Nuclear rearrangement scattering. I. Quasifree $(p,2p)$ reactions*

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A formal theory of scattering, including rearrangement processes, is presented. This theory is based on a model using completely antisymmetrized and orthogonalized channels. A detailed discussion of the $(p,2p)$ reaction which emphasizes the sequential amplitude for this process is given as an application of the formal theory. The single-hole state generated upon the first collision is discussed using an extension of the theory of intermediate structure to doorway states which are not in a space orthogonal to the space of continuum channels. The theory developed here is able to properly include off-shell and Pauli principle effects in the description of the reaction. Some of the approximations necessary to obtain the standard results for the $(p,2p)$ cross sections are also exhibited in the context of our general theory.

I. INTRODUCTION

A persistent problem arising in all attempts to formulate a Hamiltonian theory of nuclear reactions is caused by the nonorthogonality of the eigenstates used to describe the different open channels. As evidenced by the numerous reaction theories in existence, the above does not represent an insurmountable handicap. This problem has, however, led to the necessity of employing less direct, and as we shall see, for many purposes less convenient, means of achieving the required orthogonality.

Perhaps the most common approach employs the concept of asymptotic orthogonality (AO). Intuitively, AO is based upon the observation that in the asymptotic region, where the wave packets describing the various reaction products no longer overlap, different channels (i.e., different rearrangements of the particles), as well as different states within the same channel, must be orthogonal. Thus AO is a fundamentally time-dependent concept and it is in this property that its limitations reside. The thrust of this remark will become clear as we go on.

Mathematically, AO can be expressed in several ways. Perhaps the most straightforward is to use lowest-order wave-packet theory (i.e., neglecting spreading) to construct wave-packet envelopes for each of the clusters of particles defining a particular channel. For example, the state appropriate for a two-body channel would then be of the form

$$\Phi_E(t) = e^{-iEt/\hbar}\mathcal{A}\Phi_1(\vec{x}_1, \ldots, \vec{x}_A) \times \Phi_2(\vec{x}_{A+1}, \ldots, \vec{x}_{A+N}) \times G_1(\vec{R}_1 - \vec{v}_1 t) G_2(\vec{R}_2 - \vec{v}_2 t), \quad (1.1)$$

where $\vec{R}_i$ is the center-of-mass coordinate of the ith cluster, Φ_i is the wave function for the ith cluster, and $\mathcal{A}$ is the antisymmetrization operator. Asymptotically, the wave-packet functions G_i are nonoverlapping and this assures orthogonality among the various channels. Alternatively, one can use the more elegant and more convenient, but weaker, definition of AO based on techniques familiar from the formal theory of scattering and discussed, for example, by Ekstein.[1] Ekstein defines two functions, Φ_m and Φ_n, as being AO if for some weighting function $C(m)$,

$$\lim_{t\to\pm\infty}\left(\Phi_n, \int e^{-iE_m t}\Phi_m C_\pm(m)\,dm\right) = \lim_{t\to\pm\infty}\int e^{-iE_m t}(\Phi_n, \Phi_m)C_\pm(m)\,dm = \lim_{t\to\pm\infty} C_\pm(n)e^{-iE_n t}. \quad (1.2)$$

Using the Riemann-Lebesque theorem this requires only that

$$(\Phi_n, \Phi_m) = \delta(n-m) + q(n,m), \quad (1.3)$$

where $q(n,m)$ is square integrable. In fact Ekstein assumes that this is the case for the channel functions under consideration.

An alternative to the use of AO to meet the problem of nonorthogonality of channel states has been given in a series of papers by Hahn.[2] He shows how the problem can be circumvented by formulating the entire problem in a multidimensional Hilbert space, using a generalization of Feshbach's projection-operator theory. The "dimension" is given by the number of channels that are treated directly. The equations one ultimately obtains still involve overlaps between (nonorthogonal) projection operators for different channels.

We may contrast the above methods of dealing with the problem of nonorthogonality of channel states with a more direct solution to this problem.

That is the orthonormalization of the channel states themselves, where orthonormalization is now used in the usual sense. While it is easy enough to postulate such orthogonal channel states, a procedure which can be carried through in practice is much more difficult to find. However, just such an orthogonalization scheme has already been proposed for the lowest bound states and the elastic channel[3] and it is clear that similar methods can be used to extend the orthonormal subset to include other channels as well. For the time being, we will assume this has been done and examine the consequences of the existence of a set of orthonormal channel states for the formulation of reaction theory.

In the full Hilbert space, we single out a subset of channels which are of particular interest to the problem at hand. We can then immediately construct a projection operator, P, which projects onto the subspace spanned by these states. The projection operator onto the complementary subspace is $Q=1-P$. Also, we can construct a *generalized channel Hamiltonian*, $\mathcal{K}$, which has as its eigenstates those channel states of interest to us. If we let $|\Phi_c^\alpha\rangle$ be the state c in channel α, then from the above we have

$$\langle \Phi_{c'}^{\alpha'} | \Phi_c^\alpha \rangle = \delta_{\alpha\alpha'}\delta_{cc'} \tag{1.4}$$

and

$$P\mathcal{K}P = \sum_{\alpha,c} |\Phi_c^\alpha\rangle E_{c,\alpha} \langle \Phi_c^\alpha| , \tag{1.5}$$

where $E_{c,\alpha}$ is the asymptotic energy corresponding to the channel state $|\Phi_c^\alpha\rangle$. The existence of $\mathcal{K}$ distinguishes the present approach to the problem of channel orthogonalization from those described above.

The most important consequence of the existence of the generalized channel Hamiltonian $\mathcal{K}$ is that there exists a corresponding *generalized channel interaction* V, defined by

$$V = H - \mathcal{K} , \tag{1.6}$$

where H is the total Hamiltonian for the problem. The channel Hamiltonian and the channel interaction appropriate to any given channel can be obtained immediately by projecting these operators on the given channel. In contrast, the other methods of dealing with orthogonality (e.g., those described above) must define a distinct channel Hamiltonian and channel interaction for each channel.

The existence of V in turn allows one to apply, in a straightforward manner, the well-known techniques of the interaction representation to achieve a systematic perturbation theory for rearrangement scattering. The use of wave packets is obviated by applying the adiabatic approximation and taking the interaction to be

$$V = (H-\mathcal{K})e^{-\alpha|t|} , \tag{1.7}$$

where α is a small positive quantity. The Hamiltonian is then

$$H = \mathcal{K} + (H-\mathcal{K})e^{-\alpha|t|} \to \mathcal{K} \quad \text{as } t \to \pm\infty . \tag{1.8}$$

Thus the problem is in a form appropriate for the introduction of the interaction representation.

An immediate consequence is that one can define an S operator[4] which connects an asymptotic state at $t=-\infty$ with that into which it evolves at $t=+\infty$. The S operator is of course just the quantity

$$S(+\infty,-\infty) = \lim_{\substack{t'\to+\infty \\ t\to-\infty}} U_I(t',t), \tag{1.9}$$

where $U_I(t',t)$ is the evolution operator in the interaction representation and is given by

$$\begin{aligned} U_I(t',t) &= e^{i\mathcal{K}t'}e^{-iH(t'-t)}e^{-i\mathcal{K}t} \\ &= \sum_{n=0}^{\infty}(-i)^n\frac{1}{n!}\int_t^{t'}dt_1\cdots\int_t^{t'}dt_n T[V(t_1)\cdots V(t_n)] \end{aligned} \tag{1.10}$$

with

$$V(t) = e^{i\mathcal{K}t}\,V e^{-i\mathcal{K}t} \tag{1.11}$$

being the potential in the interaction representation.

The existence of an *operator* S in our formulation is in contrast to the situation in theories employing AO or other procedures to treat the channel orthogonality problem. Ekstein very clearly shows that only an S *matrix* exists in the latter theories. This feature is directly related to the fact that in those theories the channel states in distinct channels are eigenstates of different channel Hamiltonians. In our formulation *a single Hamiltonian* $\mathcal{K}$ *serves for all P-space channels.*

It is perhaps useful to summarize Ekstein's result. Asymptotically his (Schrödinger) states have the form

$$\int e^{-iE_nt}C_\pm(n)\Phi_n\,dn, \quad t\to\pm\infty , \tag{1.12}$$

where the Φ_n are channel states with

$$\mathcal{K}_n\Phi_n = E_n\Phi_n . \tag{1.13}$$

The S matrix is simply the array of coefficients connecting the $C_+(n)$ with the $C_-(n)$, i.e.

$$C_+(n) = S_{nm}C_-(m) . \tag{1.14}$$

The existence of the *matrix* $\{S_{nm}\}$ can be demon-

strated by explicit construction. These matrix elements are merely a convenient way of expressing the results of a scattering experiment. An S operator, on the other hand, would connect the asymptotic states,

$$\left(\int C_+(n)\Phi_n\,dn\right)=S\left(\int C_-(n)\Phi_n\,dn\right), \tag{1.15}$$

and such an operator does not exist if different Φ_n's have distinct channel Hamiltonians.

The formulation outlined above has advantages outside of the possibility of employing time-dependent perturbation theory. These are related to the fact that the generalized interaction

$$V=H-\mathcal{H} \tag{1.16}$$

treats all identical particles symmetrically. This is in marked contrast to the channel interactions V_α, V_β, etc., arising in traditional nuclear reaction theories.[5] The latter are obviously not symmetric. Our symmetrical version allows us to deal with T-matrix elements between completely antisymmetrized states. On the other hand, in the traditional treatment of rearrangement scattering, each relabeling of the particle indices must be treated as a separate rearrangement channel. In fact this is true even in the elastic channel where effects of exchange of the incident nucleon with the target nucleons require the introduction of separate channels.

The difference between the two approaches may be made more explicit by noting that in the standard theory the matrix element for scattering from channel β to channel α is given by[6]

$$\langle\Phi_\alpha|\,T\,|\Phi_\beta\rangle=\langle\Phi_\alpha|V_\alpha|\Psi_\beta^{(+)}\rangle=\langle\Psi_\alpha^{(-)}|V_\beta|\Phi_\beta\rangle \tag{1.17}$$

with

$$|\Psi_\beta^{(\pm)}\rangle=|\Phi_\beta\rangle+G^{(\pm)}V_\beta|\Phi_\beta\rangle, \tag{1.18}$$

where

$$G^{(\pm)}=\frac{1}{E-H\pm i\epsilon}. \tag{1.19}$$

Thus

$$\begin{aligned}\langle\Phi_\alpha|\,T\,|\Phi_\beta\rangle&=\langle\Phi_\alpha|V_\alpha+V_\alpha G^{(+)}V_\beta|\Phi_\beta\rangle\\&=\langle\Phi_\alpha|V_\beta+V_\alpha G^{(+)}V_\beta|\Phi_\beta\rangle.\end{aligned} \tag{1.20}$$

Here Φ_α and Φ_β are not antisymmetrized with respect to the particles in different clusters. (The individual bound clusters contained in the Φ_α are antisymmetrized, however.)

In our formulation all the exchange effects for a given reaction are included in expressions having the following structure:

$$\begin{aligned}\langle\Phi_c^\alpha|\,T\,|\Phi_{c'}^\beta\rangle&=\langle\Phi_c^\alpha|\,T_{\text{orth}}\,|\Phi_{c'}^\beta\rangle\\&\quad+\langle\Phi_c^\alpha|(H-\mathcal{H})+(H-\mathcal{H})G^{(+)}(H-\mathcal{H})|\Phi_{c'}^\beta\rangle,\end{aligned} \tag{1.21}$$

where

$$\begin{aligned}P|\Psi_{c,\beta}^{(+)}\rangle&=|\Phi_c^\beta\rangle+P\frac{1}{E-H+i\epsilon}(H-\mathcal{H})|\Phi_c^\beta\rangle\\&=|\Phi_c^\beta\rangle+\frac{P}{E-\mathcal{H}+i\epsilon}(H-\mathcal{H})|\Psi_{c,\beta}^{(+)}\rangle.\end{aligned} \tag{1.22}$$

In Eq. (1.21), $\langle\Phi_c^\alpha|T_{\text{orth}}\,|\Phi_{c'}^\beta\rangle$ is the part of T arising solely from the use of orthonormalized channel states.[7]

One additional point can be made with regard to the distinction between the two formulations given above. In their exact forms, the two formulations are mathematically equivalent. However, when these theories are applied the equations are never solved exactly but instead are truncated in some manner. When approximations are used, the theories are of course no longer identical, and it could easily be that for the description of certain reaction mechanisms, e.g., Pauli breakup of the deuteron, our formulation is superior. This, however, is probably a question that can only be answered by detailed calculation in each particular case.

Finally, it is interesting to compare our formulation of reaction theory with the second quantized formulation due to Villars.[8] In spirit and motivation his theory is similar to ours. However, his mathematical formulation is different. In a channel in which the cluster c is incident on the target, Villars defines an operator J_c through the commutator relation

$$[H,A_c]=-\epsilon_c A_c-J_c. \tag{1.23}$$

Here H is the total Hamiltonian, A_c is the destruction operator for the cluster c, and ϵ_c is the energy of c (including its binding energy). It is shown that appropriate matrix elements of J_c are directly related to those of the transition operator. The relation between Villars's approach and the present one will be expanded upon in a future publication dealing with deuteron elastic scattering and stripping.

II. MICROSCOPIC UNIFIED NUCLEAR-REACTION THEORY

In this section we will discuss some of the practical advantages of the present formulation. One obvious advantage occurs in the formulation of

coupled-channel equations. The use of nonorthogonal channels states always results in terms involving overlap integrals between the various channels in addition to those terms coupling the channels through the interaction potentials. Because our formulation of reaction theory involves orthogonalized channel states, terms of the former type do not occur. The coupled-channel equations to be solved are thereby simplified. The significance of this simplification is probably not very great, however, because the coupled-channel method is seldom applied to rearrangement channels. When the coupled-channels method is used to discuss the coupling between elastic and inelastic channels only, the antisymmetrization and orthonormalization of the states is no problem, in principle, and our approach and the standard one are then rather similar.

A more important advantage of our formulation is that is uses a basis of states which incorporates fully the effect of the identity of the particles comprising the projectile and the target. In addition, a workable prescription for orthogonalizing such states is provided. (This matter has been discussed in detail in Ref. 3 for the special case of elastic scattering. Similar techniques can be applied in general.) Such states can then be used as the basis for a microscopic description of various nuclear reactions, including elastic and inelastic scattering and certain types of rearrangement reactions. The resulting formulation does not, however, provide a proper description of the motion of the center of mass of the system. Techniques such as those introduced by Faddeev or those associated with the resonating group method allow for a proper treatment of the center-of-mass motion, but such techniques are completely impractical for all but systems containing a very small number of particles.

A detailed account of the techniques to be used has already been given for the case of elastic scattering.[9] Similar techniques can be applied for more general reactions. We will discuss in detail in the following sections the treatment of $(p, 2p)$ reactions. In a future publication we will discuss deuteron scattering, including the elastic, stripping, and breakup channels.

The theory that emerges provides a way of systematically describing nuclear reactions which are mediated by realistic (i.e., strong, short-range) nuclear forces. The fundamental quantity of the theory is a Brueckner reaction matrix, which is obtained by summing ladders of potential interactions. The transition amplitudes for any given reaction (or the optical potential in the case of elastic scattering) is expanded into a series of terms involving the reaction matrix, just as in the Brueckner-Bethe-Goldstone theory of nuclear matter and finite nuclei. The analogy is not complete because we deal with a *correlated* ground state from the outset and describe the reaction processes with respect to that correlated state. In many-body theories of nuclear structure, the *model* ground state is taken as the reference state. The former choice, however, is the natural way to describe nuclear-reaction processes.

Just as is the case in nuclear-structure calculations, the contributions to the scattering amplitude, obtained using the techniques described above, can be represented diagrammatically. Ideally one would like to obtain the expansion appropriate to some general amplitude (i.e., the general rules for drawing the diagrams which contribute to any reaction process) just as expansions for various quantities related to the nuclear ground state, and for various shell-model quantities, have been obtained. Unfortunately the situation is considerably more complicated for nuclear reactions because one must deal with collision partners, one or both of which are correlated in a complicated manner. Thus we propose to consider each reaction of interest separately and to apply the techniques referred to above to obtain the corresponding amplitude. In the following sections we will treat the $(p, 2p)$ reaction as an example of the application of our methods.

We feel that the approach described above, which treats the nuclear-structure and the nuclear-scattering problems from a unified point of view, and with similar techniques, will aid in achieving a better understanding of nuclear dynamics.

III. APPLICATION TO QUASIFREE $(p, 2p)$ REACTIONS

A. Introduction

While quasifree scattering initiated by electrons or protons has been rather extensively studied[10] there does not appear to exist a theory of this process which addresses itself to some of the formal questions arising in the study of the reaction. For example, there is the question of the relation of the hole states used in the description of this reaction and the actual physical channels. In the simplest description the projectile knocks a particle out of the target leaving behind a single-hole state. This description is clearly a "doorway" description in that the single-hole state is not an eigenfunction of the Hamiltonian of the residual nucleus. Further, in the case of quasifree scattering ($p + A \to p +$anything) initiated by protons or in $(p, 2p)$ reactions ($p + A \to p + p +$anything) there are questions as to which reaction matrices are to be used. Off-shell effects and the role of the Pauli principle should be clarified. In this work

we wish to discuss the aforementioned reactions in the context of the formal reaction theory discussed in the previous sections. We hope this discussion will aid in understanding the nature of the approximations necessary for the treatment of these reactions and indicate what may be learned from their study. As part of this program we will extend the theory of intermediate structure somewhat.

The usual discussions of doorway states and intermediate structure[11] have largely centered about the theory of resonance reactions (for example, as in the theory of analog resonances[12]) or photonuclear reactions.[13] In these examples, the doorway states are constructed so that they are in a Hilbert space orthogonal to the Hilbert space used to describe the continuum channels. In the reactions discussed in this work this orthogonalization of doorway and continuum channels is not a natural procedure so that the analysis proceeds somewhat differently than is usual.

B. Formal considerations

In some previous works we were able to construct formal Lippmann-Schwinger equations for nuclear reactions.[3] This was accomplished through the definition of a channel Hamiltonian, $\mathcal{H}$, and a set of orthonormal states $|X^{(+)}_{\vec{k},A}\rangle$ which could serve as channel vectors for the elastic channel. It is probably useful to summarize some of our previous results in a somewhat simplified formulation.

We had defined a set of Fermion operators $(\eta^{(\pm)\dagger}_{\vec{k}}, \eta^{\dagger}_{b})$ which when acting on the vacuum created a set of orthornormal single-particle states

$$|\chi^{(\pm)}_{\vec{k}}\rangle = \eta^{(\pm)\dagger}_{\vec{k}}|0\rangle \tag{3.1}$$

and

$$|\phi_b\rangle = \eta^{\dagger}_{b}|0\rangle . \tag{3.2}$$

The $|\phi_b\rangle$ were chosen as the bound-state eigenfunctions arising from self-consistent Brueckner-Hartree-Fock calculations, for example. The $|\chi^{(\pm)}_{\vec{k}}\rangle$ were defined so that they would be orthonormal to the $|\phi_b\rangle$, with $|\chi^{(+)}_{\vec{k}}\rangle$ ($|\chi^{(-)}_{\vec{k}}\rangle$) satisfying outgoing (incoming) boundary conditions. Indeed, they satisfied the single-particle equation

$$(\epsilon_{\vec{k}} - h_M)|\chi^{(\pm)}_{\vec{k}}\rangle = 0, \tag{3.3}$$

where

$$h_M = php + \sum_b |\phi_b\rangle \epsilon_b \langle\phi_b| . \tag{3.4}$$

Here p is a projection operator,

$$p = 1 - \sum_b |\phi_b\rangle\langle\phi_b| , \tag{3.5}$$

and h_0 is the kinetic-energy operator. Further we constructed the states

$$|\chi^{(+)}_{\vec{k},A}\rangle = \eta^{(+)\dagger}_{\vec{k}}|\Phi_A\rangle \tag{3.6}$$

and

$$|\chi_{b,A}\rangle = \eta^{\dagger}_{b}|\Phi_A\rangle , \tag{3.7}$$

where $|\Phi_A\rangle$ is the exact ground state of the target, $H|\Phi_A\rangle = E_A|\Phi_A\rangle$. The $|X^{(+)}_{\vec{k},A}\rangle$ were constructed to be an *orthonormal* set by writing them as linear combinations of the states of Eqs. (3.6) and (3.7). The states of Eqs. (3.6) and (3.7) are not orthonormal, in general. In the simplest case we may write

$$|X^{(+)}_{\vec{k},A}\rangle = \int |\chi^{(+)}_{\vec{k}',A}\rangle\langle\chi^{(+)}_{\vec{k}'}|\gamma|\chi^{(+)}_{\vec{k}}\rangle\, d\vec{k}' , \tag{3.8}$$

where we have neglected problems arising from the possible lack of orthogonality of the $|\chi^{(+)}_{\vec{k},A}\rangle$ and the $|\chi_{b,A}\rangle$, a very small effect.

In terms of the $|X^{(+)}_{\vec{k},A}\rangle$, we defined a projection operator for the elastic channel

$$P = \int |X^{(+)}_{\vec{k},A}\rangle\, d\vec{k}\, \langle X^{(+)}_{\vec{k},A}| = 1 - Q \tag{3.9}$$

and also wrote

$$\mathcal{H} = P\mathcal{H}P + Q\mathcal{H}Q \tag{3.10}$$

with the further definition

$$\overline{H}_0 = P\mathcal{H}P \equiv \int |X^{(+)}_{\vec{k},A}\rangle(\epsilon_{\vec{k}} + E_A)\, d\vec{k}\, \langle X^{(+)}_{\vec{k},A}| . \tag{3.11}$$

With these definitions it was possible to write the formal integral equation

$$|\Psi^{(+)}_{\vec{k},A}\rangle = |X^{(+)}_{\vec{k},A}\rangle + \frac{1}{E - \mathcal{H} + i\epsilon}\,(H - \mathcal{H})|\Psi^{(+)}_{\vec{k},A}\rangle . \tag{3.12}$$

The T matrix for elastic scattering, which may be obtained from the knowledge of the asymptotic form of $P|\Psi^{(+)}_{\vec{k},A}\rangle$, has been extensively studied in a previous work.[3] It is given by

$$\langle\vec{k}'|T|\vec{k}\rangle = \langle\vec{k}'|T_{\text{orth}}|\vec{k}\rangle + \langle X^{(-)}_{\vec{k}',A}|(H - \overline{H}_0)|\Psi^{(+)}_{\vec{k},A}\rangle , \tag{3.13}$$

where T_{orth} has been defined previously[3] and represents the scattering due to the use of the particular "distorted waves" of $|X^{(+)}_{\vec{k},A}\rangle$.

In this work we are particularly interested in the T matrix for the $(p, 2p)$ reaction. To discuss that T matrix it is ultimately necessary to recognize the multiplicity and complexity of the final channels, since the $(p, 2p)$ reaction is an inclusive reaction involving channels with many continuum particles. However, it is useful for the sake of conceptual and notational simplicity to consider

initially only the simplest final channel, the three-body channel consisting of two free protons and a bound residual nucleus. After treating the simple case we will consider the channels having a larger number of continuum particles and we will see that, in the desired approximation, these more complicated channels can be treated in close analogy to the simpler case treated first.

We define the bound eigenfunctions of H for the $(A-1)$ body system as $|\Phi^{\Lambda}_{A-1}\rangle$ such that

$$H|\Phi^{\Lambda}_{A-1}\rangle = E^{\Lambda}_{A-1}|\Phi^{\Lambda}_{A-1}\rangle . \tag{3.14}$$

Here Λ denotes all the quantum numbers necessary to specify the particular eigenfunction of H. Using these states we may also define the states

$$|\hat{Y}^{(+)}_{\vec{k}_1,\vec{k}_2,\Lambda}\rangle = \eta^{(+)\dagger}_{\vec{k}_1}\eta^{(+)\dagger}_{\vec{k}_2}|\Phi^{\Lambda}_{A-1}\rangle . \tag{3.15}$$

These could serve as a set of orthonormal channel states except for the identity of the nucleons created by the $(\eta^{\dagger}_{\vec{k}_1}, \eta^{\dagger}_{\vec{k}_2})$ and the nucleons of $|\Phi^{\Lambda}_{A-1}\rangle$. We note, however, that the states of Eq. (3.15) may be orthonormalized and we will denote this orthonormalized set by $|Y^{(+)}_{\vec{k}_1,\vec{k}_2,\Lambda}\rangle$. These states may be written explicitly by means of the formal equation

$$|Y^{(+)}_{\vec{k}_1,\vec{k}_2,\Lambda}\rangle = \sum_{\Lambda'}\int |\hat{Y}^{(+)}_{\vec{k}_1',\vec{k}_2',\Lambda'}\rangle \times\langle\chi^{(+)}_{\vec{k}_1'}\chi^{(+)}_{\vec{k}_2'}|\,\delta_{\Lambda',\Lambda}|\chi^{(+)}_{\vec{k}_1}\chi^{(+)}_{\vec{k}_2}\rangle d\vec{k}_1' d\vec{k}_2' . \tag{3.16}$$

We will not discuss the details of this construction here. It is also quite useful to require that the states of Eq. (3.16) be orthogonal to the $|X^{(+)}_{\vec{k},A}\rangle$ and $|X_{b,A}\rangle$. Since the $|Y^{(+)}_{\vec{k}_1,\vec{k}_2,\Lambda}\rangle$ have two free particles asymptotically and the $|X^{(+)}_{\vec{k},A}\rangle$ and $|X_{b,A}\rangle$ have at most one, this requirement will not entail great formal complication.

The notation used in Eq. (3.15) is motivated by that of our previous work.[9] We had defined the states[14]

$$|\hat{Y}^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle = \eta^{\dagger}_{\vec{k}_1}\eta^{\dagger}_{\vec{k}_2}\eta_B|\Phi_A\rangle\rho_B^{-1/2} ,$$
$$\rho_B \equiv \langle\Phi_A|\eta^{\dagger}_B\eta_B|\Phi_A\rangle , \tag{3.17}$$

where B denotes a bound state that is occupied with a high probability in $|\Phi_A\rangle$. In addition we had indicated that the states of Eq. (3.17) could be orthonormalized:

$$|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle = \sum_{B'}\int|\hat{Y}^{(+)}_{\vec{k}_1',\vec{k}_2',B'}\rangle d\vec{k}_1' d\vec{k}_2' \times\langle\chi^{(+)}_{\vec{k}_1'}\chi^{(+)}_{\vec{k}_2'}|\,\delta_{B',B}|\chi^{(+)}_{\vec{k}_1}\chi^{(+)}_{\vec{k}_2}\rangle . \tag{3.18}$$

While the states of Eqs. (3.17) and (3.18) played an important role in the discussion of the optical model for elastic scattering, they clearly cannot serve as proper channel states in a formally correct reaction theory. These states however do play a very important role in the "doorway" description of quasifree scattering. Anticipating our results somewhat we may note that the overlap of the states of Eq. (3.16) and Eq. (3.18) will be an important consideration. Therefore we define

$$\langle\chi^{(+)}_{\vec{k}_1'}\chi^{(+)}_{\vec{k}_2'}|\,\mathcal{S}_{\Lambda,B}\,|\chi^{(+)}_{\vec{k}_1}\chi^{(+)}_{\vec{k}_2}\rangle^{1/2} = \langle Y^{(+)}_{\vec{k}_1',\vec{k}_2',\Lambda}|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle . \tag{3.19}$$

To a good approximation we may write

$$\langle\chi^{(+)}_{\vec{k}_1'}\chi^{(+)}_{\vec{k}_2'}|\,\mathcal{S}_{\Lambda,B}\,|\chi^{(+)}_{\vec{k}_1}\chi^{(+)}_{\vec{k}_2}\rangle^{1/2} \approx[\delta(\vec{k}_1-\vec{k}_1')\delta(\vec{k}_2-\vec{k}_2') - \delta(\vec{k}_1-\vec{k}_2')\delta(\vec{k}_2-\vec{k}_1')] \times S_{\Lambda,B}{}^{1/2} + \cdots , \tag{3.20}$$

where

$$S_{\Lambda,B}{}^{1/2} \equiv \langle\Phi^{\Lambda}_{A-1}|\eta_B|\Phi_A\rangle\rho_B{}^{-1/2} . \tag{3.21}$$

With these definitions we may return to the consideration of the T matrix for the $(p, 2p)$ process. Let us denote the transition amplitude for the channel Λ by

$$\langle\vec{k}_1,\vec{k}_2,\Lambda|T|\vec{k}\rangle = \langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|(H-\mathcal{K})|\Psi^{(+)}_{\vec{k},A}\rangle . \tag{3.22}$$

Note that in order to evaluate Eq. (3.22) we require a definition of the projected channel Hamiltonian, $P\mathcal{K}P$, which goes beyond that of Eq. (3.11), viz.

$$P\mathcal{K}P \equiv \bar{H}_0 + \frac{1}{2}\sum_{\Lambda}\iint d\vec{k}_1 d\vec{k}_2 |Y^{(+)}_{\vec{k}_1,\vec{k}_2,\Lambda}\rangle \times(\epsilon_{\vec{k}_1}+\epsilon_{\vec{k}_2}+E^{\Lambda}_{A-1})\langle Y^{(+)}_{\vec{k}_1,\vec{k}_2,\Lambda}| . \tag{3.23}$$

It is clear that Eq. (3.22) is not very useful as it stands and simplifying assumptions are necessary for its evaluation.

C. Derivation of the factorized $(p, 2p)$ amplitude

In this section we wish to indicate how the amplitude of Eq. (3.22) may be factorized into a "doorway" part and a spectroscopic factor. This factorization follows from a specific model for the reaction mechanism (the impulse approximation) and is not derived from first principles. Inspecting the structure of Eq. (3.22) we note that we can assume that the nucleons with momenta $\vec{k}_1$ and $\vec{k}_2$ are the struck nucleon and the projectile.

For the moment we will neglect secondary production, that is, we will neglect the creation of additional continuum nucleons by the *final* nucleons

on their passage through the target. With that assumption we see that it is possible to use only a portion of $|\Psi^{(+)}_{\vec{k},A}\rangle$ to define a doorway approximation to Eq. (3.22). Indeed, let us recall that in addition to the operator P, we have previously[9] defined the projection operator Q_1,

$$Q_1 = \frac{1}{2}\sum_B \iint |Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle\, d\vec{k}_1 d\vec{k}_2 \langle Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}| \,, \tag{3.24}$$

which was useful in the theory of the optical potential. In terms of this operator our doorway model may be stated as

$$\langle \vec{k}_1,\vec{k}_2,\Lambda|T|\vec{k}\rangle \approx \langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|(H-\mathcal{K})(P+Q_1)|\Psi^{(+)}_{\vec{k},A}\rangle \,. \tag{3.25}$$

The terms contained in Eq. (3.22) but neglected in Eq. (3.25) are necessary to describe secondary production by the projectile or struck particle as they leave the nucleus. However, it is common practice to treat these features by introducing optical potentials for the outgoing particles.[15] We will return to these questions at a later stage. However, it should be emphasized that in a $(p,2p)$ experiment the experimental apparatus is usually arranged in such a manner as to reduce the influence upon the cross section of the portions of the total wave function which we have ignored in our doorway model.

From a formal point of view we may always write $1 = P + Q_1 + q$ and write the formal equation

$$|\Psi^{(+)}_{\vec{k},A}\rangle = [(P+Q_1) + \left(\frac{1}{E-qHq+i\epsilon}\right) qH(P+Q_1)]|\Psi^{(+)}_{\vec{k},A}\rangle \tag{3.26}$$

which may be approximated in lowest order by the first term

$$|\Psi^{(+)}_{\vec{k},A}\rangle \simeq (P+Q_1)|\Psi^{(+)}_{\vec{k},A}\rangle \tag{3.27}$$

leading to Eq. (3.25). More generally, therefore,

$$\langle \vec{k}_1,\vec{k}_2,\Lambda|T|\vec{k}\rangle = \langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|[(H-\mathcal{K})+(H-\mathcal{K})\frac{q}{E-qHq+i\epsilon}(H-\mathcal{K})](P+Q_1)|\Psi^{(+)}_{\vec{k},A}\rangle \,. \tag{3.28}$$

However, we will discuss the simpler form, Eq. (3.25).

Let us consider the first term of Eq. (3.25):

$$\langle Y^{(-)}_{\vec{k}_1,k_2,\Lambda}|(H-\mathcal{K})P|\Psi^{(+)}_{\vec{k},A}\rangle = \int \langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|H|X^{(+)}_{\vec{k}',A}\rangle \langle \chi^{(+)}_{\vec{k}'}|\psi^{(+)}_{\vec{k}}\rangle\, d\vec{k}' \,, \tag{3.29}$$

where $\langle \chi^{(+)}_{\vec{k}'}|\psi^{(+)}_{\vec{k}}\rangle$ is the *optical-model wave function* defined in Ref. 9 through the relation

$$P|\Psi^{(+)}_{\vec{k},A}\rangle = \int |X^{(+)}_{\vec{k}',A}\rangle\, d\vec{k}' \langle X^{(+)}_{\vec{k}',A}|\Psi^{(+)}_{\vec{k},A}\rangle = \int |X^{(+)}_{\vec{k}',A}\rangle\, d\vec{k}' \langle \chi^{(+)}_{\vec{k}'}|\psi^{(+)}_{\vec{k}}\rangle \,. \tag{3.30}$$

The matrix element of H appearing on the right-hand side of Eq. (3.29) may be evaluated by inserting a representation of unity in the space of $(A+1)$-body wave functions

$$1 = P + Q_1 + q \tag{3.31}$$

into Eq. (3.29). (Some discussion of the structure of q appears in Ref. 9 for the case in which we represent unity in terms of states characterized by the number of holes we make in the correlated target $|\Phi_A\rangle$.)

Now, since we have required $\langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|P = 0$, we have

$$\begin{aligned}\langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|H|X^{(+)}_{\vec{k}',A}\rangle &\simeq \langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|(P+Q_1)H|X^{(+)}_{\vec{k}',A}\rangle \simeq \langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|Q_1 H|X^{(+)}_{\vec{k}',A}\rangle \\ &\simeq \sum_B (S_{\Lambda,B})^{1/2}\langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,B}|H|X^{(+)}_{\vec{k}',A}\rangle \simeq \sum_B (S_{\Lambda,B})^{1/2}\langle \hat{Y}^{(-)}_{\vec{k}_1,\vec{k}_2,B}|H|\chi^{(+)}_{\vec{k}',A}\rangle \\ &\simeq \sum_{B,\beta} (S_{\Lambda,B})^{1/2}\langle \chi^{(-)}_{\vec{k}_1}\chi^{(-)}_{\vec{k}_2}|v|\chi^{(+)}_{\vec{k}'}\beta\rangle_A \langle \beta|\rho|\phi_B\rangle \rho_B^{-1/2} \,.\end{aligned} \tag{3.32}$$

Here we have used the fact that since H is at most a two-body operator its matrix elements between $|X^{(+)}_{\vec{k},A}\rangle$ and the states in q are small.

In Eq. (3.32) $\langle \chi^{(-)}_{\vec{k}_1}\chi^{(-)}_{\vec{k}_2}|v|\chi^{(+)}_{\vec{k}'}\beta\rangle_A$ is an antisymmetrized matrix element of the two-body potential v

and $\langle\beta|\rho|\phi_B\rangle$ is the density matrix associated with the target. This matrix may be assumed to be diagonal in the subspace of the bound states. Also we may take $\langle\chi^{(+)}_{\vec{k}}|\rho|\phi_B\rangle = 0$, so that we have

$$\langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|H|X^{(+)}_{\vec{k}',A}\rangle \simeq \sum_B (S_{\Lambda,B})^{1/2}\langle\chi^{(-)}_{\vec{k}_1}\chi^{(-)}_{\vec{k}_2}|v|\chi^{(+)}_{\vec{k}'}\phi_B\rangle_A\rho_B^{1/2}\,. \tag{3.33}$$

In deriving Eqs. (3.32) and (3.33) we have neglected the small quantities δ and γ which are necessary to relate the $|Y^{(-)}_{\vec{k}_1,\vec{k}_2,B}\rangle$ to the $|\hat{Y}^{(-)}_{\vec{k}_1,\vec{k}_2,B}\rangle$ and the $|X^{(+)}_{\vec{k},A}\rangle$ to the $|\chi^{(+)}_{\vec{k},A}\rangle$, respectively.

Inserting our approximate result, Eq. (3.33), into Eq. (3.29), we have

$$\langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|(H-\mathcal{K})P|\Psi^{(+)}_{\vec{k},A}\rangle \simeq \sum_B (S_{\Lambda,B})^{1/2}\langle\chi^{(-)}_{\vec{k}_1}\chi^{(-)}_{\vec{k}_2}|v_{12}|\psi^{(+)}_{\vec{k}}\phi_B\rangle_A\rho_B^{1/2}\,. \tag{3.34}$$

It is clear from Eq. (3.34) that in the case of the singular or very strong interactions appropriate to nuclear physics, we must consider additional terms in the transition amplitude. The additional terms permit us to replace v by some appropriate reaction matrix. To this end we consider the second term in Eq. (3.25),

$$\langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|(H-\mathcal{K})Q_1|\Psi^{(+)}_{\vec{k},A}\rangle\,. \tag{3.35}$$

Again this is best developed by inserting a representation of unity. However, in this case it is appropriate to use a representation built on the channel vectors, i.e.,

$$1 = P + \frac{1}{2}\sum_\Lambda\iint|Y^{(+)}_{\vec{k}_1',\vec{k}_2',\Lambda}\rangle\,d\vec{k}_1'\,d\vec{k}_2'\langle Y^{(+)}_{\vec{k}_1',\vec{k}_2',\Lambda}| + \cdots\,. \tag{3.36}$$

Noting that $PQ_1 = 0$, we see that we need quantities of the form

$$\langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|(H-\mathcal{K})|Y^{(+)}_{\vec{k}_1',\vec{k}_2',\Lambda'}\rangle$$

and

$$\langle Y^{(+)}_{\vec{k}_1',\vec{k}_2',\Lambda'}|Q_1|\Psi^{(+)}_{\vec{k},A}\rangle \simeq \sum_B (S_{\Lambda',B})^{1/2}\langle Y^{(+)}_{\vec{k}_1',\vec{k}_2',B}|\Psi^{(+)}_{\vec{k},A}\rangle\,. \tag{3.37}$$

The quantity

$$\langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,B}|(H-\mathcal{K}_0)|Y^{(+)}_{\vec{k}_1',\vec{k}_2',B'}\rangle$$

with

$$Q_1\mathcal{K}_0Q_1 \equiv \frac{1}{2}\sum_B\int|Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}\rangle(\epsilon_{\vec{k}_1}+\epsilon_{\vec{k}_2}-\epsilon_B+E_A) \times\langle Y^{(+)}_{\vec{k}_1,\vec{k}_2,B}|\,d\vec{k}_1 d\vec{k}_2 \tag{3.38}$$

has been discussed at great length in Ref. 9. The quantity $Q_1\mathcal{K}_0Q_1$ may be termed the (projected) *doorway* Hamiltonian and is to be contrasted to the *channel* Hamiltonian defined above,

$$\mathcal{K} = P\mathcal{K}P + \frac{1}{2}\sum_\Lambda\iint|Y^{(+)}_{\vec{k}_1,\vec{k}_2,\Lambda}\rangle(\epsilon_{\vec{k}_1}+\epsilon_{\vec{k}_2}+E^\Lambda_{A-1} \times\langle Y^{(+)}_{\vec{k}_1,\vec{k}_2,\Lambda}|\,d\vec{k}_1 d\vec{k}_2\,. \tag{3.39}$$

Again, dropping the matrices δ, we have, following Ref. 9:

$$\begin{aligned}\langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|(H-\mathcal{K})|Y^{(+)}_{\vec{k}_1',\vec{k}_2',\Lambda'}\rangle &\approx \delta_{\Lambda,\Lambda'}\langle\Phi^\Lambda_{A-1}|\eta_{\vec{k}_2}\eta_{\vec{k}_1}[V,\eta^\dagger_{\vec{k}_1'}\eta^\dagger_{\vec{k}_2'}]|\Phi^\Lambda_{A-1}\rangle\\ &\approx \delta_{\Lambda,\Lambda'}\langle\chi^{(-)}_{\vec{k}_1}\chi^{(-)}_{\vec{k}_2}|v|\chi^{(+)}_{\vec{k}_1'}\chi^{(+)}_{\vec{k}_2'}\rangle_A\,.\end{aligned} \tag{3.40}$$

To complete this portion of our calculation we still need the quantity $\langle Y^{(+)}_{\vec{k}_1',\vec{k}_2',B}|\Psi^{(+)}_{\vec{k},A}\rangle$ which appears in Eq. (3.37). Again we will build our development using the methods discussed in Ref. 9. We write $Q = 1-P$, and

$$Q|\Psi^{(+)}_{\vec{k},A}\rangle = \left(\frac{1}{E-QHQ+i\epsilon}\right)QHP|\Psi^{(+)}_{\vec{k},A}\rangle\,. \tag{3.41}$$

As was discussed in Ref. 9 in some detail, we may approximate

$$\frac{Q}{E-QHQ+i\epsilon} = \frac{Q_1}{E-Q_1HQ_1+i\epsilon} + \cdots\,, \tag{3.42}$$

and also put

$$G_1 \equiv \frac{Q_1}{E-Q_1HQ_1+i\epsilon} = g_0 + g_0(H-\mathcal{K}_0)G_1\,. \tag{3.43}$$

Here

$$g_0 = \left(\frac{Q_1}{E-Q_1\mathcal{K}_0Q_1+i\epsilon}\right)$$

and $Q_1\mathcal{K}_0Q_1$ is as defined in Eq. (3.38). Now we may write

$$\begin{aligned}\langle Y^{(+)}_{\vec{k}_1',\vec{k}_2',B}|\Psi^{(+)}_{\vec{k},A}\rangle &\simeq \langle Y^{(+)}_{\vec{k}_1',\vec{k}_2',B}|[g_0+g_0(H-\mathcal{K}_0)g_0+\cdots]HP|\Psi^{(+)}_{\vec{k},A}\rangle\\ &\simeq \int\langle Y^{(+)}_{\vec{k}_1',\vec{k}_2',B}|[g_0+g_0(H-\mathcal{K}_0)g_0+\cdots]H|X^{(+)}_{\vec{k}',A}\rangle \times d\vec{k}'\langle\chi^{(+)}_{\vec{k}'}|\psi^{(+)}_{\vec{k}}\rangle\,.\end{aligned} \tag{3.44}$$

It was shown in Ref. 9 that it is possible to keep a special class of terms in Eq. (3.44), in particular, just those particle-particle interaction terms necessary to renormalize the matrix elements of v. We obtain

$$\langle Y^{(+)}_{\vec{k}_1',\vec{k}_2',B}|\Psi^{(+)}_{\vec{k},A}\rangle = -\int \langle \chi^{(+)}_{\vec{k}_1'}\chi^{(+)}_{\vec{k}_2'}|\frac{Q_{12}}{e_{12}}v_{12}-\frac{Q_{12}}{e_{12}}v_{12}\frac{Q_{12}}{e_{12}}v_{12}+\cdots|\chi^{(+)}_{\vec{k}'},\phi_B\rangle_A\rho_B^{1/2}\,d\vec{k}'\langle\chi^{(+)}_{\vec{k}'}|\psi^{(+)}_{\vec{k}}\rangle \tag{3.45}$$

with Q_{12} the Pauli principle operator,

$$Q_{12}=\int |\chi^{(+)}_{\vec{k}_1''},\chi^{(+)}_{\vec{k}_2''}\rangle\, d\vec{k}_1''\,d\vec{k}_2''\,\langle\chi^{(+)}_{\vec{k}_1''}\chi^{(+)}_{\vec{k}_2''}|, \tag{3.46}$$

and

$$e_{12}=h_0(1)+h_0(2)-\epsilon_B-\epsilon_{\vec{k}}\,. \tag{3.47}$$

In Eq. (3.47), $h_0(1)$ is the kinetic-energy operator of particle 1, ϵ_B is the (renormalized) Brueckner-Hartree-Fock energy of the occupied state B, and $\epsilon_{\vec{k}}$ is the kinetic energy of the projectile.

It is now clear that Eq. (3.35) may be written [using Eqs. (3.37), (3.40), and (3.45)]:

$$\langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\Lambda}|(H-\mathcal{H})Q_1|\Psi^{(+)}_{\vec{k},A}\rangle \approx -\sum_B (S_{\Lambda,B})^{1/2}\langle\chi^{(-)}_{\vec{k}_1}\chi^{(-)}_{\vec{k}_2}|v_{12}\frac{Q_{12}}{e_{12}}v_{12}-v_{12}\frac{Q_{12}}{e_{12}}v_{12}\frac{Q_{12}}{e_{12}}v_{12}+\cdots|\psi^{(+)}_{\vec{k}}\phi_B\rangle_A\rho_B^{1/2}\,. \tag{3.48}$$

Finally, therefore, using Eqs. (3.34) and (3.48) we have for Eq. (3.25)

$$\langle\vec{k}_1,\vec{k}_2,\Lambda|T|\vec{k}\rangle=\sum_B (S_{\Lambda,B})^{1/2}\langle\chi^{(-)}_{\vec{k}_1}\chi^{(-)}_{\vec{k}_2}|K_{12}(\epsilon_{\vec{k}}+\epsilon_B)|\psi^{(+)}_{\vec{k}}\phi_B\rangle_A\rho_B^{1/2}, \tag{3.49}$$

where K_{12} is the Brueckner reaction matrix, which satisfies the Bethe-Goldstone equation

$$K_{12}=v_{12}-v_{12}\frac{Q_{12}}{e_{12}}K_{12} \tag{3.50}$$

with only kinetic energies for the particle states. Inspection of Eq. (3.49) will indicate that while the optical-model wave function $|\psi^{(+)}_{\vec{k}}\rangle$ appears on the right side of the expression, the "orthogonality scattering" states appear on the left. This "unphysical" feature has its origin in the neglect of the q space. In particular, in Ref. 9, we considered a representation of unity of the form

$$1=P+Q_1+Q_2+\cdots, \tag{3.51}$$

where Q_2 was built of (appropriately orthogonalized) states of the primary form

$$\eta^\dagger_{k_1}\eta^\dagger_{k_2}\eta^\dagger_{k_3}\eta_{B_1}\eta_{B_2}|\Phi_A\rangle\,.$$

These may be considered as "two-hole" states while Q_1 may be thought of as spanned by "one-hole" states. (This terminology is somewhat deficient since $|\Phi_A\rangle$ is correlated and therefore contains various numbers of "holes" and "particles.")

We may ask how the inclusion of the Q_2 space would modify our considerations. Without belaboring this point, we can see that the inclusion of this space will provide the possibility of including optical potentials for the final particles. Again what is required is a selective summation of interactions—see Fig. 1. In addition, one can consider final-state interactions between the two outgoing protons, but we will not treat this feature in detail here. With these remarks we are led to the simple approximation[16]

$$\langle\vec{k}_1,\vec{k}_2,\Lambda|T|\vec{k}\rangle = \sum_B (S_{\Lambda,B})^{1/2}\langle\psi^{(-)}_{\vec{k}_1}\psi^{(-)}_{\vec{k}_2}|K_{12}(\epsilon_{\vec{k}}+\epsilon_B)|\psi^{(+)}_{\vec{k}}\phi_B\rangle_A\rho_B^{1/2}\,. \tag{3.52}$$

Note that this expression treats the influence of the Pauli principle exactly. Previous theories based on the impulse approximation ignore Pauli effects completely. Such effects are, or course, of relatively greater importance at lower energies.

To make contact with previous approaches[10] this expression can be further simplified by replacing the Brueckner K matrix by the free-scattering matrix

$$K_{12}^{\text{free}}=v_{12}-v_{12}\frac{1}{e_{12}}K_{12}^{\text{free}}\,. \tag{3.53}$$

For convenience, let us define

$$\langle\vec{k}_1,\vec{k}_2,B|T_D|\vec{k}\rangle = \langle\psi^{(-)}_{\vec{k}_1}\psi^{(-)}_{\vec{k}_2}|K_{12}(\epsilon_{\vec{k}}+\epsilon_B)|\psi^{(+)}_{\vec{k}}\phi_B\rangle_A\rho_B^{1/2} \tag{3.54}$$

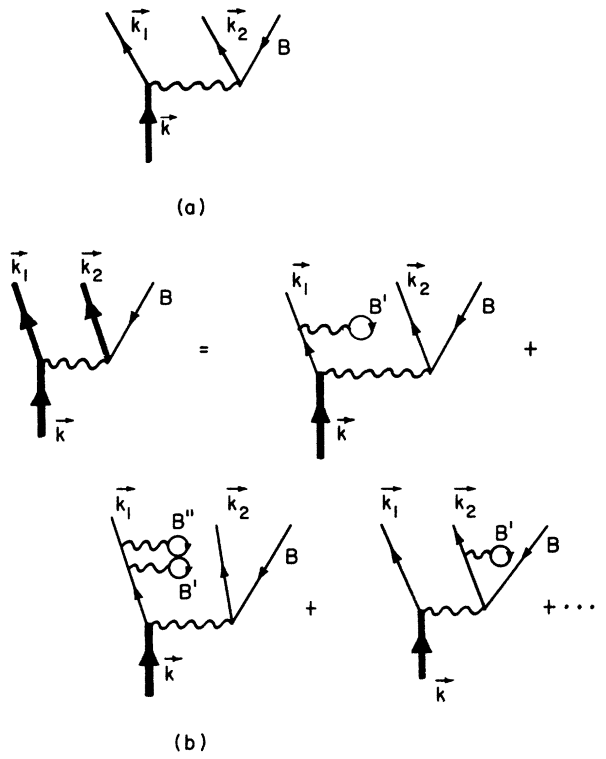

FIG. 1. In these figures a heavy upgoing line represents a particle moving in an optical potential. The light upgoing line represents a particle undergoing orthogonality scattering. Downgoing lines refer to occupied states with wave functions $|\phi_B\rangle$. Wavy lines represent K matrices. (a) This figure represents the matrix element appearing in Eq. (3.49), $\langle \chi^{(-)}_{\vec{k}_1} \chi^{(-)}_{\vec{k}_2} | K_{12} | \psi^{(+)}_{\vec{k}} \phi_B \rangle$. (b) The inclusion of the Q_2 space yields diagrams of the type shown here, where there are two holes (B, B') present in intermediate states. Consideration of this class of diagrams allows one to replace $\langle \chi^{(-)}_{\vec{k}_1} \chi^{(-)}_{\vec{k}_2} | K_{12} | \psi^{(+)}_{\vec{k}} \phi_B \rangle_A$ by $\langle \psi^{(-)}_{\vec{k}_1} \psi^{(-)}_{\vec{k}_2} | K_{12} | \psi^{(+)}_{\vec{k}} \phi_B \rangle_A$ in Eq. (3.49).

so that Eq. (3.52) may be written

$$\langle \vec{k}_1, \vec{k}_2, \Lambda | T | \vec{k} \rangle = \sum_B (S_{\Lambda,B})^{1/2} \langle \vec{k}_1, \vec{k}_2, B | T_D | \vec{k} \rangle . \tag{3.55}$$

We will speak of T_D of Eq. (3.54) as the *doorway* T matrix.

For the final states considered here (viz., those with two continuum particles and a bound residual nucleus), Eq. (3.52) represents our basic factorized result. In the next section we will generalize this result to final channels with *more than two* continuum particles in addition to a bound residual nucleus.

IV. TREATMENT OF FINAL-STATE CHANNELS CONTAINING MORE THAN TWO UNBOUND NUCLEONS

The discussion of the previous section is strictly applicable only to final states $|Y^{(+)}_{\vec{k}_1, \vec{k}_2, \Lambda}\rangle$ of the form given in Eq. (3.16). Recall that $\vec{k}_1$ and $\vec{k}_2$ refer to the two final-state protons and that Λ is a set of quantum numbers specifying a bound state of the remaining $A-1$ particles. In addition to these channels, however, contributions to the $(p, 2p)$ cross section can arise from the more complicated final-state channels which contain more than two continuum nucleons; the only requirement is that, of all the continuum nucleons in the final state, two of them are protons having momenta appropriate to the experimental situation. We will now show that these more complex channels can be treated, within the doorway approximation, in complete analogy with the treatment given in the previous section for the channels spanned by the states, $|Y^{(+)}_{\vec{k}_1, \vec{k}_2, \Lambda}\rangle$.

We will consider first the case of a final state containing three continuum nucleons and described by the channel state $|Y^{(+)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda}\rangle$. The corresponding result for a channel containing any number of continuum particles will then be obvious. The subscripts $\vec{k}_1$, $\vec{k}_2$, and $\vec{k}_3$ appearing on the channel state refer to the momenta of the continuum nucleons, while Λ now represents the quantum numbers of an $(A-2)$-particle bound state. Since we are considering only quasifree scattering we will assume that the two protons in the final state have energies significantly larger than that of the remaining free nucleon. We will denote the momenta of these two "fast" protons as $\vec{k}_1$ and $\vec{k}_2$. The states $|Y^{(+)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda}\rangle$ are fully antisymmetrized and are regarded as forming an orthonormal set with the (previously defined) states $|X^{(+)}_{\vec{k}, A}\rangle$ and $|Y^{(+)}_{\vec{k}_1, \vec{k}_2, \Lambda}\rangle$. The corresponding projection operator is

$$P_3 = \frac{1}{3!} \sum_\Lambda \int\int\int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 \, |Y^{(+)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda}\rangle \langle Y^{(+)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda}| \tag{4.1}$$

and the associated piece of the channel Hamiltonian is

$$\mathcal{H}_3 = \frac{1}{3!} \sum_\Lambda \int\int\int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 \, |Y^{(+)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda}\rangle (\epsilon_{\vec{k}_1} + \epsilon_{\vec{k}_2} + \epsilon_{\vec{k}_3} + E^\Lambda_{A-2}) \langle Y^{(+)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda}| . \tag{4.2}$$

The T-matrix element of interest now [in analogy to Eq. (3.22) for the case of two continuum nucleons] is

$$\begin{aligned}\langle \vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda | T | \vec{k} \rangle &= \langle Y^{(-)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda} | (H - \mathcal{H}) | \Psi^{(+)}_{\vec{k}, A} \rangle \\ &= \langle Y^{(-)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda} | (H - \mathcal{H}_3) | \Psi^{(+)}_{\vec{k}, A} \rangle . \end{aligned} \tag{4.3}$$

If we again make the "doorway" approximation as in the previous section we have

$$\langle \vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda | T | \vec{k} \rangle \approx \langle Y^{(-)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda} | (H - \mathcal{H}_3)(P + Q_1) | \Psi^{(+)}_{\vec{k}, A} \rangle . \tag{4.4}$$

The procedure for evaluating Eq. (4.4) is identical to that used to evaluate $\langle \vec{k}_1, \vec{k}_2, \Lambda | T | \vec{k} \rangle$. The latter was discussed in detail in the last section. We will therefore remark only on several new features that occur in the process of reducing Eq. (4.4) to a form analogous to Eq. (3.52).

Consider first the term involving P. We have

$$\begin{aligned}\langle Y^{(-)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda} | (H - \mathcal{H}_3) P | \Psi^{(+)}_{\vec{k}, A} \rangle &= \langle Y^{(-)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda} | HP | \Psi^{(+)}_{\vec{k}, A} \rangle \\ &= \langle Y^{(-)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda} | (P + Q_1 + \cdots) HP | \Psi^{(+)}_{\vec{k}, A} \rangle \\ &\approx \langle Y^{(-)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda} | Q_1 HP | \Psi^{(+)}_{\vec{k}, A} \rangle \\ &\approx \frac{1}{2} \sum_B \int d\vec{k}_1' d\vec{k}_2' d\vec{k}_3' \langle Y^{(-)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda} | Y^{(+)}_{\vec{k}_1', \vec{k}_2', B} \rangle \langle Y^{(+)}_{\vec{k}_1', \vec{k}_2', B} | H | X^{(+)}_{\vec{k}_3', A} \rangle \\ &\quad \times \langle X^{(+)}_{\vec{k}_3', A} | \Psi^{(+)}_{\vec{k}, A} \rangle \\ &\approx \frac{1}{2} \sum_B \int d\vec{k}_1' d\vec{k}_2' d\vec{k}_3' \langle \hat{Y}^{(-)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda} | \hat{Y}^{(+)}_{\vec{k}_1', \vec{k}_2', B} \rangle \langle \hat{Y}^{(+)}_{\vec{k}_1', \vec{k}_2', B} | H | \chi^{(+)}_{\vec{k}_3', A} \rangle \langle X^{(+)}_{\vec{k}_3', A} | \Psi^{(+)}_{\vec{k}, A} \rangle \\ &\approx \frac{1}{2} \sum_B \int d\vec{k}_1' d\vec{k}_2' \langle \hat{Y}^{(-)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda} | \hat{Y}^{(+)}_{\vec{k}_1', \vec{k}_2', B} \rangle \langle \chi^{(-)}_{\vec{k}_1'} \chi^{(-)}_{\vec{k}_2'} | v | \psi^{(+)}_{\vec{k}} \phi_B \rangle_A \rho_B^{1/2} . \end{aligned} \tag{4.5}$$

This result differs from the corresponding one encountered in the last section only in the factor involving the overlap

$$\langle \hat{Y}^{(-)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda} | \hat{Y}^{(+)}_{\vec{k}_1', \vec{k}_2', B} \rangle .$$

Evaluating this expression we obtain

$$\begin{aligned}\langle \hat{Y}^{(-)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda} | \hat{Y}^{(+)}_{\vec{k}_1', \vec{k}_2', B} \rangle \approx & [\delta(\vec{k}_1' - \vec{k}_1)\delta(\vec{k}_2' - \vec{k}_2) - \delta(\vec{k}_1' - \vec{k}_2)\delta(\vec{k}_2' - \vec{k}_1)](S^{\vec{k}_3}_{\Lambda, B})^{1/2} \\ & - [\delta(\vec{k}_1' - \vec{k}_1)\delta(\vec{k}_2' - \vec{k}_3) - \delta(\vec{k}_1' - \vec{k}_3)\delta(\vec{k}_2' - \vec{k}_1)](S^{\vec{k}_2}_{\Lambda, B})^{1/2} \\ & + [\delta(\vec{k}_1' - \vec{k}_2)\delta(\vec{k}_2' - \vec{k}_3) - \delta(\vec{k}_1' - \vec{k}_3)\delta(\vec{k}_2' - \vec{k}_2)](S^{\vec{k}_1}_{\Lambda, B})^{1/2} , \end{aligned} \tag{4.6}$$

where we have defined

$$(S^{\vec{k}}_{\Lambda, B})^{1/2} \equiv \langle \Phi^{\Lambda}_{A-2} | \eta_{\vec{k}} \eta_B | \Phi_A \rangle \rho_B^{-1/2} . \tag{4.7}$$

In Eq. (4.6) we have, in the interests of notational simplicity, suppressed pairs of S-matrix elements connecting the orthogonality scattering states $|\chi^{(-)}_{\vec{k}'}\rangle$ and $|\chi^{(+)}_{\vec{k}}\rangle$. These merely serve to convert the $\chi^{(+)}_{\vec{k}}$'s, which would otherwise occur on the left-hand side of the matrix element of v in Eq. (4.5), into $\chi^{(-)}_{\vec{k}'}$'s, and this fact has been used in writing Eq. (4.5) in the form given. Combining Eqs. (4.6) and (4.5) we finally obtain

$$\begin{aligned}\langle Y^{(-)}_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \Lambda} | (H - \mathcal{H}_3) P | \Psi^{(+)}_{\vec{k}, A} \rangle \approx & \sum_B (S^{\vec{k}_3}_{\Lambda, B})^{1/2} \langle \chi^{(-)}_{\vec{k}_1} \chi^{(-)}_{\vec{k}_2} | v | \psi^{(+)}_{\vec{k}} \phi_B \rangle_A \rho_B^{1/2} \\ & - \sum_B (S^{\vec{k}_2}_{\Lambda, B})^{1/2} \langle \chi^{(-)}_{\vec{k}_1} \chi^{(-)}_{\vec{k}_3} | v | \psi^{(+)}_{\vec{k}} \phi_B \rangle_A \rho_B^{1/2} \\ & + \sum_B (S^{\vec{k}_1}_{\Lambda, B})^{1/2} \langle \chi^{(-)}_{\vec{k}_2} \chi^{(-)}_{\vec{k}_3} | v | \psi^{(+)}_{\vec{k}} \phi_B \rangle_A \rho_B^{1/2} . \end{aligned} \tag{4.8}$$

Similarly, evaluating the term involving Q_1 we obtain

$$\langle Y^{(-)}_{\vec{k}_1,\vec{k}_2,\vec{k}_3,\Lambda}|(H-\mathcal{K}_3)Q_1|\Psi^{(+)}_{\vec{k},A}\rangle \approx \frac{1}{3!}\sum_{\Lambda'}\int\int\int d\vec{k}_1' d\vec{k}_2' d\vec{k}_3' \times\langle \hat{Y}^{(-)}_{\vec{k}_1,\vec{k}_2,\vec{k}_3,\Lambda}|(H-\mathcal{K}_3)|\hat{Y}^{(+)}_{\vec{k}_1',\vec{k}_2',\vec{k}_3',\Lambda'}\rangle\langle \hat{Y}^{(+)}_{\vec{k}_1',\vec{k}_2',\vec{k}_3',\Lambda'}|Q_1|\Psi^{(+)}_{\vec{k},A}\rangle \quad (4.9)$$

with

$$\langle \hat{Y}^{(+)}_{\vec{k}_1',\vec{k}_2',\vec{k}_3',\Lambda}|Q_1|\Psi^{(+)}_{\vec{k},A}\rangle \approx \frac{1}{2}\sum_B\int d\vec{k}_1''d\vec{k}_2''\langle \hat{Y}^{(+)}_{\vec{k}_1',\vec{k}_2',\vec{k}_3',\Lambda}|\hat{Y}^{(+)}_{\vec{k}_1'',\vec{k}_2'',B}\rangle\langle \hat{Y}^{(+)}_{\vec{k}_1'',\vec{k}_2'',B}|\Psi^{(+)}_{\vec{k},A}\rangle . \quad (4.10)$$

The matrix element of $H-\mathcal{K}_3$ which occurs in Eq. (4.9) involves interactions between the "slow" nucleon (i.e., that of momentum $\vec{k}_3$) and each of the two "fast" nucleons, and between the "slow" nucleon and the residual nucleus, in addition to the interactions between the two outgoing nucleons. At the present stage, we are only interested in interactions of the latter type, namely those which must be used to develop the reaction matrix from the bare v interaction appearing in Eq. (4.8). The remainder of Eq. (4.9), written explicitly in Eq. (4.10), involves the product of a spectroscopic amplitude, indentical to that given in Eq. (4.6), with the amplitude of $\Psi^{(+)}_{\vec{k},A}$ projected onto the Q_1 space. The latter was evaluated in the last section. Thus we see that Eq. (4.9) contains everything we need to renormalize the bare v interactions occurring in Eq. (4.8). Combining Eqs. (4.8) and (4.9) we then finally have for Eq. (4.4)

$$\begin{aligned}\langle \vec{k}_1,\vec{k}_2,\vec{k}_3,\Lambda|T|\vec{k}\rangle \approx{}& \sum_B (S^{\vec{k}_3}_{\Lambda,B})^{1/2}\langle \chi^{(-)}_{\vec{k}_1}\chi^{(-)}_{\vec{k}_2}|K(\epsilon_{\vec{k}}+\epsilon_B)|\psi^{(+)}_{\vec{k}}\phi_B\rangle_A\rho_B^{1/2}\\ &-\sum_B (S^{\vec{k}_2}_{\Lambda,B})^{1/2}\langle \chi^{(-)}_{\vec{k}_1}\chi^{(-)}_{\vec{k}_3}|K(\epsilon_{\vec{k}}+\epsilon_B)|\psi^{(+)}_{\vec{k}}\phi_B\rangle_A\rho_B^{1/2}\\ &+\sum_B (S^{\vec{k}_1}_{\Lambda,B})^{1/2}\langle \chi^{(-)}_{\vec{k}_2}\chi^{(-)}_{\vec{k}_3}|K(\epsilon_{\vec{k}}+\epsilon_B|\psi^{(+)}_{\vec{k}}\phi_B\rangle_A\rho_B^{1/2}. \end{aligned} \quad (4.11)$$

If we recall that we have assumed $\vec{k}_1$ and $\vec{k}_2$ to be the large momenta, we can see that the last two terms of Eq. (4.11) will be considerably smaller than the first in the energy region of interest ($\epsilon_{\vec{k}} > 150$ MeV). There are two reasons for this. First of all the factors $(S^{\vec{k}_i}_{\Lambda,B})^{1/2}$ $(i=1,2)$ are much smaller than $(S^{\vec{k}_3}_{\Lambda,B})^{1/2}$, since $|\vec{k}_3| \ll |\vec{k}_i|$ $(i=1,2)$, and the spectroscopic amplitudes decrease rapidly with increasing momentum. Secondly, momentum conservation applied to the second and third K-matrix elements requires

$$\vec{k}_i+\vec{k}_3 \simeq \vec{k}_B+\vec{k} \quad (i=1,2), \quad (4.12)$$

where $\vec{k}_B$ is a typical momentum component of ϕ_B. But $|\vec{k}_B| \ll |\vec{k}|$ and $|\vec{k}_3| \ll |\vec{k}_i|$ $(i=1,2)$, so that momentum conservation requires

$$\vec{k}_i \approx \vec{k}. \quad (4.13)$$

However, the geometry of most $(p,2p)$ experiments is such that $\vec{k}_i$ $(i=1,2)$ is quite different from the incident momentum $\vec{k}$. For such geometries, then, *both* the spectroscopic factors and the K-matrix elements occurring in Eq. (4.11) are small, and we have

$$\langle \vec{k}_1,\vec{k}_2,\vec{k}_3,\Lambda|T|\vec{k}\rangle \approx \sum_B (S^{\vec{k}_3}_{\Lambda,B})^{1/2}\langle \chi^{(-)}_{\vec{k}_1}\chi^{(-)}_{\vec{k}_2}|K(\epsilon_{\vec{k}}+\epsilon_B)|\psi^{(+)}_{\vec{k}}\phi_B\rangle_A\rho_B^{1/2}. \quad (4.14)$$

Thus we have again obtained a factorized amplitude.

It is clear that the approximations leading from Eq. (4.11) to Eq. (4.14) are valid only at high energy. This demonstrates the weakness of the usual analyses of $(p,2p)$ reactions which view the quasifree process as the formation of a hole state, and thus always arrive at a factorized amplitude. Our results show that in situations where two of the momenta are not clearly larger than the third, all three components of the T matrix in Eq. (4.11) contribute significantly. In this case the theory is considerably more complicated. This more complicated situation is indicated schematically in Fig. 2.

The extension of the results of this section to final states with four or more continuum nucleons is obvious since the considerations required to obtain the contribution of such states to the $(p,2p)$ cross section are identical to those needed for the three-continuum-nucleon case.

It is important to keep in mind that all the considerations of this section have been directed toward the description of final states in which the continuum nucleons other than the two protons of interest arise from the decay of an unstable hole state (the doorway assumption). In particular, continuum nucleons arising from multiple scattering of the two "fast" protons or from the

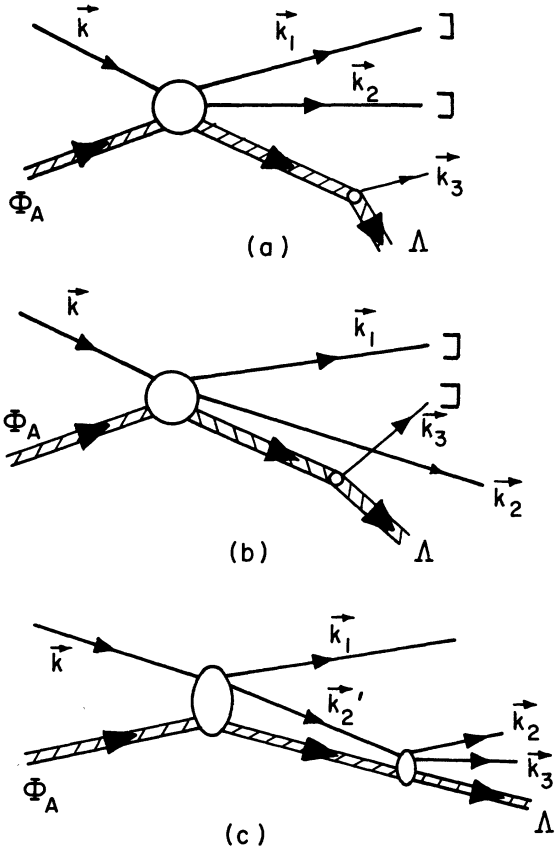

FIG. 2. (a) The amplitude associated with the factorized T matrix for $(p, 2p)$ scattering leading to a four-particle final state. Particles with the momenta $\vec{k}_1$ and $\vec{k}_2$ are detected. (b) A process which can interfere with that given in (a) and which leads to a nonfactorizable result. Here the second particle detected arises not from the quasifree collision but from some other process. (c) A pictorial representation of secondary production which provides some background to the simple diagram in (a).

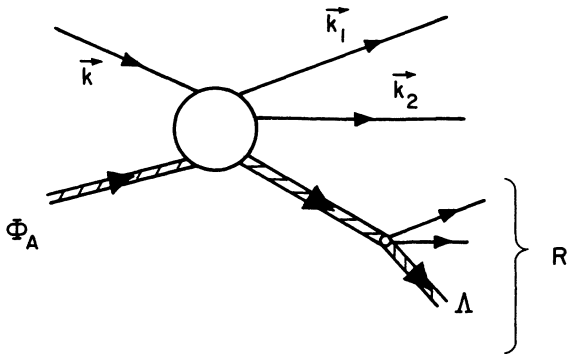

FIG. 3. The amplitude for the $(p, 2p)$ reaction in a sequential model. Here $\vec{k}_1$ and $\vec{k}_2$ denote the momenta of the fast protons in the final state. The residual system denoted by R may have several continuum particles plus a bound fragment Λ.

ejection of strongly correlated nucleons by the incident proton have been neglected. This is, of course, consistent with the quasifree model for the reaction.

V. CALCULATION OF THE CROSS SECTION

We consider using the model developed in the previous sections for a $(p, 2p)$ reaction in which the energy E_R is the total energy of the residual system—see Fig. 3. This system may be a bound state of $(A-1)$ particles, or more generally, may contain several fragments. We will also write $E_R = E_{\text{c.m.}} + E_X$, where $E_{\text{c.m.}}$ is the energy associated with the center-of-mass motion of the residual system.

We assume that we observe two nucleons (protons) with momenta $\vec{k}_1$ and $\vec{k}_2$. The conditions of energy and momentum conservation allow for the determination of E_X and $E_{\text{c.m.}}$. We have

$$\vec{k} = \vec{k}_1 + \vec{k}_2 + \vec{K}_{\text{c.m.}} \tag{5.1}$$

and

$$\epsilon_{\vec{k}} + E_A = \epsilon_{\vec{k}_1} + \epsilon_{\vec{k}_2} + E_R , \tag{5.2}$$

where $\vec{k}$ is the momentum of the incident proton, $\epsilon_{\vec{k}}$ the associated energy, and $\vec{K}_{\text{c.m.}}$ is the total momentum of the residual system, etc.

To obtain the cross section for the observation of the momenta $\vec{k}_1$ and $\vec{k}_2$ we must calculate the sum of all cross sections with *at least* two particles in the final state and integrate over the unobserved momenta. We will keep only the terms in which the interaction of the incident particle with a target nucleon leads to the final momenta $\vec{k}_1$ and $\vec{k}_2$. In these terms, if there are more than two particles in the final state, the other momenta appear in the spectroscopic amplitudes. Thus,

$$\frac{d\sigma}{d\Omega_1 d\Omega_2} \approx K \sum_{\Lambda} \sum_{B,B'} \Big[(S_{\Lambda,B})^{1/2} (S_{\Lambda,B'})^{1/2} \delta(E_\Lambda - E_R) + \int (S^{\vec{k}_3}_{\Lambda,B})^{1/2} (S^{\vec{k}_3}_{\Lambda,B'})^{1/2} \delta(E_\Lambda + \epsilon_{\vec{k}_3} - E_R) d\vec{k}_3 + \int\int (S^{\vec{k}_3\vec{k}_4}_{\Lambda,B})^{1/2} (S^{\vec{k}_3\vec{k}_4}_{\Lambda,B'})^{1/2} \delta(E_\Lambda + \epsilon_{\vec{k}_3} + \epsilon_{\vec{k}_4} - E_R) d\vec{k}_3 d\vec{k}_4 + \cdots \Big]$$
$$\times \langle \psi^{(-)}_{\vec{k}_1} \psi^{(-)}_{\vec{k}_2} | K_{12} | \psi^{(+)}_{\vec{k}} \phi_B \rangle_A \rho_B^{1/2} \langle \psi^{(-)}_{\vec{k}_1} \psi^{(-)}_{\vec{k}_2} | K_{12} | \psi^{(+)}_{\vec{k}} \phi_{B'} \rangle^*_A \rho_{B'}^{1/2} . \tag{5.3}$$

In Eq. (5.3), κ is a kinematical factor. The summation over Λ and the integration over the unobserved momenta, $\vec{k}_3, \vec{k}_4, \ldots$, serves to sum the various contributions to the $(p, 2p)$ cross section. In this equation we have also included the distorted waves $\psi^{(-)}_{\vec{k}_1}$ and $\psi^{(-)}_{\vec{k}_2}$ describing the refraction and absorption of the outgoing particles with momenta $\vec{k}_1$ and $\vec{k}_2$.[16] Energy conservation is indicated by the inclusion of the appropriate δ functions which restrict the phase space available for the unobserved particles. [The δ function $\delta(E_\Lambda - E_R)$ appears in the first term of Eq. (5.3) since the quantity E_Λ is a continuous variable containing both the center-of-mass energy and the binding energy of the fragment Λ.]

It is useful to define the quantity in the brackets in Eq. (5.3), summed over Λ, as $s_{BB'}(E_R)$. In certain kinematical regions the cross section could be dominated by the ejection of a particle from a particular occupied state $|\phi_B\rangle$. We may consider the cross section to be additively composed of such terms; that is, we may neglect interference of amplitudes containing different hole states. This may be a good approximation if the energy transferred into the excitation of the residual system is relatively small. In this approximation we may write

$$\frac{d\sigma}{d\Omega_1 d\Omega_2} \simeq \kappa \sum_B s_B(E_R) |\langle \psi^{(-)}_{\vec{k}_1} \psi^{(-)}_{\vec{k}_2} | K_{12} | \psi^{(+)}_{\vec{k}} \phi_B \rangle_A|^2 \rho_B \tag{5.4}$$

with the definition

$$s_B(E_R) \equiv s_{BB}(E_R). \tag{5.5}$$

The factor ρ_B which appears in Eq. (5.4) provides a measure of the probability of finding the struck particle in the orbit $|\phi_B\rangle$; for a nucleus with short-range correlations, this factor is expected to be about 0.8–0.9. Note the structure of $s_B(E_R)$,

$$\begin{aligned} s_B(E_R) = \frac{1}{\rho_B} \sum_\Lambda \Big[& \langle \Phi_A | \eta_B^\dagger | \Phi^\Lambda_{A-1} \rangle \langle \Phi^\Lambda_{A-1} | \eta_B | \Phi_A \rangle \, \delta(E_\Lambda - E_R) \\ & + \int d\vec{k}_3 \langle \Phi_A | \eta_B^\dagger \eta^\dagger_{\vec{k}_3} | \Phi^\Lambda_{A-2} \rangle \langle \Phi^\Lambda_{A-2} | \eta_{\vec{k}_3} \eta_B | \Phi_A \rangle \, \delta(E_\Lambda + \epsilon_{\vec{k}_3} - E_R) \\ & + \int d\vec{k}_3 d\vec{k}_4 \langle \Phi_A | \eta_B^\dagger \eta^\dagger_{\vec{k}_3} \eta^\dagger_{\vec{k}_4} | \Phi^\Lambda_{A-3} \rangle \langle \Phi^\Lambda_{A-3} | \eta_{\vec{k}_4} \eta_{\vec{k}_3} \eta_B | \Phi_A \rangle \, \delta(E_\Lambda + \epsilon_{\vec{k}_3} + \epsilon_{\vec{k}_4} - E_R) + \cdots \Big], \end{aligned} \tag{5.6}$$

which may be rewritten in the suggestive form

$$s_B(E_R) = \frac{1}{\rho_B} \sum_\Lambda \left\{ \langle \Phi_A | \eta_B^\dagger \left[|\Phi^\Lambda_{A-1}\rangle \, \delta(E_\Lambda - E_R) \langle \Phi^\Lambda_{A-1}| + \int \eta^\dagger_{\vec{k}_3} |\Phi^\Lambda_{A-2}\rangle \delta(E_\Lambda + \epsilon_{\vec{k}_3} - E_R) d\vec{k}_3 \langle \Phi^\Lambda_{A-2} | \eta_{\vec{k}_3} + \cdots \right] \eta_B | \Phi_A \rangle \right\}. \tag{5.7}$$

As a simple example, let us assume that *only* the bound states, $|\Phi^\Lambda_{A-1}\rangle$, of the $(A-1)$-body Hamiltonian have significant overlap with the doorway state $\eta_B|\Phi_A\rangle$. *On the basis of that assumption,* we may write

$$s_B(E_R) = \hat{s}_B(E_R) \equiv \langle \Phi_A | \eta_B^\dagger \delta(H - E_R) \eta_B | \Phi_A \rangle \rho_B^{-1}. \tag{5.8}$$

To the extent that Eq. (5.8) is valid, it is useful to note that the quantity $\hat{s}_B(E_R)$ satisfies an energy-weighted sum rule,[17]

$$\int (E_R - E_A) \hat{s}_B(E_R) dE_R = \langle \Phi_A | \eta_B^\dagger [H, \eta_B] | \Phi_A \rangle = -\epsilon_B, \tag{5.9}$$

where ϵ_B is the renormalized Brueckner-Hartree-Fock energy associated with the orbit $|\phi_B\rangle$ and

$$\epsilon_B = \langle \phi_B | t | \phi_B \rangle + \sum_{B'} \langle \phi_B \phi_{B'} | K_{12}(\epsilon_B + \epsilon_{B'}) | \phi_B \phi_{B'} \rangle_A \rho_{B'}. \tag{5.10}$$

In those theories[10, 18] in which one does not take into account the "doorway" character of $|\phi_B\rangle$ in the manner we have done, one assumes that the cross section is proportional to the $\hat{s}_B(E_R)$ of Eq. (5.8). It is of interest to ask what approximations need to be made to obtain a cross section proportional to $\hat{s}_B(E_R)$, rather than $s_B(E_R)$. Using our methods, we see that what is desired is a different channel Hamiltonian based on states of the type

$$\eta^\dagger_{\vec{k}_1} \eta^\dagger_{\vec{k}_2} |\Phi^\Lambda_{A-1}\rangle, \quad \eta^\dagger_{\vec{k}_1} \eta^\dagger_{\vec{k}_2} |\Phi^{(+)}_{\vec{k}_3, \Lambda(A-2)}\rangle,$$

$$\eta^\dagger_{\vec{k}_1} \eta^\dagger_{\vec{k}_2} |\Phi^{(+)}_{\vec{k}_3, \vec{k}_4, \Lambda(A-3)}\rangle, \quad \text{etc.},$$

where the states $|\Phi\rangle$ here are eigenfunctions of the total $(A-1)$-body Hamiltonian, including *both* continuum and bound states. The use of such channel states is possible if we consider states of the P space with *only* two very large momenta (which may be denoted as $\vec{k}_1$ and $\vec{k}_2$). In that limit the

new channel states would be approximately orthonormal; for example

$$\langle \Phi^{(+)}_{\vec{k}_3',\vec{k}_4',\Lambda'(A-3)} | \eta_{\vec{k}_2'}\eta_{\vec{k}_1'}\eta^{\dagger}_{\vec{k}_1}\eta^{\dagger}_{\vec{k}_2} | \Phi^{(+)}_{\vec{k}_3,\vec{k}_4,\Lambda(A-3)} \rangle$$
$$\simeq [\,\delta(\vec{k}_1-\vec{k}_1')\delta(\vec{k}_2-\vec{k}_2') - \delta(\vec{k}_1-\vec{k}_2')\delta(\vec{k}_2-\vec{k}_1')]$$
$$\times \langle \Phi^{(+)}_{\vec{k}_3',\vec{k}_4',\Lambda'(A-3)} | \Phi^{(+)}_{\vec{k}_3,\vec{k}_4,\Lambda(A-3)} \rangle . \quad (5.11)$$

Since, by assumption, the $|\Phi\rangle$ are eigenfunctions of H, then

$$\langle \Phi^{(+)}_{\vec{k}_3',\vec{k}_4',\Lambda'(A-3)} | \Phi^{(+)}_{\vec{k}_3,\vec{k}_4,\Lambda(A-3)} \rangle$$
$$= \frac{\delta_{\Lambda,\Lambda'}}{2} [\,\delta(\vec{k}_3-\vec{k}_3')\delta(\vec{k}_4-\vec{k}_4') - \delta(\vec{k}_3-\vec{k}_4')\delta(\vec{k}_4-\vec{k}_3')] . \quad (5.12)$$

Using this latter scheme, in which there is a clear distinction between the large momenta and the small, we could obtain a cross section proportional to the $s_B(E_R)$. However, this approach would make a discussion of the production of secondary particles by the fast particles leaving the nucleus extremely difficult, and therefore, it is not appropriate to a more complete treatment of the $(p, 2p)$ reaction.

To the extent that Eq. (5.4) provides a reasonable description of the $(p, 2p)$ cross section one may learn something about the momentum content of $|\phi_B\rangle$. If we keep only a single term of Eq. (5.4) we have

$$\frac{d\sigma}{d\Omega_1 d\Omega_2} \simeq \kappa s_B(E_R) \left| \int \langle \psi^{(-)}_{\vec{k}_1}\psi^{(-)}_{\vec{k}_2} | K_{12} | \psi^{(+)}_{\vec{k}}\vec{k}' \rangle_A \right.$$
$$\left. \times d\vec{k}' \langle \vec{k}' | \phi_B \rangle \right|^2 \rho_B . \quad (5.13)$$

In the high-energy limit

$$\frac{d\sigma}{d\Omega_1 d\Omega_2} \to \kappa s_B(E_R) |\langle \vec{k}_1\vec{k}_2 | K_{12} | \vec{k}\vec{k}' \rangle_A|^2 P(\vec{k}')\rho_B \quad (5.14)$$

with

$$P(\vec{k}') = |\langle \vec{k}' | \phi_B \rangle|^2 \quad (5.15)$$

and

$$\vec{k}' = \vec{k}_1 + \vec{k}_2 - \vec{k} . \quad (5.16)$$

It is clear that Eq. (5.14) can only be obtained if all continuum wave functions are replaced by plane waves. Otherwise the quantum-mechanical interference effects between the various momentum components of $\langle \vec{k}' | \phi_B \rangle$ enter as in Eq. (5.13). There is no way to obtain a factorized form involving the momentum distribution and the free nucleon-nucleon cross section without serious approximations in the treatment of distortion effects. For a highly simplified treatment of distortion yielding a factorized form analogous to Eq. (5.14), see Ref. 10.

VI. SUMMARY AND CONCLUSIONS

Several of the advantages of our approach to rearrangement reactions have been discussed in the first two sections of this paper. In this section we wish to summarize some of the novel features of our methods when applied to the description of the $(p, 2p)$ reaction. We will also indicate several open problems.

To some extent previous discussions of the $(p, 2p)$ reaction have not been placed within the context of a reaction theory capable, in principle, of dealing with the full complexity of the actual physical process. It is useful to be able to understand the series of approximations which lead to the factorized (or sequential) amplitude for this process. In the theory developed in this paper we have carried through a derivation of the factorized amplitudes while indicating the nature of the approximations necessary to obtain the standard result. Some of the novel features of our approach are presented below:

(i) In developing our theory we have extended the usual "doorway" concept of the theory of intermediate structure to include doorway states which are in the *same* Hilbert space as the continuum channels of interest. This may be placed in contrast to the procedure in which the doorway space and the continuum space are explicitly orthogonalized.[11-13] The overlap of the doorway state and the continuum channels allows us to introduce a generalized energy-dependent spectroscopic factor for the doorway.

(ii) Since we deal with a theory in which all the particles are treated on the same footing, we are able to discuss various exchange amplitudes that can contribute to the $(p, 2p)$ cross section. We are also able to see how in the limit where the observed final-state particles have very high momenta, the identity of the particles becomes less important and we see how, in this limit, the factorized amplitude becomes a reasonable approximation.

(iii) The formulation used, being based on that previously given for the calculation of the optical-model potential and wave functions, allows one to define distorted waves from the point of view of a fundamental theory. For example, in our previous work, the optical wave functions were defined in a Hilbert space orthogonal to the bound-state wave functions. The use of such wave functions avoids the introduction of some spurious (Pauli principle violating) terms in Eq. (5.3), a feature which is of somewhat less importance in the very high energy limit.

(iv) The formulation used here also leads to the introduction of off-shell reaction matrices. While

we have not expanded upon this point here it is worth noting that if we had considered the $(p, 2p)$ process in which a strongly correlated particle was struck by the projectile we would have had to use K matrices which would be very far off shell.[19] The methods developed here allow for a mathematical description of such events; however, this feature has not been fully explored. In general such events may lead to the presence of three or more *fast* particles in the final state and need to be considered along with the discussion of the "background" due to secondary production.

Our work, while clarifying some matters, still leaves many unanswered questions which should be explored in order that one may feel confident in the validity of the interpretation of the $(p, 2p)$ reaction. We list a few of these questions below:
(i) To what extent can one be sure that the observed particles are the projectile and its first collision partner?
(ii) Can one obtain reliable momentum distributions free of uncertainties due to distortion effects?
(iii) To what degree is it possible to measure the binding energy of the hole states? [To the extent that we appeal to the energy-weighted sum rule, Eq. (5.9), for an answer to this question, we need to have some general idea of the dependence of the generalized spectroscopic factor on energy.]

There are additional questions concerning the importance of final-state interactions between the ejected particles as well as questions as to the off-shell nature of the reaction matrices.[20]

In this paper we have explored only a very limited number of the questions that pertain to the complete description of the $(p, 2p)$ reaction. If we can give satisfactory answers to the other questions we have raised, we can expect that the $(p, 2p)$ reaction will be important for a more detailed understanding of the properties of the nucleus.

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[1]H. Ekstein, Phys. Rev. 101, 880 (1956). More modern approaches exist but this paper is sufficient to illustrate the point we wish to make.
[2]Y. Hahn, Phys. Rev. 142, 603 (1966); Nucl. Phys. A132, 353 (1969); Phys. Lett. 30B, 595 (1969); Phys. Rev. C 3, 1432 (1971).
[3]R. R. Scheerbaum, C. M. Shakin, and R. M. Thaler, Ann. Phys. (N.Y.) 76, 333 (1973).
[4]The operator S is unitary only if P spans the space of all open channels.
[5]N. Austern, *Direct Nuclear Reaction Theories* (Wiley, New York, 1970).
[6]M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964).
[7]This feature of our formalism, which leads to nonvanishing *off-diagonal* matrix elements of T_{orth} between channel states, will be fully discussed in a future publication.
[8]F. Villars, in *Fundamentals in Nuclear Theory* (International Atomic Energy Agency, Vienna, 1967).
[9]C. M. Shakin and R. M. Thaler, Phys. Rev. C 7, 494 (1973).
[10]G. Jacob and Th. A. J. Maris, Rev. Mod. Phys. 38, 121 (1966); 45, 6 (1973). These papers each contain an extensive list of additional references.
[11]H. Feshbach, A. K. Kerman, and R. H. Lemmer, Ann. Phys. (N.Y.) 41, 230 (1967).
[12]N. Auerbach, J. Hüfner, A. K. Kerman, and C. M. Shakin, Rev. Mod. Phys. 44, 48 (1972).
[13]W. L. Wang and C. M. Shakin, Phys. Rev. C 5, 1898 (1972).
[14]In the interest of notational simplicity we have dropped the designation of the boundary conditions from $\eta_k^{\pm}$'s.
[15]Recently detailed consideration has been given to the influence of some of the lowest-order secondary processes on the interpretation of $(p, 2p)$ measurements—see S. Pittel and N. Austern, Phys. Rev. Lett. 29, 1403 (1972).
[16]Note that $\psi_{k_1}^{(-)}$ and $\psi_{k_2}^{(-)}$ must be calculated differently from $\psi_k^{(+)}$ because they correspond to propagation in a complex final state configuration of the excited $(A-1)$-particle system.
[17]C. M. Shakin and J. Da Providência, Phys. Rev. Lett. 27, 1069 (1971).
[18]D. H. E. Gross and R. Lipperheide, Nucl. Phys. A150, 449 (1970); U. Wille, D. H. E. Gross, and R. Lipperheide, Phys. Rev. C 4, 1070 (1971); U. Wille and R. Lipperheide, Nucl. Phys. A189, 113 (1972).
[19]R. R. Scheerbaum, C. M. Shakin, and R. M. Thaler, Phys. Rev. C 7, 2346 (1973).
[20]E. F. Redish, G. J. Stephenson, Jr., and G. M. Lerner, Phys. Rev. C 2, 1665 (1970); G. J. Stephenson, Jr., E. F. Redish, and G. M. Lerner, Phys. Rev. C 6, 1559 (1972).