Semidileptonic decays of the light vector mesons in the light-front quark model

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We study the transition form factors of the light vector to pseudoscalar mesons as functions of the momentum transfer q^2 within the light-front quark model. With these form factors, we calculate the decay branching ratios of all possible modes for $V \to P \ell^+ \ell^-$ ($V = \omega$ and ϕ, $P = \pi^0$, η, and η', and $\ell = e$ and μ). We find that our numerical results fit with the data, such as those of $\omega \to \pi^0 \ell^+ \ell^-$ and $\phi \to \pi^0 e^+ e^-$ from the NA60 Collaboration and $\phi \to \eta e^+ e^-$ with the Spherical Neutral Detector. We also predict that the branching ratios of $\phi \to \pi^0 \mu^+ \mu^-$, $\omega \to \eta e^+ e^-$, $\omega \to \eta \mu^+ \mu^-$, $\phi \to \eta \mu^+ \mu^-$, and $\phi \to \eta' e^+ e^-$ to be around 3.48×10^{-6}, 3.22×10^{-6}, 1.81×10^{-9}, 6.86×10^{-6}, and 2.97×10^{-7}, respectively.

I. INTRODUCTION

Investigations of the transition processes between light vectors ($V = \omega$ and ϕ) and pseudoscalars ($P = \pi^0$, η, and η') mesons are helpful to infer the internal physics properties of mesons, in particular, the nonperturbative QCD effects. These processes can be described by transition form factors, denoted $f_{V \to P}$, from the parametrizations of the hadronic matrix elements. Some nonperturbative QCD approaches are available to evaluate these elements, such as the lattice QCD, QCD sum rule, and vector meson dominant (VMD) methods. Phenomenologically, the relativistic light-front quark model (LFQM) also provides a convenient method to study these form factors.

Motivated by the recent accurate data on $\omega \to \pi^0 \mu^+ \mu^-$ from the NA60 Collaboration (NA60) [1], we would like to study the transition form factors of $f_{V \to P}$ within the framework of the LFQM. In particular, we concentrate on the decay processes of $V \to P \ell^+ \ell^-$ with $V = (\omega, \phi)$, $P = (\pi^0, \eta, \eta')$ and $\ell = (e, \mu)$. In the LFQM, one can calculate the form factors in the frame where the momentum transfer is purely longitudinal, i.e, $p_\perp = 0$ and $p^2 = p^+ p^-$, which covers the whole allowed kinematic region of $0 \leqslant p^2 \leqslant p_{max}^2$. We use the phenomenological light-front (LF) meson wave functions [2–4] to evaluate $f_{V \to P}$ in the LFQM. The LF wave functions can be constructed by the simplest structures of the meson constituents in terms of quark-antiquark ($Q \bar{Q}$) Fock states [4], which have been widely applied to study the form factors of Dalitz decays [2,4,5].

The paper is organized as follows: In Sec. II, we present our formalism for the transition form factors of $V \to P$ within the LFQM and the decay branching ratios of $V \to P \ell^+ \ell^-$. In Sec. III, we perform our numerical calculations on these processes. We also compare our results in the LFQM with the data as well as other theoretical predictions. We give our conclusions in Sec. IV.

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II. FORMALISM

We start with the decay process,

$$V(p_v) \to P(p_p)\gamma^*(q) \to P(p_p)\ell^+(p_1)\ell^-(p_2), \quad (1)$$

where V represents the vector meson of ω or ϕ, P stands for the pseudoscalar of π^0 or η or η', and $\ell = e$ or μ, while p_v, p_p, q, p_1, and p_2 are the corresponding momenta. The reason that we only consider the process in Eq. (1) is that the dominant contributions to the dilepton channels of the vector mesons are through the exchanges of the virtual photon. The decay amplitude for the process in Eq. (1) is given by [6]:

$$A = ie^2 f_{V \to P}(q^2) \varepsilon_{\mu\nu\rho\sigma} \epsilon^\mu p_p^\nu q^\rho \frac{1}{q^2} \bar{u}(p_2)\gamma^\sigma v(p_1), \quad (2)$$

where $q = p_1 + p_2$, ϵ^μ is the polarization vector of the vector meson, and $f_{V \to P}(q^2)$ is the transition form factor, defined by

$$\langle P(p_p)|J_\sigma^{em}|V(p_v)\rangle = f_{V \to P}(q^2)\varepsilon_{\mu\nu\rho\sigma}\epsilon^\mu p_p^\nu q^\rho. \quad (3)$$

To calculate the form factor of $f_{V \to P}(q^2)$, we use the quark-flavor mixing scheme to express the vector mesons of ϕ and ω as two orthogonal flavor states of $|\psi_q\rangle$ and $|\psi_s\rangle$ with one mixing angle scenario, parameterized as [7,8]

$$\begin{pmatrix} |\phi\rangle \\ |\omega\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta^V & -\sin\theta^V \\ \sin\theta^V & \cos\theta^V \end{pmatrix} \begin{pmatrix} |\psi_q\rangle \\ |\psi_s\rangle \end{pmatrix}, \quad (4)$$

where $|\psi_q\rangle = \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle$, $|\psi_s\rangle = |s\bar{s}\rangle$, and θ^V is the mixing angle, which has been extensively studied in the literature [7]. In this paper, we set its value to be $\theta^V \simeq -3.18°$ in the quark-flavor basis. Under this scheme, the physical states of ω and ϕ can be written as combinations of $Q\bar{Q}$ states:

$$|\phi\rangle = \frac{\cos\theta^V}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle - \sin\theta^V|s\bar{s}\rangle,$$

$$|\omega\rangle = \frac{\sin\theta^V}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle + \cos\theta^V|s\bar{s}\rangle. \quad (5)$$

For the outgoing pesudoscalar mesons, we use $|\pi^0\rangle = |u\bar{u} - d\bar{d}\rangle/\sqrt{2}$, while the states of η and η' can be written in terms of

the two orthogonal states of $|\eta_q\rangle$ and $|\eta_s\rangle$ in the quark-flavor mixing scheme, given by [9,10]

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{pmatrix}, \qquad (6)$$

where $|\eta_q\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$, $|\eta_s\rangle = |s\bar{s}\rangle$, and β is the mixing angle, constrained to be $\beta \simeq 37° \sim 42°$ [10]. Consequently, the valence states of $\eta^{(\prime)}$ can be presented as

$$\begin{aligned} |\eta\rangle &= \cos\beta \frac{|u\bar{u} + d\bar{d}\rangle}{\sqrt{2}} - \sin\beta |s\bar{s}\rangle, \\ |\eta'\rangle &= \sin\beta \frac{|u\bar{u} + d\bar{d}\rangle}{\sqrt{2}} + \cos\beta |s\bar{s}\rangle. \end{aligned} \qquad (7)$$

Combined with Eqs. (1), (3), and (5), the transition form factor of $V \to \pi^0$ can be found by summing up the relevant Fock states to be

$$f_{V \to \pi^0}(q^2) = \frac{\cos\theta^V}{\sqrt{2}} f_{|\psi_q\rangle \to |\pi^0\rangle}(q^2) = \frac{\cos\theta^V}{\sqrt{2}} f_{V_{q\bar{q}} \to \pi^0}(q^2). \qquad (8)$$

Similarly, the transition from factors of $V \to \eta, \eta'$ have the forms

$$\begin{aligned} f_{\phi \to \eta} &= \cos\theta^V \cos\beta f_{|\psi_q\rangle \to |\eta_q\rangle} + \sin\theta^V \sin\beta f_{|\psi_s\rangle \to |\eta_s\rangle}, \\ f_{\phi \to \eta'} &= \cos\theta^V \sin\beta f_{|\psi_q\rangle \to |\eta_q\rangle} - \sin\theta^V \cos\beta f_{|\psi_s\rangle \to |\eta_s\rangle}, \\ f_{\omega \to \eta} &= \sin\theta^V \cos\beta f_{|\psi_q\rangle \to |\eta_q\rangle} - \cos\theta^V \sin\beta f_{|\psi_s\rangle \to |\eta_s\rangle}, \\ f_{\omega \to \eta'} &= \sin\theta^V \sin\beta f_{|\psi_q\rangle \to |\eta_q\rangle} + \cos\theta^V \cos\beta f_{|\psi_s\rangle \to |\eta_s\rangle}, \end{aligned} \qquad (9)$$

where $f_{|\psi_q\rangle \to |\eta_q\rangle}$ and $f_{|\psi_s\rangle \to |\eta_s\rangle}$ can be replaced by $f_{V_{Q\bar{Q}} \to P_{Q\bar{Q}}}(Q = u, d, s)$.

In the LFQM, a neutral meson wave function is constructed by the simple structure of $Q\bar{Q}$ in terms of its constituent quark Q and antiquark $\bar{Q}$ with the total momentum p and spin S as [4]

$$\begin{aligned} &|M(p, S, S_z)\rangle \\ &= \int [dk_1][dk_2] 2(2\pi)^3 \delta^3(p - k_1 - k_2) \\ &\quad \times \sum_{\lambda_1 \lambda_2} \Phi_M(k_1, k_2, \lambda_1, \lambda_2) b_Q^+(k_1, \lambda_1) d_{\bar{Q}}^+(k_2, \lambda_2) |0\rangle, \quad (10) \end{aligned}$$

where

$$[dk] = \frac{dk^+ d^2 k_\perp}{2(2\pi)^3}, \qquad (11)$$

Φ_M is the amplitude of the corresponding $Q\bar{Q}$, and $k_{1(2)}$ ($\lambda_{1(2)}$) is the on-mass shell LF momentum (helicity) of the internal quark. In the momentum space, the wave function Φ_M can be expressed as a covariant form [2,3],

$$\begin{aligned} \Phi_M(z, k_\perp) &= \left(\frac{k_1^+ k_2^+}{2[M_0^2 - (m_Q - m_{\bar{Q}})^2]} \right)^{\frac{1}{2}} \\ &\quad \times \bar{u}(k_1, \lambda_1) \Gamma v(k_2, \lambda_2) \phi_M(z, k_\perp), \quad (12) \\ M_0^2 &= \frac{m_{\bar{Q}}^2 + k_\perp^2}{z} + \frac{m_Q^2 + k_\perp^2}{1 - z}, \end{aligned}$$

where Γ stands for

$$\begin{aligned} \Gamma &= \gamma_5 && \text{(pseudoscalar;} \quad S = 0), \\ \Gamma &= -\hat{\not{\varepsilon}}(S_z) + \frac{\hat{\varepsilon} \cdot (k_1 - k_2)}{M_0 + m_Q + m_{\bar{Q}}} && \text{(vector;} \quad S = 1), \end{aligned}$$

(13)

and

$$\begin{aligned} \hat{\varepsilon}^\mu(\pm 1) &= \left[\frac{2}{p^+} \vec{\varepsilon}_\perp(\pm 1) \cdot \vec{p}_\perp, 0, \vec{\varepsilon}_\perp(\pm 1) \right], \\ \vec{\varepsilon}_\perp(\pm 1) &= \mp(1, \pm i)/\sqrt{2}, \\ \hat{\varepsilon}^\mu(0) &= \frac{1}{M_0} \left(\frac{-M_0^2 + p_\perp^2}{p^+}, p^+, p_\perp \right). \end{aligned} \qquad (14)$$

The LF relative momentum variables $(z, k_\perp)$ are defined by

$$\begin{aligned} k_1^+ &= z p^+, \quad k_2^+ = (1 - z) p^+, \\ k_{1\perp} &= z p_\perp - k_\perp, \quad k_{2\perp} = (1 - z) p_\perp + k_\perp. \end{aligned} \qquad (15)$$

In principle, the momentum distribution amplitude $\phi_M(z, k_\perp)$ can be obtained by solving the LF QCD bound-state equation [2]. However, before such first-principle solutions are available, we would have to consider phenomenological ones. One example that has been widely used is the the Gaussian-type wave function [5], given by

$$\phi_M(z, k_\perp) = N \sqrt{\frac{1}{N_c} \frac{dk_z}{dz}} \exp\left(-\frac{\vec{k}^2}{2\omega_M^2} \right), \qquad (16)$$

where $N = 4(\pi/\omega_M^2)^{\frac{3}{4}}$, $\vec{k} = (k_\perp, k_z)$, and k_z is defined through

$$z = \frac{E_Q + k_z}{E_Q + E_{\bar{Q}}}, \quad 1 - z = \frac{E_{\bar{Q}} - k_z}{E_Q + E_{\bar{Q}}}, \quad E_i = \sqrt{m_i^2 + \vec{k}^2}$$

(17)

by

$$k_z = \left(z - \frac{1}{2} \right) M_0 + \frac{m_{\bar{Q}}^2 - m_Q^2}{2M_0}, \quad M_0 = E_Q + E_{\bar{Q}}, \qquad (18)$$

and $dk_z/dz = E_Q E_{\bar{Q}}/z(1 - z) M_0$. From Eq. (3), one has

$$\begin{aligned} &\langle P_{Q\bar{Q}}(p_p) | J_\sigma^{\text{em}} | V_{Q\bar{Q}}(p_v) \rangle \\ &= N_c \int \frac{d^4 k_3}{(2\pi)^4} \Lambda_P \left\{ \text{Tr} \left[\left(-\hat{\not{\varepsilon}} + \frac{\hat{\varepsilon} \cdot (k_1 - k_3)}{M_0 + m_Q + m_{\bar{Q}}} \right) \right. \right. \\ &\quad \left. \left. \times \frac{i(-\not{k}_3 + m_{\bar{Q}})}{k_3^2 - m_{\bar{Q}}^2 + i\epsilon} \not{\varepsilon}_\sigma \frac{i(\not{k}_2 + m_Q)}{k_2^2 - m_Q^2 + i\epsilon} \gamma_5 \frac{i(\not{k}_1 + m_Q)}{k_1^2 - m_Q^2 + i\epsilon} \right] \right\} \\ &\quad + (k_{1(3)} \leftrightarrow k_{3(1)}, m_Q \leftrightarrow m_{\bar{Q}}). \end{aligned} \qquad (19)$$

The hadronic matrix element of the $V \to P$ transition can be newly parametrized in terms of the initial and final meson momenta, given by

$$\begin{aligned} &\langle P_{Q\bar{Q}}(p_p) | J_\sigma^{\text{em}} | V_{Q\bar{Q}}(p_p + q) \rangle \\ &= f_{V_{Q\bar{Q}} \to P_{Q\bar{Q}}}(q^2) \varepsilon_{\mu\nu\rho\sigma} \epsilon^\mu p_p^\nu q^\rho, \end{aligned} \qquad (20)$$

where we have used the LF momentum variables $(x, k_\perp)$ and worked in the frame where the transverse momentum is purely longitudinal, i.e., $q_\perp = 0$. We note that $q^2 = q^+ q^- \geqslant 0$ covers the entire range of momentum transfers. Therefore, the trace in

Eq. (19) can be easily carried out. The form factor $f_{V_{Q\bar{Q}} \to P_{Q\bar{Q}}}$ is then found to be

$$f_{V_{Q\bar{Q}} \to P_{Q\bar{Q}}}(q^2) = N_c \int_0^r dx \int \frac{d^2k_\perp}{2(2\pi)^3} \frac{2x'\phi_V^*(x',k_\perp)\phi_P(x,k_\perp)}{m_Q^2 + k_\perp^2} \left\{ m_Q + \frac{1}{W_V}\left[rk_\perp^2 + (1-r)\left(2xM_{0_P}k_z - \frac{x'k_\perp^2}{1-x'} \right) \right] \right\}, \quad (21)$$

where $N_c = 3$ is the number of colors and

$$x = rx', \quad r = \frac{m_V^2 + m_P^2 - q^2 + \sqrt{(m_V^2 + m_P^2 - q^2)^2 - 4m_V^2 m_P^2}}{2m_V^2}. \quad (22)$$

At $q^2 = 0$, the form factor in Eq. (21) is evaluated to be

$$f_{V_{Q\bar{Q}} \to P_{Q\bar{Q}}}(0) = N_c \int dx \int \frac{d^2k_\perp}{2(2\pi)^3} \frac{2x\phi_V^*(x,k_\perp)\phi_P(x,k_\perp)}{m_Q^2 + k_\perp^2} \left(m_Q + \frac{k_\perp^2}{W_V} \right). \quad (23)$$

It has been noted [11] that in the purely longitudinal $q^+ > 0$ frame, the nonvalence contribution [12] to the form factor has to be included and the frame dependence should be checked [13]. To avoid the nonvalence contribution, one may use the Drell-Yan-West frame, i.e., $q^+ = 0$, which contains only the valence part [11]. In this frame, we obtain

$$f_{V_{Q\bar{Q}} \to P_{Q\bar{Q}}}(q^2) = N_c \int_0^1 dx \int \frac{d^2k_\perp}{2(2\pi)^3} \frac{2x\phi_V^*(x,k_\perp)\phi_P(x,k'_\perp)}{m_Q^2 + k_\perp^2} \left\{ m_Q + \frac{2}{W_V}\left[k_\perp^2 + \frac{(k_\perp q_\perp)^2}{q^2} \right] \right\}, \quad (24)$$

with $k_\perp = (x-1)q_\perp + k'_\perp$, which also agrees with Eq. (D2) in Ref. [11] and Eq. (4.13) in Ref. [14], respectively. At $q^2 = 0$, the formula for $f_{V_{Q\bar{Q}} \to P_{Q\bar{Q}}}(0)$ from Eq. (24) is the same as that in Eq. (23), which is a frame-independent quantity as expected. For $q^2 \neq 0$, we remark that the results in the two frames should not be very different in our cases of light-to-light transitions [11].

The interaction between the photon and the leptons is given by the conventional QED [6]. One easily obtains the differential decay rates normalized to the radiative decay widths of $V \to P\gamma$ as

$$\frac{d\,\Gamma(V \to P\ell^+\ell^-)}{\Gamma(V \to P\gamma)\,dq^2} = \frac{\alpha}{3\pi} \frac{1}{q^2} \left(1 - \frac{4m_\ell^2}{q^2} \right)^{1/2} \left(1 + \frac{2m_\ell^2}{q^2} \right)$$
$$\times \left[\left(1 + \frac{q^2}{m_V^2 - m_P^2} \right)^2 - \frac{4q^2 m_V^2}{(m_V^2 - m_P^2)^2} \right]^{3/2} \left| \frac{f_{V \to P}(q^2)}{f_{V \to P}(0)} \right|^2. \quad (25)$$

III. NUMERICAL RESULTS

To numerically calculate the transition form factor of $f_{V \to P}$, we need to specify the quark masses of $m_{u,d,s}$ and the parameters appearing in $\phi_M(x,k_\perp)$ ($M = V$ or P). Here, we have chosen the quark masses as $m_q = m_{u,d} = 0.25$ and $m_s = 0.42$ (in GeV). To constrain the meson scale parameter of ω_M in Eq. (16), we use the branching ratio of $V \to P\gamma$, given by

$$\mathcal{B}(V \to P\gamma) = \frac{\alpha(M_V^2 - M_P^2)^3}{24 M_V^3 \Gamma_V} |f(0)_{V \to P}|^2. \quad (26)$$

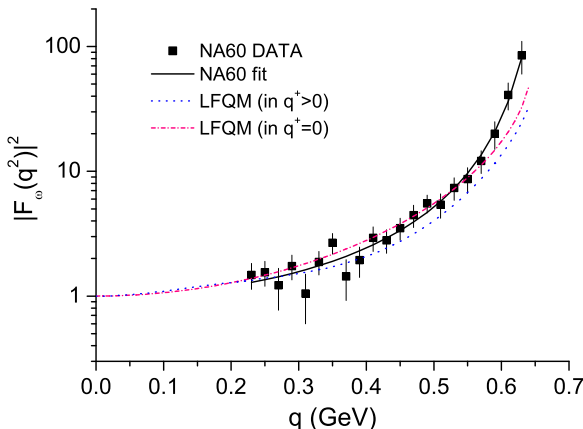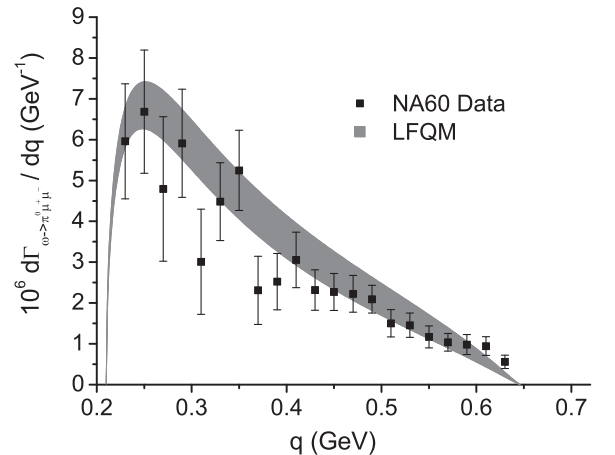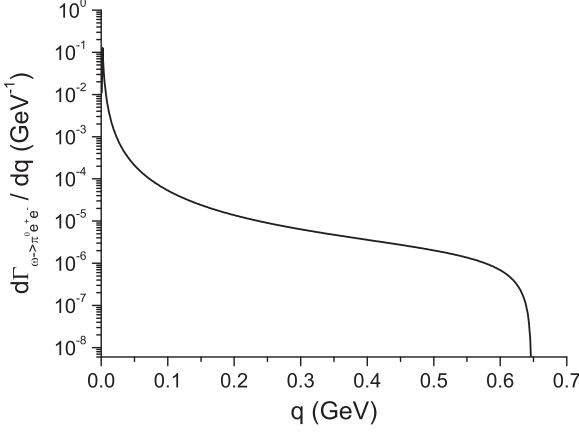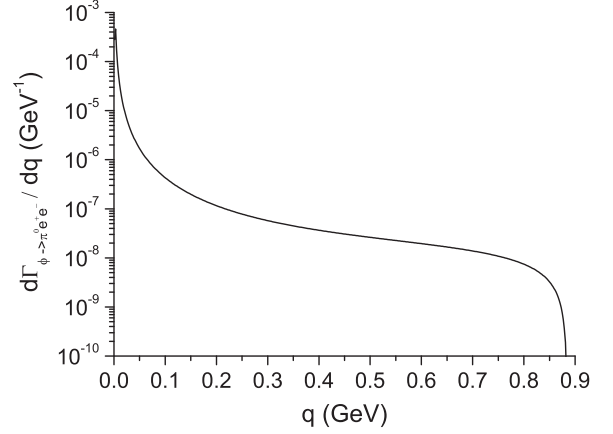FIG. 1. (Color online) $F_\omega(q^2) \equiv f_{\omega \to \pi^0}(q^2)/f_{\omega \to \pi^0}(0)$ in the LFQM.

FIG. 2. Differential decay width of $\omega \to \pi^0 \mu^+ \mu^-$ as a function of q in the LFQM, where the small band is due to the uncertainty of the data in Eq. (26).

FIG. 3. Same as Fig. 2, but for the *e* mode.

FIG. 5. Same as Fig. 4, but for the *e* mode.
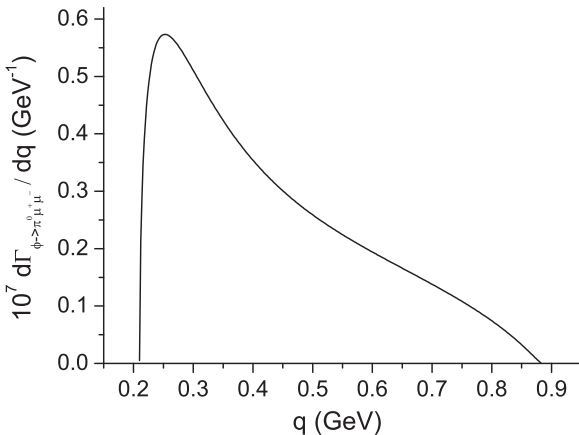
Explicitly, we take [15]

$$\mathcal{B}^{exp}(\omega \to \pi^0 \gamma) = (8.28 \pm 0.28) \times 10^{-2}, \qquad (27)$$

$$\mathcal{B}^{exp}(\phi \to \pi^0 \gamma) = (1.27 \pm 0.06) \times 10^{-3}. \qquad (28)$$

which lead to $|f_{\omega \to \pi^0}(0)| = (2.297 \pm 0.05) \times 10^{-3}$ and $|f_{\phi \to \pi^0}(0)| = (1.33 \pm 0.038) \times 10^{-4}$ (in MeV^{-1}), respectively. We can then fit the parameters of $w_{\omega,\phi}$ from Eq. (23) if we input the quark masses.

The numerical results for $F_\omega(q^2) \equiv f_{\omega \to \pi^0}(q^2)/f_{\omega \to \pi^0}(0)$ in the $q^+ > 0$ and $q^+ = 0$ frames of the LFQM are shown in Fig. 1. From the figure, we see that our results in the LFQM, particularly the one in $q^+ = 0$, fit well with the NA60 experimental data [1] even though they are slightly lower for the large-q^2 region. Note that the numerical values in the $q^+ = 0$ frame are slightly larger than those in $q^+ > 0$ in the large-q^2 region. The difference may result from the nonvalence part in the $q^+ > 0$ frame [11].

In the following calculations, we only use the form factors evaluated in the $q^+ = 0$ frame. As seen from Eq. (25), the differential decay widths of $V \to P\ell^+\ell^-$ depend on the factor of $1/q^2$, highly suppressed by the phase space. In Figs. 2–5, we display the differential decay widths of $V \to \pi^0\ell^+\ell^-$ ($V = \omega$,

ϕ and $\ell = e, \mu$) as functions of q in the LFQM. From Fig. 2, we find that our result for $\omega \to \pi^0\mu^+\mu^-$ is consistent with the experimental data from NA60 [1]. Integrating over q^2 in Eq. (25), we can obtain the branching ratios of $V \to \pi^0\ell^+\ell^-$. Our results in the LFQM are listed in Table I. In the table, we also give the experimental data listed by the Particle Data Group (PDG) [15] as well as the theoretical predictions based on the VMD method [16] and DR [17]. In Table I, we observe that our predicted values for $\omega \to \pi^0 e^+e^-$ and $\pi^0\mu^+\mu^-$ in the LFQM agree with those of the PDG [15] in errors and also are close to the theoretical values for the VMD method [16] and DR [17], respectively. On the other hand, our results for the $\phi \to \pi^0\ell^+\ell^-$ decay modes are slightly smaller than those in Refs. [16,17]. However, it is interesting to note that the decay branching ratio of $\phi \to \pi^0 e^+e^-$ in the LFQM is consistent with the experimental data [15].

We now consider the processes of $V \to \eta^{(\prime)}$ ($V = \omega$, ϕ). In Fig. 6, we show the transition form factor of $F_\phi \equiv f_{\phi \to \eta}(q^2)/f_{\phi \to \eta}(0)$ in the LFQM. In the figure, we also plot the experimental data from the Spherical Neutral Detector (SND) at the VEPP-2M collider [18]. Our result is consistent with the current data. Since the data contain large errors when $q > 0.2$ GeV, more precise experimental measurements are clearly needed to test our model. In Table II, we list all possible decay branching ratios of $V \to \eta^{(\prime)}\ell^+\ell^-$ in the LFQM. In the table, we also list the experimental data of the PDG [15] and the theoretical results based on the VMD method [16] and large-N_c [19] calculations. Note that most of these decay modes have not been measured yet except

FIG. 4. Differential decay width of $\phi \to \pi^0\mu^+\mu^-$ as a function of q in the LFQM.

TABLE I. Decay branching ratios of $V \to \pi^0\ell^+\ell^-$ ($V = \omega$, ϕ and $\ell = e, \mu$) in the LFQM, where the experimental data listed by the Particle Data Group (PDG) [15] as well as the theoretical predictions based on the vector meson dominant method (VMD) [16] and dispersion relations (DR) [17] are alsolisted.

Model	LFQM	PDG [15]	VMD [16]	DR [17]
$10^4 \mathcal{B}_{\omega \to \pi^0 e^+ e^-}$	7.82 ± 0.39	7.7 ± 0.6	7.9	7.6...8.1
$10^4 \mathcal{B}_{\omega \to \pi^0 \mu^+ \mu^-}$	1.21 ± 0.15	1.3 ± 0.4	0.92	0.94...1.0
$10^5 \mathcal{B}_{\phi \to \pi^0 e^+ e^-}$	1.35 ± 0.12	1.12 ± 0.28	1.6	1.39...1.51
$10^6 \mathcal{B}_{\phi \to \pi^0 \mu^+ \mu^-}$	3.48 ± 0.57	–	4.8	3.7...4.0

FIG. 6. $F_\phi(q^2) \equiv f_{\phi \to \eta}(q^2)/f_{\phi \to \eta}(0)$ in the LFQM.

TABLE II. Decay branching ratios of $V \to \eta^{(\prime)} \ell^+ \ell^-$ ($V = \omega, \phi$ and $l = e, \mu$), along with the experimental data on PDG [15] and the theoretical results based on the vector meson dominant method (VMD) [16] and large-N_c [19] calculations.

Model	LFQM	PDG [15]	VMD [16]	Large N_c [19]
$10^6 \mathcal{B}_{\omega \to \eta e^+ e^-}$	3.22 ± 0.28	–	6.0	3.20 ± 0.10
$10^9 \mathcal{B}_{\omega \to \eta \mu^+ \mu^-}$	1.81 ± 0.23	–	1.8	1.00 ± 0.00
$10^4 \mathcal{B}_{\phi \to \eta e^+ e^-}$	1.07 ± 0.02	1.15 ± 0.10	1.1	1.09 ± 0.06
$10^6 \mathcal{B}_{\phi \to \eta \mu^+ \mu^-}$	6.86 ± 0.18	<9.4	6.8	6.44 ± 0.69
$10^7 \mathcal{B}_{\phi \to \eta' e^+ e^-}$	2.97 ± 0.10	–	–	–

$\phi \to \eta e^+ e^-$. In the table, we see that our results are close to those in the large-N_c calculations [19] except the one for $\omega \to \eta \mu^+ \mu^-$. The prediction of $\omega \to \eta e^+ e^-$ in the VMD model is about two times larger than ours, but those of $\omega \to \eta \mu^+ \mu^-$ and $\phi \to \eta \ell^+ \ell^-$ are consistent with the ones in the LFQM. Compared with the only experimental value in $\phi \to \eta e^+ e^-$, our result fits well with the data in error.

IV. CONCLUSIONS

We have studied the form factors of light vector mesons (ω and ϕ) into pseudoscalar mesons (π^0, η, and η') in the LFQM. In our calculations, we have adopted the Gaussian-type wave function and evaluated the form factors for the momentum dependences in all allowed q^2 regions. Our numerical results for $f_{\omega \to \pi^0}(q^2)/f_{\omega \to \pi^0}(0)$ are close to the experimental data

of NA60. Similarly, our values for $\phi \to \eta e^+ e^-$ compare well with the data with the SND at the VEPP-2M collider. With these form factors, we have calculated all possible decay branching ratios of $V \to P \ell^+ \ell^-$ with $V = \omega$ and ϕ, $P = \pi^0$, η, and η', and $\ell = e$ and μ. Explicitly, we have found that our numerical results can fit with the data, such as those for $\omega \to \pi^0 \ell^+ \ell^-$ and $\phi \to \pi^0 e^+ e^-$ by NA60 and $\phi \to \eta e^+ e^-$ by the SND. We also predict the branching ratios of $\phi \to \pi^0 \mu^+ \mu^-$, $\omega \to \eta e^+ e^-$, $\omega \to \eta \mu^+ \mu^-$, $\phi \to \eta \mu^+ \mu^-$, and $\phi \to \eta' e^+ e^-$ to be $(3.48 \pm 0.57) \times 10^{-6}$, $(3.22 \pm 0.28) \times 10^{-6}$, $(1.81 \pm 0.23) \times 10^{-9}$, $(6.86 \pm 0.18) \times 10^{-6}$ $(2.97 \pm 0.10) \times 10^{-7}$ respectively. Some of these modes could be measured in future experiments.

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