

## Three-body structure of the $nn\Lambda$ system with $\Lambda N - \Sigma N$ coupling

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The structure of the three-body  $nn\Lambda$  system, which has been observed recently by the HypHI Collaboration, is investigated taking  $\Lambda N - \Sigma N$  coupling explicitly into account. The  $YN$  and  $NN$  interactions employed in this work reproduce the binding energies of  ${}^3_{\Lambda}\text{H}$ ,  ${}^4_{\Lambda}\text{H}$ , and  ${}^4_{\Lambda}\text{He}$ . We do not find any  ${}^3_{\Lambda}n$  bound state, which contradicts the interpretation of the data reported by the HypHI Collaboration.

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### I. INTRODUCTION

In 2013 the HypHI Collaboration [1] reported evidence that the neutron-rich system  $nn\Lambda$  was bound (the hypernucleus  ${}^3_{\Lambda}n$ ), based upon observation of the two-body and three-body decay modes. This claim is very significant for hypernuclear physics for the following reason: One research goal in hypernuclear physics is to study new dynamical features obtained by adding a  $\Lambda$  particle to a nucleus. In this vein it is interesting to explore the resulting structure of neutron-rich  $\Lambda$  hypernuclei. Core nuclei corresponding to neutron-rich systems can be weakly bound halo states or even resonant states. When a  $\Lambda$  particle is added to such a nuclear core, the resultant hypernuclei will become more stable against neutron decay, due to the attraction of the  $\Lambda N$  interaction. The  ${}^3_{\Lambda}n$  system would be the lightest such example that might be bound, one in which a  $\Lambda$  is bound to a di-neutron ( $nn$ ) pair.

Another important goal of studying  $\Lambda$  hypernuclei is the extraction of information about the effect of  $\Lambda N - \Sigma N$  coupling. For this purpose many authors have performed few-body calculations: Miyagawa *et al.* [2], performed Faddeev calculations for  ${}^3_{\Lambda}\text{H}$  using realistic  $YN$  interactions—the Nijmegen soft core 89 (NSC89) [3] and the Juelich potential [4]; these authors confirmed that  $\Lambda N - \Sigma N$  coupling plays a crucial role in obtaining a bound state of  ${}^3_{\Lambda}\text{H}$ . To further investigate  $\Lambda N - \Sigma N$  coupling,  ${}^4_{\Lambda}\text{He}$  and  ${}^4_{\Lambda}\text{H}$  are perhaps most useful because both of the spin-doublet states have been observed. To study this feature in the  $A = 4$  hypernuclei, the authors of Ref. [5] utilized a coupled-channel two-body model of  ${}^3\text{He}(3\text{H}) + \Lambda/\Sigma$ , and later Akaishi *et al.* [6] analyzed the role of the  $\Lambda N - \Sigma N$  coupling for the  $0^+ - 1^+$  splitting within the same framework. It is necessary to perform four-body coupled-channel calculations to investigate the role of  $\Lambda N - \Sigma N$  coupling. Four-body coupled-channel calculations with separable potentials that were central in nature were performed by the authors of Ref. [7]. Carlson then carried out four-body calculations with the NSC89 potential model using a Monte Carlo variational method [8], and he obtained binding energies with statistical errors of 100 keV. Later, Hiyama *et al.* performed four-body coupled-channel calculations [9] using a  $\Lambda N - \Sigma N$  coupled-channel  $YN$  potential [10] with central, spin-orbit, and tensor terms which simulates the scattering

phase shifts given by NSC97f [11]. A four-body calculation of  ${}^4_{\Lambda}\text{He}$  and  ${}^4_{\Lambda}\text{H}$  and a five-body calculation of  ${}^5_{\Lambda}\text{He}$  using the same  $YN$  interaction were performed by Nemura *et al.* [12]. More sophisticated four-body calculations of  ${}^4_{\Lambda}\text{He}$  and  ${}^4_{\Lambda}\text{H}$  using realistic  $NN$  and  $YN$  interactions were performed by Nogga *et al.* [13]. In the  $A = 4$  system  $\Lambda N - \Sigma N$  coupling plays a crucial role in charge symmetry breaking as well as in the spin of the ground states being  $0^+$ . However, the nature of  $\Lambda N - \Sigma N$  mixing is not fully understood. The hypertriton  ${}^3_{\Lambda}\text{H}$  is bound. If  ${}^3_{\Lambda}n$  were also bound, the pair would provide complementary information about  $\Lambda N - \Sigma N$  coupling.

It is thought that  $\Lambda N - \Sigma N$  coupling may also play an important role in the structure of heavier neutron-rich  $\Lambda$  hypernuclei, because of the increasing total isospin. For example, a recent FINUDA experiment [14] reported a heretofore unobserved bound state of the superheavy hydrogen- $\Lambda$  hypernucleus  ${}^6_{\Lambda}\text{H}$ . Furthermore, in 2013, another light neutron-rich  $\Lambda$  hypernucleus bound state,  ${}^7_{\Lambda}\text{He}$ , was observed at the Thomas Jefferson National Accelerator Facility (JLab) [15]. Among the observed neutron-rich  $\Lambda$  hypernuclei,  ${}^3_{\Lambda}n$  would be a unique neutron-rich  $\Lambda$  hypernucleus, one containing no protons. Thus, measuring the binding energy of a bound  ${}^3_{\Lambda}n$  system would contribute directly to understanding the structure of neutron-rich  $\Lambda$  hypernuclei and to understanding the nature of  $\Lambda N - \Sigma N$  coupling. However, no binding energy was reported for  ${}^3_{\Lambda}n$  by the HypHI Collaboration.

Given the current situation, an important theoretical issue to address is whether a bound state of  ${}^3_{\Lambda}n$  can exist. In addition, it is imperative that an estimate of the binding of any such bound system be calculated. In such an analysis it is important that the interactions employed be realistic and constrained by the known data for the  $s$ -shell  $\Lambda$  hypernuclei.  $\Lambda N - \Sigma N$  coupling should be a component of the  $YN$  interactions used in the investigation. Previously, in Ref. [16], a three-body calculation of  ${}^3_{\Lambda}n$  was performed using a variational method and a  $\Lambda N$  interaction without  $\Lambda N - \Sigma N$  coupling; it was concluded that no bound state existed. Recently, a Faddeev calculation for this system was performed taking  $\Lambda N - \Sigma N$  coupling explicitly into account [17]; again, no bound state was found. Thus, previous theoretical predictions are inconsistent with the conclusion based upon the new observation of the decay modes for a bound  ${}^3_{\Lambda}n$  as reported by the HYPHI Collaboration [1].

Here, motivated by experimental and theoretical studies, the goal of this work is to investigate the possibility of the existence of a bound state in this system using realistic  $YN$  interactions with  $\Lambda N$ - $\Sigma N$  coupling explicitly taken into account. We use the Gaussian expansion method in our calculations. It is important that we employ a  $YN$  interaction which reproduces the binding energies of the observed  $s$ -shell  $\Lambda$  hypernuclei such as  ${}^3_\Lambda\text{H}$ ,  ${}^4_\Lambda\text{H}$ , and  ${}^4_\Lambda\text{He}$  in addition to a realistic  $NN$  interaction. For this purpose, we employ the NSC97f simulated  $YN$  interaction [10] which was used in Refs. [9,12], because this interaction reproduces reasonably the binding energies of those  $\Lambda$  hypernuclei in combination with the AV8  $NN$  interaction.

This article is organized as follows: In Sec II, the method and interactions used in the three-body calculation for the  $nn\Lambda$  system are described. The numerical results and a corresponding discussion are presented in Sec. III. A summary is given in Sec. IV.

## II. METHOD AND INTERACTION

The total three-body wave functions for  ${}^3_\Lambda\text{H}$  and  ${}^3_\Lambda n$  are described as a sum of amplitudes for all rearrangement channels ( $c = 1$ –3) of Fig. 1 in the  $LS$  coupling scheme:

$$\begin{aligned} \Psi_{JM}({}^3_\Lambda\text{He}, {}^3_\Lambda n) &= \sum_{Y=\Lambda, \Sigma} \sum_{c=1}^3 \sum_{n\ell, NL, I} \sum_{sS, tY} \mathcal{A} \\ &\times \left\{ \left[ \left[ \phi_{nl}^{(c)}(\mathbf{r}_c) \psi_{NL}^{(c)}(\mathbf{R}_c) \right]_I \right. \right. \\ &\times \left. \left[ \left[ \chi_{\frac{1}{2}}(N_1) \chi_{\frac{1}{2}}(N_2) \right]_s \chi_{\frac{1}{2}}(Y) \right]_S \right\}_{JM} \\ &\times \left\{ \left[ \left[ \eta_{\frac{1}{2}}(N_1) \eta_{\frac{1}{2}}(N_2) \right]_t \eta_{n_r}(Y) \right]_T \right\}. \quad (1) \end{aligned}$$

Here,  $\mathcal{A}$  is the two-nucleon antisymmetrization operator and the  $\chi$ 's and  $\eta$ 's are the spin and isospin functions, respectively, with the isospin  $t_{Y=0(1)}$  for  $Y = \Lambda(\Sigma)$ .  $T$  is total isospin, 0 for  ${}^3_\Lambda\text{H}$  and 1 for  ${}^3_\Lambda n$ .  $J$  is the total spin,  $1/2^+$ , for both hypernuclei. In addition, to investigate the contribution of the  ${}^3S_1$  state in the  $YN$  interaction, we calculate the binding energy for the  $J = 3/2^+$  of  ${}^3_\Lambda\text{H}$ . The functional form of  $\phi_{nl}^{(c)}(\mathbf{r}_c)$  is taken as  $\phi_{nl}^{(c)}(\mathbf{r}_c) = r^\ell e^{-r/r_n} Y_{\ell m}(\hat{\mathbf{r}})$ , where the Gaussian range parameters are chosen to satisfy a geometrical progression ( $r_n = r_1 a^{n-1}$ ;  $n = 1 \sim n_{\text{max}}$ ), and similarly for  $\phi_{NL}(\mathbf{R})$ . Three basis functions were verified to be sufficient for describing both the short-range correlations and the long-range tail behavior

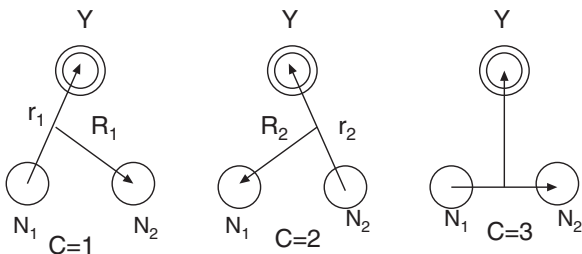


FIG. 1. Jacobi coordinates of the  $nn\Lambda$  three-body system. Antisymmetrization of the two neutrons is to be made.

TABLE I. Parameters of the  $YN$  interaction defined in Eq. (2). Range parameters are in fm and strengths are in MeV.

$i$	1	2
$\beta_i$	0.5	1.2
${}^1V_{\Lambda\Lambda-N\Lambda}^C$	732.08	-99.494
${}^1V_{\Lambda\Lambda-N\Sigma}^C$	61.223	-15.977
${}^1V_{\Sigma\Sigma-N\Sigma}^C$	1708.0	80.763
${}^1U_{\Lambda\Sigma-N\Sigma}^C$	695.39	-109.37
${}^3V_{\Lambda\Lambda-N\Lambda}^C$	1068.8	-45.490
${}^3V_{\Lambda\Lambda-N\Sigma}^C$	-770.21	68.274
${}^3V_{\Sigma\Sigma-N\Sigma}^C$	863.76	28.284
${}^3U_{\Lambda\Sigma-N\Sigma}^C$	-181.08	23.282
${}^3V_{\Lambda\Lambda-N\Lambda}^T$	-243.31	-10.413
${}^3V_{\Lambda\Lambda-N\Sigma}^T$	287.54	62.438
${}^3U_{\Lambda\Sigma-N\Sigma}^T$	333.05	22.234
${}^3V_{\Lambda\Lambda-N\Lambda}^{LS}$	1023.8	-17.195
${}^3V_{\Lambda\Lambda-N\Sigma}^{LS}$	-19.930	22.299
${}^3U_{\Lambda\Sigma-N\Sigma}^{LS}$	-462.31	0.0023

of the few-body systems [18–20]. The details of the four-body calculation,  ${}^4_\Lambda\text{H}$  and  ${}^4_\Lambda\text{He}$ , can be found in Ref. [9].

The  $YN$  interaction employed in the three- and four-body systems is the same as in Refs. [9,12]. Namely, the  $YN$  potential simulates the scattering phase shifts given by NSC97f. The  $YN$  interaction is represented as

$$\begin{aligned} {}^{2S+1}V_{NY-NY'}(r) &= \sum_i \left( {}^{2S+1}V_{NY-NY'}^C e^{-(r/\beta_i)^2} \right. \\ &\quad \left. + {}^{2S+1}V_{NY-NY'}^T S_{12} e^{-(r/\beta_i)^2} \right. \\ &\quad \left. + {}^{2S+1}V_{NY-NY'}^{LS} \mathbf{L}\mathbf{S} e^{-(r/\beta_i)^2} \right) \end{aligned}$$

for  $T = 1/2$ ,

$$\begin{aligned} {}^{2S+1}U_{N\Sigma-N\Sigma}(r) &= \sum_i \left( {}^{2S+1}U_{N\Sigma-N\Sigma}^C e^{-(r/\beta_i)^2} \right. \\ &\quad \left. + {}^{2S+1}U_{N\Sigma-N\Sigma}^T S_{12} e^{-(r/\beta_i)^2} \right. \\ &\quad \left. + {}^{2S+1}U_{N\Sigma-N\Sigma}^{LS} \mathbf{L}\mathbf{S} e^{-(r/\beta_i)^2} \right) \end{aligned}$$

for  $T = 3/2$ ,

(2)

with  $Y, Y' = \Lambda$  or  $\Sigma$ . Here,  $C, T, LS$  mean central, tensor, and spin-orbit terms with two-range Gaussian forms. The potential parameters are listed in Table I.

The interaction reproduces the observed binding energy of  ${}^3_\Lambda\text{H}$ : the calculated  $\Lambda$  binding energy  $B_\Lambda$ , 0.19 MeV, is consistent with the observed data [ $B_\Lambda({}^3_\Lambda\text{H}) = 0.13 \pm 0.05$  MeV]. Furthermore, the calculated energies of  ${}^4_\Lambda\text{H}$  and  ${}^4_\Lambda\text{He}$  are 2.33 and 2.28 MeV, respectively. For the  $NN$  interaction we employ the AV8 potential [21].

## III. RESULTS AND DISCUSSION

Before discussing results for the  ${}^3_\Lambda n$  system, we consider two possibilities: (1) We investigate first the possibility of having a bound state in the  $nn\Lambda$  system by tuning the  $YN$  potential while maintaining consistency with the binding energies of  ${}^3_\Lambda\text{H}$  and  ${}^4_\Lambda\text{H}$  and  ${}^4_\Lambda\text{He}$ . (2) We investigate second

TABLE II. Expectation values of the  $YN$  interaction for (a)  $J = 1/2^+$  state and (b)  $J = 3/2^+$  state of  ${}^3_\Lambda\text{H}$ , and (c)  $J = 1/2^+$  state of  ${}^3_\Lambda n$ . In the case of (a), the calculated  $B_\Lambda = 0.19$  MeV. In (b), the calculated  $B_\Lambda = 0.43$  MeV using  ${}^3V_{N\Lambda-N\Sigma}^T \times 1.20$ . In (c), the calculated  $B_\Lambda = 0.054$  MeV using  ${}^3V_{N\Lambda-N\Sigma}^T \times 1.20$ .

(a) $J = 1/2^+$ for ${}^3_\Lambda\text{H}$	$\langle V_{\Lambda N-\Lambda N} \rangle$	$\langle V_{\Lambda N-\Sigma N} \rangle$	$\langle V_{\Sigma N-\Sigma N} \rangle$	$\langle V_{YN} \rangle$
$S = 0$	-2.12	-0.04	0.02	-2.14
$S = 1$	0.04	-1.49	-0.03	-1.48
all	-2.08	-1.53	-0.01	-3.62
(b) $J = 3/2^+$ for ${}^3_\Lambda\text{H}$	$\langle V_{\Lambda N-\Lambda N} \rangle$	$\langle V_{\Lambda N-\Sigma N} \rangle$	$\langle V_{\Sigma N-\Sigma N} \rangle$	$\langle V_{YN} \rangle$
$S = 0$	-0.11	0.02	0.08	-0.01
$S = 1$	7.87	-34.69	2.29	-24.53
all	7.76	-34.67	2.37	-24.54
(c) $J = 1/2^+$ for ${}^3_\Lambda n$	$\langle V_{\Lambda N-\Lambda N} \rangle$	$\langle V_{\Lambda N-\Sigma N} \rangle$	$\langle V_{\Sigma N-\Sigma N} \rangle$	$\langle V_{YN} \rangle$
$S = 0$	-0.65	-0.01	0.00	-0.66
$S = 1$	2.45	-11.71	0.45	-8.81
all	1.80	-11.72	0.45	-9.47

the possibility of having a bound state in the  $nn\Lambda$  system by tuning the  $nn$   ${}^1S_0$  state while maintaining consistency with the binding energies of  ${}^3\text{H}$ .

First, let us consider case (1). In  ${}^3_\Lambda n$ , the  $nn$  pair is in the spin-singlet state ( $s = 0$ , spin antiparallel), while in the  ${}^3_\Lambda\text{H}$  system, the  $np$  pair is in the spin-triplet state ( $s = 1$ , spin parallel). The difference in the spin value of  $[NN]_{s=1\text{or}0}$  leads to different contributions of the  $\Lambda N$  spin-spin interaction to the doublet splitting. In the  ${}^3_\Lambda n$  system, the  $1/2^+$  ground state includes  $YN$  spin-singlet and spin-triplet states; while in the  ${}^3_\Lambda\text{H}$  system, the  $1/2^+$  ground state is dominated by the  $YN$  spin-singlet state. To investigate the contributions of the spin-singlet and spin-triplet states, in Table II(a) we list expectation values of the  $S = 0$  and  $S = 1$  states of  $V_{\Lambda N-\Lambda N}$ ,  $V_{\Lambda N-\Sigma N}$ , and  $V_{\Sigma N-\Sigma N}$ . We find that the  $S = 0$  state in the  $V_{\Lambda N-\Lambda N}$  term dominates in the binding energy of  ${}^3_\Lambda\text{H}$ . Also, it is found that the contribution of  $V_{\Lambda N-\Sigma N}$  coupling in the  $S = 1$  state to the binding energy is large. For the  $S = 1$  state the calculated expectation value of the tensor component in  $V_{\Lambda N-\Sigma N}$  is  $-0.47$  MeV. This means that the  $S = 1$  state of the central  $V_{\Lambda N-\Sigma N}$  component dominates in the binding energy of  ${}^3_\Lambda\text{H}$ . Therefore, we tune the spin-triplet state of the  $YN$  interaction in a manner that does not affect the binding energy of  ${}^3_\Lambda\text{H}$  significantly. To accomplish this, we multiply the strength of the tensor part of the  $\Lambda N-\Sigma N$  coupling by a factor, because the tensor part of the  $\Lambda N-\Sigma N$  coupling acts in the spin-triplet state of the  $YN$  interaction. To verify the consistency of the effect due to the spin-triplet state of the  $YN$  interaction, we also investigated the binding energy of the  $J = 3/2^+$  state in  ${}^3_\Lambda\text{H}$ , which has not been observed experimentally.

The calculated energy of  ${}^3_\Lambda n$  with  $T = 1$  and  $J^\pi = 1/2^+$  is illustrated in Fig. 2. In Fig. 3 we show the binding energies of the  $J = 1/2^+$  and  $3/2^+$  states in  ${}^3_\Lambda\text{H}$ . The energy of  ${}^3_\Lambda n$  in Fig. 2(a) is obtained using the  $YN$  interaction which reproduces the binding energy of the ground state in  ${}^3_\Lambda\text{H}$ ,

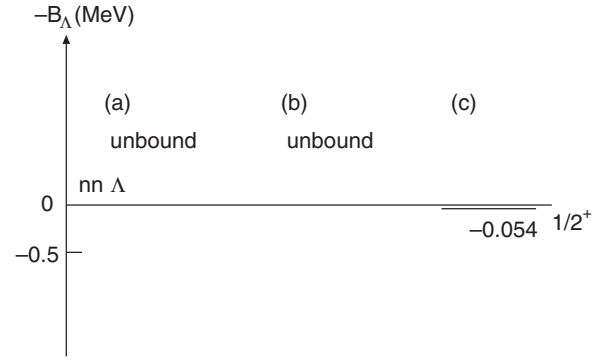


FIG. 2. Calculated  $\Lambda$ -separation energy for  ${}^3_\Lambda n$  with (a)  ${}^3V_{N\Lambda-N\Sigma}^T \times 1.00$ , (b)  ${}^3V_{N\Lambda-N\Sigma}^T \times 1.10$ , and (c)  ${}^3V_{N\Lambda-N\Sigma}^T \times 1.20$ . The energy is measured with respect to the  $nn\Lambda$  three-body breakup threshold.

the  $J = 1/2^+$  state. In Figs. 2(b) and 2(c), we multiply the tensor part of the  $\Lambda N-\Sigma N$  coupling by the factors 1.10 and 1.20. In Fig. 2(b), we still obtain no bound state in the  $nn\Lambda$  system. When  ${}^3V_{N\Lambda-N\Sigma}^T$  is multiplied by 1.20, then we obtain a very weakly bound state ( $-0.054$  MeV) with respect to the  $nn\Lambda$  three-body breakup threshold. To judge whether the adjusted  ${}^3V_{N\Lambda-N\Sigma}^T$  is reasonable, we calculated the binding energies of the ground and excited states in  ${}^3_\Lambda\text{H}$  as shown in Fig. 3. In Fig. 3(a) the binding energy of the ground state,  $J = 1/2^+$ , is in good agreement with the observed data. As one should anticipate, we find the energy of  ${}^3_\Lambda\text{H}$  becomes deeper with increasing strength of  ${}^3V_{N\Lambda-N\Sigma}^T$ . When the strength of  ${}^3V_{N\Lambda-N\Sigma}^T$  is multiplied by the factor 1.20, which leads to a bound state in the  $nn\Lambda$  system, the energy of  ${}^3_\Lambda\text{H}$  is over bound ( $-0.7$  MeV) compared with the observed data ( $-0.13$  MeV). In addition, we find a bound  $J = 3/2^+$  excited state ( $-0.4$  MeV) of  ${}^3_\Lambda\text{H}$ , for which there is no experimental evidence. To make clear the contribution of the  $YN$  potential in the binding energies of  ${}^3_\Lambda n$  and  ${}^3_\Lambda\text{H}$ , we show the expectation values of

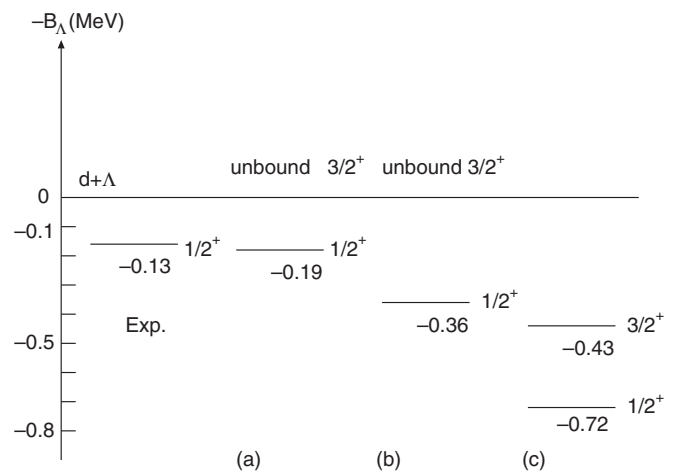


FIG. 3. Calculated  $\Lambda$ -separation energy for  ${}^3_\Lambda\text{H}$  with (a)  ${}^3V_{N\Lambda-N\Sigma}^T \times 1.00$ , (b)  ${}^3V_{N\Lambda-N\Sigma}^T \times 1.10$ , and (c)  ${}^3V_{N\Lambda-N\Sigma}^T \times 1.20$ . The energy is measured with respect to the  $np\Lambda$  three-body breakup threshold.

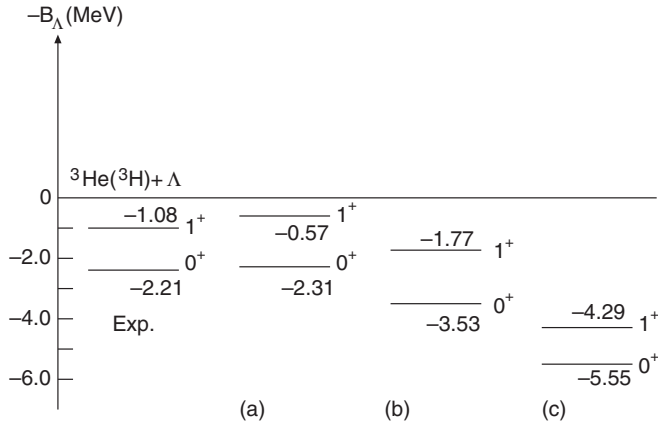


FIG. 4. Calculated  $\Lambda$ -separation energy of ground state in  ${}^4_\Lambda\text{He}$  with (a)  ${}^3V_{N\Lambda-N\Sigma}^T \times 1.00$ , (b)  ${}^3V_{N\Lambda-N\Sigma}^T \times 1.10$ , and (c)  ${}^3V_{N\Lambda-N\Sigma}^T \times 1.20$ . The energy is measured with respect to the  ${}^3\text{He} + \Lambda$  breakup threshold. In parentheses is the energy of  ${}^4_\Lambda\text{H}$ .

the  $YN$  potential in Table II, section (b) and (c). We find that with the tensor term of  $V_{\Lambda N-\Sigma N}$  enhanced by 20%, the corresponding contribution to the potential expectation values is much larger.

To further investigate the reliability of the employed  $YN$  interaction, we calculate the binding energies of  ${}^4_\Lambda\text{H}$  and  ${}^4_\Lambda\text{He}$ . In these two  $\Lambda$  hypernuclei, we also see evidence of the important effect of charge symmetry breaking (CSB) in the  $\Lambda N$  interaction. The CSB effects appear in the ground-state and excited-state differences  $\Delta_{\text{CSB}} = B_\Lambda({}^4_\Lambda\text{He}) - B_\Lambda({}^4_\Lambda\text{H})$ , the experimental values of which are  $0.35 \pm 0.06$  and  $0.24 \pm 0.06$  MeV, respectively. The  $\Lambda$ -separation energies of the ground states in  ${}^4_\Lambda\text{He}$  and  ${}^4_\Lambda\text{H}$  using the present  $YN$  interaction are 2.28 and 2.33 MeV, respectively, which do not reproduce the CSB effect. (To investigate CSB in greater detail, it is planned to measure the  $\Lambda$ -separation energy of  ${}^4_\Lambda\text{H}$  at Mainz and JLab.) Here, it is not our purpose to explore the CSB effect in  $A = 4$  hypernuclei. Therefore, we adopt the average value of these hypernuclei. That is, we adopt  $B_\Lambda = 2.21$  and 1.08 MeV as experimental data for the ground state and the excited state, respectively.

In Fig. 4 we illustrate the average binding energies of the  $A = 4$  hypernuclei. In the case of Fig. 4(a), the calculated ground-state energy reproduces the data nicely, while the excited state is less bound than the observed data. Then, as was done in the case of  ${}^3_\Lambda\text{H}$ , we tuned  ${}^3V_{N\Lambda-N\Sigma}^C$ . As shown in Figs. 4(b) and 4(c), increasing the strength of  ${}^3V_{N\Lambda-N\Sigma}^C$  means that both the  $0^+$  and  $1^+$  states become overbound by 1–3 MeV.

In addition, we adjusted other parts of the  $YN$  potential such as  ${}^3V_{N\Lambda-N\Lambda}^C$ ,  ${}^3V_{N\Lambda-N\Sigma}^C$ , etc. However, we could not find any modification of the  $YN$  potential that produces a bound state in  ${}^3_\Lambda n$  while maintaining consistency with the binding energies of  $A = 3$  and 4  $\Lambda$  hypernuclei.

Next, we investigate in case (2) the possibility of having a bound state in  $nn\Lambda$  by tuning the strength of the  $nn T = 1, {}^1S_0$  interaction. It has been suggested that *if* this channel has a bound state, i.e., a di-neutron state, then it may be possible to describe the anomalies in neutron-deuteron elastic scattering

TABLE III. Calculated binding energies of  $nn\Lambda$ ,  $E_{\Lambda n}^3$ , in the case that the  ${}^1S_0$  component is multiplied by the factor  $x$ . The scattering length  $a_{nn}$ , effective range  $r_{\text{eff}}$ , and energies of the di-neutron system  $\epsilon_{nn}$  and of  ${}^3\text{H}$   $E_{3\text{H}}$  are also listed for each  $x$  factor.

$x$	$a_{nn}$ (fm)	$r_{\text{eff}}$ (fm)	$\epsilon_{nn}$ (MeV)	$E_{3\text{H}}$ (MeV)	$E_{\Lambda n}^3$ (MeV)
1.0	-23.7	2.78	Unbound	-7.77	Unbound
1.13	25.1	2.40	-0.066	-9.75	Unbound
1.35	6.88	1.96	-1.269	-13.93	-1.272

and the deuteron breakup reaction above threshold [22]. (However, we note that  $pp$  scattering is well described by standard methods which do not admit a bound di-proton. Thus, the hypothesis of a bound di-neutron suggests strong CSB in the  $NN$  spin-singlet interaction.) This  $nn$  spin-singlet channel does not contribute to the binding energy of  ${}^3_\Lambda\text{H}$  since the spin of the core nucleus, the deuteron, has spin 1. On the other hand, the  ${}^1S_0$  state of the  $nn$  pair contributes to the binding energy of  ${}^3_\Lambda n$ . It also contributes to the energy of  ${}^3\text{H}$ . The observed energies of  ${}^3\text{H}$  and  ${}^3\text{He}$  are  $-8.48$  MeV and  $-7.72$  MeV. Then, it is interesting to ask what is the energy of  $nn\Lambda$  as one tunes the strength of the  $nn T = 1, {}^1S_0$  together with the predicted binding energy of  ${}^3\text{H}$ .

In Table III, we illustrate the component of the  ${}^1S_0$  state when multiplied by the factor  $x$ , scattering length, effective range, energy of di-neutron, and energy of  ${}^3_\Lambda n$ . When we use a  ${}^1S_0$  component multiplied by the factor 1.13 and 1.35, we have a bound state in the di-neutron system. However, the  $nn$  interaction multiplied by a factor of 1.13 does not produce any bound state in  ${}^3_\Lambda n$ . We find that the  ${}^1S_0$  component, when multiplied by a factor of 1.35, leads to a bound state in  ${}^3_\Lambda n$ . However, we see that as we increase the factor  $x$  in the  ${}^1S_0$  component, that is, to 1.13 and 1.35, which produces a di-neutron bound state, the energy of  ${}^3\text{H}$  is overbound compared with the observed data. Then, we do not have a bound state in  ${}^3_\Lambda n$  while maintaining consistency with the observed data for  ${}^3\text{H}$ , *unless* we introduce a large repulsive  $nnp$  three-body force. It is known that one needs a small (about 0.7 MeV) attractive  $nnp$  three-body force to obtain agreement with the  ${}^3\text{He}$  binding energy; our model value for the  ${}^3\text{He}$  binding energy is  $-7.12$  MeV. Thus, the hypothesis of a bound di-neutron would require a very large CSB in the  $NNN$  three-body force, which is not easily understood.

#### IV. SUMMARY

Motivated by the reported observation of data suggesting a bound  ${}^3_\Lambda n$  by the HypHI Collaboration, we have calculated the binding energy of this hypernucleus taking into account  $\Lambda N-\Sigma N$  explicitly. We consider it important to reproduce the observed data for  ${}^3_\Lambda\text{H}$ ,  ${}^4_\Lambda\text{H}$ , and  ${}^4_\Lambda\text{He}$ , and to be consistent with the energies of  ${}^3\text{H}$  and  ${}^3\text{He}$ . For this purpose, we used simulated NSC97f  $YN$  and AV8  $NN$  potentials which maintain consistency with the data mentioned above. However, we did not find any bound state in the  ${}^3_\Lambda n$  system. Then, we investigated the possibility to produce such a bound state in  ${}^3_\Lambda n$  (i) by tuning the strength of the  $YN$  NSC97f potential and (ii) by tuning the  $nn$  component of the  ${}^1S_0$  potential.

When the strengths of the  $YN$  NSC97f potential and the  $nn$   $^1S_0$  potential are multiplied by 1.2, and 1.35, respectively, we obtain a very weakly bound state in the  $^3_\Lambda n$  system. However, in both cases, the calculated binding energies of the  $s$ -shell  $\Lambda$  hypenuclei  $^3_\Lambda\text{H}$ ,  $^4_\Lambda\text{H}$ , and  $^4_\Lambda\text{He}$ , and the  $s$ -shell nucleus  $^3\text{H}$  are overbound by  $0.6 \sim 3$  MeV in comparison with the observed data. That is, we did not find any possibility to have a bound state in  $^3_\Lambda n$ . However, the HypHI Collaboration reported evidence for a bound state in this system; such a finding is inconsistent with the present result. To corroborate the HypHI result, we should consider additional missing elements in the present calculation. Unfortunately, the HypHI data provide information on the lifetime of this system but no binding energy. If the experimentalists can provide a  $^3_\Lambda n$  binding energy, it would be very helpful in explicating the mechanism

that would produce such a bound state. Further experimental study is urgently needed.

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