentum transfer and k_ϵ is the on-shell momentum. Substituting Eqs. (6) and (7) into Eq. (3) and regrouping terms one obtains

$$\begin{aligned} t_{\text{acm}}(\epsilon_k; k, k', \cos\theta_k) = {} & ak_\epsilon^2 + bk_\epsilon^2(\alpha_0+\alpha_2) \\ & + b\alpha_1 kk'\cos\theta_k \\ & + b\alpha_2(-q^2 + q^4/4k_\epsilon^2). \end{aligned} \tag{8}$$

It is interesting to note that since $(\alpha_0+\alpha_2) \simeq 0$ and $\alpha_1 \simeq 1$ the main effect of the transformation comes from the last term in Eq. (8), and we examine the resulting modifications for the two most commonly employed potentials:

(i) The "modified Kisslinger" potential. The off-shell extrapolation of the $kk'\cos\theta_k$ term in Eq. (8) is assumed to give rise to a gradient potential. Combining Eqs. (1) and (8) one finds that the corresponding potential to be used in conjunction with the modified Klein-Gordon equation[8] is given by

$$\begin{aligned} U_{\text{MK}}(r) &= 2\epsilon_k V(r) \\ &= -Ak_\epsilon^2[b_0 + b_1(\alpha_0+\alpha_2)]\rho(r) \\ &\quad + Ab_1\alpha_1\vec{\nabla}\rho(r)\cdot\vec{\nabla} - Ab_1\alpha_2[\nabla^2\rho(r) + \nabla^4\rho(r)/4k_\epsilon^2], \end{aligned} \tag{9}$$

where $\rho(r)$ is the target density normalized to one and all other quantities are as previously defined.

(ii) The "modified local Laplacian" potential. It arises from the following off-shell extrapolation:

$$kk'\cos\theta_k = k_\epsilon^2 - \tfrac{1}{2}q^2.$$

Proceeding as indicated one finds

$$\begin{aligned} U_L(r) &= 2\epsilon_k V(r) \\ &= -Ak_\epsilon^2[b_0 + b_1(\alpha_0+\alpha_1+\alpha_2)]\rho(r) \\ &\quad - \tfrac{1}{2}Ab_1\alpha_1\nabla^2\rho(r) - Ab_1\alpha_2[\nabla^2\rho(r) + \nabla^4\rho(r)/4k_\epsilon^2]. \end{aligned} \tag{10}$$

We then see that the angular transformation presented in this work may readily be applied to modify the usual Kisslinger[7] and local Laplacian[6] potentials, the main effects being explicitly exhibited in Eqs. (9) and (10). It should be pointed

TABLE I. Values of α_0, α_1, α_2 as defined by Eqs. (6b)–(6d) for several values of the π lab kinetic energy.

T_π (MeV)	α_0	α_1	α_2
120	−0.195	0.957	0.184
150	−0.212	0.950	0.199
180	−0.228	0.943	0.213
200	−0.238	0.939	0.221
260	−0.267	0.925	0.244

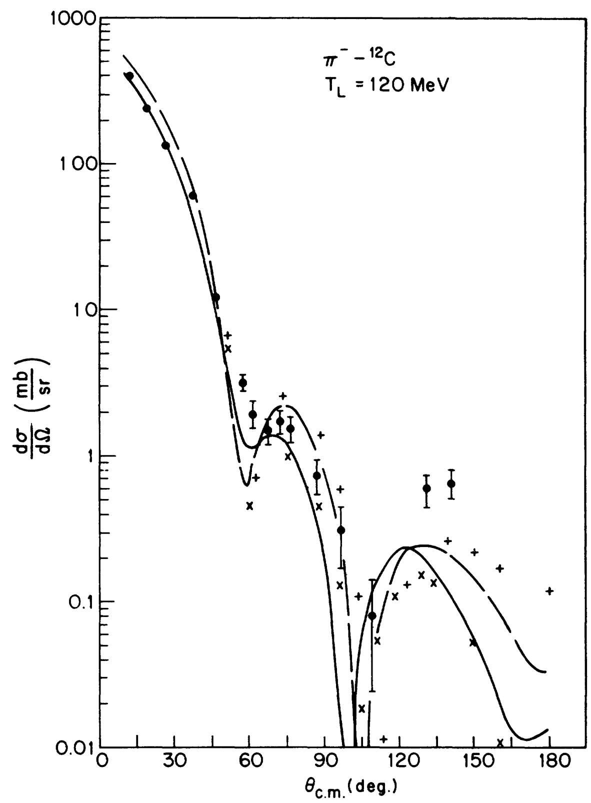

FIG. 1. Elastic differential cross sections for $\pi^- - {}^{12}C$ at 120-MeV lab kinetic energy; ⧳ : experimental points of Binon *et al.* (Ref. 2); —— : results obtained using the "modified Kisslinger" potential given by Eq. (9); – – – : results obtained using the "modified local Laplacian" potential given by Eq. (10); × : results obtained using the usual Kisslinger potential ($\alpha_0 = \alpha_2 = 0$, $\alpha_1 = 1$); +: results obtained using the usual local Laplacian potential ($\alpha_0 = \alpha_2 = 0$, $\alpha_1 = 1$).

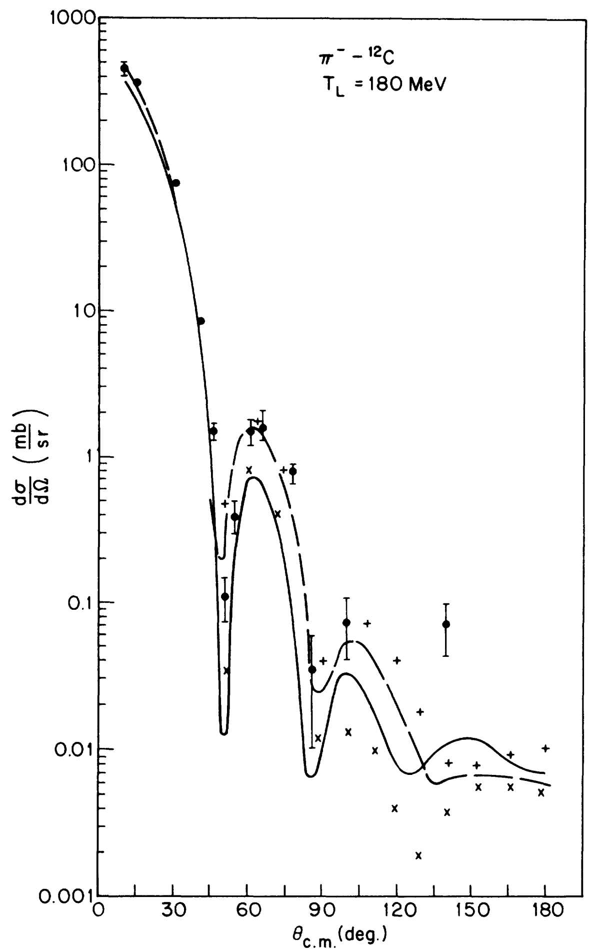

FIG. 2. Same as Fig. 1 but at 180 MeV.

out that $\nabla^2\rho$ corrections to the usual Kisslinger potential have also been obtained by Kisslinger and Tabakin.[5]

To investigate the importance of the above modifications we consider $\pi^- - {}^{12}C$ elastic scattering. For b_0 and b_1 we use the Fermi averaged values given by Sternheim and Auerbach,[8] and $\rho(r)$ is assumed to be the appropriate harmonic oscillator density.[3] The differential cross sections were obtained by solving the Klein-Gordon equation including the Coulomb potential. In Figs. 1 and 2 we compare the results obtained with the potentials given by Eqs. (9) and (10) with the standard Kisslinger[7] and local Laplacian[6] potentials ($\alpha_1 = 1$, $\alpha_0 = \alpha_2 = 0$) and the corresponding experimental data at 120- and 180-MeV lab kinetic energy.[2] The effects of the angle transformation presented in this work are significant. While it enhances the large-angle scattering for the standard Kisslinger potential, it reduces it for the standard local Laplacian potential bringing the two off-shell extrapolations in much closer agreement than previously believed.[6] This is a remarkable feature of the above angle transformation, and it leads to a general improvement for both potentials throughout the angular range. With the exception of the backward angles where the data is sparse and higher-order effects are likely to be important, the fits provided by the rather simple potentials given by Eqs. (9) and (10) are very reasonable.

We conclude by stating that the angle transformation presented in this work does not introduce any anomalies, and it can be applied in conjunction with any off-shell extrapolation of the π-nucleon T matrix. For the two simple off-shell extrapolations examined in this note it seems to provide the necessary features needed to improve agreement with the data.

The authors gratefully acknowledge several stimulating discussions with F. Tabakin. They also wish to thank R. Sorensen for helpful comments concerning the manuscript.

---

*Supported in part by the National Science Foundation.

[1]It would be impractical to attempt to list all the recent works dealing with π-nucleus scattering. Instead we refer the reader to the review given by C. Wilkin at the Fifth International Conference on High Energy Physics and Nuclear Structure held at Uppsala, June, 1973 (unpublished).

[2]Binon *et al.*, Nucl. Phys. B17, 168 (1970); B33, 42 (1971).

[3]R. H. Landau, S. C. Phatak, and F. Tabakin, Ann. Phys. (N. Y.) 78, 299 (1973); Phys. Rev. C 7, 1803 (1973).

[4]M. G. Piepho and G. E. Walker, to be published.

[5]L. S. Kisslinger and F. Tabakin, to be published.

[6]G. Fäldt, Phys. Rev. C 5, 400 (1972).

[7]L. S. Kisslinger, Phys. Rev. 98, 761 (1955).

[8]M. M. Sternheim and E. H. Auerbach, Phys. Rev. Lett. 25, 1550 (1970); E. H. Auerbach, D. M. Fleming, and M. M. Sternheim, Phys. Rev. 162, 1683 (1967); 171, 1781 (1968).

[9]A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N. Y.) 8, 551 (1959).

[10]L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley, Reading, Mass., 1962).