# Limitation to complete fusion during a collision between two complex nuclei

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Experimental results on the limits observed for complete fusion between heavy nuclei are discussed. It is shown that there is no critical angular momentum independent of the bombarding energy and of the entrance channel. Excitation energy and total angular momentum of the compound system do not appear as good parameters for determining the complete fusion cross section. All the available results can be presented in terms of a critical distance of approach  $R_{\rm cr}$ . Such a distance is obtained when the kinetic energy of incoming nucleons equals the interaction potential and is related to the nuclear-matter density of both nuclei. For very heavy partners, a fusion barrier occurs, in addition to the interaction barrier.

NUCLEAR REACTIONS Heavy-ion complete fusion; calculated critical distance of approach,  $R_{cr} = (1.0 \pm 0.07)(A_1^{1/3} + A_2^{1/3})$ .

## 1. COMPLETE FUSION AND CRITICAL ANGULAR MOMENTUM

When nuclear reactions are induced by heavy ions, an important question is which part of the total reaction cross section goes into complete fusion and then leads to a compound nucleus. It has important consequences for the yield for the production of new isotopes and in itself it might give information on the dynamical properties of nuclear matter.

A number of experimental results<sup>1-8</sup> have been obtained during the last decade, mainly on the ratio ( $\sigma_{CF}/\sigma_R$ ) of the complete-fusion cross section as compared to the total reaction cross section. The first conclusion was that the completefusion cross section did not always represent the largest fraction of the total cross section and a reasonable explanation was given by considering the large angular momenta brought into the compound system by heavy projectiles of large momenta. The concept of a critical value of angular momentum was stated by several authors.<sup>1,9</sup>

However, more recent data have shown that such a critical limit is certainly not characterized by only the properties of the compound system. In particular, experiments by Zebelman *et al.*<sup>10, 11</sup> have very clearly demonstrated that the entrance channel had to be considered since, for the same compound nucleus at the same excitation energies ( $^{170}_{70}$  Yb) four different critical  $l\hbar$ were deduced from  $\sigma_{CF}$  measurements, when four different projectiles ( $^{11}B$ ,  $^{12}C$ ,  $^{16}O$ ,  $^{20}Ne$ ) were used. Similar results have been obtained at Orsay<sup>8</sup> with a completely different experimental method of determination of  $l_{crit}\hbar$ . For the formation of  $^{117}Te$  at the same excitation of 107 MeV energy,  $l_{crit}$  was found equal to 70 when the projectile was <sup>40</sup>Ar, and only 50 with <sup>14</sup>N.

A second set of results has also very clearly shown that for a given system,  $l_{crit}$  does not stay at a constant value whatever the bombarding energy and, therefore, the excitation energy of the compound system. If a limiting of  $l\hbar$  is found at a given energy and if such a critical  $l\hbar$  is a constant, then by increasing the energy,  $\sigma_{\rm CF}/\sigma_R$ should decrease at the same rate as  $(l_{crit}/l_{max})^2$ . Although the first experiments<sup>1</sup> could give the feeling that this was the case, new data from Natowitz, Chulick, and Namboodiri<sup>3</sup> with C, N, O, and Ne ions have demonstrated that  $\sigma_{\rm CF}/\sigma_R$ did not decrease so much. New results obtained at Orsay with Ar ions' have shown that  $l_{crit}/\hbar$  was indeed increasing at a similar rate as  $l_{\max}\hbar$  for a given system.

Therefore, it is obvious that an explanation of the limit to complete fusion should not only imply a static model on the basis of the shape and properties of the compound nucleus, but should take account of dynamical aspects of the colliding process. Then the excitation energy and the angular momentum of the compound system are not the most relevant parameters. The important variable should rather be the velocity of the approaching nucleons as a function of the distance between nuclei.

Recently, Wilczynski<sup>12</sup> has made an attempt to describe the limitation to complete fusion on the basis of a two-body contact configuration. The nucleus-nucleus force is then derived from simple surface-energy considerations. More precisely, this author makes the assumption that the derivative of the surface energy represents the force acting between two spherical liquid drops, with-

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out taking account of the diffuseness of the nuclear surface. Then  $l_{crit}$  is obtained when the force

$$F(R_1 + R_2) = (\gamma_1 + \gamma_2) \frac{2\pi R_1 R_2}{R_1 + R_2}$$

[where  $\gamma_1$  and  $\gamma_2$  are the surface-tension coefficients (in MeV per cm<sup>2</sup>) for drop 1 and drop 2, respectively] is exactly balanced by Coulomb and centrifugal forces

$$\frac{2\pi(\gamma_1 + \gamma_2)R_1R_2}{R_1 + R_2} = \frac{V_{\text{Coul}}}{R_1 + R_2} + \frac{(l_{\text{crit}}\hbar)^2}{2\mu(R_1 + R_2)^3}$$

In order to deduce  $l_{crit}\hbar$  from this expression, it is necessary to make an *a priori* choice of the distance  $(R_1+R_2)$  at which the derivative of the potential is equal to zero. Wilczynski has used the liquid-drop parameters and assumed that  $R_1+R_2$  is obtained when the matter density in both drops become equal to  $\frac{1}{2} \rho_{max}$ , if  $\rho_{max}$  is the nuclear-matter density in the core. Therefore, the above expression does not include any velocity or kinetic energy dependence. A single value of  $l_{crit}$ is obtained whatever the energy. As has been said previously, this is in total disagreement with most of the experimental results.<sup>3, 6, 7, 8</sup>

In the present paper, we should like to describe a rather different approach in which we do not try to define a priori the conditions for fusing two nuclei. On the contrary, we wish first to express, as a function of the distance of approach, the interaction potential between two colliding nuclei V(r), and then study how the original kinetic energy is diminished by such a potential  $[V_{(r)} = V(nu$ clear) + V(Coulomb) + V(centrifugal)]. For each partial wave of order l we build a potential curve of the system, as it will be described in the next section. Suppose that at a given kinetic energy (in the center-of-mass system), a critical  $l\hbar$  $(l_{crit}\hbar)$  has been determined. It corresponds to a particular curve of the potential, where the centrifugal contribution is equal to

$$\frac{l_{\rm crit}(l_{\rm crit}+1)\hbar^2}{29}$$

Suppose a center-of-mass kinetic energy  $E_{\rm I}$  at the infinite. It decreases as the two nuclei become closer and closer and becomes equal to zero for a given partial wave  $l_1$  at a distance  $\dot{R}_{\rm cr}$ where  $E_{\rm I} = V(r, l_1)$ , since all the kinetic energy has been transformed into potential energy. A glance at the schematic drawing of Fig. 1 shows that for a higher bombarding energy  $E_{\rm II}$ , the distance of approach where  $E_{\rm II} = V(r, l_1)$  would be smaller if the same potential were used; i.e., if  $l_{\rm crit}$  had not been changed. On the contrary if, for  $E_{\rm II}$ , another value  $l_2$  were taken, one would have to compare  $E_{\rm II}$  to another potential curve  $V'(r, l_2)$ . We have made a systematic survey of

all the experimental results on  $l_{crit}$  determinations, except the very first results from Ref. 1 which show a decrease of  $J_{crit}$  with increasing energy but which are in disagreement with the new results on a similar system by Natowitz, Chulick, and Namboodiri.<sup>3</sup> The comparison, as described above, of calculated V(r) and kinetic energy  $E_{b}$ has given the following conclusion: For a given system where different  $l_{crit}$  were measured at various energies, a single distance of approach  $R_{cr}$  was found for which  $V(R_{cr}) = E_k$ . For two different systems at the same excitation energy we could also explain why  $l_{crit}\hbar$  values were found different because neither the bombarding energies E, nor the potential curves V(r) were the same and therefore the intersection at a critical distance of approach  $R_{cr}$  between V(r) and  $E_k$  did not define the same centrifugal potential.

#### 2. CALCULATION OF THE INTERACTION POTENTIAL BETWEEN TWO APPROACHING NUCLEI

The potential energy as a function of the distance between the two centers r is calculated as usual:

$$V(r) = V_{\text{nucl}}^{(r)} + V_{\text{Coul}}^{(r)} + \frac{l(l+1)\hbar^2}{2g(r)} .$$
 (1)



FIG. 1. Potential energy curves for the interaction of two complex nuclei (see text).  $E_{IB}$  is the interaction barrier;  $E_{I}$  and  $E_{II}$  are two c.m. kinetic energies at the infinite.

The centrifugal part is deduced for a given l assuming a momentum of inertia  $\mathcal{I} = \mu r^2$  of a rigid body where  $\mu$  is the reduced mass and r the distance between the two centers.

The Coulomb term is calculated on the basis of the charge distribution in each nucleus. The difficult point is, however, the nuclear term. For a given distance r it is equal to the difference between the nuclear energy of the system of two nuclei interacting and the binding energies of the two nuclei entirely apart one from the other. We shall discuss in the conclusion the reasons for our choice. Let us, for the moment, give the approximation we have used.<sup>13</sup> The nuclear potential has been taken as a functional of the nuclear-matter density  $\rho(r)$ , according to a method proposed by Bruckner et al.<sup>14</sup> For a given nucleus. Bruckner et al. calculate the binding energy at a particular point of the nucleus as a function of the nuclearmatter density  $\rho$  at this point. The total binding energy is obtained by integrating over all the nuclear volume. A large variety of binding energies have been calculated using the nuclear-matter densities available from experimental data. They are in good agreement with binding energies deduced from experiment.

Let us have two nuclei with a separation distance between centers r. For calculating the interaction nuclear potential at a particular point M(located at a distance  $r_1$  from the center of nucleus 1 and a distance  $r_2$  from nucleus 2) the difference is

$$E[\rho_1(r_1) + \rho_2(r_2)] - E[\rho_1(r_1)] - E[\rho_2(r_2)],$$

where  $\rho_1(r_1)$  and  $\rho_2(r_2)$  represent the local nuclearmatter densities of nucleus 1 and 2, respectively, at distances  $r_1$  and  $r_2$  from the centers. The local binding energy for the composite system of two nuclear-matter densities is  $E[\rho_1(r_1) + \rho_2(r_2)]$ , while for a separate nucleus it is  $E[\rho_1(r_1)]$ .

The integration over the volume of the two nuclei gives:

$$V_{\text{nucl}}(r) = \int \left\{ E[\rho_1(r_1) + \rho_2(r_2)] - E[\rho_1(r_1)] - E[\rho_2(r_2)] \right\} dr^3$$

Nuclear-matter densities and parameters of the functionnals were given by Beiner and Lombard.<sup>15</sup>

It should be emphasized that such a calculation implies the *sudden approximation*; i.e., the structure of each nucleus is entirely conserved during the contact and nuclear-matter densities overlap in a reversible process without any rearrangements. Therefore, the potential curves certainly do not describe the interaction after the nucleus has fused. However, they might be considered as a rough model for studying the approach interaction and for estimating how the kinetic energy is being decreased by the interaction potential as a function of the distance r.

## 3. APPLICATION OF THE INTERACTING POTENTIAL CALCULATIONS TO VARIOUS SYSTEMS

The method described in Sec. 2 has been applied to a number of systems of two colliding nuclei for which experimental results have been published on critical angular momentum determinations. Most of these results were obtained by measuring  $\sigma_{\rm CF}/\sigma_R$  and applying the sharp cutoff approximation

$$\frac{\sigma_{\rm CF}}{\sigma_R} = \left(\frac{l_{\rm crit}}{l_{\rm max}}\right)^2.$$

However, there are also a few determinations which were deduced from angular distribution of  $\alpha$  particles emitted by the compound nucleus,<sup>8</sup> since the magnitude of the anisotropy is related to the angular momentum distribution of the emitting system.

Among all the available results, the systems of particular interest are those for which a dependence of  $l_{crit}$  on the bombarding energy has been observed, and we have made calculations in these

 $\begin{bmatrix} 100 \\ 50 \\ 50 \\ 50 \\ -50 \\ -50 \\ 50 \\ -50 \\$ 

FIG. 2. Potential energy curves for l = 0, l = 50, and l = 70 in the case of the system <sup>77</sup>Se + <sup>40</sup>Ar.

cases. A typical example is given in Fig. 2 which is devoted to the system  ${}^{40}Ar + {}^{77}Se$ . The potential curve V(r) is given as a function of r for a few partial waves. For each kinetic energy in the center of mass  $(E_{b})$  a distance of approach  $R_{cr}$ can be defined as the distance where the ordinate  $E_{k}$  intersects the curve  $V(r, l_{crit})$  where  $l_{crit}\hbar$  is the experimentally determined critical angular momentum. Any curve V(r, l) for a centrifugal potential calculated with  $l > l_{crit}$  would intersect  $E_k$  at a distance larger than  $R_{cr}$ . It means that the velocity of colliding nucleons would become zero (when all the kinetic energy is transformed into potential energy) at a distance larger than  $R_{\rm cr}$ , i.e., too large to permit a fusion process. Any curve V(r, l) for  $l < l_{crit}$  would intersect  $E_k$  at a distance closer than  $R_{cr}$ . Then the incoming nucleons would still have some velocity at the distance  $R_{\rm cr}$  and they could enter deeper into the target nucleus so the fusion would occur.

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MeV (c.m.)  $l_{crit}$  was found<sup>8</sup> equal to 70 and  $V(r, l_{crit})$ = 132 MeV for a distance  $R_{cr}$  = 7.0 fm. At a lower kinetic energy (96 MeV)  $l_{crit}$  was measured,  $l_{crit}$ = 50, and  $V(r, l_{crit})$  = 96 MeV is again at a distance  $R_{cr}$  = 7.0 fm.

On Fig. 1, it is interesting to consider the general behavior of the curves V(r, l) for different l values. For s waves, V(r, 0) exhibits a potential well which characterizes quasimolecular states. If the kinetic energy  $E_{IB}$  is high enough to overcome V(r, 0) such a well exerts attractive forces even at distances as large as the sum of the radii of the two nuclei. For low  $l\hbar$ , this is still true and therefore,  $l_{crit}$  is not very different from  $l_{max}$ . There is no well-defined critical distance  $R_{cr}$ , since at any distance closer than the interaction distance  $R_0$  (where dV/dr = 0) there is the possibility for a quasimolecular potential and later on for fusing into a compound-system potential.

For higher l values, the potential dip has a tendency to become very shallow and vanishes

In '	the	present	case,	at :	a	kinetic	energy	of	132
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System	E <sub>k</sub> (c.m. energy) (MeV)	Experim. <i>l</i> <sub>crit</sub>	Exp. Refs.	R <sub>cr</sub> deduced in this work (fm)	$\langle \boldsymbol{r}_{\rm cr} \rangle = \langle \boldsymbol{R}_{\rm cr} \rangle / (\boldsymbol{A}_1^{1/3} + \boldsymbol{A}_2^{1/3})$
$Ni + {}^{12}C$	37	24	3	5.8	
	53	29		6	· · · · · · · · · · · · · · · · · · ·
or	67	33		6	0.96
$^{12}C + Cu$	81	36-40		6	
	150	51-53		6	)
$^{27}Al + ^{12}C$	30	20	3		ý
	44	25		5.2	0.98
	69	29			
	125	40			)
$Ti + {}^{12}C$	65	30		5.1	)
	78	33	3	5.7	0.96
	144	50		6.0	<b>)</b>
${}^{152} m Tb + {}^{11} m B$	107	$40 \pm 3$	10		
$^{158}$ Gd + $^{12}$ C	117	$46 \pm 4$	10	7.3	0.95
$^{154}$ Sm + $^{16}$ O	124	$58 \pm 4$	10	7.5	0.96
$^{150}$ Nd + $^{20}$ Ne	127	$70 \pm 6$	11	7.7	1.00
$^{103}$ Rh + $^{14}$ N	71	$40\pm 5$	8	7.3	1.02
	107	$52 \pm 5$		7.3	
77Se + $40$ Ar	96	$50 \pm 5$	8	7.0	0.92
	132	$70 \pm 5$		7.0	
$^{107}$ Ag + Ar	144	$70 \pm 5$	6	7.6	0.08
	210	$115 \pm 7$	6	8.0	0.98
	120	$29 \pm 3$	7		)
40	134	$57 \pm 5$	7	$7.9 \pm 1$	
Sb + 40 Ar	148	$80\pm 8$		8.9	$(1.07 \pm 0.1)$
	168	$91 \pm 9$	7	8.7	(
	195	$114 \pm 12$	7	10.	
	226	$132 \pm 13$	7	$10.2 \pm 1$	)

TABLE I. Critical distance of approach  $R_{cr}$  deduced from experimental measurements of  $l_{crit}$ .

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entirely for very high l values. However, we believe that the potential curve V(r, l) is still useful for describing the approach of the two colliding nuclei. In Wilzynski's treatment, such curves would predict no fusion since the limit is given by the partial wave for which V(r, l) has a derivative equal to zero.

In Table I we give the critical angular momenta observed in a large number of experiments, as well as the critical distance of approach  $R_{\rm cr}$  which can be deduced from our analysis. Also, shown in Table II are  $l_{\rm crit}$  values deduced by Wilczynski with the liquid-drop model<sup>12</sup> as well as  $l_{\rm crit}$  deduced with Bruckner's formalism but according to Wilczynski's criteria; i.e., for the first curve where the potential dip is replaced by a shoulder on which dV/dr = 0. Both  $l_{\rm crit}$  values are similar, indicating that the nuclear model is not determining, but they both disagree with experimental results. On the contrary the concept of a single critical distance for a given system seems to fit very well the experiments.

Moreover, using the expression  $R_{\rm cr} = r_{\rm cr} (A_1^{1/3} + A_2^{1/3})$ , it was found that the parameter  $r_{\rm cr}$  is more or less constant throughout the Periodic Table, with an average value around  $r_{\rm cr} = 1.0 \pm 0.07$  fm.

Figures 3 and 4 illustrate these results for the cases of the results of Natowitz, Chulick, and Namboodiri<sup>3</sup> on  $(Ti + {}^{12}C)$ ,  $(Cu + {}^{12}C)$ , and  $(Ni + {}^{12}C)$ . Also the influence of the entrance channel stipulated by Miller *et al.*<sup>10,11</sup> is well reproduced. For the four systems studied by these authors, the

TABLE II. Comparison of  $l_{crit}$  calculated with two different nuclear models but with the same criteria (dV/dr = 0) according to Ref. 12.

		Calculated $l_{\text{crit}}$ at $\frac{dV}{dr} = 0$		
System	Experimental	Liquid drop	Bruckner	
	• crit	(1101.12)	potential	
$Ni + {}^{12}C$	24 to 53	33	35	
r or $r$ $r$ $r$				
Cu + C				
$^{27}Al + ^{12}C$	20 to 40	24	22	
$Ti + {}^{12}C$	30 to 50	30	30	
$^{152}$ Tb + $^{11}$ B	40	43		
${}^{158}$ Gd + ${}^{12}$ C	46	47	48	
$^{154}Sm + {}^{16}O$	58	55	60	
$^{150}$ Nd + $^{20}$ Ne	70	62	70	
$^{103}$ Rh + $^{14}$ N	40 to 52	46	45	
$^{77}$ Se + $^{40}$ Ar	50 to 70	70	72	
$^{107}$ Ag + $^{40}$ Ar	70 to 115	77	88	
$Sb + {}^{40}Ar$	29 to 132	79	90	

compound nucleus was produced at the same excitation energy. However, the center-of-mass bombarding energies and, of course, the V(r, l) curves were different. Therefore there is no reason to find the same  $l_{crit}$  values since neither the excitation energy nor the angular momentum are good parameters for determining limits for fusion. If one follows our conclusions, the limit between complete and incomplete fusion would be determined by a critical distance of approach corresponding to critical conditions of overlapping of the nuclear matter of the two nuclei. Such a concept is certainly very strongly related to viscosity and damping effects and should be reproduced in friction calculations, as they are being made by K. Dietrich and also by Bondorf and Sperber.16

#### 4. FUSION BARRIER FOR VERY HEAVY PROJECTILES AND TARGETS

Interaction potentials have been drawn for the cases of very heavy systems. For example, on Fig. 5 (Se+Gd) and (Gd+Gd) have been considered. If we assume that  $R_{\rm cr}$  is given by the expression  $r_{\rm cr}(A_1^{1/3}+A_2^{1/3})$  as shown previously, critical angu-



FIG. 3. Potential energy curves for different l values in the case of  ${}^{48}\text{Ti} + {}^{12}\text{C}$ .



FIG. 4. Potential energy curves for different l values in the case of Cu +<sup>12</sup>C and Ni +<sup>12</sup>C.

lar momenta can be deduced at different energies. For l=0, the interaction barrier is given by the energy needed to overcome V(r, 0) (Fig. 1) at the distance  $R_0$  where dV/dr = 0. Such an energy is found at 138 MeV for (Ar+Gd), 239 for (Se+Gd). and 410 for (Gd+Gd). But one can notice that the situation is very different for the second system, where the curve V(r, 0) can be intersected at a distance r roughly equal to smaller than  $R_{cr}$  (then fusion is allowed), and for the system (Gd + Gd)where V(r, 0) is intersected at a distance larger than  $R_{\rm crit}$  (then fusion is not possible even for s waves). In other words, for very heavy projectiles, a head-on collision even at an energy higher than the interaction barrier might not lead to a complete fusion. An additional energy is needed, as shown on Fig. 5 which is of the order of 40 MeV in the case of (Gd+Gd). At the inverse of what has been always assumed for lighter ions, the first channels open when nuclear interaction becomes possible are not those leading to compound nuclei.

Because the calculations are very approximative (and particularly the nuclear potentials with the Brueckner's method), it is difficult to evaluate precisely the magnitude of such a fusion barrier. However, it can be predicted that for the case of Kr + Bi, the fusion process should occur only with



FIG. 5. Potential energy curves for l = 0 in the case of <sup>158</sup>Gd + <sup>74</sup>Se (dashed curve) and of <sup>158</sup>Gd + <sup>158</sup>Gd (full curve).

a very low cross section in the range of 10-50MeV above the barrier, in agreement with the experimental results obtained by Lefort *et al*.<sup>17</sup> For higher energies the contributions of low-*l* partial waves becomes relatively small, and although fusion might become possible, the cross section will still be very low, since  $R_{\rm cr}$  will be reached only for low *lh*.

### 5. CONCLUDING REMARKS ON THE INTERACTION NUCLEAR POTENTIAL

There are many controversial arguments concerning the use of a nuclear potential with a strong repulsive part.

As far as the two partner nuclei do not overlap along their relative motion, the total energy of the system is equal to the sum of the energies of the two nuclei and the dynamical aspect of the interaction is well defined. But when there is a partial overlapping of the nuclear matter from both partners, the question of the time of collision arises.

(i) According to the sudden approximation hypothesis, the structure of each nucleus is conserved during the contact and nuclear matter densities overlap in a reversible process.

(ii) According to the adiabatic approximation, at low velocities, there is a slow approach and a continuous exchange of energy between the incoming nucleus and the target nucleus. A composite structure is built up through the surface of contact, and the nuclear potential is continuously changing. An intermediate shape is formed and there is no repulsive nuclear potential while a compound system is formed.

In the actual nuclear reactions that have been considered, the relative average velocity of the nuclei is probably around 1 or 2 MeV per nucleon since the Coulomb barrier has slowed down the projectile. Even so, it corresponds to a time of collision of the order of  $2.10^{-22}$  sec if one assumes that the interaction occurs along 2 fm. Such a time is probably longer than the duration for individual level excitation, but is shorter than the period of relaxation of a nucleus or than the period of intrinsic motion. Then the sudden approximation is perhaps acceptable at least for the first step of the interaction during which dynamical processes drift the system either into complete fusion or into incomplete fusion, depending how strong the friction forces are.

This question of the validity of the nuclear potential and of the comparison between adiabatic and sudden approximations have been discussed by Greiner and Scheid.<sup>18</sup> Perhaps it is not a very determining clue since the problem in which we are interested is to find the distance of approach

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 $R_{\rm crit}$  where the incoming velocity has decreased so much that the strongly repulsive behavior of the two-body potential V(r) is changed into an attractive well corresponding to a single composite nucleus.

The critical distance parameter  $r_{\rm cr}$  was found around  $1.0\pm0.07$  fm. It might appear as a rather close distance, since the two cores of maximum nuclear density are in contact along the direction between centers for a separation of  $(A_1^{1/3} + A_2^{1/3})$ . But the integration is made over all directions and the overlapping has an axial symmetry while the two nuclei are spherical. Then the  $\sin\theta$  effect makes, as an average, only the tails of nuclear densities overlap although the surface of contact is rather large.

Although all the above results are very crude estimates, they focus attention on the fact that, in our opinion, the crucial parameter for limitation to complete fusion is a critical distance between the two colliding nuclei. Such a distance can be reached depending on the bombarding energy and on the potential of interaction.

Note added in proof: During the preparation of the manuscript, Beck and Gross<sup>19</sup> published a theoretical treatment of the friction forces which seems to be in good agreement with our results.

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