$(sd)^2$ 0^+ states coupled to p-shell cores

H. T. Fortune
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

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Energies of the two lowest $(sd)^2$ 0^+ states in ^{10,12}Be and ^{14,16}C are examined in a simple model that works surprisingly well. Energies are predicted for the second $(sd)^2$ 0^+ state in ^{10,12}Be, where they are unknown.

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In the (t,p) reaction on ^{10}Be, ^{12,14}C [1–4], a set of three states are especially strong in all three final $A+2$ nuclei. These are 0^+, 2^+, and 4^+ states whose dominant configuration is two sd-shell neutrons coupled to the 0^+ ground state (g.s.) of the target A. Similar states are also known [5] in ^{10}Be, even though they cannot be reached in a (t,p) reaction. I recently noted the regular behavior [6] of the 4^+ energies, in terms of energies computed from the positions of the lowest $5/2^+$ states [7–10] in the $A+1$ nuclei.

It seems reasonable to also investigate the behavior of the 0^+ states. That is the purpose of the present Brief Report. It is already known that the 2^+ behaviors are more complex, probably because of varying amounts of configuration mixing between p-shell and $(sd)^2$ states, and perhaps because of the importance of structures built on 2^+ cores. Thus, the 2^+ states are not candidates for such an analysis.

The procedure here is similar to the one used for the 4^+ states: an $(sd)^2$ shell-model calculation with "local" single-particle energies (SPEs) and global two-body residual interaction matrix elements 2BME. The 4^+ calculations depended primarily on the energies of the $5/2^+$ states, and very slightly on the assumed $d_{5/2}$-$d_{3/2}$ splitting. The 0^+ evaluation will depend on both the $1/2^+$ and $5/2^+$ energies.

Table I lists the 2BMEs for 0^+ [11]. Table II contains the $1/2^+$ and $5/2^+$ energies in the relevant nuclei and the results of the diagonalization. In this simple model, there are only two $(sd)^2$ 0^+ states, because I have ignored the $d_{3/2}$ orbital. It is present at some level, but the bulk of its strength lies 10–14 MeV above the first $(sd)^2$ 0^+ state. Furthermore, the 0^+ 2BMEs that I am using were derived [11] for the $d_{5/2}$, $s_{1/2}$ space.

We note that the calculated energies of the lowest 0^+ $(sd)^2$ states agree reasonably well with the experimental ones. The signs of the differences are easy to understand for ^{10,12}Be and ^{14}C. In ^{10}Be and ^{14}C, a p-shell 0^+ state (the normal g.s.) lies below the lower $(sd)^2$ 0^+ state. Slight mixing between the two will lower the energy of the predominantly p-shell one and raise the energy of the other. Because I have not included any such p/sd mixing, the calculated energy of the $(sd)^2$ 0^+ state is slightly below the experimental energy of the second 0^+ state in ^{10}Be and ^{14}C. In ^{12}Be, the situation is just the opposite. The $(sd)^2$ 0^+ state lies below the predicted p-shell ^{12}Be (g.s.). Mixing between the two lowers the lower one and raises the other. Thus, it is not surprising that the calculated $(sd)^2$ 0^+ energy is slightly high in ^{12}Be.

TABLE I. 2BMEs (MeV [11]) for $(sd)^2$ 0^+ states.

Configuration	Value
d^2, d^2	−2.78
d^2, s^2	−1.72
s^2, s^2	−1.54

Table III lists the energies of both 0^+ states in all four nuclei. The second $(sd)^2$ 0^+ state is not known in ^{10,12}Be. It will have very little $L=0$ $2n$ cluster strength because the lower $(sd)^2$ 0^+ state contains most of it. These cluster factors are listed in the last column of Table III (normalized to a sum of unity in our space). For this reason, the second $(sd)^2$ 0^+ state was not observed in ^{10}Be(t,p). In ^{10}Be, its very small $L=0$ $2n$ cluster strength may also produce a very small alpha spectroscopic factor for $^{10}\text{Be} \rightarrow {}^{6}\text{He} + \alpha$. The second $(sd)^2$ 0^+ state will be very difficult to populate in any reaction. In ^{12,14}C(t,p), the second $(sd)^2$ 0^+ states had only about 5% of the strength of the first one—in approximate agreement with the present results.

In ^{14}C, the presence of the normal p-shell 0^+ state and its mixing with the lowest $(sd)^2$ 0^+ state do not appear to affect the energy or strength of the second $(sd)^2$ 0^+ state, as evidenced by the fact that this latter state (the third one in ^{14}C) has nearly identical characteristics to the second 0^+ state in ^{16}C, where no p-shell 0^+ is present. I thus expect the third 0^+ states in ^{10,12}Be [the second $(sd)^2$ ones] to have energies near the predicted values.

Candidates for other nuclei to which to apply the current procedure would include ^{14}Be, ^{10}He, and ^{8}He. However, all three nuclei present serious complications. First, the s and d SPEs in the core $+1n$ nuclei (^{13}Be, ^{9}He, and ^{7}He) are not well determined. Second, for ^{14}Be the ^{12}Be core is not of pure p-shell character, and for ^{10}He the nature of the ^{8}He g.s. core is poorly known. Furthermore, all three nuclei involve the difficulty of treating an unbound s-wave neutron. For all these reasons, the current procedure cannot be applied to those nuclei at the present time.

In summary, the present simple model works very well for the lowest $(sd)^2$ 0^+ states in ^{10,12}Be and ^{14,16}C, and for the second $(sd)^2$ 0^+ in ^{14,16}C, where they are known. The model offers predicted energies for the second $(sd)^2$ 0^+ in ^{10,12}Be, where they are not known. The suggestion [12] that the probable (3^-) state at 4.56 MeV [1,13,14] might instead be 0^+ has been addressed in a recent Comment [15] and a Reply [16]. As we see from Table III, this predicted 0^+ state would have been very weak in (t,p), whereas the 4.56-MeV state is quite strong in that reaction [1].

TABLE II. Energies (MeV) of relevant states.

		Core + 1n				Core + 2n				
		E_x		E_n		E_{2n} (g.s.)	E_{2n} Calc.	E_x Calc.	E_x Expt.	
Core	E_n (g.s.)	$1/2^+$	$5/2^+$	$1/2^+$	$5/2^+$					$P(s^2)$
^{8}Be	−1.665	1.684	3.049	0.019	1.384	−8.478	−2.631	5.847	6.179	0.70
^{10}Be	−0.503	0	1.783	−0.503	1.280	−3.673	−3.460	0.213	0	0.78
^{12}C	−4.946	3.089	3.854	−1.857	−1.092	−13.12	−6.835	6.287	6.590	0.54
^{14}C	−1.218	0	0.740	−1.218	−0.478	−5.469	−5.580	−0.111	0	0.53

TABLE III. Calculated and experimental energies (MeV) of $(sd)^2$ 0^+ states in selected nuclei and calculated $2n$ transfer strengths S_0.

Nucleus	State	Excitation energy		S_0
		Calculated	Experiment	
^{10}Be	$(sd)^2$ 0_1	5.85	6.18	1.0
	$(sd)^2$ 0_2	9.60	Unknown	0.0007
^{12}Be	$(sd)^2$ 0_1	0.21	0	0.98
	$(\mathrm{sd})^2$ 0_2	4.36	Unknown	0.014
^{14}C	$(sd)^2$ 0_1	6.29	6.59	0.98
	$(sd)^2$ 0_2	9.74	9.75	0.019
^{16}C	$(sd)^2$ 0_1	−0.11	0	0.98
	$(sd)^2$ 0_2	3.33	3.03	0.023

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