Neutron skin thickness of heavy nuclei with α-particle correlations and the slope of the nuclear symmetry energy

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The formation of α-particle clusters on the surface of heavy nuclei is described in a generalized relativistic mean-field model with explicit cluster degrees of freedom. The effects on the size of the neutron skin of Sn nuclei and ^{208}Pb are investigated as a function of the mass number and the isospin-dependent part of the effective interaction, respectively. The correlation of the neutron skin thickness with the difference of the neutron and proton numbers and with the slope of the nuclear symmetry energy is modified as compared to the mean-field calculation without α-cluster correlations.

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I. INTRODUCTION

Correlations are an essential feature in interacting many-body systems, which can have a strong impact on particular observables. In dilute nuclear matter, the strong interaction leads to the appearance of clusters, i.e., correlated states of nucleons, below the nuclear saturation density $n_{\text{sat}} \approx 0.15\ \text{fm}^{-3}$. Such conditions are found in the debris of heavy-ion collisions when the hot compressed baryonic matter expands, cools, and fragments of different sizes emerge; see, e.g., Refs. [1–3]. In the postbounce evolution of core-collapse supernovae large abundancies of light clusters might affect the neutrino absorption and heating of the low-density matter behind the shock front [4–6]. On the surface of nuclei, the formation of clusters is a prerequisite for cluster radioactivity [7] and in particular the α-decay of heavy nuclei [8]. Here, the tunneling through the Coulomb barrier is well understood but the preformation of the α particle is a challenge for theoretical model descriptions. Clustering phenomena are expected to affect the density dependence of the symmetry energy of nuclear matter [9] and the structure of nuclei, in particular skin and halo phenomena; see, e.g., the review article [10]. The natural emergence of clusters is still a difficult task in many nuclear structure models.

In this work, the effect of α clustering on the neutron skin thickness of heavy nuclei is investigated. This quantity is defined as the difference $r_{\text{skin}} = r_n - r_p$ of the root-mean-square (rms) radii of neutrons, r_n, and protons, r_p. A strong correlation of the neutron skin thickness with the density dependence of the neutron-matter equation of state [11,12] and that of the symmetry energy of nuclear matter (see, e.g., Refs. [13,14]) was found in mean-field calculations. At present, the density dependence of the symmetry energy is intensively studied in theory and experiment by using different approaches (see the articles in the topical issue [15]). It can be quantified with the so-called slope coefficient L that appears in the expansion of the energy per baryon in nuclear matter (see, e.g., Ref. [9]). A precise knowledge of the relation between r_{skin} and L and consequently the density dependence of the symmetry energy is essential for predicting the structure of neutron stars, in particular their radii [16]. Hence, there are a large number of experimental attempts in recent years in order to determine either the neutron skin thickness r_{skin} directly, e.g., by parity violation in electron scattering on Pb nuclei in the PREX experiment [17,18], or the slope coefficient L by indirect methods (see, e.g., Refs. [14,19,20] and references therein). Until now, the quantitative correlation between r_{skin} and L relies on the description of nuclei in self-consistent mean-field approaches [21] such as nonrelativistic Skyrme Hartree–Fock and relativistic mean-field (RMF) calculations. These models are based on the picture of independent nucleonic quasiparticles. They do not consider residual cluster correlations beyond pairing in the most simple applications.

II. THEORETICAL DESCRIPTION OF CLUSTER CORRELATIONS

In the external region of a nucleus low-density nuclear matter properties are tested. Such dilute matter at finite temperature is described by models for the equation of state that are designated for astrophysical applications [22]. Many-body correlations have to be taken into account in order to describe correctly the thermodynamic properties and chemical composition of the system; most notably the formation of light clusters, such as deuterons or α particles. Therefore, similar effects can be anticipated in the vicinity of the nuclear surface.

In Refs. [9,23,24] an extended RMF model with density-dependent couplings was developed that treats few-body correlations as explicit degrees of freedom. The formation and dissolution of clusters are a result of medium-dependent mass shifts, which are taken from a quantum statistical approach to describe clusters in dilute matter. These shifts originate mainly from the action of the Pauli exclusion principle that prohibits the formation of few-body bound and resonant states with increasing density of the medium. The model was applied to the description of finite nuclei in warm matter by applying fully self-consistent calculations in an extended relativistic Thomas–Fermi (RTF) approximation within spherical Wigner–Seitz cells [25]. It was found that a heavy nucleus is formed in the center of the cell, which is surrounded by a low-density gas of nucleons and light clusters. A particular observation was the enhanced probability of finding clusters on the nuclear surface (see Fig. 11 in Ref. [25]). This behavior is caused by an attractive pocket in the effective cluster potentials at the nuclear surface due to a finite range of the interaction. The attractive scalar potential S_i extends further out than the repulsive vector potential V_i of a cluster i. Typical values of 1–2 fm and 10–20 MeV for the width and the depth, respectively, of the potential pocket are obtained in the present calculations. In contrast, the appearance of clusters inside the heavy nucleus is strongly suppressed because of the large positive mass shift in the scalar potential.

A similar approach can be used to describe heavy nuclei in the vacuum at zero temperature in order to study the significance of few-body correlations at the nuclear surface. However, a few modifications have to be taken into account. The α particle with the highest binding energy of the light clusters emerges as the only relevant correlation. In contrast to nucleons, which are fermions and can be treated in the Thomas–Fermi approximation, α particles are bosons. They populate only the ground-state wave function, which has to be determined explicitly. This "condensation" is one foundation of the very successful THSR description of dilute excited nuclei [26], e.g., the Hoyle state in ^{12}C, where the many-body wave function is constructed from α particles occupying the same quantum state. In the present calculation, a Wentzel–Kramers–Brillouin (WKB) approximation is used to obtain the α-particle wave function self-consistently with the nucleon distributions. The resulting density distribution of ^{4}He has a maximum at the position of the pocket in the effective potential (see below). The amount of α clustering is determined such that the effective position-dependent α energy $E_\alpha(\vec{r}) = m_\alpha + V_\alpha(\vec{r}) - S_\alpha(\vec{r})$ does not exceed the α-particle chemical potential $\mu_\alpha = 2\mu_n + 2\mu_p$ (including rest masses). The latter is given by the neutron and proton chemical potentials μ_n and μ_p that are found from the extended RTF description of the nucleon distributions. The number of α particles is a result of the self-consistent solution of the coupled equations with nucleon, α particles, and meson fields as degrees of freedom. The radial distribution $n_\alpha(r)$ is given by the modulus square $n_\alpha = |\psi_\alpha|^2$ of the α-particle wave function $\psi_\alpha(\vec{r})$ and the absolute number is found in a variational calculation.

The density-dependent DD2 parametrization was introduced in the extended RMF model of Ref. [23]. It was obtained by fitting the parameters to properties of finite nuclei by using the usual mean-field Hartree approximation. It can be directly applied to the description of homogeneous matter with clusters as in Refs. [23,24]; however, the calculation of nuclear properties in the extended RTF approximation will give slightly different results for energies and radii. In order to compensate, at least partly, for these differences, in the present calculations the mass of the σ meson was increased from the original value $m_\sigma^{(\mathrm{orig})}$ to $m_\sigma^{(\mathrm{mod})} = 577.9$ MeV and the σ meson coupling Γ_σ was multiplied by the factor $m_\sigma^{(\mathrm{mod})}/m_\sigma^{(\mathrm{orig})}$. This rescaling does not affect the results for uniform matter but it improves the description of finite nuclei. Although the extended TF calculations will give smaller neutron skin thicknesses than the full Hartree calculations (see below), the general trends due to the α-particle correlations can be studied in such an approach.

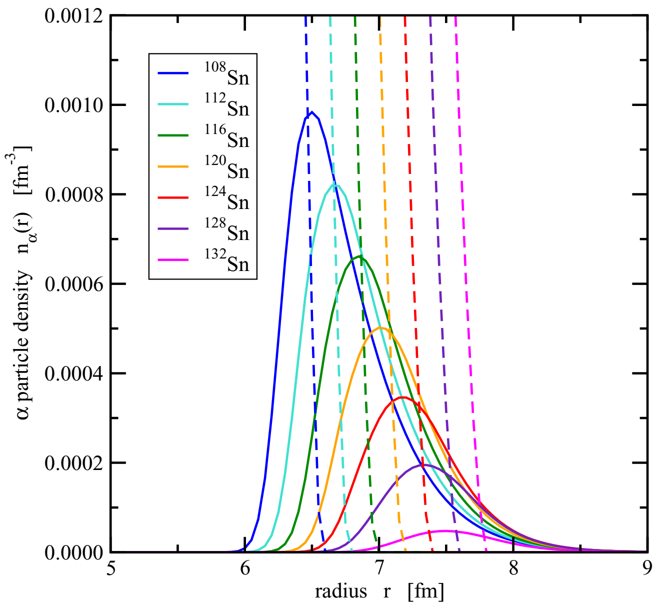

FIG. 1. (Color online) Radial density distribution of α particles (full lines) and neutrons (dashed lines) for a selected set of isotopes of the Sn chain from ^{108}Sn (leftmost curve) to ^{132}Sn (rightmost curve).

III. α-PARTICLE CORRELATIONS ON THE SURFACE OF Sn NUCLEI

The radial distributions of α particles for seven isotopes of the Sn chain is shown in Fig. 1 by full lines. The corresponding distributions for neutrons are indicated by dashed lines with a very steep decrease with increasing radius. It is obvious that the α-particle densities are much smaller than those of the nucleons. The position of the maximum in the α-particle density $n_\alpha(r)$ moves to larger radii in accordance with the extension of the neutron distribution when the neutron excess of the Sn nuclei increases. At the same time, the height of the maximum decreases significantly such that the total amount of α particles at the surface becomes smaller.

In Fig. 2 the evolution of the neutron and proton rms radii in the chain of Sn isopotes is depicted when the mass number A increases. At mass numbers $A \approx 107$ the neutron and proton distributions of a nucleus have almost identical rms radii without forming a neutron skin. At even lower mass numbers a proton skin develops with a size that could also be affected by α clustering. However, for lower A the α particle becomes unbound in the present model and the α-particle dripline is crossed. With increasing neutron number, the neutron rms radius rises stronger than the proton rms radius and a neutron skin appears. With α-particle correlations, however, the rms radii are smaller for a given nucleus than in the model without α correlations. This is a consequence of the larger diffuseness of the total neutron and proton density distribution. It requires smaller rms radii to keep the total number of neutrons and protons for a given nucleus constant. For $A \approx 133$ the differences between the rms radii in the model calculations without and with α particles practically vanish.

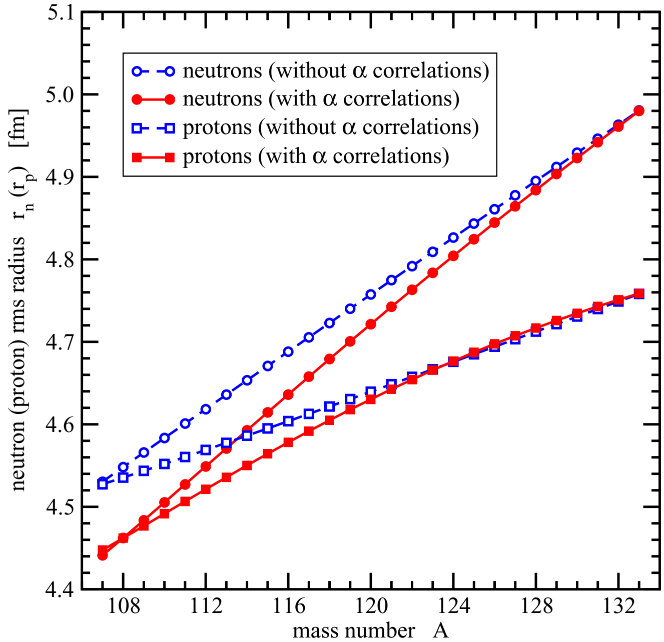

FIG. 2. (Color online) Dependence of the rms radii of neutrons (circles) and protons (squares) on the mass number A of Sn nuclei in the extended RTF calculation with the modified DD2 parametrization. Full red (open blue) symbols denote the results with (without) α-particle correlations.

The dependence of the resulting neutron skin thicknesses r_{skin} on A for the same chain of Sn nuclei is shown in Fig. 3. Without α correlations, r_{skin} increases almost linearly with the mass number A. The values of the present calculation and their mass-number dependence are comparable to the results of the Hartree–Fock-Bogoliubov calculations with the models BSk24, BSk25, and BSk26 in Ref. [27]. However, the consideration of α-cluster correlations leads to a substantial reduction of the neutron skin, in particular in the middle of

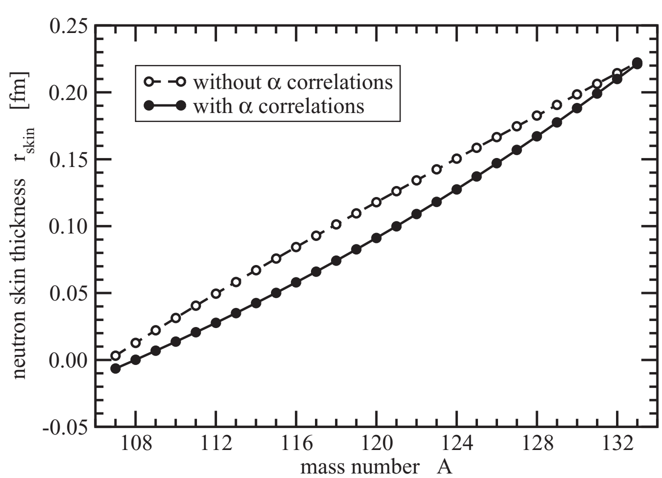

FIG. 3. Dependence of neutron skin thickness on mass number A of Sn nuclei in the extended RTF calculation with the rescaled DD2 parametrization. Full (open) symbols denote the results with (without) α-particle correlations.

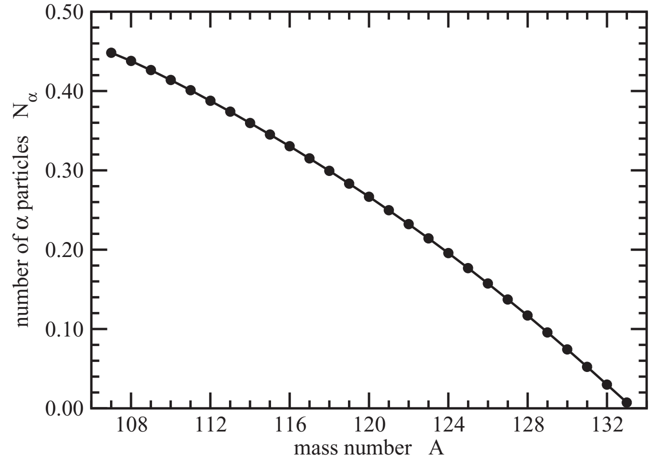

FIG. 4. Dependence of the number of α particles N_α on the mass number A of Sn nuclei in the extended RTF calculation with the rescaled DD2 parametrization.

the chain. This can be well understood because the appearance of α particles on the nuclear surface pushes the abundancies of neutrons and protons (including those bound in clusters) towards a more symmetric distribution. For small A with almost the same neutron and proton numbers in the nucleus, there is no effect and no neutron skin develops. For a large neutron excess, α particles cannot be formed efficiently in the neutron-rich low-density matter on the nuclear surface and the effect vanishes again.

The effects observed in Figs. 2 and 3 correlate with the amount of α particles that appear on the surface of the nucleus. The effective number N_α of α particles in the nuclei of the Sn chain is illustrated in Fig. 4. Since the present approach is based on a statistical description, N_α is not an integer number. For small A the effective α-particle number is largest. With decreasing A, the binding energy of an α cluster reduces and finally the α-particle drip line will be reached, indicating the possibility of α decay. By increasing the mass number A in the chain of Sn nuclei, the effective number of α particles at the nuclear surface decreases continuously until it finally vanishes for large A. Here, a sizable neutron skin develops but the four-nucleon correlations have no effect on its size since α particles do not form in a significant amount in such a neutron-rich environment.

IV. CORRELATION OF THE NEUTRON SKIN THICKNESS WITH THE SYMMETRY ENERGY SLOPE COEFFICIENT

The formation of α-particle correlations at the nuclear surface will modify the universal relation between the neutron skin thickness r_{skin} and the symmetry energy slope coefficient L that was established in mean-field descriptions of nuclei and nuclear matter. The size of the neutron skin of heavy nuclei is strongly affected by the density dependence of the symmetry energy that reflects the isospin dependence of the nuclear interaction. In RMF models the isovector ρ meson usually represents the only contribution to the isospin dependence of the interaction. Earlier versions with nonlinear meson

TABLE I. Isovector parameters for the variations of the DD2 parametrization of the RMF model with density-dependent meson-nucleon couplings.

Parametrization	Symmetry energy J [MeV]	Slope coefficient L [MeV]	ρ-meson coupling $\Gamma_\rho(n_{\mathrm{ref}})$	ρ-meson parameter a_ρ
$\mathrm{DD2}^{+++}$	35.34	100.00	4.109251	0.063577
$\mathrm{DD2}^{++}$	34.12	85.00	3.966652	0.193151
$\mathrm{DD2}^{+}$	32.98	70.00	3.806504	0.342181
DD2	31.67	55.04	3.626940	0.518903
$\mathrm{DD2}^{-}$	30.09	40.00	3.398486	0.742082
$\mathrm{DD2}^{--}$	28.22	25.00	3.105994	1.053251

self-interactions considered only a single parameter, the ρ meson coupling strength Γ_ρ. In the RMF approach with density dependent meson-nucleon couplings and parametrizations such as TW99 [28], DD2 [23], ..., the ρ meson coupling

$$\Gamma_\rho(n) = \Gamma_\rho(n_{\mathrm{ref}}) \exp\left[-a_\rho\left(\frac{n}{n_{\mathrm{ref}}} - 1\right)\right] \quad (1)$$

depends on the total baryon density n with three parameters: the coupling $\Gamma_\rho(n_{\mathrm{ref}})$ at a reference density n_{ref} (usually taken as the saturation density n_{sat}) and a parameter a_ρ that regulates the strength of the density dependence. A variation of $\Gamma_\rho(n_{\mathrm{ref}})$ and a_ρ modifies the symmetry energy at saturation density J and in particular the density dependence characterized by the slope coefficient L.

In order to study the correlation between the neutron skin thickness and the slope coefficient, variations of the original DD2 parametrization were created by fixing L to particular values and refitting J to properties of finite nuclei. In this process the isoscalar part of the effective interaction, i.e., the σ- and ω-meson couplings and their density dependence were not touched. In Table I, the parameters of these new effective interactions are given. Parametrizations with L values larger than the original DD2 value are denoted by $\mathrm{DD2}^{+++}$, $\mathrm{DD2}^{++}$, and $\mathrm{DD2}^{+}$. In contrast, the parametrizations $\mathrm{DD2}^{-}$ and $\mathrm{DD2}^{--}$ have smaller values for L. The correlation of J and L is obvious. Larger slope coefficients are accompanied by larger symmetry energies at saturation. It has to be mentioned that the quality of the description of finite nuclei deteriorates when L deviates strongly from the value of the original DD2 parametrization, but the variation covers the range of typical mean-field model calculations.

The correlation between the neutron skin thickness r_{skin} of the ^{208}Pb lead nucleus and the slope coefficient L of the nuclear symmetry energy is depicted in Fig. 5 for the six different parametrizations of Table I. A distinct correlation between r_{skin} and L is observed that is well known from previous mean-field calculations. The green open squares show the correlation in the original mean-field Hartree calculation that was used to fit the parameters of the interactions. The neutron skin thickness rises with increasing L. Since the parameter sets with L values departing from that of the original DD2 parametrization are not optimal fits to all considered properties of finite nuclei and the isoscalar part of the effective interaction

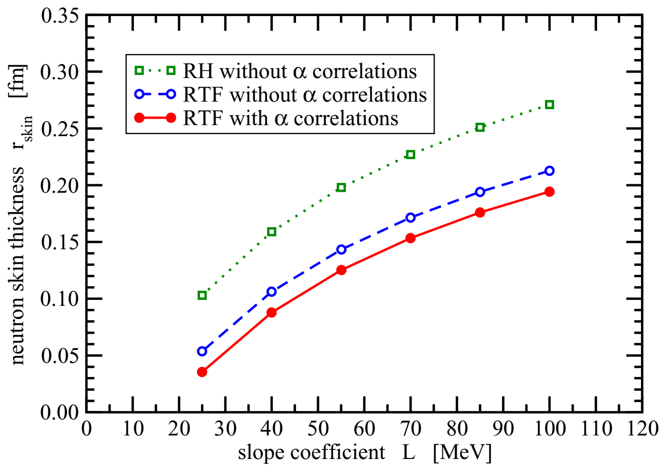

FIG. 5. (Color online) Dependence of the neutron skin thickness of ^{208}Pb on the slope parameter L. Squares denote the results of the RMF calculation with the original DD2 parametrization in the Hartree approximation and full (open) circles are those of the relativistic TF model with the rescaled DD2 parametrization with (without) α-particle correlations.

is not modified, a curvature of the correlation is found in contrast to the almost linear correlation that is observed for models with best fit parameters. The results of the extended RTF model with the rescaled σ-meson mass and coupling are given by the open blue circles. Because this calculation cannot describe sufficiently well the extended neutron density distribution at large radii, the neutron skin thicknesses are systematically smaller than the mean-field Hartree results with the same isovector interaction. However, the general trend is the same. Including the α-particle correlation leads to a further reduction of the neutron skin thickness in the order of 0.02 fm, which can be a substantial fraction of the total neutron skin thickness. Thus, the correlation between r_{skin} and L is modified when α-cluster formation is taken into account.

V. SUMMARY

In conclusion, it was shown that an extended RTF model with explicit α-cluster degrees of freedom predicts an appearance of α particles on the surface of heavy nuclei and a reduction of the neutron skin thickness depending on the neutron excess of the nucleus. This behavior affects the r_{skin}-L correlation observed in conventional mean-field models. Therefore, the extraction of the parameter L from measuring r_{skin} needs some caution and the clusterization effect increases the systematic error. Obviously, for more precise quantitative results on the amount of α clustering on the nuclear surface and the change of the neutron skin thickness due to α-particle correlations, improved calculations beyond the extended RTF approximation, which also take pairing and shell effects into account, have to be performed in the future. The systematic variation of α-particles abundancies on the nuclear surface should be studied experimentally, e.g., by quasifree $(p,p\alpha)$ reactions [29].

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