

Systematic law for half-lives of double- β decay with two neutrinos

Yuejiao Ren¹ and Zhongzhou Ren^{1,2,*}

¹*Department of Physics, Nanjing University, Nanjing 210093, China*

²*Center of Theoretical Nuclear Physics, National Laboratory of Heavy-Ion Accelerator, Lanzhou 730000, China*

(Received 5 March 2014; revised manuscript received 14 April 2014; published 6 June 2014)

Nuclear double- β decay with two neutrinos is a rare and important process for natural radioactivity of unstable nuclei. The experimental data of nuclear double- β^- decay with two neutrinos are analyzed and a systematic law to calculate the half-lives of this rare process is proposed. It is the first analytical and simple formula for double- β -decay half-lives where the leading effects from both the Coulomb potential and the nuclear structure are included. The systematic law shows that the logarithms of the half-lives are inversely proportional to the decay energies for the ground-state transitions between parent nuclei and daughter nuclei. The calculated half-lives are in agreement with the experimental data of ground-state transitions of all known 11 nuclei with an average factor of 3.06. The half-lives of other possible double- β -decay candidates with two neutrinos are predicted and these can be useful for future experiments. The law, without introducing any extra adjustment, is also generalized to the calculations of double- β -decay half-lives from the ground states of parent nuclei to the first 0^+ excited states of daughter nuclei, and the calculated half-lives agree very well with the available data. The calculated half-lives from the ground states of parent nuclei ^{48}Ca and ^{150}Nd to the first 0^+ excited states of daughter nuclei are the first theoretical results as far as we know. The similarity and difference between the law of α decay and that of double- β^- decay are also analyzed and discussed.

DOI: [10.1103/PhysRevC.89.064603](https://doi.org/10.1103/PhysRevC.89.064603)

PACS number(s): 23.40.Bw, 23.40.Hc, 21.10.Tg

I. INTRODUCTION

The discovery of natural radioactivity by Becquerel changed our views on the structure of matter and promoted the development of modern physics and modern chemistry. Rutherford identified that there are three kinds of natural radioactivity and named them α , β , and γ rays [1–4]. The structure of an atom is clear when Rutherford proposed the existence of a nucleus in an atom based on α -scattering experiments with α particles from natural radioactivity. Research on the problem of energy conservation in a β -decay process led to Pauli's suggestion of the existence of a new particle, the neutrino. Based on this idea, Fermi proposed the basic formulation of the weak interaction for the description of β decay of a nucleus. In 1956, Lee and Yang proposed that parity cannot be conserved for a weak process such as β decay [5]. Wu and collaborators carried out the β -decay experiment with polarized ^{60}Co nuclei and observed that parity symmetry was broken [6]. The vector and axial-vector theory (V-A) of the weak interaction for four fermions was founded in 1958 [7,8] and is still widely used for the calculation of β -decay half-lives and double- β -decay half-lives [9–11]. The unification of weak and electromagnetic interactions was reached with the discovery of intermediate bosons, and the standard model is well founded with the experimental confirmation of the Higgs particle. Currently new progress on physics processes with neutrinos or without neutrinos is being made. Some important processes with neutrinos are the measurement of θ_{13} in neutrino oscillations and the weak processes such as the neutrino-induced reaction and the double- β decay with neutrinos. An important process without neutrinos is the search of the nuclear double- β decay

without emission of neutrinos. In this article we focus on the research of half-lives of double- β decay with two neutrinos.

There are many experimental and theoretical studies on double- β decay with two neutrinos and without neutrinos [10–29]. A list of available experimental data on decay energies and half-lives can be found in recent review articles [15,16] and in nuclear data tables [17,18]. Some recent experimental results on the half-lives of ^{130}Te are published in Ref. [14] and those of ^{136}Xe are published in Refs. [12,13]. Very recent data for ^{76}Ge are reported in Ref. [22]. As experimental data are accumulated for 11 double- β -decay nuclei with neutrinos, to make a systematic analysis on them and to find a systematic law of data are useful for future research.

II. SYSTEMATIC LAW FOR HALF-LIVES OF DOUBLE- β DECAY

Theoretically there are two kinds of calculations on double- β -decay half-lives and nuclear matrix elements [10,21,23]. One is based on the nuclear shell model with an effective interaction from a renormalized g matrix obtained with the Bonn potential [10,21,23,25]. Another is based on the quasi-particle random-phase approximation (QRPA) or self-consistent renormalized random-phase approximation (RQRPA) [10,21,23] where the collective particle-hole excitation of nuclei is included and the intermediate-nucleus states are represented as a set of phonon states [10,21,23]. A recent review can be found in Ref. [21]. These calculations have been very successful in describing double- β -decay half-lives and nuclear matrix elements thanks to the efforts of many theoretical physicists. They are in general very complicated and require both sophisticated computer codes and experiences in the specific problem to generate results which can be compared with those obtained in the laboratory. And in many cases different models generate

*Corresponding author: zren@nju.edu.cn

TABLE I. Experimental data of double- β -decay half-lives $T_{1/2}(\text{expt})$ and decay energies $Q_{2\beta}$ of 11 even-even nuclei. To analyze the law of the data we also list the logarithm of double- β -decay half-lives and the square root of the multiplication between the logarithm of experimental half-life and the decay energy in columns 3 and 5, respectively. The units of the half-lives are Ey (10^{18} years). The experimental half-lives and decay energies are from Refs. [16] and [18], respectively.

Nuclei	$T_{1/2}(\text{expt})(\text{Ey})$	$\log_{10} T_{1/2}(\text{expt})$	$Q_{2\beta}(\text{MeV})$	$\sqrt{\log_{10} T_{1/2}(\text{expt})} \sqrt{Q_{2\beta}(\text{expt})}$
^{48}Ca	44_{-5}^{+6}	1.643	4.267	2.649
^{76}Ge	1840_{-100}^{+140}	3.265	2.039	2.580
^{82}Se	92_{-7}^{+7}	1.964	2.996	2.424
^{96}Zr	23.5 ± 0.21	1.371	3.349	2.143
^{100}Mo	7.1 ± 0.4	0.851	3.034	1.606
^{116}Cd	28 ± 2	1.447	2.813	2.017
^{128}Te	$(1.9 \pm 0.4) \times 10^6$	6.279	0.8665	2.333
^{130}Te	700 ± 140	2.845	2.528	2.682
^{136}Xe	2300 ± 120	3.362	2.458	2.876
^{150}Nd	9.11 ± 0.68	0.960	3.371	1.799
^{238}U	2000 ± 600	3.301	1.144	1.944

different predictions which can be not easily reconciled. In the cases of the two neutrino double- β -decay, after decades of research there are plenty of theoretical half-lives for many possible candidates based on the calculations with the nuclear shell model or with the various quasi-particle random-phase approximations. Although plenty of theoretical half-lives have been reported for many nuclei, some recent experimental data are not covered by the calculations [16]. For example, it is observed that there is double- β decay from the ground state of ^{150}Nd to the first excited 0^+ state of its daughter nucleus and there is no theoretical half-life on this [16]. Therefore further calculation on double- β -decay half-lives is needed. Because there are plenty of calculations based on the the nuclear shell model and the various quasi-particle random-phase approximations, we will not carry out calculations similar to those, but instead we will try a different way to investigate the double- β -decay half-lives in this article. It is believed that the different approach to double- β decay is useful for future development of the field and is also useful for future experimental research. It is widely accepted that experimental data themselves can test the reliability of the prediction of the different approaches in physics.

At first we make a systematic analysis on the available data of double- β decay to the ground state of daughter nuclei. Up to now there are 11 nuclei which have been observed to have double- β decay with two neutrinos, and their decay energies and half-lives are listed in Table I. In Table I, the first column denotes the parent nucleus and the second column represents the experimental double- β -decay half-life for the ground-state transition between the parent nucleus and the daughter nucleus. The experimental double- β -decay half-lives are mainly presented in Refs. [15–17] and here the data are from Ref. [16]. We also list the logarithm of the half-life in column 3. Column 4 is the experimental double- β decay energy of the nucleus from the ground state of the parent nucleus to the ground state of the daughter nucleus where the data are from the nuclear mass table by Audi *et al.* and Wang *et al.* [17,18]. The units of half-lives are Ey (10^{18} years). The fifth column represents the square root of the multiplication between the logarithm of experimental half-life and the decay energy.

It is seen from Table I that the shortest half-life is 7.1 Ey for ^{100}Mo and the longest is 1.9×10^6 Ey for ^{128}Te . The half-lives vary from 7.1 Ey (^{100}Mo) to 1.9×10^6 Ey (^{128}Te) although the change of the decay energies is less than four times (from 3.034 MeV for ^{100}Mo to 0.8665 MeV for ^{128}Te). This shows that the half-life is very sensitive to the decay energy and the relationship between them may be very complex by a glance of the data. To describe quantitatively the sensitivity and to find a law among the data, we introduce the logarithm of the half-life and also make a square root for the multiplication between the logarithm of the half-life and the decay energy. The square root of the multiplication is listed in the last column of Table I. It is clearly seen from the last column that the variation of the square root of the multiplication is in a very narrow range from the minimum 1.606 to the maximum 2.876. This fact strongly suggests that a constant value can be a good approximation for the multiplication of the logarithm of the half-lives and decay energies. The physics behind this will be discussed later in this article. This is the starting point of our research and we simply write it in the following mathematical equation between the logarithm of half-life and the decay energy,

$$\log_{10} T_{1/2}(\text{Ey}) = a / Q_{2\beta}(\text{MeV}). \quad (1)$$

Here a is a constant and its value is to be determined. We will discuss the meaning of this constant later and will point out that the constant a is from the universal properties of the weak interaction for the decay process of different nuclei.

For a better description of the double- β -decay half-lives, the effects from the Coulomb potential and from nuclear structure should be taken into account. It is well known from β -decay theory [2–4] that the effect from the Coulomb potential can be derived from quantum mechanics and its expression in the nonrelativistic case is given in some textbooks [2–4] as

$$\rho(Z, \epsilon) = 2\pi\eta / (1 - e^{-2\pi\eta}), \quad (2)$$

where $\eta = (Z/137)(\epsilon/cp)$ and ϵ is the energy of the electron and p is the magnitude of the electron momentum.

Usually the correction of the Coulomb potential should multiply the square of the matrix element to obtain the probability of β decay [2–4]. This correction factor is close

to unity for light nuclei but it has an evident effect for heavy nuclei. In the usual β decay or the double- β decay of this article, the speed of an electron is very close to the speed of light ($v_e \approx 0.86\text{--}0.95 c$) and therefore (ϵ/cp) is close to unity. For the denominator of Eq. (2) it is also a good approximation to choose it to be unity for heavy nuclei. Therefore we only keep the leading term of Eq. (2) for simplicity, and a correction of the Coulomb potential for double- β -decay half-lives is approximately $-2 \log_{10}[(2\pi Z)/137]$ to the numerator of Eq. (1) where Z is chosen to be the charge number of the parent nuclei and this is also for the convenience of calculations. This correction is approximately zero for light double- β -decay nuclei such as ^{48}Ca and it can make a significant contribution for heavy nuclei such as ^{238}U .

Now let us take the nuclear structure effect into account to some extent. We keep with the same spirit as for the correction of the Coulomb potential and only include the leading effect from the strong interaction. For nuclear structure, the most important effect from the strong interaction of nucleons is the existence of magic numbers and this corresponds to the appearance of a nuclear shell structure in nuclei. The nuclei with magic numbers are more stable than nonmagic nuclei. For the numerical calculations of both the shell model and mean-field model, the first step is the choice of the number of valence nucleons. This choice is with reference to magic numbers: The number of valence neutrons of a nucleus is zero if its neutron number corresponds to a magic number; and it is not zero if the neutron number does not correspond to a magic number. The parameters of the effective mean-field potential in a self-consistent mean-field model are often obtained with the fitting of ground-state properties of several magic nuclei such as ^{16}O , $^{40,48}\text{Ca}$, and ^{208}Pb , and the residual interaction such as the pairing force is included for valence nucleons when it is used for research of open shell nuclei. For the double- β -decay process it involves a change of two neutrons into two protons for nuclei with the same nucleon number (A). It can be observed if other decay modes such as a single β decay are forbidden in energy. Based on this we simulate the leading effect of nuclear structure (i.e., the shell effect) by introducing an addition quantity S to the numerator of Eq. (1). $S = 2$ when the neutron number of a parent nucleus is a magic number and $S = 0$ when the neutron number is nonmagic. Whether this choice is suitable can be tested by the numerical results of this article. In this way we do not touch the details of the complex nuclear potential and we also avoid solving the very complicated nuclear many-body problem. So a new systematic law of double- β -decay half-lives is proposed to be

$$\log_{10} T_{1/2}(\text{Ey}) = [a - 2 \log_{10}(2\pi Z/137) + S]/Q_{2\beta} \text{ (MeV)}, \quad (3)$$

where the constant a is obtained by fitting the experimental data of Table I and its value is $a = 5.843$. We will point out later that the physical meaning of a is related to the square of the strength of the weak interaction which leads to the instability of a nucleus. Z is the charge number of the parent nucleus. $S = 2$ when the neutron number of parent nuclei is a magic number and $S = 0$ when it is not a magic number. The number of parameters of Eq. (3) is two if both a and S are considered to

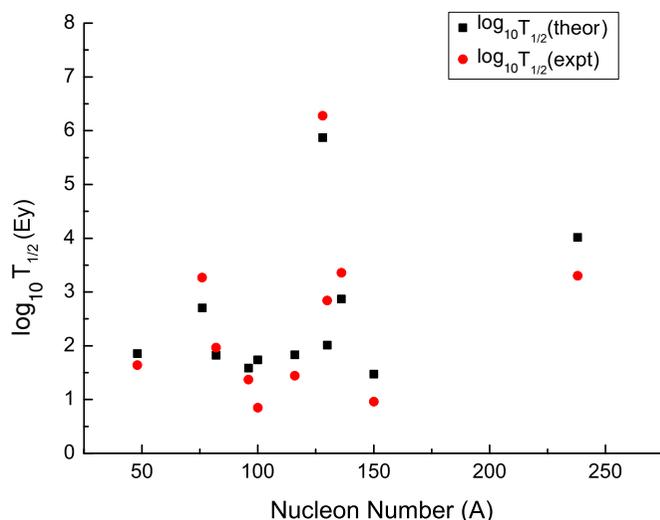


FIG. 1. (Color online) Logarithms of the experimental double- β -decay half-lives and theoretical ones for ground-state transitions of 11 even-even nuclei from ^{48}Ca to ^{238}U . The calculated half-lives are in agreement with the experimental data of all known 11 nuclei with an average factor of 3.06. The maximum deviation between theoretical half-life and experimental one is a factor of 7.709 and this corresponds to the case of ^{100}Mo .

be adjusting parameters. The number of parameters of Eq. (3) is one if only a is considered to be an adjusting parameter and if S is accepted to be from the shell effect (in the shell model and in the mean-field model people naturally choose the valence number with reference to the magic numbers). In a word, the number of adjusting parameters in Eq. (3) is the least as compared to a model with an effective potential because one needs two parameters (depth and force range) to define a central potential and a third parameter to define the strength of the spin-orbit potential. Because Eq. (3) is very clear and simple, any physicist including experimental physicists can use it to calculate the half-lives of double- β decay with inputs of the decay energies. A pocket calculator with a logarithm function is enough to get numerical results.

The numerical results from Eq. (3) are drawn in Fig. 1 and also listed in Table II, together with experimental half-lives and decay energies. In Fig. 1, the x axis is the nucleon number and the y axis is the logarithm of the double- β -decay half-lives. Experimental data and calculated results with Eq. (3) are denoted with different symbols in the figure. It is seen that the calculated results are close to experimental data and reasonable agreement is achieved for the 11 nuclei which have been observed to have double- β decay with two neutrinos. We make a detailed discussion on the calculated results of Table II. In Table II columns 1–4 are experimental data and they have the same meaning as columns 1–4 of Table I. Column 5 is the logarithm of the calculated double- β -decay half-lives with Eq. (3). The last column is the logarithm of the ratio between experimental double- β -decay half-lives and calculated ones and it shows the deviation between experimental half-lives and calculated ones. When the deviation is less than 0.301, the agreement between the experimental half-life and the calculated one is within a factor of 2 ($\log_{10} 2 = 0.301$) and

TABLE II. Logarithms of double- β -decay half-lives of even-even isotopes calculated with new law [$\log_{10} T(\text{theor})$] and the corresponding experimental ones [$\log_{10} T(\text{expt})$]. The units of the half-lives are Ey (10^{18} years). The experimental decay energies of nuclei [$Q_{2\beta}$ (MeV)] are also listed where the decay energies are from the nuclear mass table [18]. The calculated half-lives are in agreement with the experimental data of all known 11 nuclei with an average factor of 3.06. The maximum deviation between theoretical half-life and experimental one is a factor of 7.709 and this corresponds to the case of ^{100}Mo .

Nuclei	$T_{1/2}(\text{expt})(\text{Ey})$	$\log_{10} T_{1/2}(\text{expt})$	$Q_{2\beta}(\text{MeV})$	$\log_{10} T_{1/2}(\text{theor})$	$\log_{10}[T_{1/2}(\text{expt})/T_{1/2}(\text{theor})]$
^{48}Ca	44_{-5}^{+6}	1.643	4.267	1.856	-0.213
^{76}Ge	1840_{-100}^{+140}	3.265	2.039	2.702	+0.563
^{82}Se	92_{-7}^{+7}	1.964	2.996	1.822	+0.142
^{96}Zr	23.5 ± 0.21	1.371	3.349	1.587	-0.216
^{100}Mo	7.1 ± 0.4	0.851	3.034	1.738	-0.887
^{116}Cd	28 ± 2	1.447	2.813	1.834	-0.378
^{128}Te	$(1.9 \pm 0.4) \times 10^6$	6.279	0.8665	5.872	0.407
^{130}Te	700 ± 140	2.845	2.528	2.013	0.832
^{136}Xe	2300 ± 120	3.362	2.458	2.871	0.491
^{150}Nd	9.11 ± 0.68	0.960	3.371	1.473	-0.513
^{238}U	2000 ± 600	3.301	1.144	4.015	-0.714

this corresponds to the cases for ^{48}Ca , ^{82}Se , ^{96}Zr . When the deviation is less than 0.477, the agreement between the experimental half-life and the calculated one is within a factor of 3 ($\log_{10} 3 = 0.477$) and this is the case for ^{116}Cd and ^{128}Te . It is seen from Table II that experimental half-lives can be reproduced by calculations within a factor of 4 for eight nuclei ($\log_{10} 4 = 0.602$) (such as the case of ^{76}Ge , ^{136}Xe , and ^{150}Nd). The deviation between experimental half-lives and the calculated results is beyond a factor of 4 for three nuclei ^{100}Mo , ^{130}Te , and ^{238}U . For ^{238}U the agreement for half-lives is approximately a factor of 5.2 ($\log_{10} 5.2 = 0.716$). For ^{100}Mo the calculation can reproduce its experimental half-life within a factor of 8 ($\log_{10} 8 = 0.903$) and for ^{130}Te the calculation can reproduce that within a factor of 7 ($\log_{10} 7 = 0.845$). To see the total agreement for the 11 nuclei, we define an average deviation $\Delta = \{\sum_{i=1}^{11} |\log_{10}[T_{1/2}(\text{expt})/T_{1/2}(\text{theor})]|\}/11 = 0.486$. This corresponds to a factor of 3.06 between experimental half-lives and calculated ones ($\log_{10} 3.06 = 0.486$). This agreement between double- β -decay half-lives and calculations with Eq. (3) is good compared with the calculations of single β -decay half-lives [30–34] and α -decay half-lives of unstable nuclei [35–38]. Especially the calculations of this article are with the minimum number of adjusting parameters and cover a wide range of nuclear mass from $A = 48$ (^{48}Ca) to $A = 238$ (^{238}U). This means it can be used to predict the double- β -decay half-lives of other nuclei in nuclide charts.

In Table III we predict the double- β -decay half-lives with two neutrinos for 11 nuclei. This corresponds to the decays between the ground states of parent and daughter nuclei, and their mass numbers lie in a very wide range from ^{46}Ca to ^{244}Pu . They are good candidates for observing double- β decay with two neutrinos. In Table III, the first column denotes the parent nuclei and the second column is the experimental decay energies where the experimental data are from the nuclear mass tables [17,18]. The third column corresponds to the logarithm of calculated half-lives with Eq. (3). We also list the double- β -decay half-lives in the last column and this is convenient for making a direct comparison with future double- β -decay experiments. The decay energies of these

nuclei vary approximately from 1 to 2 MeV and their half-lives range approximately from 10^2 to 10^6 Ey. These will be tested by future measurements.

Recently it has been also observed that double- β decay with two neutrinos can occur from the ground state of parent nuclei to the first 0^+ excited state of daughter nuclei [16]. There are two experimental results on the decay half-lives of ^{100}Mo and ^{150}Nd to the first 0^+ excited state of the daughter nuclei [16]. Previous theoretical calculations only cover the decay of ^{100}Mo to the first 0^+ excited state of its daughter nucleus [16]. This clearly shows that more theoretical calculations are needed for the decays to the first 0^+ excited state of the daughter nuclei. Based on the successful description of the double- β -decay half-lives of ground-state transitions with Eq. (3), we now generalize Eq. (3) to the calculation of double- β -decay half-lives to the first 0^+ excited state of daughter nuclei. We list our numerical results in column 4 of Table IV, together with experimental decay energies and two experimental half-lives

TABLE III. Double- β -decay half-lives [$T_{1/2}(\text{theor})$] of even-even isotopes calculated with the new systematic law and the corresponding logarithms [$\log_{10} T_{1/2}(\text{theor})$]. The units of the half-lives are Ey (10^{18} years). The experimental decay energies of nuclei [$Q_{2\beta}$ (MeV)] are also listed where the decay energies are from the nuclear mass table [18].

Nuclei	$Q_{2\beta}(\text{MeV})$	$\log_{10} T_{1/2}(\text{theor})$	$T_{1/2}(\text{theor})(\text{Ey})$
^{46}Ca	0.989	5.984	9.64×10^5
^{86}Kr	1.258	5.889	7.74×10^5
^{94}Zr	1.142	4.655	4.52×10^4
^{104}Ru	1.301	4.023	1.05×10^4
^{110}Pd	2.017	2.576	3.77×10^2
^{148}Nd	1.928	2.575	3.76×10^2
^{154}Sm	1.251	3.945	8.81×10^3
^{160}Gd	1.731	2.835	6.84×10^2
^{198}Pt	1.049	4.515	3.27×10^4
^{124}Sn	2.291	2.236	1.72×10^2
^{244}Pu	1.35	3.388	2.44×10^3

TABLE IV. Double β -decay from the ground state of parent nuclei to the first 0^+ excited state of daughter nuclei (denoted with a symbol *). Column 1 represents the parent nuclei and column 2 denotes the experimental double- β -decay half-lives [16]. Our calculated half-lives from the systematic law are listed in column 4. The units of the calculated half-lives in column 4 are Ey (10^{18} years). The experimental decay energies of nuclei [$Q_{2\beta}^*$ (MeV)] are also listed where the experimental data are from Ref. [16]. The calculated results of ^{48}Ca and ^{150}Nd are the first theoretical results according to Table III of the newest review article [16]. The calculated results (Ey) from other groups are also listed in the last two columns for comparison and these are also taken from Table III of the review article [16].

Nuclei	$T_{1/2}^*(\text{expt})(\text{Ey})$	$Q_{2\beta}^*(\text{MeV})$	$T_{1/2}^*(\text{theor})(\text{Ey})$	$T_{1/2}^*(\text{other 1})(\text{Ey})$	$T_{1/2}^*(\text{other 2})(\text{Ey})$
^{48}Ca		1.275	1.63×10^6		
^{76}Ge		0.917	1.02×10^6	$(7.5-310) \times 10^3$ [39,40]	4.5×10^3 [41]
^{82}Se		1.506	4.21×10^3	$(1.5-3.3) \times 10^3$ [39,40]	
^{96}Zr		2.203	2.59×10^2	$(24-27) \times 10^2$ [39,40]	38×10^2 [41]
^{100}Mo	$5.9_{-0.6}^{+0.8} \times 10^2$	1.904	5.89×10^2	16×10^2 [42]	21×10^2 [41]
^{116}Cd		1.048	8.36×10^4	1.1×10^4 [39,40]	0.11×10^4 [41]
^{130}Te		0.735	8.38×10^6	$(5.1-14) \times 10^4$ [39,40,43]	
^{150}Nd	$1.33_{-0.26}^{+0.45} \times 10^2$	2.627	0.776×10^2		

from the decay of ^{100}Mo and ^{150}Nd to the first 0^+ excited state of the daughter nuclei [16]. The calculated results from other groups are listed in the last two columns for comparison [16]. For the half-life from ^{100}Mo , the calculated result is approximately the same as the experimental one. For that of ^{150}Nd , the calculated result agrees with the experimental one within a factor of 2. It is seen without introducing any extra adjustment in the law that very perfect agreement is reached for the available data when the law is extended to the decay to the first 0^+ excited state of daughter nuclei. This confirms the validity of the law. According to the newest review article of the double- β decay with two neutrinos [16], no theoretical half-life is reported for the decay from ^{150}Nd to the first 0^+ excited state of the daughter nucleus. Therefore the calculated half-life of ^{150}Nd in Table IV is the first theoretical result. For the decay of ^{48}Ca to the first 0^+ excited state of its daughter nucleus, there is no other theoretical result and our result is the first prediction. Besides the experimental half-lives of ^{100}Mo and ^{150}Nd , there are no definite half-lives from the measurement of other nuclei in Table IV [16]. Our calculated results are above the low limit of the half-lives for other nuclei [16]. It is also interesting to note from the last two columns that the differences of half-lives from different groups is large. The differences of half-lives of ^{76}Ge from different groups have a factor ranging approximately from 2 to 70. For the case of ^{116}Cd , the difference of theoretical results from different groups is 10 times. This clearly shows again that further research on this kind of decay is strongly needed from both experimental and theoretical sides.

After presenting the numerical results for double- β decay both to the ground state and to the first 0^+ excited state of daughter nuclei, it is useful to discuss the physics behind the new law. For this purpose we compare the differences and common points between α decay and double- β decay because they are two important decay modes of unstable nuclei. Both α decay and double- β decay are natural phenomena and they also obey the same exponential decay law: $N = N_0 e^{-\lambda t}$. They often occur for ground-state transitions of even-even nuclei and they obey the laws of quantum mechanics and quantum field theory. For both decays among the ground states of even-even nuclei,

the changes of quantum numbers between parent nuclei and daughter nuclei are very similar, $\delta l = 0$ and $\delta P = +$ for the transition from 0^+ to 0^+ . The long-range Coulomb repulsive potential among protons leads to the appearance of α decay and the very short-range weak interaction among nucleons leads to double- β decay.

It is known that the half-lives of α decay can be calculated by the Geiger-Nuttall law or the Viola-Seaborg formula with a few parameters [35,37,38]. A new version of the Geiger-Nuttall law is proposed [37] and it can well reproduce the experimental half-lives of both α decay and cluster radioactivity [35,37,38]. The new Geiger-Nuttall law between α -decay half-lives (in seconds) and α -decay energies (in MeV) of ground-state transitions of even-even nuclei is [37]

$$\log_{10} T_{1/2}^\alpha(\text{seconds}) = a\sqrt{\mu}Z_c Z_d / \sqrt{Q^\alpha} + b\sqrt{\mu}\sqrt{Z_c Z_d} + c + S. \quad (4)$$

In this equation the values of the three parameters are $a = 0.39961$, $b = -1.31008$, and $c_{e-e} = -17.00698$ for even-even (e-e) nuclei [37]. $T_{1/2}^\alpha$ (seconds) is the half-life of α decay and Q^α (MeV) is the corresponding decay energy. Z_c and Z_d are the charge numbers of the cluster and the daughter nucleus, respectively. $\mu = A_c A_d / (A_c + A_d)$ is the reduced mass and A_c , A_d are the mass numbers of the cluster and daughter nucleus, respectively. For α decay, $Z_c = 2$ and $A_c = 4$. The three parameters, a, b, c , are obtained by fitting the data of even-even nuclei with $Z \geq 84$ and $N \geq 128$ [35]. S is a new quantum number to mock up the shell effect of $N = 126$ on α -decay half-lives. The value of S for ground-state transitions of even-even nuclei is $S = 0$ for $N \geq 128$ and $S = 1$ for $N \leq 126$ [37].

When we compare the new Geiger-Nuttall law of α -decay half-lives [Eq. (4)] with the new systematic law of double- β -decay half-lives [Eq. (3)], we find they are very similar although they are governed by different interactions. For a clear comparison, one can keep the first term of both laws temporarily because it is the leading term and the last two terms are the corrections to the leading term (the constant c in the new Geiger-Nuttall law can be approximately eliminated if we change the units of the half-life from seconds to 10^{17}

seconds). A common point between α decay and double- β decay is that their half-lives are very sensitive to the decay energy and the equations on their half-lives look alike. The first term in the new Geiger-Nuttall law [Eq. (4)] is dependent on both charge numbers and decay energies because the Coulomb repulsive potential leads to the appearance of α decay (a quantum tunneling effect) and the total effect from the Coulomb potential is related to the charge numbers (similarly the total effect of the strong interaction is also directly related to the nucleon number of a nucleus). For the systematic law of double- β -decay half-lives [Eq. (3)], the first term is only dependent on the decay energy because the weak interaction is universal for natural decay processes and the total effect from the weak interaction is not very sensitive to the change of nucleon numbers (such as proton numbers). This agrees with our knowledge of the weak interaction that the strength of the weak interaction of a free-neutron β decay is approximately the same as that of a μ decay although two decay systems are very different and the difference of their decay energies is approximately 100 times (of course the strength of a weak interaction can be different if different generation of quarks or different generation of leptons in standard models are involved.) It is due to the difference of this total effect between the weak interaction and the Coulomb interaction that β decay occurs for the ground state of many unstable nuclei from very light ones (such as a decay from a neutron or from a triton) to heavy ones and α decay occurs for ground states of medium and heavy nuclei.

Another important difference between the new systematic law and the Geiger-Nuttall law is from the difference of the perturbation approximation in quantum mechanics. Double- β decay is a second-order process of the weak interaction with the V-A four-fermion theory where a single- β decay is forbidden in many double- β decay nuclei. For the Geiger-Nuttall law, α decay is a first-order process of the electromagnetic interaction and there are significant influences from the strong interaction. Before ending the discussion, we would like to point out that the right side of Eq. (3) can be dimensionless if one would like to replace the decay energy $Q_{2\beta}$ by $Q_{2\beta}/(2m_e c^2)$. In this case the accuracy of calculated half-lives with the systematic law is almost kept due to $2m_e c^2 = 1.022 \approx 1$ MeV.

III. CONCLUSIONS

In summary, a systematic law for the calculation of double- β -decay half-lives is proposed where the leading effects of the decay energy, the Coulomb potential and the nuclear structure, are naturally taken into account. This is the first analytical formula for the half-lives of the complex double β -decay with two neutrinos where only two parameters are used in the formula. By including these leading effects, the available data of double- β -decay half-lives of ground-state transitions in even-even nuclei are reasonably reproduced. Without introducing extra adjustment on the two parameters of the law, the law is generalized to the double- β decay between the ground state of parent nuclei and the first 0^+ excited state of daughter nuclei and perfect agreement between the calculated half-lives and the data is reached. The calculated decay half-lives from the ground states of parent nuclei ^{48}Ca and ^{150}Nd to the first 0^+ excited states of daughter nuclei are the first theoretical results as far as we know. The existence of these terms in the systematic law is based on the available data and on the quantum description of the double- β -decay process. Because it is very difficult to derive this simple law from theory, this law is an empirical law with justification. To derive this law will be very interesting and it is beyond the scope of this article. We will try to derive this law in the future. The universal behavior of the weak interaction manifests itself in the formula of double- β -decay half-lives by comparing the similarity and difference between the systematic law of double- β -decay half-lives and the famous Geiger-Nuttall law of α decay. The half-lives of the two-neutrino double- β -decay candidates are predicted and they will be useful in future experiments.

ACKNOWLEDGMENTS

This work is supported by National Natural Science Foundation of China (Grants No. 11035001, No. 10975072, 11120101005, No. 11175085), by the Research Fund of Doctoral Point (RFDP), No. 20070284016, and by the Priority Academic Program Development of Jiangsu Higher Education Institutions.

-
- [1] J. S. Lilley, *Nuclear Physics* (Wiley, Chichester, 2001).
 [2] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley, Reading, MA, 1962).
 [3] A. De Shalit and H. Feshbach, *Theoretical Nuclear Physics*, Vol. 1 (Wiley, New York, 1974).
 [4] J. D. Walecka, *Theoretical Nuclear and Subnuclear Physics* (World Scientific, Singapore, 2004).
 [5] T. D. Lee and C. N. Yang, *Phys. Rev.* **104**, 254 (1956).
 [6] C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, *Phys. Rev.* **105**, 1413 (1957).
 [7] R. P. Feynman and M. Gell-Mann, *Phys. Rev.* **109**, 193 (1958).
 [8] E. C. G. Sudarshan and R. E. Marshak, *Phys. Rev.* **109**, 1860 (1958).
 [9] *International Conference on Physics Since Parity Symmetry Breaking, in Memory of Professor C. S. Wu, Nanjing, 16-18 August, 1997* (World Scientific, Singapore, 1998).
 [10] W. C. Haxton and G. J. Stephenson, *Prog. Part. Nucl. Phys.* **12**, 409 (1984).
 [11] H. Ejiri, *Phys. Rep.* **338**, 265 (2000).
 [12] N. Ackerman *et al.*, *Phys. Rev. Lett.* **107**, 212501 (2011).
 [13] A. Gando *et al.*, *Phys. Rev. C* **85**, 045504 (2012).
 [14] R. Arnold *et al.*, *Phys. Rev. Lett.* **107**, 062504 (2011).
 [15] A. S. Barabash, *Phys. Rev. C* **81**, 035501 (2010).
 [16] R. Saakyan, *Annu. Rev. Nucl. Part. Sci.* **63**, 503 (2013).
 [17] G. Audi, F. G. Kondev, M. Wang, B. Pfeiffer, X. Sun, J. Blachot, and M. MacCormick, *Chin. Phys. C* **36**, 1157 (2012).

- [18] M. Wang, G. Audi, A. H. Wapstra, F. G. Kondev, M. MacCormick, X. Xu, and B. Pfeiffer, *Chin. Phys. C* **36**, 1603 (2012).
- [19] H. V. Klapdor and K. Grotz, *Phys. Lett. B* **142**, 323 (1984).
- [20] A. A. Raduta and C. M. Raduta, *Phys. Lett. B* **647**, 265 (2007).
- [21] F. T. Avignone, III, S. R. Elliott, and J. Engel, *Rev. Mod. Phys.* **80**, 481 (2008).
- [22] M. Agostini *et al.*, *J. Phys. G* **40**, 035110 (2013).
- [23] A. Faessler, *Prog. Part. Nucl. Phys.* **21**, 183 (1988).
- [24] A. Faessler and F. Simkovic, *J. Phys. G* **24**, 2139 (1998).
- [25] E. Caurier, F. Nowacki, and A. Poves, *Phys. Lett. B* **711**, 62 (2012).
- [26] J. Suhonen, *Phys. Rev. C* **87**, 034318 (2013).
- [27] J. Suhonen and O. Civitarese, *Phys. Rep.* **300**, 123 (1998).
- [28] P. Vogel and M. R. Zirnbauer, *Phys. Rev. Lett.* **57**, 3148 (1986).
- [29] J. D. Vergados, *Phys. Rep.* **361**, 1 (2002).
- [30] Dongdong Ni and Zhongzhou Ren, *J. Phys. G* **39**, 125105 (2012).
- [31] Dongdong Ni and Zhongzhou Ren, *J. Phys. G* **41**, 025107 (2014).
- [32] Hantao Li and Zhongzhou Ren, *J. Phys. G* **40**, 105110 (2013).
- [33] Xiaoping Zhang and Zhongzhou Ren, *Phys. Rev. C* **73**, 014305 (2006).
- [34] P. Moeller, J. R. Nix, and K. Kratz, *At. Data Nucl. Data Tables* **66**, 131 (1997).
- [35] Dongdong Ni, Zhongzhou Ren, Tiekuan Dong, and Chang Xu, *Phys. Rev. C* **78**, 044310 (2008).
- [36] Dongdong Ni and Zhongzhou Ren, *Phys. Rev. C* **81**, 064318 (2010).
- [37] Yuejiao Ren and Zhongzhou Ren, *Phys. Rev. C* **85**, 044608 (2012).
- [38] Zhongzhou Ren, Chang Xu, and Zaijun Wang, *Phys. Rev. C* **70**, 034304 (2004).
- [39] M. Aunola and J. Suhonen, *Nucl. Phys. A* **602**, 133 (1996).
- [40] J. Toivanen and J. Suhonen, *Phys. Rev. C* **55**, 2314 (1997).
- [41] S. Stoica and I. Mihut, *Nucl. Phys. A* **602**, 197 (1996).
- [42] J. G. Hirsch, O. Castaños, P. O. Hess, and O. Civitarese, *Phys. Rev. C* **51**, 2252 (1995).
- [43] A. S. Barabash *et al.*, *Eur. Phys. J. A* **11**, 143 (2001).