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πN P_{11} wave amplitude in the Skyrme model

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The quantized Skyrme model is applied to the πN scattering problem, especially the P_{11} wave amplitude. The field fluctuation and the zero mode are consistently taken into account, and the quantum correction to the Born term is calculated. It is found that this $1/N_c$ correction is comparable with the Born term and is crucially important for the πN scattering problem in the Skyrme model.

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I. INTRODUCTION

The Skyrme model is one of the effective models of QCD and describes the nucleon structure in a completely different manner from the constituent quark model [1]. Based on the spontaneous breakdown of the $SU_R(2) \times SU_L(2)$ chiral symmetry, this model is a nonlinear field theory of the pseudoscalar Nambu-Goldstone pion. The static soliton solution with the hedgehog configuration satisfies the nonlinear Euler-Lagrange equation for the classical pion field. The isospin rotation of this soliton is quantized in order to produce a nucleon state built on a single soliton vacuum. This model was succeeded in reproducing the static property of a nucleon with less than 30% accuracy [2].

The effective model has a purpose to get the understanding of low-lying baryons and their resonances. Although the static property is an important source, much more information can be extracted from the dynamical matter such as meson-baryon interactions. Therefore the validity of the Skyrme model should be judged by applying it, for example, to the πN scattering problem. The pion field must be quantized in addition to the isospin rotation of the classical soliton so as to deal with the pion-nucleon interaction.

The method of quantization is well established for the nonlinear field theory with a nontrivial classical solution [3–5]. By defining the field fluctuation on the classical solution, the Lagrangian is expanded with respect to this fluctuation. The second-order term becomes the basis for the canonical method of quantization.

When the field fluctuation is built on the spatially localized solution, there inevitably appears the zero mode. The field fluctuation should be treated separately from this zero mode; otherwise the canonical quantization does not work. As for the zero mode, a collective coordinate is properly introduced and treated as the quantum mechanical operator.

In the Skyrme model, the quantization is carried out for the zero mode by Ref. [2] and brings out the static property of a nucleon. The field fluctuation is quantized in a series of works by Walliser *et al.* [6–8], and has been energetically investigated in its application to the pion nucleon interaction [9–14].

It is not an easy task, however, to show that the Skyrme model is really available for the dynamical property of a nucleon in spite of these examinations. There has been a bias that this model is not suitable for the πN scattering problem, e.g., the Yukawa problem and the missing of Born term.

But this is not the case. The possibility of the Skyrme model as an effective tool to study the pion-nucleon interaction was not argued enough exhaustively. The field fluctuation and the zero mode have still not been considered in a fully consistent manner. In fact it is possible to show that the Skyrme model safely reproduces the Born term and the Yukawa coupling of the πN scattering [15].

There exist many experimental data for which the conventional picture of the constituent quark model does not work, e.g., the light mass of $N(1440)$ and $\Lambda(1405)$, the mass difference between $N(1535)$ and $N(1520)$, the strong coupling of $N(1535)$ with the η meson, and so on [16]. New data of the pion photoproduction is now available, and systematic analyses are energetically performed so as to know more about the nature of baryons and mesons [17,18]. A well-established model is desired, and we really expect that the Skyrme model is a candidate for the effective model of baryons.

In this paper, we evaluate a quantum correction to the Born term in the Skyrme model, which emerges from the canonical quantization of the field fluctuation. The zero mode for the isospin rotation and the spatial translation of the classical soliton is consistently taken into account together with the field fluctuation. Employing the chiral reduction formula [19], we examine the πN scattering problem by using this quantized Skyrme model.

We especially choose the P_{11} wave amplitude because it occupies a major part of the πN scattering amplitude. We consider this partial wave as a test ground of our attempt developed here.

In the next section we briefly describe the static aspect of the Skyrme model of Ref. [2]. We develop the canonical quantization in Sec. III and apply it to the calculation of the πN P_{11} wave amplitude in Sec. IV. We show our results in Sec. V and summarize this work in the last section.

II. SKYRME MODEL: BRIEF REVIEW

The Skyrme model is a nonlinear σ model characterized by the classical soliton with the hedgehog configuration. The collective coordinate method of quantization is applied to this soliton so as to bring a nucleon state out of the pion field.

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The $SU_R(2) \times SU_L(2)$ chiral symmetry is spontaneously broken to the $SU_V(2)$ vector symmetry. The representative ξ of the left coset $SU_R(2) \times SU_L(2)/SU_V(2)$ and its proper combinations $U = \xi^2$, $L_\mu = U^\dagger \partial_\mu U$ are employed to write a Lagrangian density [1,20–22]

$$\mathcal{L} = -\frac{f_\pi^2}{4}\mathrm{tr}L_\mu L^\mu + \frac{1}{32e^2}\mathrm{tr}[L_\mu, L_\nu][L^\mu, L^\nu] + \frac{m_\pi^2 f_\pi^2}{4}\mathrm{tr}(U + U^\dagger), \quad (1)$$

where m_π is the pion mass, f_π is the pion decay constant, and e is the model parameter. The first term comes from the nonlinear σ model, and the second term (the Skyrme term) is introduced to stabilize a static soliton solution. Explicit breaking of the chiral symmetry is taken into account in the third term.

The pion field, denoted by the isovector pseudoscalar field $\vec{\varphi}$, is introduced by the exponential mapping $\xi = \exp(i\vec{\varphi} \cdot \vec{\tau}/2f_\pi)$ where $\vec{\tau}$ is the Pauli matrix. The classical Euler-Lagrange equation has the static soliton solution $\vec{\varphi}_c$ with the hedgehog configuration

$$\vec{\varphi}_c = f_\pi F(r)\hat{r}, \quad (2)$$

where $\hat{r} = \vec{r}/|\vec{r}\,|$ [23,24]. With this configuration the Lagrangian (1) becomes

$$\mathcal{L}_c[F] = -\frac{f_\pi^2}{2}\left(F'^2 + 2\frac{\sin^2 F}{r^2}\right) - \frac{1}{2e^2}\frac{\sin^2 F}{r^2}\left(2F'^2 + \frac{\sin^2 F}{r^2}\right) + m_\pi^2 f_\pi^2 \cos F, \quad (3)$$

and $F(r)$ satisfies the nonlinear differential equation

$$\left(1 + \frac{2}{(f_\pi e)^2}\frac{\sin^2 F}{r^2}\right)F'' + \frac{2}{r}F' - \frac{\sin 2F}{r^2} + \frac{1}{(f_\pi e)^2}\frac{\sin 2F}{r^2}F'^2 - \frac{1}{(f_\pi e)^2}\frac{\sin 2F \sin^2 F}{r^4} - m_\pi^2 \sin F = 0, \quad (4)$$

where $F' = dF/dr$ and $F'' = d^2F/dr^2$. The boundary condition, $F(0) = \pi$ and $F(\infty) = 0$, provides a unit winding number to the classical soliton. Asymptotic behavior of $F(r)$ is $\exp(-m_\pi r)/r$ when $r \to \infty$, and $\pi - ar$ around $r = 0$ with a positive constant a [25].

By using the collective coordinate method of quantization, an isospin eigenstate is built on the classical soliton. Owing to the hedgehog configuration, the isospin rotation is exactly compensated by the spatial rotation. Then the simultaneous eigenstate is obtained both for the isospin and spin operators, and this eigenstate is identified as a nucleon state.

The rotation of $U_c(\vec{r}\,) = \exp(iF(r)\hat{r} \cdot \vec{\tau})$ in the isospin space is given by

$$U(t,\vec{r}\,) = A(t)U_c(\vec{r}\,)A^\dagger(t) \quad (5)$$

where $A(t)$ is the SU(2) matrix parametrized by three Euler angles depending on time: $[\alpha(t),\beta(t),\gamma(t)]$,

$$A(t) = e^{\frac{i}{2}\gamma(t)\tau_3}e^{\frac{i}{2}\beta(t)\tau_2}e^{\frac{i}{2}\alpha(t)\tau_3}. \quad (6)$$

By using Eq. (5), the Lagrangian (1) becomes

$$L = \frac{1}{2}\mathcal{I}\,\vec{\Omega} \cdot \vec{\Omega} + \int \mathcal{L}_c[F]\,d^3r, \quad (7)$$

where $\vec{\Omega}$ is the angular velocity,

$$\vec{\Omega} = -i\mathrm{tr}\vec{\tau}\dot{A}A^\dagger = \begin{pmatrix} -\dot{\alpha}\sin\beta\cos\gamma + \dot{\beta}\sin\gamma \\ \dot{\alpha}\sin\beta\sin\gamma + \dot{\beta}\cos\gamma \\ \dot{\alpha}\cos\beta + \dot{\gamma} \end{pmatrix}, \quad (8)$$

and $\mathcal{I}$ is the moment of inertia of the classical soliton,

$$\mathcal{I} = \frac{2f_\pi^2}{3}\int \sin^2 F\left\{+\frac{1}{(f_\pi e)^2}F\left(F'^2 + \frac{\sin^2 F}{r^2}\right)\right\}d^3r. \quad (9)$$

The canonical quantization is carried out by treating these Euler angles as quantum mechanical operators; $\alpha(t) \to \alpha$, $\partial L/\partial\dot{\alpha} \to -i\partial/\partial\alpha$, and so on. The isospin $\hat{I}_a$ defined by

$$\hat{I}_a = -\frac{\partial L}{\partial \Omega_a} = -\mathcal{I}\Omega_a \quad (10)$$

becomes the differential operator satisfying the SU(2) Lie algebra. The spin operator is similarly defined as

$$\hat{J}_a = i\mathcal{I}\mathrm{tr}\tau_a\dot{A}^\dagger A, \quad (11)$$

which satisfies the same commutation relation as $\hat{I}_a$. Since $\hat{J}^2 = \hat{I}^2$ and $\sum_b D_{ab}\hat{J}_b = -\hat{I}_a$, where D_{ab} is the three-dimensional representation of the SU(2) element, the simultaneous eigenstate of $\hat{J}^2 = \hat{I}^2$, $\hat{I}_3$, and $\hat{J}_3$ in "the Euler angle representation" is given by the $\mathcal{D}$ function

$$\langle\alpha\beta\gamma|J = I, I_3, J_3\rangle = (-1)^{-I_3}\sqrt{\frac{2J+1}{8\pi^2}}\mathcal{D}^{J=I}_{-I_3,J_3}(\alpha,\beta,\gamma), \quad (12)$$

which is normalized in the parameter space of SU(2). This eigenstate is a nucleon state in the Skyrme model.

III. QUANTIZATION: FIELD FLUCTUATION

The field fluctuation is defined as $\vec{\chi}(t,\vec{r}) = \vec{\varphi}(t,\vec{r}) - f_\pi F(r)\hat{r}$. Taking account of the isospin rotation, we write U in the Lagrangian (1) as

$$U(t,\vec{r}\,) = A(t)\exp\left(i\left(F(r)\hat{r} + \frac{1}{f_\pi}\vec{\chi}(t,\vec{r}\,)\right)\cdot\vec{\tau}\right)A(t)^\dagger. \quad (13)$$

Making a Taylor expansion up to the second order of $\vec{\chi}$, we obtain the Lagrangian

$$L = \frac{1}{2}\mathcal{I}\vec{\Omega} \cdot \vec{\Omega} + \int \mathcal{L}_c[F]\,d^3r + \int\left(g_F(r)\frac{\partial\vec{\chi}}{\partial t} \cdot \frac{\partial\vec{\chi}}{\partial t} - \vec{\chi} \cdot \mathcal{H}_f\vec{\chi}\right)d^3r, \quad (14)$$

where $g_F(r) = [\sin F(r)/F(r)]^2$ and $\mathcal{H}_f$ is the linear differential operator. We note here that this operator depends on $F(r)$ instead of showing a complicated expression of $\mathcal{H}_f$. The first-order term with respect to $\vec{\chi}$ is not there because the classical soliton gives the minimum of action integral.

The conjugate field to $\vec{\chi}$ is

$$\pi_i(t,\vec{r}) = \frac{\delta L}{\delta(\partial\chi_i/\partial t)} = g_F(r)\frac{\partial\chi_i}{\partial t}, \tag{15}$$

and the Hamiltonian becomes

$$H = \frac{1}{2\mathcal{I}}\hat{I}\cdot\hat{I} - \int \mathcal{L}_c[F]d^3r + \frac{1}{2}\int\left\{\frac{1}{g_F(r)}\vec{\pi}\cdot\vec{\pi} + \vec{\chi}\cdot\mathcal{H}_f\vec{\chi}\right\}d^3r. \tag{16}$$

Following the canonical procedure, we treat $\vec{\chi}$ and $\vec{\pi}$ as the quantum field operators and apply the equal time commutation relation for them:

$$[\hat{\chi}_i(t,\vec{r}\,),\hat{\pi}_j(t,\vec{r}\,')] = i\delta_{ij}\delta(\vec{r}-\vec{r}\,')\,. \tag{17}$$

The linear differential equation satisfied by $\hat{\vec{\chi}}$ is obtained as

$$g_F(r)\frac{\partial^2\hat{\vec{\chi}}(t,\vec{r}\,)}{\partial t^2} + \mathcal{H}_f\hat{\vec{\chi}}(t,\vec{r}\,) = 0, \tag{18}$$

through the Heisenberg equation.

In order to make the normal mode expansion for $\hat{\vec{\chi}}$, we need the eigenfunction $\vec{\chi}_p$ of $\mathcal{H}_f$

$$\mathcal{H}_f\vec{\chi}_p(\vec{r}) = g_F(r)\omega_p^2\vec{\chi}_p(\vec{r}). \tag{19}$$

Because $\mathcal{H}_f$ is the self-adjoint operator in spite of the complicated dependence on $F(r)$, the eigenfunctions span the complete and orthonormal set. Since $F(r)\to 0$ in $r\to\infty$, the asymptotic form of Eq. (19) becomes

$$\left(-\nabla^2 + m_\pi^2\right)\vec{\chi}_p(\vec{r}\,) = \omega_p^2\vec{\chi}_p(\vec{r}\,) \tag{20}$$

and $\vec{\chi}_p$ approaches $\exp(i\vec{p}\cdot\vec{r})$ with $\omega_p^2 = m_\pi^2 + p^2$.

The classical soliton takes the hedgehog configuration in which the isovector field is directed in $\hat{r}$. This configuration is invariant under the simultaneous rotation in the coordinate space and the isospin space. Because the differential equation (19) is also invariant under this rotational symmetry, the vector spherical harmonics ($Y_{JM}^{(E)}$, $Y_{JM}^{(L)}$, $Y_{JM}^{(M)}$) is useful to extract the radial dependence of Eq. (19) [26,27].

There are three types of eigenfunctions of Eq. (19): two linear combinations of $Y_{JM}^{(E)}$ and $Y_{JM}^{(L)}$

$$\chi_{pJM}^{(k)}(\vec{r}\,) = R_{pJ}^{(E)}(r)Y_{JM}^{(E)}(\theta,\varphi) + R_{pJ}^{(L)}(r)Y_{JM}^{(L)}(\theta,\varphi) \tag{21}$$

for $k = 1,2$, and

$$\chi_{pJM}^{(3)}(\vec{r}\,) = R_{pJ}^{(M)}(r)Y_{JM}^{(M)}(\theta,\varphi). \tag{22}$$

The spatial angular momentum is $L = J\pm 1$ for $Y_{JM}^{(E)}$ and $Y_{JM}^{(L)}$ and their parity is $(-1)^{J\pm1}$, while $L = J$ for $Y_{JM}^{(M)}$ and the parity is $(-1)^J$. By substituting Eq. (21) in Eq. (19), we obtain the coupled ordinary differential equation for $R_{pJ}^{(E)}$ and $R_{pJ}^{(L)}$,

$$\begin{aligned}
&-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_{pJ}^{(L)}}{dr}\right) + \frac{J(J+1)}{r^2}R_{pJ}^{(L)} + \left(2\frac{g_F}{r^2} + \frac{d^2g_F}{dF^2}\frac{F^2}{r^2} + 4\frac{dg_F}{dF}\frac{F}{r^2}\right)R_{pJ}^{(L)} - \left(2g_F + \frac{dg_F}{dF}F\right)\frac{\sqrt{J(J+1)}}{r^2}R_{pJ}^{(E)}\\
&+\frac{g_F}{(f_\pi e)^2}\left\{-2\frac{F^2}{r^2}\frac{d^2R_{pJ}^{(L)}}{dr^2} + \sqrt{J(J+1)}\frac{FF'}{r^2}\frac{dR_{pJ}^{(E)}}{dr} - \sqrt{J(J+1)}\left(\frac{F'^2}{r^2} - \frac{2FF''}{r^2}\right)R_{pJ}^{(E)} + J(J+1)\frac{F^2}{r^4}R_{pJ}^{(L)}\right\}\\
&+\frac{g_F}{(f_\pi e)^2}\frac{\sin 2F}{F}\left(-2\sqrt{J(J+1)}\frac{F^2}{r^4}R_{pJ}^{(E)}\right) + \frac{1}{(f_\pi e)^2}\frac{\sin 2F}{F}\left(\sqrt{J(J+1)}\frac{F'^2}{r^2}R_{pJ}^{(E)} - 2\frac{FF'}{r^2}\frac{dR_{pJ}^{(L)}}{dr} - 2\frac{FF''}{r^2}R_{pJ}^{(L)}\right)\\
&+\frac{2}{(f_\pi e)^2}\cos F\left(-\frac{F'^2}{r^2} + g_F\frac{F^2}{r^4}\right)R_{pJ}^{(L)} + \frac{1}{(f_\pi e)^2}\frac{\sin^2 2F}{r^4}R_{pJ}^{(L)} + \left(m_\pi^2\cos F - g_F\omega_p^2\right)R_{pJ}^{(L)} = 0.
\end{aligned} \tag{23}$$

According to the asymptotic forms in $r\to\infty$, the orthogonality relation is given by

$$\int d^3r g_F(r)\vec{\chi}_{p'J'M'}^{(k')*}(\vec{r}\,)\cdot\vec{\chi}_{pJM}^{(k)}(\vec{r}\,) = \frac{(2\pi)^3}{p^2}\delta(p'-p)\delta_{J'J}\delta_{M'M}\delta_{k'k} \tag{24}$$

and the completeness relation becomes

$$\sum_{kJM}\int_0^\infty \frac{dpp^2}{(2\pi)^3}g_F(r)\chi_{pJM,i'}^{(k)*}(\vec{r}\,')\chi_{pJM,i}^{(k)}(\vec{r}\,) = \delta_{i'i}\delta^3(\vec{r}\,'-\vec{r}\,). \tag{25}$$

We write the normal mode expansion of $\hat{\vec{\chi}}(\vec{r}\,)$ as

$$\hat{\vec{\chi}}(\vec{r}\,) = \sum_{kJM}\int_0^\infty \frac{dpp^2}{\sqrt{(2\pi)^3 2\omega_p}}\left\{\hat{a}_{pJM}^{(k)}e^{-i\omega_p t}\vec{\chi}_{pJM}^{(k)}(\vec{r}\,) + \hat{a}_{pJM}^{(k)\dagger}e^{i\omega_p t}\vec{\chi}_{pJM}^{(k)*}(\vec{r}\,)\right\}, \tag{26}$$

where $\hat{a}_{pJM}^{(k)}$ ($\hat{a}_{pJM}^{(k)\dagger}$) is the annihilation (creation) operator for the dynamical particle in the Skyrme model (we use a term "the dynamical pion").

Although the eigenfunction $\vec{\chi}_{pJM}^{(k)}$ is distorted by the classical soliton, the creation and annihilation operators for the dynamical pion satisfy the free-particle commutation relation,

$$\left[\hat{a}^{(k)}_{pJM}, \hat{a}^{(k')\,\dagger}_{p'J'M'}\right] = \frac{(2\pi)^3 2\omega_p}{p^2}\delta(p-p')\delta_{JJ'}\delta_{MM'}\delta_{kk'}. \quad (27)$$

For the P_{11} wave amplitude considered in this work, the spatial angular momentum is $L = 1$ and relevant value of J is 0 and 2. We focus on the eigenfunction with $J = 0$ and $L = 1$ (a purely longitudinal wave) because it takes the same functional form as the hedgehog configuration; $\chi^{(L)}_{p0M} = R^{(L)}_{p0}(r)\hat{r}/\sqrt{4\pi}$. The linear differential equation satisfied by $R^{(L)}_{p0}(r)$ is

$$\begin{aligned}&\left(1+\frac{2}{(f_\pi e)^2}\frac{\sin^2 F}{r^2}\right)\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR^{(L)}_{p0}}{dr}\right)-\frac{2}{r^2}\cos 2F R^{(L)}_{p0}\\&-\frac{2}{(f_\pi e)^2}\frac{1}{r^2}\left(\frac{2\sin^2 F}{r}-F'\sin 2F\right)\frac{dR^{(L)}_{p0}}{dr}\\&+\frac{2}{(f_\pi e)^2}\frac{1}{r^2}\left\{F''\sin 2F-\cos F\left(\frac{\sin^2 F}{r^2}-F'^2\right)\right.\\&\left.-\frac{\sin^2 2F}{2r^2}+\frac{1}{r}\left(\frac{2\sin^2 F}{r}-F'\sin 2F\right)\right\}R^{(L)}_{p0}\\&-\left(m_\pi^2\cos F-g_F\omega_p^2\right)R^{(L)}_{p0}=0\,. \quad (28)\end{aligned}$$

This peculiar form of the excitation corresponds to the breathing mode of the classical soliton, i.e., the excitation, without changing its shape.

Here we comment on the zero mode in connection with the vector spherical harmonics. Two types of the zero mode are there in the Skyrme model. The one is due to the spatial translation. This zero mode is obtained by the gradient of the hedgehog configuration,

$$\nabla_m F(r)\hat{r} = \sqrt{\frac{4\pi}{3}}\frac{dF}{dr}Y^{(L)}_{1m} + \sqrt{\frac{8\pi}{3}}\frac{F}{r}Y^{(E)}_{1m}\,. \quad (29)$$

The other one is due to the isospin rotation. The infinitesimal isospin rotation is given by the corresponding spatial rotation,

$$L_m F(r)\hat{r} = -\sqrt{\frac{8\pi}{3}}F Y^{(M)}_{1m}, \quad (30)$$

where L_m is the angular momentum operator. Since these zero modes are orthogonal to $\vec{\chi}^{(k)}_{pJM}$ in Eq. (26) owing to the orthogonality of $Y^{(k)}_{JM}$ and the nonzero value of ω_p, they do not mix with our field fluctuation.

Before ending this section, we summarize our view for the pion and the nucleon in our treatment. The canonical quantization provides us these particles. Both are emerged from the classical soliton; the nucleon appears by quantizing the zero mode while the pion is embedded in the field fluctuation.

IV. AXIAL VECTOR CURRENT AND AMPLITUDE

The quantized Skyrme model developed so far is now used to calculate the πN P_{11} wave amplitude. We employ the chiral reduction formula developed by Yamagishi and Zahed [19]. This formula provides us a practical way of analysis for the scattering phenomena in the effective theory with the chiral symmetry. The S matrix is written as the time-ordered product of relevant currents.

The S matrix for the elastic πN scattering is given by

$$\begin{aligned}&\langle \pi_b(k_2)N(p_2)|\hat{S}-1|\pi_a(k_1)N(p_1)\rangle\\&=-\frac{i}{f_\pi}m_\pi^2\delta_{ab}\int d^4x e^{-(k_1-k_2)\cdot x}\langle N(p_2)|\sigma(x)|N(p_1)\rangle\\&\quad-\frac{1}{f_\pi^2}\int d^4x_1 d^4x_2 e^{-ik_1\cdot x_1+ik_2\cdot x_2}\\&\quad\times\langle N(p_2)|\mathrm{T}(k_1\cdot j_{A,a}(x_1)\,k_2\cdot j_{A,b}(x_2))|N(p_1)\rangle\\&\quad+\frac{1}{f_\pi^2}\int d^4x e^{-i(k_1-k_2)\cdot x}\epsilon_{abc}\\&\quad\times\langle N(p_2)|k_1\cdot j_{V,c}(x)|N(p_1)\rangle, \quad (31)\end{aligned}$$

where $j_{A,a}$ and $j_{V,c}$ are the axial-vector and the vector currents, respectively; a, b, and c are the pion isospin index; and σ is the scalar current. Because the P_{11} wave amplitude is related with the second term of the right-hand side of Eq. (31), we consider the axial-vector current in the following calculations.

The symmetry transformation by $g_L(\vec{\alpha}) \in \mathrm{SU}_L(2)$ and $g_R(\vec{\alpha}) \in \mathrm{SU}_R(2)$ is

$$U \to U' = g_L(\vec{\alpha})Ug_R^\dagger(\vec{\alpha}), \quad (32)$$

where $\vec{\alpha}$ parametrizes the group element g_L. For an infinitesimal $\vec{\alpha}$, the corresponding Nöther current (the left current) is

$$J^\mu_{L,a} = -\frac{\partial\delta\mathcal{L}}{\partial(\partial_\mu\alpha_a)}, \quad (33)$$

and the right current $J^\mu_{R,a}$ is derived in a similar way as $J^\mu_{L,a}$. We obtain the axial vector current $J^\mu_{A,a} = J^\mu_{R,a} - J^\mu_{L,a}$ which satisfies the Partially Conserved Axial-vector Current (PCAC) relation

$$\partial_\mu J^\mu_{A,a} = f_\pi m_\pi^2\pi_a, \quad (34)$$

where the right-hand side is due to the explicit breaking of the chiral symmetry, and π_a is the pseudoscalar-isovector field.

Before using this axial vector current in the reduction formula, we must subtract the pion pole contribution mixed in through the PCAC relation [19,28]. The pole-free axial vector current is defined by

$$j^\mu_{A,a} = J^\mu_{A,a} + f_\pi\partial^\mu\pi_a. \quad (35)$$

By using the expressions

$$J^\mu_{A,a} = -\frac{if_\pi^2}{4}\mathrm{tr}\tau_a(U\partial^\mu U^\dagger - U^\dagger\partial U) \quad (36)$$

and

$$\pi_a = \frac{if_\pi}{4}\mathrm{tr}\tau_a(U^\dagger - U), \quad (37)$$

the pole-free axial vector current is given as a function of U. Using Eq. (13) and making a Taylor expansion about $\hat{\vec{\chi}}$ up to its first order, we write the spatial component of Eq. (35) as

$$\vec{j}_{A,a} = \vec{j}^{(c)}_{A,a} + \vec{j}^{(q)}_{A,a}, \quad (38)$$

where the first term is the "classical" part

$$j^{(c)}_{A,ia} = f_\pi^2 (1-\cos F)\left\{\frac{\sin F}{r}\delta_{ij} - \left(F' + \frac{\sin F}{r}\right)\hat{r}_i\hat{r}_j\right\}\frac{1}{2}\mathrm{tr}\tau_a A\tau_j A^\dagger \tag{39}$$

and the second term is the "quantum" part depending linearly on $\hat{\vec{\chi}}$

$$\begin{aligned}\hat{j}^{(q)}_{A,ia} = f_\pi\Bigg\{&\frac{1}{r}\left[\left(\cos F - \frac{\sin F}{F}\right)(1-\cos F) + \sin^2 F\right](\delta_{ij} - \hat{r}_i\hat{r}_j)\hat{r}\cdot\hat{\vec{\chi}} - F'\sin F\hat{r}\cdot\hat{\vec{\chi}}\hat{r}_i\hat{r}_j + \frac{\sin F}{F}(1-\cos F)\nabla_i\hat{\chi}_j\\ &-\frac{1}{F}\left(1+\frac{\sin F}{F}\right)(1-\cos F)\left[F\hat{r}\cdot\nabla_i\hat{\vec{\chi}}\hat{r}_j + \left(F' - \frac{F}{r}\right)\hat{r}\cdot\hat{\vec{\chi}}\hat{r}_i\hat{r}_j + \frac{F}{r}\hat{\chi}_i\hat{r}_j + F'\hat{r}_i\hat{\chi}_j\right]\\ &+\frac{2}{F}\left(1+\frac{\sin F}{F}\right)(1-\cos F)F'\hat{r}\cdot\hat{\vec{\chi}}\hat{r}_i\hat{r}_j\Bigg\}\frac{1}{2}\mathrm{tr}\tau_a A\tau_j A^\dagger.\end{aligned} \tag{40}$$

The time component of the axial vector current (35) depends on $1/\mathcal{I}$ and is smaller than the spatial components by $1/N_c$. We therefore consider only the spatial part in the following calculations.

The second term of Eq. (31) becomes

$$\begin{aligned}&\langle\pi_b(k_2)N(p_2)|\hat{S}-1|\pi_a(k_1)N(p_1)\rangle_{\mathrm{2nd}}\\ &= -\frac{1}{f_\pi^2}\int d^4x_1 d^4x_2 e^{-ik_1x_1+ik_2x_2}\langle N(p_2)|\mathrm{T}\big(\vec{k}_1\cdot\vec{j}^{(c)}_{A,a}[\vec{r}_1-\vec{R}(t_1)]\vec{k}_2\cdot\vec{j}^{(c)}_{A,b}[\vec{r}_2-\vec{R}(t_2)]\big)|N(p_1)\rangle\\ &\quad-\frac{1}{f_\pi^2}\int d^4x_1 d^4x_2 e^{-ik_1x_1+ik_2x_2}\langle N(p_2)|\mathrm{T}\big(\vec{k}_1\cdot\hat{\vec{j}}^{(q)}_{A,a}[\vec{r}_1-\vec{R}(t_1)]\vec{k}_2\cdot\hat{\vec{j}}^{(q)}_{A,b}[\vec{r}_2-\vec{R}(t_2)]\big)|N(p_1)\rangle\\ &= i(2\pi)^4\delta^4(k_1+p_1-k_2-p_2)(\mathcal{M}_{\mathrm{Born}}(\vec{k}_1,\vec{k}_2) + \mathcal{M}_{Qc}(\vec{k}_1,\vec{k}_2)),\end{aligned} \tag{41}$$

where $\mathcal{M}_{\mathrm{Born}}$ depends on the "classical" part of the axial-vector current, which is free from the field fluctuation, and $\mathcal{M}_{Qc}$ depends on the "quantum" part.

In Eq. (41), the intermediate state consists of the nucleon [described by Eq. (12) with $I = J = 1/2$] and the dynamical pion. We consider the partial wave $J = 0$ (and $L = 1$) in Eq. (26) because this excitation takes the same form as the hedgehog configuration. $\mathcal{M}_{\mathrm{Born}}$ is the Born term, which has no contribution from the dynamical pion, and is made up of the nucleon pole term (s channel) and its crossing (u channel), and $\mathcal{M}_{Qc}$ represents the quantum correction due to the dynamical pion propagating in the intermediate state.

We introduce the central position of the soliton as the time-dependent coordinate $\vec{R}(t)$ in Eq. (41). When the static property of a nucleon is considered, this coordinate is usually set to be 0 and no reference is paid to it. As for the πN S matrix, because $\vec{R}(t)$ is related with the zero mode for the translational invariance of the classical soliton, we should quantize it by the collective coordinate method.

We substitute the expressions (38)–(40) to Eq. (41) and calculate $\mathcal{M}_{\mathrm{Born}}$ and $\mathcal{M}_{Qc}$. Since the classical soliton is treated as a heavy object in the canonical quantization, the nucleon state built on this soliton has necessarily large mass. Following this nonrelativistic view of the nucleon state, we approximate the nucleon energy by its mass.

For the elastic P_{11} wave amplitude, we obtain $\mathcal{M}_{\mathrm{Born}}$ as

$$\mathcal{M}_{\mathrm{Born}}(\vec{k},\vec{k}) = -\frac{8f_\pi^2}{9}\frac{4\pi k^2}{3}\frac{J(k)^2}{\omega_k}, \tag{42}$$

where $\vec{k}$ and (ω_k) is the pion momentum (energy). We define

$$\begin{aligned}J(q) = 4\pi\int_0^\infty dr r^2\Bigg\{&j_0(qr)\left(f_{A1}(r) + \frac{1}{3}f_{A2}(r)\right)\\ &-\frac{2}{3}j_2(qr)f_{A2}(r)\Bigg\},\end{aligned} \tag{43}$$

where $j_0(qr)$ and $j_2(qr)$ are the spherical Bessel function, and

$$f_{A1}(r) = \frac{\sin F}{r}(1-\cos F), \tag{44}$$

$$f_{A2}(r) = -\left(F' + \frac{\sin F}{r}\right)(1-\cos F). \tag{45}$$

The result for $\mathcal{M}_{Qc}$ is

$$\begin{aligned}\mathcal{M}_{Qc}(\vec{k},\vec{k}) &= -\frac{4\pi k^2}{3}\int_0^\infty \frac{dq q^2}{(2\pi)^3 2\omega_q}\frac{\tilde{J}(k,q)^2}{\omega-\omega_q+i\epsilon}\\ &\quad+\frac{1}{9}\frac{4\pi k^2}{3}\int_0^\infty\frac{dq q^2}{(2\pi)^3 2\omega_q}\frac{\tilde{J}(k,q)^2}{\omega+\omega_q}\\ &= \mathcal{M}^s_{Qc}(\vec{k},\vec{k}) + \mathcal{M}^u_{Qc}(\vec{k},\vec{k}).\end{aligned} \tag{46}$$

We write the s-channel and u-channel parts as $\mathcal{M}^s_{Qc}$ and $\mathcal{M}^u_{Qc}$, separately. We define

$$\begin{aligned}\tilde{J}(k,q) = \sqrt{4\pi}\int_0^\infty & dr r^2 j_0(kr)\\ &\times\left(R^{(L)}_{q0}(r)f_{A3}(r) + \frac{dR^{(L)}_{q0}}{dr}f_{A5}(r)\right)\end{aligned}$$

$$-\frac{2\sqrt{4\pi}}{3}\int_0^\infty drr^2 j_2(kr) \times \left(R_{q\,0}^{(L)}(r)f_{A4}(r)+\frac{dR_{q\,0}^{(L)}}{dr}f_{A5}(r)\right) \quad (47)$$

where

$$f_{A3}(r) = -\frac{F'\sin F}{3}+\frac{2}{3r}(1-\cos F)(1+2\cos F), \quad (48)$$

$$f_{A4}(r) = -F'\sin F-\frac{1}{r}(1-\cos F)(1+2\cos F), \quad (49)$$

$$f_{A5}(r) = -(1-\cos F). \quad (50)$$

$\mathcal{M}_{Qc}$ is the $1/N_c$ correction to $\mathcal{M}_{\rm Born}$ because $f_\pi \sim \sqrt{N_c}$. The explicit form of the πN P_{11} wave amplitude is now completed. In the next section we show the numerical results for this amplitude.

V. RESULTS AND DISCUSSION

Since our purpose is not the parameter search, we use the parameter value of Ref. [2] without any change. The Skyrme model has two parameters, $f_\pi = 64.5$ MeV and $e = 5.45$, which are fixed to reproduce the $N\Delta$ mass difference in the static treatment. No other additional parameters are introduced in this work.

Figure 1 shows $F(r)$ obtained by solving Eq. (4) with the proper boundary conditions. $F(r)$ is used to calculate the radial function $R_{p0}^{(L)}$. This function does not couple with $R_{pJ}^{(E)}$ because $J = 0$ [see Eq. (28)]. The r dependence of $R_{p0}^{(L)}$ at small r and its asymptotic form at large r is determined by L not by J. Although the centrifugal barrier is apparently absent in Eq. (23) when $J = 0$, the "potential" $-2/r^2$ (same as the P-wave centrifugal barrier) emerges out of the $F(r)$-dependent term. This characteristic behavior of the "potential" has been also pointed out in Ref. [29].

Figure 2 shows $R_{p0}^{(L)}$ at $p = 296$ MeV/c ($\omega_p = 330$ MeV) given by Eq. (28) and compares it with the spherical Bessel function $j_1(pr)$. Both functions are multiplied by r for convenience in this figure. The dynamical pion in this partial wave feels attractive force from the nucleon state through the $F(r)$-dependent "potential." This attractive force is observed up to $\omega_p \sim 600$ MeV and reaches its maximum value around $\omega_p = 330$ MeV. Although this radial function does not directly correspond to the pion scattering wave, the πN P_{11} wave amplitude is influenced by $R_{p0}^{(L)}$ through Eq. (47).

Now we calculate the πN P_{11} wave amplitude. Figure 3 shows $\mathcal{M}_{\rm Born}$, $\mathcal{M}_{Qc}^s$, and $\mathcal{M}_{Qc}^u$ as functions of the pion energy. Both $\mathcal{M}_{\rm Born}$ and $\mathcal{M}_{Qc}^u$ are real valued amplitude, while $\mathcal{M}_{Qc}^s$ is complex valued one because the πN channel opens in the intermediate state [see the propagator in Eq. (46)]. $\mathcal{M}_{\rm Born}$ dominates the amplitude for wide energy range. While

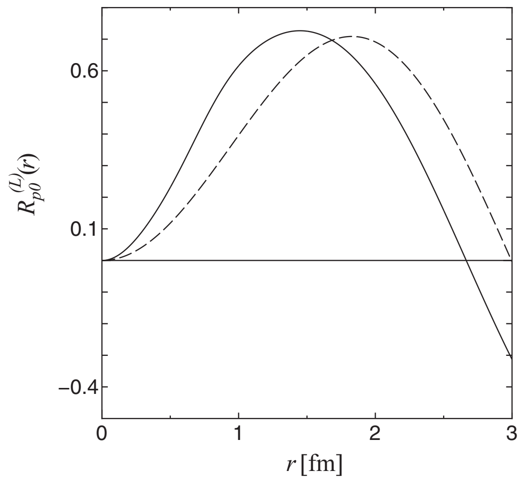

FIG. 2. The radial function $R_{p0}^{(L)}$ for the dynamical pion (solid curve) at $p = 296$ MeV/c ($\omega_p = 330$ MeV), and the spherical Bessel function $j_1(pr)$ (dashed curve).

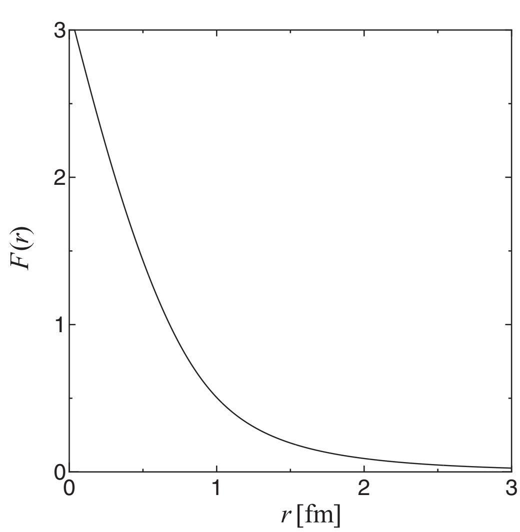

FIG. 1. $F(r)$ for the hedgehog configuration.

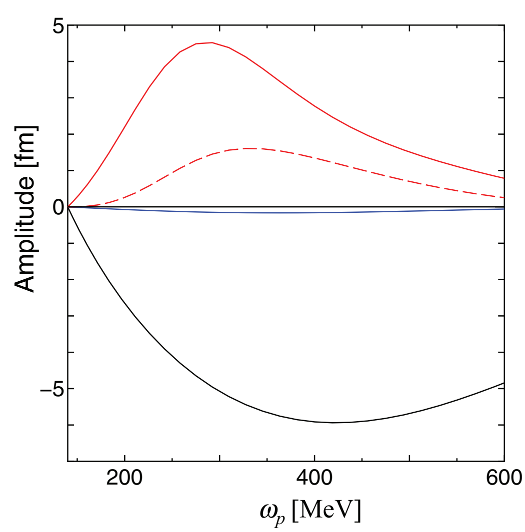

FIG. 3. (Color online) The πN P_{11} amplitudes $\mathcal{M}_{\rm Born}$ (black solid curve), Re $\mathcal{M}_{Qc}^s$ (red [gray] solid curve), Im $\mathcal{M}_{Qc}^s$ (red [gray] dashed curve), and $\mathcal{M}_{Qc}^u$ (blue [gray] solid curve) are shown as functions of the pion energy ω_p.

the contribution of $\mathcal{M}^u_{Qc}$ is small, the strength of $\mathcal{M}^s_{Qc}$ is compatible with that of $\mathcal{M}_{\text{Born}}$ around $\omega_p = 300$ MeV.

We consider this results from the large-N_c viewpoint on which the Skyrme model is grounded. The large-N_c limit is indeed important to classify various contributions that appeared in the quantization procedure. Here we find that it is not straightforward to understand our results in the large-N_c argument.

Although $\mathcal{M}_{Qc}$ is merely a small $1/N_c$ correction to $\mathcal{M}_{\text{Born}}$ when $N_c \to \infty$, both amplitudes have comparable strength in the real world with $N_c = 3$. This result suggests that the higher order contributions in the $1/N_c$ expansion are not always "small" when $N_c = 3$. In fact the zero mode for the isospin rotation, which is no more than the $O(1/N_c)$ contribution on the mass of baryons, is indispensable to reproduce the correct form of $\mathcal{M}_{\text{Born}}$. And this zero mode is of critical importance for the dynamical problems in the Skyrme model together with the field fluctuation which is the $O(N_c^0)$ contribution.

This serious problem for the $1/N_c$ expansion in the chiral soliton model has already been pointed out in the detailed analysis of the strangeness [30–32]. The well-known methods of quantization for the SU(3) Skyrme model, the bound-state approach [29] and the rigid rotor approach [33], are equivalent in the large-N_c limit. There appears, however, apparent difference between them when $N_c = 3$ in the calculation of observables, such as the baryon mass. These considerations argue that we must be careful in using any estimation based on the $1/N_c$ expansion in the chiral soliton model. Our observation obtained here is in accordance with this argument.

Note the peak in Im $\mathcal{M}^s_{Qc}$ at $\omega_p \sim 330$ MeV which reflects the attractive feature observed in $R^{(L)}_{p0}$. Although we expect that it might be related with the nucleon structure, it is too early to draw a decisive conclusion before doing more quantitative analysis, e.g., the parameter search in comparison with the experimental data. It is, however, important that the structure information about the nucleon can be extracted from the quantum correction due to the field fluctuation. Our results argue that the quantization is neccesary to examine the nucleon and its resonances in the Skyrme model.

VI. SUMMARY

Considering the field fluctuation and the zero mode in a fully consistent manner, we have applied the quantized Skyrme model to the πN scattering problem. We have introduced the dynamical pion in the single soliton vacuum. We have properly taken account of the $F(r)$-dependent effect on the dynamical pion, and we have also safely separated the zero mode from the field fluctuation.

In our calculation of the πN P_{11} wave amplitude, the amplitude obtained by quantizing the field fluctuation has comparable strength with the Born amplitude. It is necessary for the Skyrme model to deal with the field fluctuation in addition to the zero mode so as to consider the πN scattering problem.

Since the fluctuation considered here is the breathing mode in the P_{11} wave amplitude, it is interesting if this peak might be related with the Roper resonance. Although we need more examination in order to draw a decisive conclusion, we consider that the Skyrme model is really available for the practical study of the dynamical matter of the nucleon structure.

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