

Study of the low-lying states of ${}^8\text{Be}$

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An extended version of the α -particle model which takes into account the internal motion of the nucleons is proposed for ${}^8\text{Be}$. A two-center harmonic-oscillator potential is assumed for the nucleons. It is found that such a potential is not realistic enough to yield agreement with experiment. It is suggested that a two-center potential of the Woods-Saxon type may be a better choice.

I. INTRODUCTION

Several forms of the cluster model¹ have been proposed and enjoyed much success in their application to light nuclei. On the other hand, the methods of molecular physics² have been shown to be capable of predicting many energy levels in nuclei with $A = 4n$. These investigations seem to indicate that there is a quite large probability of nucleon clustering in light nuclei. Therefore it is perhaps most appropriate to describe single-particle motion in light nuclei in the framework of a shell model with many centers. A shell model with two centers has been studied by several investigators.³⁻⁶ Recently single-particle wave functions with two and three centers were used in Hartree-Fock calculations on ${}^8\text{Be}$ and ${}^{12}\text{C}$.⁷ In this paper we shall propose an extended version of the α -particle model to account for the low-lying energy levels of ${}^8\text{Be}$.

The low-lying levels of ${}^8\text{Be}$ include levels at 0 MeV ($0+$), 2.9 MeV ($2+$), 7.55 MeV ($0+$), 10 MeV ($2+$), and 10.8 MeV ($4+$). If ${}^8\text{Be}$ is treated as a rigid rotator, formed of two α particles, the rotational band consists of $0+$, $2+$, $4+$, ... states. The levels at 0, 2.9, and 10.8 MeV may be identified as the first three members of the rotational band. The experimental value for the energy ratio $E(4+)/E(2+)$ is 3.7, whereas the theoretical value is $10/3$. If the rotator is not rigid, then its moment of inertia increases with its rotational energy, and the theoretical energy ratio thus becomes smaller. Morinaga⁸ has suggested that the level at 7.55 MeV is a rotationless state of a strongly deformed configuration. In Sec. III we shall discuss the plausibility that this level is the first vibrational excited state of a system of two α clusters. It can then be speculated that the level at 10 MeV is a state of simultaneous rotation and vibration.

II. EXTENDED α -PARTICLE MODEL FOR ${}^8\text{Be}$

An extension of the α -particle model for ${}^8\text{Be}$ is the inclusion of the internal motion of the nucleons.

In constructing the single-particle states it is appropriate to assume a two-center potential for the nucleons. By analogy with the treatment of a diatomic molecule,⁹ we shall assume an adiabatic approximation. Thus it is assumed that the nuclear wave function can be approximated by a function of the form

$$u_{\mathbf{R}}(x_1, x_2, \dots, x_8)w(\vec{\mathbf{R}}), \quad (1)$$

where x_i denotes the space and spin coordinates of the i th nucleon, and $\vec{\mathbf{R}}$ is a vector connecting the two centers of the potential. In the approximation of independent particle motion, $u_{\mathbf{R}}(x_1, x_2, \dots, x_8)$ is in general a linear combination of antisymmetrized products of single-particle wave functions $u_{\mathbf{R}}(x_i)$ satisfying the equation

$$\left(-\frac{\hbar^2}{2m}\nabla_i^2 + V_{\mathbf{R}}\right)u_{\mathbf{R}}(x_i) = E_i(R)u_{\mathbf{R}}(x_i), \quad (2)$$

where m is the nucleon mass, $V_{\mathbf{R}}$ is a two-center potential common to all the nucleons,¹⁰ and R is the separation of the two centers; the total internal energy is a sum of single-particle energies. $w(\vec{\mathbf{R}})$ describes the relative motion of the two clusters and has the form

$$w(\vec{\mathbf{R}}) = \frac{S(R)}{R} Y_{JM}(\vec{\mathbf{R}}/R).$$

$Y_{JM}(\vec{\mathbf{R}}/R)$ is a spherical harmonic. $S(R)$ satisfies the equation

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + U(R) + \frac{J(J+1)\hbar^2}{2\mu R^2}\right]S(R) = ES(R), \quad (3)$$

where $U(R) = \sum_{i=1}^8 E_i(R)$, and μ is the reduced mass of the two α clusters. As we shall be concerned only with low-lying states, the effective potential

$$W_J(R) = U(R) + \frac{J(J+1)\hbar^2}{2\mu R^2}$$

can be expanded around its minimum at $R = R_J$.

$$W_J(R) = U(R_J) + \frac{J(J+1)\hbar^2}{2\mu R_J^2} + \frac{1}{2} \left(\frac{d^2 W_J}{dR^2} \right)_{R=R_J} (R - R_J)^2 + \dots \quad (4)$$

If only terms to second order in $(R - R_J)$ are kept, Eq. (3) becomes identical in form to the equation for a linear harmonic oscillator. Also, the boundary conditions in the two cases are practically equivalent,¹¹ so that the energy E has the approximate expression

$$E = U(R_J) + \frac{J(J+1)\hbar^2}{2\mu R_J^2} + (v + \frac{1}{2})\hbar\omega_J \quad (5)$$

in which

$$\omega_J = \left[\frac{1}{\mu} \left(\frac{d^2 W_J}{dR^2} \right)_{R=R_J} \right]^{1/2}.$$

The allowed values of the quantum numbers J and v are

$$J = 0, 2, 4, \dots,$$

$$v = 0, 1, 2, \dots$$

When $(R_J - R_0)/R_0 \ll 1$, the expression (5), excluding the vibrational energy, takes the approximate form

$$E = U(R_J) + \frac{J(J+1)\hbar^2}{2\mu R_0^2} \left[1 - 2 \left(\frac{R_J - R_0}{R_0} \right) \right]. \quad (6)$$

The term $2(R_J - R_0)/R_0$ represents the coupling between rotation and vibration. The value of $U(R_J)$ is minimum when $J=0$, and should increase as J increases. If we assume that the "stiffness"¹² of the structure of two α clusters is an increasing function of R_J for $J=0, 2$, and 4 , then it is possible that $U(R_4) - U(R_0)$ is much larger than $U(R_2) - U(R_0)$, while R_2 and R_4 are close in value. If this is the case, the observed energy ratio $E(4+)/E(2+) (=3.7)$ may be accounted for.

III. RESULTS AND DISCUSSION

We have applied a two-center harmonic-oscillator potential¹³ to ${}^8\text{Be}$. The potential V_R , in cylindrical coordinates, takes the form

$$V_R(\rho_i, z_i) = \frac{1}{2}m\omega^2 (\rho_i^2 + z_i^2 - R|z_i| + \frac{1}{4}R^2), \quad (7)$$

The condition that the nuclear volume remains constant during deformation provides a relationship between the oscillator frequency ω and R .¹⁴ Assuming a constant nuclear volume of $\frac{4}{3}\pi(4 \text{ fm})^3$, we calculated the effective potential $W_J(R)$ corresponding to the ground-state configuration of ${}^8\text{Be}$. In Fig. 1, $W_J(R)$ is plotted versus R for $J=0, 2$, and 4 . From the figure we obtained the following

values of the equilibrium separations: $R_0 = 2.7 \text{ fm}$, $R_2 = 3.3 \text{ fm}$, and $R_4 = 3.6 \text{ fm}$, and the corresponding values of $\hbar\omega$ are 23.8, 24.3, and 24.6 MeV, respectively. Equation (5) then yields,¹⁵ assuming $v=0$, the excitation energies of the first two rotational excited states: $E(2+) = 7.11 \text{ MeV}$ and $E(4+) = 19.39 \text{ MeV}$, and the corresponding ratio $E(4+)/E(2+)$ is 2.73, as compared with the experimental value of 3.7.

Although we failed to achieve any quantitative agreement with the experimental data, the following points may be of some value in the search of a more realistic model for ${}^8\text{Be}$:

(1) The calculated excitation energies for the 2+ and 4+ rotational states are too high by a factor of about 2. Agreement with the experimental values can be obtained if the equilibrium separations R_J are increased by a factor of about $\sqrt{2}$. In order to get equilibrium separations of such magnitudes, an unreasonably large nuclear volume has to be assumed. Thus the two-center harmonic-oscillator potential cannot provide a realistic description for ${}^8\text{Be}$. A two-center potential of Woods-Saxon type may be a better choice, from which $U(R)$ is to be derived.

(2) In the adiabatic approximation the total energy of ${}^8\text{Be}$ is the sum of three kinds of energies, namely, single-particle energy, rotational energy,

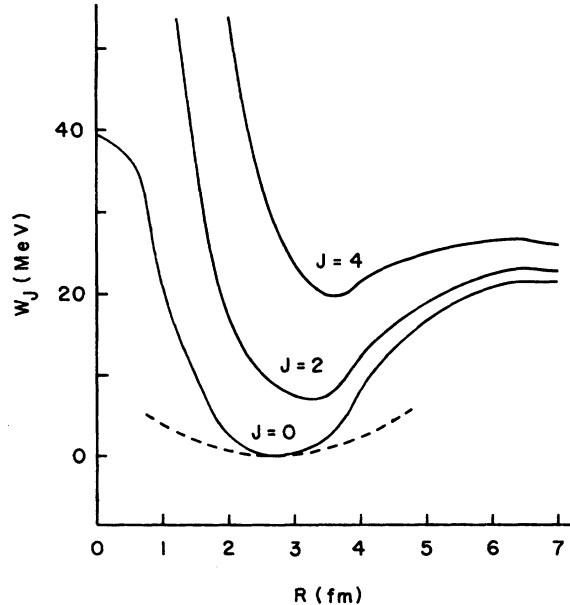


FIG. 1. The effective potential $W_J(R)$ corresponding to the ground-state configuration of ${}^8\text{Be}$ vs the separation R of the two α clusters, for $J=0, 2$, and 4 . The energy scale is chosen such that $W_0(R_0) = 0$. The dashed curve represents $\frac{1}{2}\mu\omega'^2(R - R_0)^2$ with $\hbar\omega' = 7.55 \text{ MeV}$.

and vibrational energy. Our calculations show that the inclusion of the single-particle energies increases the energy ratio $E(4+)/E(2+)$. As pointed out before, the theoretical value of this ratio according to the model of a nonrigid rotator, composed of two α particles, is too small as compared with the experimental value. It appears plausible that the dependence of the single-particle energies on rotation may account for this discrepancy.

(3) In accordance with Morinaga's suggestion, we may interpret the $0+$ state at 7.55 MeV as the first vibrational excited state in a harmonic-oscillator potential. In Fig. 1 a harmonic-oscillator potential $\frac{1}{2}\mu\omega'^2(R-R_0)^2$ (for a system of two α clusters) with $\hbar\omega' = 7.55$ MeV is represented by the

dashed curve. By comparison, the potential $U(R)$ would yield a first vibrational excited state at a much higher energy. Moreover, it is tempting to interpret the $2+$ state at 10 MeV as a state of simultaneous rotation and vibration with $J=2$ and $\nu=1$. If we simply add the energies of the $2+$ state at 2.9 MeV and the $0+$ state at 7.55 MeV, a state at 10.45 MeV is predicted. However, before we try to fit the empirical levels this way, we need a potential for a system of two α clusters which can yield equilibrium separations of the order of 4.5 fm as suggested by the rigid rotator model. When such a potential is obtained, we should then include residual two-body interactions in the next approximation.

¹D. Brink, in *Many-Body Description of Nuclear Structure and Reactions, Proceedings of the International School of Physics, "Enrico Fermi," Course XXXVI*, edited by C. Bloch (Academic, New York, 1967), p. 247; F. C. Chang, *Phys. Rev.* **178**, 1725 (1969).

²P. Hauge, S. Williams, and G. Duffey, *Phys. Rev. C* **4**, 1044 (1971).

³P. Holzer, U. Mosel, and W. Greiner, *Nucl. Phys.* **A138**, 241 (1969).

⁴D. Scharnweber, W. Greiner, and U. Mosel, *Nucl. Phys.* **A164**, 257 (1971).

⁵J. Eisenberg and W. Greiner, *Nuclear Theory* (North-Holland, Amsterdam, 1972), Vol. 3, p. 426.

⁶T. Johansson, *Nucl. Phys.* **A183**, 33 (1972).

⁷K. Kubodera and K. Ikeda, *Prog. Theor. Phys.* **42**, 740 (1969); Y. Abe, J. Hiura, and H. Tanaka, *ibid.* **49**, 800 (1973).

⁸H. Morinaga, *Phys. Rev.* **101**, 254 (1956).

⁹L. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1968), 3rd ed., p. 445.

¹⁰In the case of the two-center harmonic-oscillator potential to be assumed [see Eq. (7)], one parameter R is sufficient to describe the deformation of the potential. In other cases several deformation parameters may be necessary. The approach proposed here is somewhat similar to that of the strong coupling

theory of collective motion, where two deformation parameters, β and γ , are used; the single-particle states in the deformed potential are known to be well described by the Nilsson model. [See, for instance, K. Hecht, in *Selected Topics in Nuclear Spectroscopy* (Wiley, New York, 1964), p. 63.]

¹¹L. Pauling and E. Wilson, *Introduction to Quantum Mechanics* (McGraw-Hill, New York, 1935), p. 267.

¹²In the approximation of a harmonic oscillator, the restoring force of a system of two α clusters is proportional to $|R-R_0|$. To be more realistic, the restoring force is expected to increase more rapidly with $|R-R_0|$ when the clusters are being more strongly deformed. "Stiffness" is here used to describe this effect.

¹³We use the two-center harmonic-oscillator model developed by Holzer, Mosel, and Greiner (see Ref. 3). The single-particle energies E_i are obtained from Eq. (25) of Ref. 3. It is assumed that the centers of mass of the α clusters coincide with the centers of the potential.

¹⁴The last two equations on p. 243 of Ref. 3 should read: $\omega = \omega_0(R/r)$; $2r^3 + 3r^2z_0 - z_0^3 - 2R^3 = 0$ (in the notation of the reference).

¹⁵From Fig. 1 we estimated that $\omega_0 \cong \omega_2 \cong \omega_4$.