Pairing effects on spinodal decomposition of asymmetric nuclear matter

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We investigate the impact of pairing correlations on the behavior of unstable asymmetric nuclear matter at low temperature. We focus on the relative role of the pairing interaction, coupling nucleons of the same type (neutrons or protons), with respect to the symmetry potential, which enhances the neutron-proton attraction, along the clusterization process driven by spinodal instabilities. It is found that, especially at the transition temperature from the normal to the superfluid phase, pairing effects may induce significant variations in the isotopic content of the clusterized matter. This analysis is potentially useful for gathering information on the temperature dependence of nuclear pairing and, in general, on the properties of clusterised low-density matter, which are of interest also in the astrophysical context.

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Investigations of many-body interacting systems have always been an exciting and attractive field in different domains of physics. Indeed, understanding the properties of complex systems in terms of their constituent particles and the interaction among them is a true challenge. The original quantal many-body problem is often approached by adopting the mean-field approximation, yielding a so-called effective interaction [1–4]. However, suitable extensions of mean-field models have been introduced to take explicitly into account the effects of relevant interparticle correlations. This is the case, for instance, of pairing correlations, which occur, under suitable conditions, in fermionic systems [5].

Many efforts are currently focused on the study of the properties of complex nuclei and infinite nuclear matter. It is well known that nucleons can form paired states, analogous to the way electrons pair in metals, yielding a superfluid/superconducting phase [5]. Pairing effects on nuclear masses are widely investigated nowadays, also in connection with astrophysical applications requiring knowledge of the mass of very neutron rich nuclei, which play a crucial role in the *r* process of nucleosynthesis [6]. Moreover, the presence of neutron superfluidity in the crust and the inner part of neutron stars is considered well established in the physics of these compact stellar objects and has a significant effect on cooling processes [7] and glitch phenomena [8,9].

Complex many-body systems are also characterized by the possible occurrence of different kinds of phase transitions. For nuclear matter at subsaturation density and relatively low temperature ($T \lesssim 15$ MeV) liquid-gas phase transitions are expected to appear, being driven by the unstable mean field. Such a process is closely linked to the multifragmentation mechanism experimentally observed in nuclear reactions [10] and to the occurrence of clustering phenomena in the inner crust of neutron stars [11,12]. Owing to the two-component structure of nuclear matter, a central role in this mechanism pertains to the density behavior of the isovector part of the effective interaction and the corresponding term in the nuclear equation of state, the symmetry energy [13,14], on which many investigations are concentrated [15–19]. Indeed, as also pointed out for the pairing interaction, this information is essential for the understanding of nuclear structure [20,21] and of the properties of neutron stars [11,22]. Along a phase separation process, the symmetry energy influences significantly the so-called isospin distillation mechanism, which leads to a different species concentration in the two phases, namely a more symmetric (with respect to the initial system) liquid phase and a more asymmetric gas phase [14,23].

The aim of this Brief Report is to evaluate the impact of pairing correlations, which are mostly active at low density, on mechanical (spinodal) instabilities of asymmetric nuclear matter. Since pairing between protons and neutrons is strongly quenched in asymmetric matter [24], here we consider a pairing interaction acting only between identical nucleons in a spin-singlet state.

Our study is undertaken in the framework of the Hartree-Fock-Bogolyubov approach, which includes in a unified formalism the pairing and the mean-field effective interactions [4]. For the pairing interaction, we adopt a local interaction of strength $v_{\pi q}$, where $q = n$ or p denotes neutrons or protons, respectively.

Though the density dependence of the pairing force is still poorly known, it is often assumed that $v_{\pi q}$ depends only on the density ρ_q of the considered species [25,26]; thus we write

$$v_{\pi q}(\rho_n,\rho_p) \equiv v_\pi(\rho_q) = V_\pi\left[1 - \eta\left(\frac{2\rho_q}{\rho_0}\right)^\alpha\right], \tag{1}$$

where ρ_0 is the nuclear saturation density. The parameters in Eq. (1), V_π, η and α, are usually fitted directly to experimental data, by taking as reference the pairing gap in finite nuclei. However, in order to establish a connection with realistic nucleon-nucleon forces, it has been recently proposed that the parameters of the pairing interaction can be determined by fitting the $^1S_0$ paring gaps in infinite nuclear matter as obtained in the BCS approximation [27] or in Brueckner calculations [28]. Then the density dependence of the pairing strength can be calculated exactly, for any given $^1S_0$ pairing-gap function $\Delta(\rho_q)$, by inverting the gap equation

$$I_\Delta \equiv -v_\pi(\rho_q)\int \frac{d\mathbf{p}}{(2\pi\hbar)^3}\frac{1}{2\xi}[1 - 2F_q(\mathbf{p})] = 1, \tag{2}$$

where

$$F_q(\mathbf{p}) = \frac{1}{2}\left[1 - \frac{\xi}{E_\Delta}\tanh\left(\frac{\beta E_\Delta}{2}\right)\right] \quad (3)$$

represents the particle occupation number.

In the previous equations β indicates the inverse of the temperature T and $E_\Delta = \sqrt{\xi^2 + \Delta^2}$, where $\xi = (\frac{\mathbf{p}^2}{2m} - \mu_q^*)$, with m the nucleon mass. The reduced chemical potential μ_q^* can be obtained by fixing the particle number density:

$$I_\rho \equiv \int \frac{d\mathbf{p}}{(2\pi\hbar)^3}\left[1 - \frac{\xi}{E_\Delta}\tanh\left(\frac{\beta E_\Delta}{2}\right)\right] = \rho_q. \quad (4)$$

It may be useful to recall that, in the absence of pairing, the reduced chemical potential μ_q^* coincides with that of a noninteracting Fermi gas, $\mu_{q,F}^*$. It should be noticed that, owing to the zero range of the pairing interaction, a cutoff has to be introduced in the gap equations to avoid divergences (see, for instance, [29]). Here we adopt the energy cutoff $\epsilon_\Lambda = 16$ MeV [26].

We will take as reference the pairing gap in pure neutron matter at zero temperature, as obtained in Brueckner calculations employing the realistic Argonne v_{14} potential (see Ref. [28]). The corresponding values are plotted in Fig. 1 (black plus signs) as a function of the neutron density ρ_n. The maximum value of the gap ($\Delta \approx 3$ MeV) is reached at the density $\rho_M = 0.02\ \mathrm{fm}^{-3}$. By knowing Δ, from Eqs. (2)–(4) it is possible to extract the pairing interaction v_π as a function of the neutron matter density. The results obtained can be fitted by the functional dependence expressed by Eq. (1), with the parameters $V_\pi = -1157.51$ MeV, $\eta = 0.884$, and

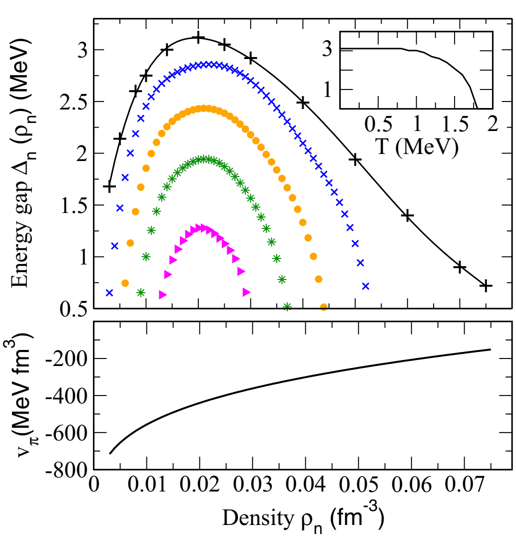

FIG. 1. (Color online) Top panel: The energy gap, as obtained in Brueckner calculations of pure neutron matter at zero temperature, as a function of the neutron density (black plus signs). The figure also shows the energy gap obtained, by solving Eqs. (2)–(4), at several T values: 1 MeV (crosses), 1.3 MeV (dots), 1.5 MeV (stars), and 1.65 MeV (triangles). The inset displays the energy gap as a function of the temperature, for the density $\rho_M = 0.02\ \mathrm{fm}^{-3}$. Bottom panel: The strength of the pairing interaction v_π as a function of the neutron matter density.

$\alpha = 0.256$ and the corresponding behavior is shown in the bottom panel of Fig. 1. Once the strength of the pairing interaction is fixed, Eqs. (2)–(4) allow one to evaluate the pairing gap at finite temperature T. Results are shown in Fig. 1 (top panel). The critical temperature $[T_c(\rho_q)]$ of the transition to the superfluid/superconducting phase (where the energy gap Δ starts to appear) depends on the density considered and is equal to $T_c = 1.8$ MeV at ρ_M [see the inset in Fig. 1 (top panel)]. The results discussed above are extended to the pp case, by assuming that the pairing strength is the same as in the nn case, just depending on the density of the species considered.

By adopting a simplified Skyrme-like effective interaction for the mean field [30], the nuclear energy density functional can be written as follows [31]:

$$\rho\frac{E}{A} = 2\sum_q \int \frac{d\mathbf{p}}{(2\pi\hbar)^3} F_q(\mathbf{p})\frac{\mathbf{p}^2}{2m} + \frac{1}{4}\sum_q v_\pi(\rho_q)\tilde{\rho}_q^*\tilde{\rho}_q + \rho\left[\frac{\mathcal{A}}{2}\left(\frac{\rho}{\rho_0}\right) + \frac{\mathcal{B}}{\sigma+1}\left(\frac{\rho}{\rho_0}\right)^\sigma + C_{\mathrm{sym}}^{\mathrm{pot}}(\rho)I^2\right], \quad (5)$$

where $\rho = \rho_n + \rho_p$, $I = \frac{(\rho_n - \rho_p)}{\rho}$ is the asymmetry parameter, and $\tilde{\rho}_q = 2\Delta(\rho_q)/v_\pi(\rho_q)$ denotes the so-called anomalous density. The coefficients $A = -356$ MeV and $B = 303$ MeV and the exponent $\sigma = \frac{7}{6}$, characterizing the isoscalar part of the mean field, are fixed by requiring that the saturation properties of symmetric nuclear matter, with a compressibility of 200 MeV, are reproduced. For the isovector part of the nuclear interaction, we consider, as a possible description of the local density (ρ) dependence of the symmetry energy coefficient $C_{\mathrm{sym}}^{\mathrm{pot}}$, two representative parametrizations: one with a linearly increasing behavior with density (asy-stiff), $C_{\mathrm{sym}}^{\mathrm{pot}}(\rho) = 112.5\rho$ (MeV), and one which saturates above normal density (asy-soft), $C_{\mathrm{sym}}^{\mathrm{pot}}(\rho) = \rho(241 - 819\rho)$ (MeV) [14,30].

The mean-field potential U_q can be derived from the potential part of the energy density functional, Eq. (5): $U_q = [\frac{\partial(\rho E_{\mathrm{pot}}/A)}{\partial\rho_q}]_{\tilde{\rho}_q}$ [32]. We notice that, because of the density dependence of the pairing strength, U_q gets a contribution also from the pairing energy density [second term on the right-hand side of Eq. (5)]: $U_q^\pi = \Delta^2/v_\pi^2\,\partial v_\pi/\partial\rho_q$.

We now turn to an examination of the possible occurrence of mechanical (spinodal) instabilities in asymmetric nuclear matter at a fixed temperature T. The spinodal region is defined as the ensemble of unstable points for which the free-energy surface is concave in at least one direction. This is determined by the curvature matrix

$$C = \begin{pmatrix} a & c/2 \\ c/2 & b \end{pmatrix}, \quad (6)$$

where $a = \partial\mu_p/\partial\rho_p$, $b = \partial\mu_n/\partial\rho_n$, and $c = 2\partial\mu_p/\partial\rho_n$. $\mu_q = \mu_q^* + U_q$ denotes the chemical potential. The lower eigenvalue gives the minimal free-energy curvature. If the latter is negative, the associated eigenvector gives the direction of phase separation [13,14].

In asymmetric nuclear matter ($\rho_n \neq \rho_p$) at low density, instabilities correspond to isoscalar-like density oscillations: The two species move in phase but with different amplitudes, according to the eigenvector components, $(\delta\rho_p,\delta\rho_n)$. In particular, defining the angle θ as $\tan\theta = \delta\rho_n/\delta\rho_p$, from the diagonalization of the C matrix one obtains [33]

$$\tan 2\theta = \frac{c}{a-b}. \tag{7}$$

It is generally observed that the asymmetry of the instability direction, $\delta I = (\delta\rho_n - \delta\rho_p)/(\delta\rho_n + \delta\rho_p)$, is smaller than the system initial asymmetry, leading to the formation of more symmetric nuclear clusters. This is the so-called isospin distillation mechanism, which is mainly ruled by the effect of the symmetry potential, which enhances the neutron-proton attraction. Indeed, a stiffer symmetry energy tends to reduce the difference between proton and neutron chemical potential derivatives, i.e., the term $(a-b)$ in Eq. (7), and the angle θ gets closer to 45°. Thus neutrons and protons oscillate with close amplitudes, in spite of the system initial asymmetry.

Here our aim is to investigate how pairing correlations may affect these features. Indeed, as seen in Fig. 1, the pairing energy gap is maximum at neutron and/or proton low densities compatible with the nuclear spinodal zone ($\rho < 0.1$ fm^{-3} at $T = 0$). The strength of the instability, i.e., the amplitude of the negative eigenvalue of C, is mainly determined by the isoscalar part of the nuclear mean-field potential, which is by far the dominant term of the interaction. Thus the pairing interaction has practically no effect on it. On the other hand, the distillation mechanism, which is connected to the strength of the symmetry potential, may be affected by the paring interaction, which couples nucleons of the same type.

To undertake this analysis we need to evaluate the expression of the elements of the C matrix in the presence of pairing correlations. We notice that, for the pairing interaction considered here, the term c gets no contribution from the pairing interaction. The calculation of a and b requires knowledge of the following derivatives: $\partial\mu_q^*/\partial\rho_q$ and $\partial\Delta/\partial\rho_q$ (with the latter appearing in the derivative of the potential U_q^π), which can be obtained by solving the following set of equations, derived from Eqs. (2) and (4):

$$\begin{aligned} \frac{\partial I_\Delta}{\partial\rho_q} + \frac{\partial I_\Delta}{\partial\mu_q^*}\frac{\partial\mu_q^*}{\partial\rho_q} + \frac{\partial I_\Delta}{\partial\Delta}\frac{\partial\Delta}{\partial\rho_q} &= 0, \\ \frac{\partial I_\rho}{\partial\mu_q^*}\frac{\partial\mu_q^*}{\partial\rho_q} + \frac{\partial I_\rho}{\partial\Delta}\frac{\partial\Delta}{\partial\rho_q} &= 1. \end{aligned} \tag{8}$$

One relevant quantity used to evaluate the width of the isospin distillation effect is the difference $\gamma = (a-b)$ of Eq. (7). Thus we consider its percentage variation in superfluid/superconducting nuclear matter, with respect to normal nuclear matter, at different global densities and asymmetries and zero temperature. Results are displayed in Fig. 2, in the case of the asy-stiff symmetry potential. One can see that it is possible to reach an effect of 20% for the variation of γ, at total densities of around 0.08 fm^{-3}. We recall that the features of the unstable modes are determined by derivatives of proton and neutron chemical potentials; thus a deeper insight into the amplitude of the pairing effect can be obtained by looking directly at the quantity $\delta_q = \partial\mu_q^*/\partial\rho_q + \partial U_q^\pi/\partial\rho_q$. The latter is displayed in the inset of Fig. 2, as a function of the density ρ_q, together with the corresponding values of normal

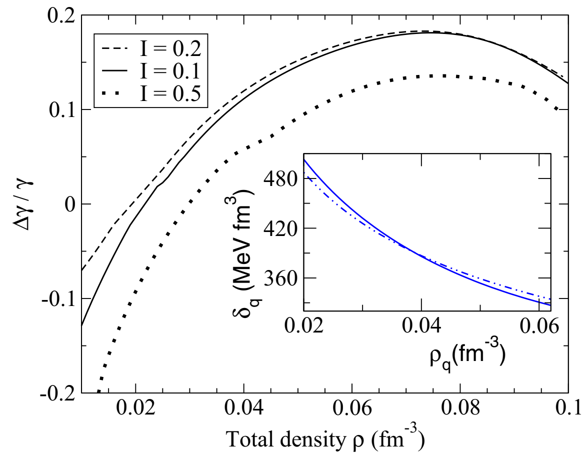

FIG. 2. (Color online) The percentage variation of the quantity γ (see text) as a function of the matter density, at zero temperature and for three values of the asymmetry I. The inset shows the density behavior of the quantity δ_q (see text), in normal (dashed line) and superfluid (full line) matter.

nuclear matter (which are nothing but the density derivatives of $\mu_{q,F}^*$). The two curves exhibit a different slope mostly around $\rho_q \approx 0.04$ fm^{-3}, thus pointing to the region of total density ($\rho \approx 2\rho_q = 0.08$ fm^{-3}) where one expects the largest influence of pairing correlations on the difference $(a-b)$, as already evidenced by the results shown in Fig. 2.

Let us move to consider nuclear matter at finite temperature T. The quantity δ_q is represented in Fig. 3, as a function of T, for three values of the density ρ_q ($\rho_M/2$, ρ_M, and $2\rho_M$). Calculations including pairing correlations (blue circles) are compared with the results of normal nuclear matter (red dashed lines). As a rather interesting effect, we clearly observe the appearance of discontinuities, at the critical temperature, in the behavior of δ_q.

It is well known that at the critical temperature $T_c(\rho_q)$ the heat capacity exhibits a discontinuity [5]. Hence, together with this widely discussed effect, one has to notice that discontinuities also appear in the behavior of the density derivative of the chemical potential, $\partial\mu_q/\partial\rho_q$, which is connected to matter compressibility. It is also rather interesting

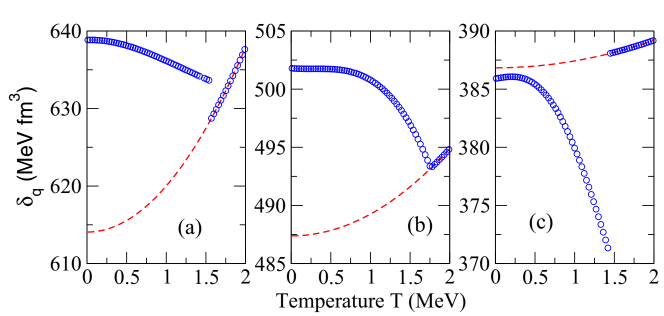

FIG. 3. (Color online) The quantity δ_q (see text) represented as a function of the temperature, for three ρ_q density values: $0.5\rho_M$ (a), ρ_M (b), and $2\rho_M$ (c). Circles indicate the calculations including pairing effects, whereas dashed lines are for normal nuclear matter.

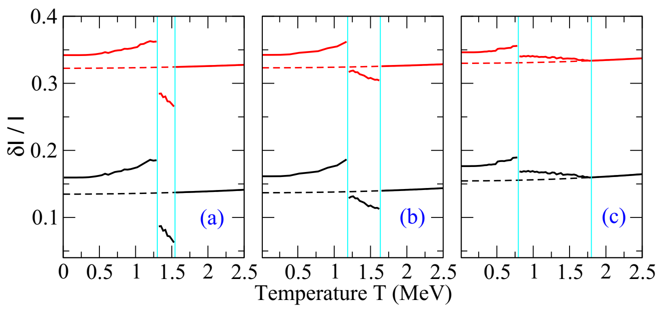

FIG. 4. (Color online) Full lines: The asymmetry of the unstable oscillations plotted as a function of the temperature, for nuclear matter at total density $\rho = 0.08$ fm^{-3} and three asymmetry values: $I = 0.1$ (a), 0.2 (b), and 0.5 (c). Results are shown for the asy-stiff (black) and the asy-soft (red) parametrizations. Dashed line: The same quantity but for normal nuclear matter. In each panel the two vertical lines indicate the position of neutron and proton critical temperatures.

to observe that, owing to the features of the gap function in nuclear matter (see Fig. 1), the jump disappears at the density $\rho_q = \rho_M$ [Fig. 1(b)], where the energy gap Δ is maximum at all temperatures.

Finally, we turn to a discussion of the impact of pairing correlations directly on the asymmetry δI of the instability direction. Results are displayed in Fig. 4, for nuclear matter at $\rho = 0.08$ fm^{-3}, where the largest effects have been observed at zero temperature (see Fig. 2), and three values of the asymmetry I. The two parametrizations of the symmetry energy introduced above are considered. First, we notice that the overall effect of the isospin distillation mechanism is rather important. Indeed, $\delta I/I$ is much lower than 1 in all cases. The effect is larger in the asy-stiff case, which is characterized by a steeper variation of the symmetry energy with density, in agreement with previous studies [14,34]. The asymmetry δI is more sensitive to the symmetry energy parametrization (black versus red lines) than to the introduction of pairing correlations (full versus dashed lines). Thus our results essentially confirm the leading role of the symmetry energy in the isospin distillation mechanism. However, new interesting effects appear at moderate temperatures. Owing to the trend followed by the chemical potential derivatives (see Fig. 3), the calculations including the pairing interaction exhibit two discontinuities, corresponding to neutron and proton critical temperatures, which may cause significant variations of δI. As observed in Fig. 3, the δ_q discontinuity is more pronounced at densities above ρ_M, around $\rho_q \approx 0.04$ fm^{-3}. This explains why the largest effect for δI is seen at small asymmetries [see Fig. 3(a)], i.e., for neutron and proton densities ρ_q close to $2\rho_M$. Hence, under suitable density and temperature conditions, pairing correlations may lead to significant deviations of the asymmetry from its average.

To conclude, we have presented an analysis framed in the general context of two-component fermionic systems subject to pairing correlations. We have focused on the interplay between the pairing force, coupling particles of the same type, and the isovector interaction, which on the contrary enhances the attraction between particles of different kinds. This study has been conducted for unstable asymmetric nuclear matter at low temperature, where it is shown that pairing correlations may have non-negligible effects, especially around the transition temperature to the superfluid/superconducting phase, on the isotopic features of the density fluctuations leading to cluster formation. These results are relevant to the general context of nuclear fragmentation and clustering mechanisms [12,35–37]. In particular, the pairing effects discussed here could influence the properties of supernova matter and of the crust of (proto-)neutron stars, where low-density clustering phenomena take place at rather low temperature [12,38–40].

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