Low-lying Ω states with negative parity in an extended quark model with Nambu–Jona-Lasinio interaction

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Here we investigate mixing of the low-lying three- and five-quark Ω states with spin-parity quantum numbers $1/2^-$ and $3/2^-$, employing the quark-antiquark creation triggered by Nambu–Jona-Lasinio (NJL) interaction. Wave functions of the three- and five-quark configurations are constructed by using the extended constituent quark model, within which the hyperfine interaction between quarks is also taken to be the NJL-induced one. Numerical results show that the NJL-interaction-induced pair creation results in vanishing mixing between three- and five-quark Ω configurations with spin-parity $1/2^-$, but mixing between three- and five-quark $3/2^-$ Ω states should be very strong. The mixing decreases energy of the lowest $3/2^-$Ω state to be 1785 ± 25 MeV, which is lower than energy of the lowest $1/2^-$ state in this model. This is consistent with our previous predictions within the instanton-induced quark-antiquark creation model.

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I. INTRODUCTION

Recently, the spectrum of low-lying Ω resonances with negative parity was investigated by employing an extended constituent quark model [1,2], within which the Ω resonances were considered as admixtures of three- and five-quark components, and the hyperfine interaction between quarks was taken to be of three different kinds, namely, one gluon exchange (OGE) [3–6], Goldstone boson exchange (GBE) [7], and instanton-induced interaction (INS) [8–11]. In Ref. [2], mixing of three- and five-quark Ω states was calculated by treating the $q\bar{q}$ creation mechanism as the one induced by instanton interaction. It is shown that the mixing between three- and five-quark components in Ω resonances with spin-parity $1/2^-$ is very small and negligible, but in the $3/2^-$ Ω resonances the mixing is very strong, and the mixing decreases the energy of the lowest $3/2^-$ state to be around 1750 ± 50 MeV. It is very interesting that this energy is lower than the energy of the lowest spin-parity $1/2^-$ Ω resonance.

As shown in [2], the instanton quark-antiquark pair creation precludes transitions between s^3 and $s^4\bar{s}$ configurations, while the instanton-induced hyperfine interaction between a quark and an antiquark could lead to mixing between five-quark Ω configurations with a light quark-antiquark pair and an $s\bar{s}$ pair [1]. Therefore, once we take the instanton-induced hyperfine interaction and quark-antiquark pair creation simultaneously into account, mixing between s^3 and $s^4\bar{s}$ configurations will not vanish. But if the hyperfine interaction between quarks is chosen as OGE or GBE, the instanton-induced quark-antiquark pair creation mechanism cannot result in mixing between s^3 and $s^4\bar{s}$ configurations. Generally, even if the probability of having an $s^4\bar{s}$ component in Ω resonances is small, it should not be exactly 0. This may indicate that, once we take the instanton-induced quark-antiquark pair creation into account, we have to keep the hyperfine interaction between quarks based on the same model.

In the present work, we try to calculate mixing between three- and five-quark components in low-lying Ω resonances with negative parity using the Nambu—Jona-Lasinio (NJL) approach [12,13], which was originally constructed for nucleons that interact via an effective two-body contact interaction and later developed to include the quark freedom [14]. Analogous to the instanton interaction [15–17], the NJL model can describe various aspects of QCD related to the dynamical and explicit breaking of chiral symmetry and the axial anomaly very well [18]. As discussed above, here we take the hyperfine interactions between quarks to be also the NJL-induced one for model consistency.

The present paper is organized as follows. In Sec. II, we present our theoretical framework, which includes explicit forms of the NJL-induced quark-quark hyperfine interactions and the quark-antiquark pair creation mechanism. Numerical results for the spectrum of the states under study and the mixing of three- and five-quark configurations in our model are shown in Sec. III. Finally, Sec. IV contains a brief conclusion.

II. THEORETICAL FRAMEWORK

In the present model, the Hamiltonian is almost the same as that used in [1,2] except for the parts describing the quark-quark hyperfine interaction and the mechanism for transition between three- and five-quark components in Ω resonances. For completeness, here we also repeat the same parts. The Hamiltonian describing Ω resonances as admixtures of three- and five-quark components is of the following form:

$$H = \begin{pmatrix} H_3 & T_{\Omega_3\leftrightarrow\Omega_5} \\ T_{\Omega_3\leftrightarrow\Omega_5} & H_5 \end{pmatrix}, \quad (1)$$

where H_3 and H_5 are the Hamiltonian for three-quark and five-quark systems, respectively, and $T_{\Omega_3\leftrightarrow\Omega_5}$ denotes the transition between three- and five-quark systems. Here we discuss the

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diagonal and nondiagonal terms of the Hamiltonian (1) in Secs. II A and II B, respectively.

A. Diagonal terms of the Hamiltonian

The Hamiltonian for a N-particle system in the constituent quark model can be written as the follows:

$$H_N = H_o + H_{\text{hyp}} + \sum_{i=1}^{N} m_i, \tag{2}$$

where H_o and H_{hyp} represent the Hamiltonians for the quark orbital motion and for the hyperfine interactions between quarks, respectively, and m_i denotes the constituent mass of the ith quark. The first term H_o can be written as a sum of the kinetic energy term and the quark confinement potential as

$$H_o = \sum_{i=1}^{N} \frac{\vec{p}_i^{\,2}}{2m_i} + \sum_{i<j}^{N} V_{\text{conf}}(r_{ij}). \tag{3}$$

In [1] the quark confinement potential was taken to be

$$V_{\text{conf}}(r_{ij}) = -\frac{3}{8}\lambda_i^C \cdot \lambda_j^C \left[C^{(N)}(\vec{r}_i - \vec{r}_j)^2 + V_0^{(N)}\right], \tag{4}$$

where $C^{(N)}$ and $V_0^{(N)}$ are constants. In principle these two constants can differ for three- and five-quark configurations. H_{hyp} denotes the hyperfine interaction between quarks; here we take H_{hyp} to be the NJL-induced one. The NJL interaction between quarks can be described by

$$\mathcal{L}_{\text{NJL}} = \frac{1}{2} g_s \sum_{a=0}^{8} [(\bar{q}\lambda^a q)^2 + (\bar{q} i \lambda^a \gamma_5 q)^2], \tag{5}$$

where λ^a ($a = 1, \ldots, 8$) are the Gell-Mann matrices in the flavor SU(3) space, and $\lambda^0 = \sqrt{\frac{2}{3}}\mathcal{I}$, with $\mathcal{I}$ the unit matrix in the three-dimensional flavor space. In the nonrelativistic approximation, the NJL-induced quark-quark interaction can be obtained as

$$H_{qq}^{\text{NJL}} = \sum_{i<j}^{N} \sum_{a=0}^{8} \hat{g}_{ij} \lambda_i^a \lambda_j^a \times \left[1 + \frac{1}{4m_i m_j} \hat{\sigma}_i \cdot (\vec{p}_i^{\,\prime} - \vec{p}_i) \hat{\sigma}_j \cdot (\vec{p}_j^{\,\prime} - \vec{p}_j)\right], \tag{6}$$

where $\hat{g}_{ij}$ is an operator which distinguishes the coupling strength between two light quarks, g_{qq}, that between one light quark and one strange quark, g_{qs}, and that between two strange quarks, g_{ss}. In principle, once the SU(3)-breaking effects are taken into account, the three coupling strengths should be different. One may find that the present hyperfine interaction is very similar to the one mediated by Goldstone boson exchange, which includes pseudoscalar and scalar meson exchange [7]. Therefore, here we take the relationship of the three different coupling strengths as in [7]:

$$g_{qq} : g_{qs} : g_{ss} = 1 : \frac{m}{m_s} : \frac{m^2}{m_s^2}, \tag{7}$$

where m and m_s represent the constituent masses of the light and strange quarks, respectively. The empirical value for the constituent mass of the light quark is in the range 300 ± 50 MeV, and that for the strange quark is $\sim$120–200 MeV higher than m. In the traditional qqq constituent quark model, m is often taken to be 320–340 MeV [7]. In the present case, since we take the baryons to be admixtures of three- and five-quark components, in general, the constituent quark mass should be lower than that in the three-quark model. Accordingly, here we take the value $m = 310$ MeV for the light quarks and $m_s = 460$ MeV for the strange quark.

Generally, one can divide the interaction (6) by different spin dependencies [19]. If we neglect the tensor term in the quark-quark interaction, the NJL-induced hyperfine interaction between quarks, $H_{\text{hyp}}^{\text{NJL}}$, should be of the following form:

$$H_{\text{hyp}}^{\text{NJL}} = \sum_{i<j}^{N} \sum_{a=0}^{8} \hat{g}_{ij} \lambda_i^a \lambda_j^a \left(1 - \frac{1}{12}\hat{\sigma}_i \cdot \hat{\sigma}_j\right); \tag{8}$$

hereafter we will use Eq. (8) as the hyperfine interaction operator in the following calculations, since the hyperfine interactions between quarks are treated as perturbations, and contributions from the tensor term should be smaller by an order of magnitude than those from the terms in Eq. (8).

Accordingly, one can obtain the three coupling strengths by reproducing the mass splitting between Σ and Λ baryons:

$$g_{qq} = 69 \text{ MeV}, \quad g_{qs} = 51 \text{ MeV}, \quad g_{ss} = 38 \text{ MeV}. \tag{9}$$

As discussed in [2], there are two low-lying Ω resonances with negative parity in the $N \leqslant 2$ band within the qqq three-quark model [4,7,20], one with spin 1/2 and the other with spin 3/2, corresponding to the first two orbitally excited states of $\Omega(1672)$. These two states should be degenerate in a given hyperfine interaction model if the L-S coupling hyperfine interaction is not taken into account. The matrix elements of the submatrix H_3 in (1) obtained by using the OGE, GBE, and INS hyperfine interactions in [2] are $\langle H_3^{\text{OGE}}\rangle_{\frac{1}{2}^-} = \langle H_3^{\text{OGE}}\rangle_{\frac{3}{2}^-} = 2020$ MeV, $\langle H_3^{\text{GBE}}\rangle_{\frac{1}{2}^-} = \langle H_3^{\text{GBE}}\rangle_{\frac{3}{2}^-} = 1991$ MeV, and $\langle H_3^{\text{INS}}\rangle_{\frac{1}{2}^-} = \langle H_3^{\text{INS}}\rangle_{\frac{3}{2}^-} = 1887$ MeV, respectively. In the present work, with the above given NJL-induced hyperfine interaction strength, by reproducing the mass of the ground state $\Omega(1672)$, one can obtain that $V_0^{(3)} = -188$ MeV, which is smaller than the value -140 MeV in the GBE model. With these parameters, we obtain the matrix elements of H_3 in the present model as $\langle H_3^{\text{NJL}}\rangle_{\frac{1}{2}^-} = \langle H_3^{\text{NJL}}\rangle_{\frac{3}{2}^-} = 1942$ MeV.

On the other hand, to get the value of the parameter for $V_0^{(5)}$, we fit the lowest five-quark Ω configuration to be $\sim$1810 MeV, the value of which was proposed to be the energy of the lowest $K\Xi$ bound state with spin-parity $1/2^-$ [21]; this method yields $V_0^{(5)} = -294$ MeV, which is also smaller than the value -269 MeV in the GBE model.

B. Nondiagonal terms of Hamiltonian

The nondiagonal term $T_{\Omega_3 \leftrightarrow \Omega_5}$ depends on the explicit quark-antiquark pair creation mechanism. In the present work, we take the quark-antiquark pair creation mechanism to be the one based on a nonrelativistic reduction of the amplitudes found from the NJL interaction. One finds that the two terms in Eq. (5) just correspond to the quark-antiquark pair creation with quantum numbers 0^+ and 0^-, respectively. Accordingly, in the present case, only the second term will contribute. In the nonrelativistic limit, the second term in Eq. (5) reduces to

$$\hat{T}_{q\bar{q}} = -\frac{2}{3m_s} g_{qs} \xi_f^\dagger \hat{\sigma} \cdot (\vec{p}_i - \vec{p}_f) \xi_i \xi_q \mathcal{I} \eta_{\bar{q}} \tag{10}$$

for light $q\bar{q}$ creation and to

$$\hat{T}_{s\bar{s}} = \frac{1}{3m_s} g_{ss} \xi_f^\dagger \hat{\sigma} \cdot (\vec{p}_i - \vec{p}_f) \xi_i \xi_s \mathcal{I} \eta_{\bar{s}} \tag{11}$$

for $s\bar{s}$ creation, where $\xi_{f(i)}$ and $\vec{p}_{f(i)}$ denote the final (initial) spin and momentum operators of the quark that emits a $q\bar{q}$ or $s\bar{s}$ pair, $\xi_{q(s)}$ is the spin operator of the created light (strange) quark, and $\eta_{\bar{q}(\bar{s})}$ is the spin operator of the created light (strange) antiquark.

If we treat the other two quarks as spectators, then the nondiagonal term $T_{\Omega_3 \leftrightarrow \Omega_5}$ in Hamiltonian (1) can be obtained as

$$T^{q\bar{q}}_{\Omega_3 \leftrightarrow \Omega_5} = -\frac{2}{3m_s} g_{qs} \sum_{i=1}^{3} \sum_{j \neq i}^{4} \mathcal{C}_\mathcal{F} \mathcal{C}_\mathcal{S} \mathcal{C}_\mathcal{C} \mathcal{C}_\mathcal{O} \times \xi_i'^\dagger \hat{\sigma} \cdot (\vec{p}_i - \vec{p}_i') \xi_i \xi_j \mathcal{I} \eta_{\bar{q}}, \tag{12}$$

$$T^{s\bar{s}}_{\Omega_3 \leftrightarrow \Omega_5} = \frac{1}{3m_s} g_{ss} \sum_{i=1}^{3} \sum_{j \neq i}^{4} \mathcal{C}_\mathcal{F} \mathcal{C}_\mathcal{S} \mathcal{C}_\mathcal{C} \mathcal{C}_\mathcal{O} \times \xi_i'^\dagger \hat{\sigma} \cdot (\vec{p}_i - \vec{p}_i') \xi_i \xi_j \mathcal{I} \eta_{\bar{s}} \tag{13}$$

for transitions $sss \longleftrightarrow sssq\bar{q}$ and $sss \longleftrightarrow ssss\bar{s}$, respectively, where $\mathcal{C}_\mathcal{F}$, $\mathcal{C}_\mathcal{S}$, $\mathcal{C}_\mathcal{C}$, and $\mathcal{C}_\mathcal{O}$ are operators for the calculation of the corresponding flavor, spin, color, and orbital overlap factors, respectively.

III. NUMERICAL RESULTS

As we have done in [1,2], to show our numerical results clearly, here we denote the two three-quark configurations as $|3, \frac{1}{2}^-\rangle$ and $|3, \frac{3}{2}^-\rangle$, respectively, five-quark configurations with spin-parity quantum number $1/2^-$ as

$$\begin{aligned}
\left|5, \tfrac{1}{2}^-\right\rangle_1 &= |s^3 q([4]_X [211]_C [31]_{FS} [31]_F [22]_S) \otimes \bar{q}\rangle, \\
\left|5, \tfrac{1}{2}^-\right\rangle_2 &= |s^3 q([4]_X [211]_C [31]_{FS} [31]_F [31]_S) \otimes \bar{q}\rangle, \\
\left|5, \tfrac{1}{2}^-\right\rangle_3 &= |s^3 q([4]_X [211]_C [31]_{FS} [4]_F [31]_S) \otimes \bar{q}\rangle, \\
\left|5, \tfrac{1}{2}^-\right\rangle_4 &= |s^4([4]_X [211]_C [31]_{FS} [4]_F [31]_S) \otimes \bar{s}\rangle,
\end{aligned} \tag{14}$$

and those with spin-parity quantum number $3/2^-$ as

$$\begin{aligned}
\left|5, \tfrac{3}{2}^-\right\rangle_1 &= |s^3 q([4]_X [211]_C [31]_{FS} [31]_F [31]_S) \otimes \bar{q}\rangle, \\
\left|5, \tfrac{3}{2}^-\right\rangle_2 &= |s^3 q([4]_X [211]_C [31]_{FS} [31]_F [4]_S) \otimes \bar{q}\rangle, \\
\left|5, \tfrac{3}{2}^-\right\rangle_3 &= |s^3 q([4]_X [211]_C [31]_{FS} [4]_F [31]_S) \otimes \bar{q}\rangle, \\
\left|5, \tfrac{3}{2}^-\right\rangle_4 &= |s^4([4]_X [211]_C [31]_{FS} [4]_F [31]_S) \otimes \bar{s}\rangle,
\end{aligned} \tag{15}$$

where $[4]_X$, $[211]_C$, $[31]_F$ ($[4]_F$), and $[22]_S$ ($[31]_S$ or $[4]_S$) are the Young tableaux for orbital, color, flavor, and spin wave functions of the four-quark subsystem, and $[31]_{FS}$ denotes the flavor-spin combined wave function.

The main parameters involved in the transitions between three- and five-quark Ω states are the ratio of harmonic oscillator parameters, $R_{35} = \omega_5/\omega_3$, and the NJL interaction strength g_{qs} and g_{ss}. We present the numerical results by taking the parameters to be empirical values in Sec. III A and those by treating the ratio R_{35} and interaction strength g_{qs} and g_{ss} to be free parameters in Sec. III B.

A. Numerical results with fixed parameters

First, we take a tentative value $R_{35} = \sqrt{5/6}$ [2] and the values for NJL interaction strength given in Sec. II A to show the mixing between three- and five-quark Ω states within the NJL-interaction-induced quark-antiquark pair creation model. With the notation in Eqs. (14) and (15), the matrix elements of the Hamiltonian (1) including H_3, H_5, and $T_{\Omega_3 \leftrightarrow \Omega_5}$ are

$$\langle H^{\rm NJL} \rangle_{1/2} = \begin{pmatrix} 1942.0 & 0 & 0 & 0 & 0 \\ 0 & 1809.9 & 0 & 0 & 0 \\ 0 & 0 & 1816.2 & -6.1 & 0 \\ 0 & 0 & -6.1 & 2254.8 & 0 \\ 0 & 0 & 0 & 0 & 2474.7.0 \end{pmatrix}, \tag{16}$$

$$\langle H^{\rm NJL} \rangle_{3/2} = \begin{pmatrix} 1942.0 & 38.5 & -60.9 & -27.2 & 20.3 \\ 38.5 & 1816.2 & 0 & -6.1 & 0 \\ -60.9 & 0 & 1821.5 & 0 & 0 \\ -27.2 & -6.1 & 0 & 2254.8 & 0 \\ 20.3 & 0 & 0 & 0 & 2474.7 \end{pmatrix}. \tag{17}$$

The numbers in the above equations are in units of MeV.

As we can see in Eq. (16), the NJL-interaction-induced quark-antiquark pair creation does not contribute to transitions between three- and five-quark Ω states with spin 1/2. This is because of the quark-antiquark pairs in corresponding five-quark Ω states have quantum number ${}^3S_1$, while the created quark-antiquark pair in the NJL approach should have quantum number ${}^1S_0$; therefore, transitions between the studied spin 1/2 three- and five-quark configurations vanish in the NJL-interaction-induced pair creation model. One may find this to be consistent with the results obtained in [2] using the instanton pair creation model. In Ref. [2], although the obtained mixing between three- and five-quark configurations with spin 1/2 is not 0, it is very small and negligible; in fact, the mixing is proportional to $1/m - 1/m_s$, with m and m_s being the constituent masses of the light and strange quarks, so the mixing will also be 0 in the flavor SU(3) limit. For the nondiagonal matrix elements from transitions between three- and five-quark spin 3/2 Ω states, Eq. (17) shows that the

TABLE I. Energies and the corresponding probability amplitudes of three- and five-quark configurations for the obtained Ω states in the NJL-induced hyperfine interaction model. The upper and lower panels are for states with quantum numbers $1/2^-$ and $3/2^-$, respectively, and for each panel, the first row shows the energies in MeV and others show the probability amplitudes.

$\frac{1}{2}^-$	1810	1816	1942	2255	2475
$\lvert 3,\frac{1}{2}^-\rangle$	0.0000	0.0000	1.0000	0.0000	0.0000
$\lvert 5,\frac{1}{2}^-\rangle_1$	1.0000	0.0000	0.0000	0.0000	0.0000
$\lvert 5,\frac{1}{2}^-\rangle_2$	0.0000	0.9999	0.0000	−0.0140	0.0000
$\lvert 5,\frac{1}{2}^-\rangle_3$	0.0000	0.0140	0.0000	0.9999	0.0000
$\lvert 5,\frac{1}{2}^-\rangle_4$	0.0000	0.0000	0.0000	0.0000	1.0000
$\frac{3}{2}^-$	1786	1818	1972	2257	2475
$\lvert 3,\frac{3}{2}^-\rangle$	0.4227	−0.0354	−0.9002	0.0905	0.0389
$\lvert 5,\frac{3}{2}^-\rangle_1$	−0.5385	−0.8135	−0.2185	0.0217	0.0023
$\lvert 5,\frac{3}{2}^-\rangle_2$	0.7286	−0.5803	0.3635	−0.0127	−0.0036
$\lvert 5,\frac{3}{2}^-\rangle_3$	0.0175	−0.0136	−0.0916	−0.9955	−0.0049
$\lvert 5,\frac{3}{2}^-\rangle_4$	−0.0125	0.0011	0.0364	−0.0085	0.9992

transition matrix elements are smaller than those caused by instanton-induced quark-antiquark creation [2].

On the other hand, as shown in Eqs. (16) and (17), the NJL-induced hyperfine interaction between quarks leads to only two very small nonvanishing nondiagonal matrix elements for both spin 1/2 and 3/2 cases. This is the same as the results obtained within the GBE hyperfine interaction model [1]. As we have discussed in Sec. II, the present hyperfine interaction model is very similar to the GBE model.

Diagonalization of Eqs. (16) and (17) results in the numerical results of the energies and corresponding probability amplitudes of three- and five-quark configurations for the obtained Ω states shown in Table I. As shown in this table, mixing between three- and five-quark spin 3/2 Ω configurations is very strong, but since the transition matrix elements listed in Eq. (17) are not large, the mixing does not decrease the energy of the lowest state as much as that obtained in [2]. Nevertheless, the obtained energy of the lowest spin 3/2 state is lower than the energy of the lowest spin 1/2 state; this is consistent with our previous results obtained within the instanton-induced quark-antiquark pair creation model [2]. One finds that the mixing between the three-quark Ω state and the $s^4\bar{s}$ is very small, with the largest mixing appearing in the third state with a probability of $P_{s\bar{s}} = 0.0364^2 \simeq 0.1\%$. This is because the transition matrix element between s^3 and $s^4\bar{s}$ configurations is smaller than the other ones but the energy of the $s^4\bar{s}$ configuration is much larger than that of the three-quark Ω state.

B. Dependence of numerical results on parameters

To show the dependence of mixing between three- and five-quark Ω states with spin 3/2, we present the energies of the obtained states in Figs. 1 and 2, the former one showing M_Ω as functions of the NJL interaction strength g_{qs} with g_{qs} varying from 25 to 75 MeV, and the latter showing the dependence of M_Ω on the ratio R_{35} with R_{35} being in the range 0.5–2. Note that here we just show the dependence of mixing effects on parameters; since there is no mixing between three- and five-quark configurations in the obtained Ω states with spin 1/2, we only give the numerical results for Ω states with spin 3/2 in this section. In addition, we keep the diagonal terms as constants when varying the parameters.

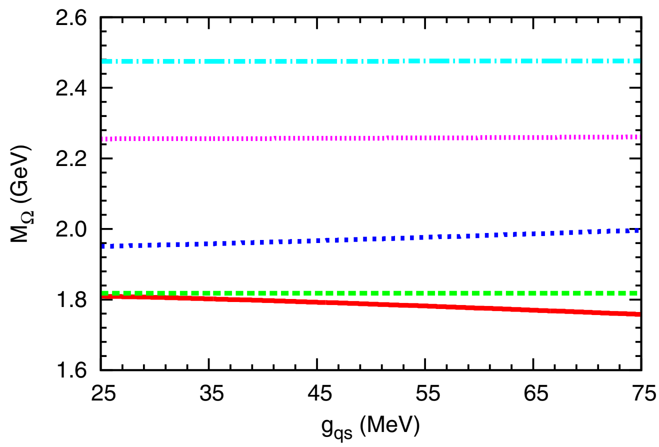

FIG. 1. (Color online) Energies of Ω resonances with spin 3/2 as functions of g_{qs}; the five curves correspond to the spin 3/2 states listed in Table I.

As we can see in Fig. 1, the energy of the lowest state shows little sensitivity on g_{qs}; $M_\Omega^{\rm lowest}$ falls in the range 1785 ± 25 MeV when g_{qs} varies from 25 to 75 MeV. This energy is higher than $M_\Omega^{\rm lowest} = 1750 \pm 50$ MeV in [2], which is found by taking the quark-antiquark pair creation mechanism to be the instanton-induced one, since the transition matrix elements between three- and five-quark Ω configurations in the present work are smaller than those obtained in [2], as we have discussed in Sec. III A. The obtained value for $M_\Omega^{\rm lowest}$ is decreased to be lower than the energy of the lowest spin 1/2 state in the present model; this conclusion is consistent

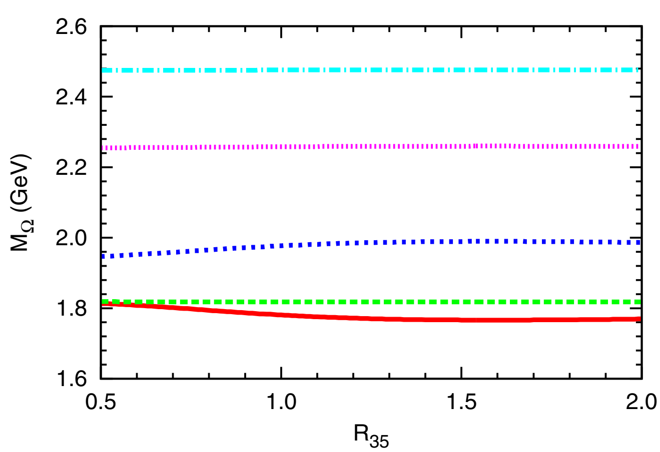

FIG. 2. (Color online) Energies of Ω resonances with spin 3/2 as functions of R_{35}; the five curves correspond to the spin 3/2 states listed in Table I.

with that in the instanton-induced interaction model. On the other hand, energies of the other states are not so sensitive to g_{qs}, especially the highest two states, whose energies are even constants; this is because mixing effects are very small in these two states, as shown in Table I. In fact, the energy of the next-to-lowest state is also insensitive to g_{qs}, as we can see in Fig. 1. This is because the main components is this state are the first two five-quark configurations, whose masses are very close to each other, lying at ∼1820 MeV, and mixing between three- and five-quark configurations in this state is not strong, as shown in Table I, as there is only a ∼0.1% three-quark component in this state.

Figure 2 shows almost the same features as Fig. 1; only the lowest and the third states are sensitive to R_{35}, and the lowest energy falls in the range 1790 ± 20 MeV.

Comparing to the numerical results in Ref. [2], one can find that the present obtained energies for spin 3/2 states are very different from those obtained by taking both the quark-quark hyperfine interaction and quark-antiquark pair creation to be the instanton-induced ones, with almost all the present obtained energies being lower than those in [2]. This is because the instanton-induced hyperfine interaction is very different from the present NJL-induced one, with the most obvious differences being that the instanton-induced hyperfine interaction can only exist between two quarks whose flavor wave function is antisymmetric and that the instanton-induced hyperfine interaction leads to strong mixing between five-quark configurations with light and strange quark-antiquark pairs. On the other hand, the transition matrix elements between three- and five-quark spin 3/2 states in the instanton-induced quark-antiquark pair creation model are larger than the present ones.

Mixing between three- and five-quark spin 1/2 Ω states is very small and negligible in the INS model and is 0 in the NJL model, while both INS and NJL-interaction-induced quark-antiquark pair creation mechanisms result in strong mixing between three- and five-quark spin 3/2 Ω states, and the mixing decreases the energy of the lowest spin 3/2 state to lower than that of the lowest spin 1/2 state.

IV. CONCLUSION

In this work, we investigated the mixing of the low-lying three- and five-quark Ω configurations with negative parity in an NJL-interaction-induced quark-antiquark pair creation model. Hyperfine interaction between quarks is also taken to be based on the NJL interaction for model consistency. Numerical results show that the three- and five-quark configurations with spin 1/2 do not mix with each other in the present model, because the spin structure of the five-quark Ω states with spin 1/2 results in vanishing matrix elements for the transition $sss \leftrightarrow sssq\bar{q}$. This is consistent with the results obtained in the instanton-induced quark-antiquark pair creation model, within which mixing between three- and five-quark Ω states with spin-parity $1/2^-$ is very small and negligible [2].

The NJL-interaction-induced quark-antiquark pair creation results in strong mixing between three- and five-quark spin 3/2 Ω states, and the mixing decreases the energy of the lowest state to 1785 ± 25 MeV, which is lower than the energy of the lowest spin 1/2 state. On the other hand, mixing between s^3 and $s^4\bar{s}$ configurations is very limited. This is also consistent with our previous predictions within the instanton-induced quark-antiquark pair creation model.

Our results with $q\bar{q}$ pair creation from both $^3P_0$ and $^1S_0$ are quite different from those with the commonly used $^3P_0$ model. With the nonrelativistic $^3P_0$ model, there would be no mixing between the three- and five-quark configurations for the low-lying $1/2^-$ and $3/2^-$ states; the $1/2^-$ state is the lowest state of negative parity for various hyperfine interactions except for the OGE hyperfine interaction. In our model, the $3/2^-$ state has large mixing between the three- and five-quark configurations and is always the lowest state of negative parity. The decay patterns for a pure five-quark state and a mixing state of both three- and five-quark configurations are also expected to be different.

At the present time, the experimental data for the Ω resonance spectrum is very poor. No Ω states with negative parity have been observed yet, so we cannot say which model is more appropriate. Recently, the BESII Collaboration at the Beijing Electron Positron Collider (BEPC) reported an interesting result that $\psi(2S) \to \Omega\bar{\Omega}$ was observed with a branch fraction of $(5 \pm 2) \times 10^{-5}$ [22]. Now with the upgraded BEPC, i.e., BEPCII, the BESIII Collaboration [23] is going to record billions of $\psi(2S)$ events, which is two orders of magnitude higher than what the BESII experiment recorded. We hope the BESIII experiment [23] will provide us more information through the $\psi(2S) \to \bar{\Omega}\Omega^*$ reaction.

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