Extracting baryon-antibaryon strong-interaction potentials from $p\bar{\Lambda}$ femtoscopic correlation functions

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The STAR experiment has measured $p\Lambda$, $\bar{p}\bar{\Lambda}$, $\bar{p}\Lambda$, and $p\bar{\Lambda}$ femtoscopic correlation functions in central Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The system size extracted for $p\Lambda$ and $\bar{p}\bar{\Lambda}$ is consistent with model expectations and results for other pair types, while for $p\bar{\Lambda}$ and $\bar{p}\Lambda$ it is not consistent with the other two and significantly lower. In addition an attempt was made to extract the unknown parameters of the strong interaction potential for this baryon-antibaryon ($B\bar{B}$) pair. In this work we reanalyze the STAR data, taking into account residual femtoscopic correlations from heavier $B\bar{B}$ pairs. We obtain different estimates for the system size, consistent with the results for $p\Lambda$ and $\bar{p}\bar{\Lambda}$ pairs and with model expectations. We give these estimates for the strong interaction potential parameters for $p\bar{\Lambda}$ and show that similar constraints can be given for parameters for other, heavier $B\bar{B}$ pairs.

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I. INTRODUCTION

Strong interaction in a two-baryon system is one of the fundamental problems in QCD [1,2]. Such processes are measured in dedicated experiments [3–5] and a significant body of data exists for baryon-baryon (BB) interactions [6]. Baryon-antibaryon ($B\bar{B}$) interaction includes a contribution from matter-antimatter annihilation. This process for $p\bar{p}$ was studied in great detail theoretically [7–10] and is measured with good precision [6]. However, no measurement exist for any $B\bar{B}$ system other than $p\bar{p}$, $p\bar{n}$, and $\bar{p}d$. There is also little theoretical guidance on what to expect for $B\bar{B}$ interaction for other baryon types. The standard hadronic rescattering code used in heavy-ion collision modeling, UrQMD [11], assumes that any $B\bar{B}$ interaction has the same parameters as the $p\bar{p}$, expressed either as a function of relative momentum or $\sqrt{s}$ of the pair.

Correlations for $p\bar{\Lambda}$ system have been studied in heavy-ion collisions [12]. The STAR experiment has measured $p\bar{\Lambda}$ femtoscopic correlation [13] in Au + Au collisions at $\sqrt{s_{NN}} =$ 200 GeV. In that work a method was proposed to determine the parameters of the strong interaction potential for $B\bar{B}$ pairs, using such correlations [14]. An estimate for the real and imaginary parts of the scattering length f_0 was given, showing a significant imaginary component, reflecting $B\bar{B}$ annihilation in this channel. At the same time femtoscopic system size (radius) was extracted. Surprisingly it was 50% lower than the one for regular BB pairs at similar pair transverse mass m_T. It was also inconsistent with hydrodynamic model predictions, which give approximate scaling of the radii with $1/\sqrt{m_T}$. This scaling is in agreement with all other femtoscopic measurements performed at Relativistic Heavy Ion Collider (RHIC), for meson and baryon pairs. Seen in this light, the validity of the $p\bar{\Lambda}$ analysis should be reconsidered if any significant, previously unaccounted for effects contributing to such functions are identified.

The issue of the residual correlations (RC) in femtoscopic correlations of BB pairs is mentioned in Ref. [13], but the work explicitly states that it is not addressed and acknowledges this fact as a weak aspect of the analysis method. In this work we show that proper treatment of RC is of central importance for any BB measurement, but in particular in the $B\bar{B}$ analysis. On the example of the STAR data we show how the extracted radius and scattering length change when RC are properly taken into account. We reanalyze the STAR data with the formalism which includes the RC contribution. We test whether the extracted radius is then compatible with other measurements and model expectations. In the process we make assumptions on the strong interaction parameters for several $B\bar{B}$ pairs and show if the extracted values are sensitive to those assumptions. As a result we put constraints on the $B\bar{B}$ strong interaction parameters, particularly on the imaginary part of the scattering length, which parametrizes the $B\bar{B}$ annihilation process at low relative momentum.

The paper is organized as follows. In Sec. II we describe the femtoscopic formalism, including the RC treatment. In Sec. III we discuss various theoretical assumptions needed for the reanalysis of the data and define four reasonable parameter sets for the theoretical description of the $B\bar{B}$ interaction. In Sec. IV we examine the STAR data from Ref. [13] and show how they should be reanalyzed in the frame of the formalism taking into account the RC. In Sec. V we apply the formalism to the STAR data and discuss the results. In Sec. VI we provide the conclusions and give recommendations for future measurements.

II. FEMTOSCOPIC FORMALISM

The femtoscopic correlation function is defined as a ratio of the conditional probability to observe two particles together, divided by the product of probabilities to observe each of them separately. Experimentally it is measured by dividing the distribution of relative momentum of pairs of particles detected in the same collision (event) by an equivalent distribution for pairs where each particle is taken from a different collision.

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This is the procedure used by STAR; details are given in Ref. [13]. The femtoscopy technique focuses on the mutual two-particle interaction. It can come from wave-function (anti-)symmetrization for pairs of identical particles, the measurement in this case is sometimes referred to as "HBT correlations." Another source is the final-state interaction (FSI), that is Coulomb or strong. In this work the Coulomb FSI is only present for $p\bar{p}$ pairs; all others are correlated due to the strong FSI only. The interaction for $p\bar{p}$ system is measured in detail and well described theoretically; we use existing calculations for this system and do not vary any of its parameters in the fits. For details please see Ref. [15]. For all other pairs the strong FSI is the only source of femtoscopic correlation. Below we describe the formalism for the strong interaction only, as it is the focus of this work.

In femtoscopy an assumption is made that the FSI of the pairs of particles is independent from their production. The two-particle correlation can then be written as [14]

$$C(\boldsymbol{k^*}) = \frac{\int S(\mathbf{r}^*,\boldsymbol{k^*})\left|\Psi^{S(+)}_{-k^*}(\mathbf{r}^*,\boldsymbol{k^*})\right|^2}{\int S(\mathbf{r}^*,\boldsymbol{k^*})}, \tag{1}$$

where $\mathbf{r}^* = \mathbf{x}_1 - \mathbf{x}_2$ is a relative space-time separation of the two particles at the moment of their creation. $\boldsymbol{k}^*$ is the momentum of the first particle in the pair rest frame (PRF), so it is half of the pair relative momentum in this frame. S is the source emission function and can be interpreted as a probability to emit a given particle pair from a given set of emission points with given momenta. The source of the correlation is the Bethe-Salpeter amplitude $\Psi^{S(+)}_{-k^*}$, which in this case corresponds to the solution of the quantum scattering problem taken with the inverse time direction. When particles interact with the strong FSI only it can be written as

$$\Psi^{S(+)}_{-k^*}(\mathbf{r}^*,\boldsymbol{k^*}) = e^{-i\boldsymbol{k^*}\cdot\boldsymbol{r^*}} + f^S(k^*)\frac{e^{ik^*r^*}}{r^*}, \tag{2}$$

where f^S is the S-wave strong interaction amplitude. In the effective range approximation it can be expressed as

$$f^S(k^*) = \left(\frac{1}{f_0} + \frac{1}{2}d_0 k^{*2} - ik^*\right)^{-1}, \tag{3}$$

where f_0 is the scattering length and d_0 is the effective radius of the strong interaction. These are the essential parameters of the strong interaction, which can be extracted from the fit to the experimental correlation function. Both are complex numbers; the imaginary part of f_0 is especially interesting as it corresponds to the annihilation process. In the relative momentum range where the effective range approximation is valid they are also directly related to the interaction cross section: $\sigma = 4\pi|f^S|^2$. Therefore, knowledge of them is of fundamental importance.

For one-dimensional correlation function the source function S has one parameter. Usually a spherically symmetric source in PRF with size r_0 is taken:

$$S(\boldsymbol{r^*}) \approx \exp\left(-\frac{r^{*2}}{4r_0^2}\right), \tag{4}$$

which gives the final form of the analytical correlation function depending on the strong FSI only [13,14]:

$$C(\boldsymbol{k^*}) = 1 + \sum_S \rho_S\left[\frac{1}{2}\left|\frac{f^S(k^*)}{r_0}\right|^2\left(1 - \frac{d_0^S}{2\sqrt{\pi}r_0}\right) + \frac{2\Re f^S(\boldsymbol{k^*})}{\sqrt{\pi}r_0}F_1(Qr_0) - \frac{\Im f^S(\boldsymbol{k^*})}{r_0}F_2(Qr_0)\right], \tag{5}$$

where $Q = 2k^*$, $F_1(z) = \int_0^z dx e^{x^2-z^2}/z$, and $F_2 = (1 - e^{-z^2})/z$. Summation is done over possible pair spin orientations, with ρ_S being the corresponding pair spin fractions. Since the data considered in this work are always for unpolarized pairs, the spin dependence of the correlation is neglected. In this formula the dependence of the correlation function on the real and imaginary part of the scattering length f_0 is expressed directly. For pairs where only the strong FSI contributes to the correlation, such as $p\Lambda$ and $p\bar{\Lambda}$, this formula can be fitted directly to extract the source size r_0 as well as the scattering length and effective radius. In realistic scenarios it is rarely possible to independently determine all parameters. In particular, in the case of the STAR data discussed here, the d_0 was fixed at zero and only the remaining three were fitted.

A. Residual correlations

In experiments conducted at colliders such as STAR experiment at RHIC, all particles propagate to the detector radially from the interaction point located in the center of the detector. A baryon coming from a weak decay often travels in a direction very similar to the parent baryon. The particle's trajectory does not point precisely to the interaction point, but this difference (called the distance of the closest approach, or DCA) is often comparable to the spatial resolution of the experiment. As a result a significant number of particles identified as protons in STAR are not primary and come from the decay of heavier baryons. The same mechanism applies to Λ baryons. In particular protons can come from a decay of Λ and Σ^+ baryons, while Λ baryons can come from decays of Σ^0 or Ξ^0. The STAR experiment has applied the DCA cut to reduce the number of such secondaries and has estimated its effectiveness based on the Monte Carlo simulation of the detector response. The fraction of true primary pairs, as well as a fraction of all other parent particle pair combinations, is taken from Ref. [13] and given in Table I. In addition to the effect mentioned above it is also possible that a primary proton is randomly associated to a pion and reconstructed as a fake Λ baryon. For that reason a pair of two protons also appears in Table I.

The strong FSI affects the behavior of the two particles in the pair just after their production, on a time scale of fm/c. For particles coming from a weak decay, which occurs on time scales of 10^{-10} s, the FSI applies to the parent pair, not the daughter. However, it is the daughters that are measured in the detector. For such a scenario Eq. (1) cannot be used directly. In this case Ψ must be taken for the parent pair and calculated for k^* and r^* between the parent particles. Then one or both of the parent particles must decay and a

TABLE I. List of possible parent pairs for the $p\Lambda$ (and $p\bar{\Lambda}$) system, with their relative contribution to the STAR sample [13] and the decay momenta values.

Pair	Fraction	Decay momenta (MeV/c)
$p\Lambda$	15%	0
$\Lambda\Lambda$	10%	101
$\Sigma^+\Lambda$	3%	189
$p\Sigma^0$	11%	74
$\Lambda\Sigma^0$	7%	101, 74
$\Sigma^+\Sigma^0$	2%	189, 74
$p\Xi^0$	9%	135
$\Lambda\Xi^0$	5%	101, 135
$\Sigma^+\Xi^0$	2%	189, 135
pp	7%	101

new k^* must be calculated for the daughter pair. This one is measured in the detector; the correlation is measured as a function of this relative momentum. Such scenario is called "residual correlations" (RC) [16,17]. Obviously the random nature of the weak decay will dilute the original correlation. However, if the decay momentum is comparable to the width of the correlation effect in relative momentum, some correlation might be preserved for the daughter particles. The RC are important if three conditions are met simultaneously: (a) the original correlation for parent particles is large, (b) the fraction of daughter pairs coming from a particular parent pair is significant, and (c) the decay momentum (or momenta) are comparable to the expected correlation width in k^*. For $p\bar{\Lambda}$ pairs all three conditions are met. The strong FSI for baryon-antibaryon pairs is dominated by annihilation, which appears in Eq. (5) as $\Im f^S$. It causes a negative correlation (anticorrelation) [17], which is wide in k^*, even on the order of 300 MeV/c. Decay momenta for all residual pairs listed in Table I are of that order or smaller. Comparing contributions to the sample from all pairs one can see that all listed are of the same order as primary $p\bar{\Lambda}$ pairs, which constitute only 15% of the sample. As for the strength of the correlation, it is in principle unknown for all pairs, except $p\bar{p}$. Estimating its strength is one of the goals of this work. However, it is often assumed that at least the annihilation cross section for $B\bar{B}$ pairs is very similar for all pairs, comparable to $p\bar{p}$ [11]. In that case it is certainly strong enough to induce RC, and contributions from all pairs listed in Table I must be considered in the analysis of the $p\bar{\Lambda}$ correlations.

The RC can be calculated for any combination of parent and daughter pairs. The correlation is expressed as a function of the relative momentum of the daughter pair; in our case it is $k^*_{p\Lambda}$. However, Eq. (1) is then used for the parent pair (let us call it XY) and gives the correlation as a function of k^*_{XY}. Baryon X is a proton or decays into a proton, and baryon Y is a Λ or decays into a Λ. The daughter momenta will differ from the parents' by the decay momentum, listed in Table I. The direction of the decay momentum is random in the parents' rest frame, and it is independent from the direction of the k^* of the pair. Therefore $k^*_{p\Lambda}$ will differ, in a random way for each pair, from k^*_{XY}. The difference is limited by the value of the decay momentum and is a nontrivial consequence of the decay kinematics.

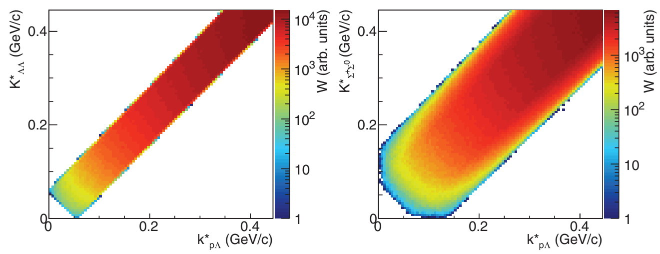

FIG. 1. (Color online) The unnormalized transformation matrix W for $\Lambda\Lambda$ (left) and $\Sigma^+\Sigma^0$ (right) pairs decaying into $p\Lambda$ pairs, as a function of relative momentum of both pair types.

One can determine what is the probability that a parent particle pair with a given k^*_{XY} will decay into a daughter pair with a given $k^*_{p\Lambda}$. Let us call such a probability distribution $W(k^*_{XY},k^*_{p\Lambda})$. In this work we have calculated it for all pairs listed in Table I. We have used the Therminator model [18,19], with parameters describing central Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. All the pairs of type XY in a given event were found and their relative momentum k^*_{XY} was calculated. Then both baryons X and Y were allowed to decay and $k^*_{p\Lambda}$ was calculated for the daughters. The pair was then inserted in a two-dimensional histogram. As a result an unnormalized probability distribution W was obtained for each pair type. Figure 1 shows two examples of this function, one for a pair where only one particle decays and the other for a pair where both particles decay. In the first case the function has a characteristic rectangular shape at low relative momentum [16]. It touches both axes at the value roughly equal to half of the decay momentum. The vertical width of the function is roughly equal to the decay momentum, as discussed above. In the second case the shape at low momentum is not as sharp, and the width is equal to the sum of decay momenta. W depends only on decay kinematics, so it is the same for BB and the corresponding $B\bar{B}$ pair.

Having defined W one can write the formula for the RC for any type of the parent pair $X\bar{Y}$, contributing to the $p\bar{\Lambda}$ correlation function:

$$C^{X\bar{Y}\to p\bar{\Lambda}}(k^*_{p\bar{\Lambda}}) = \frac{\int C^{X\bar{Y}}(k^*_{X\bar{Y}})W(k^*_{X\bar{Y}},k^*_{p\bar{\Lambda}})dk^*_{X\bar{Y}}}{\int W(k^*_{X\bar{Y}},k^*_{p\bar{\Lambda}})dk^*_{X\bar{Y}}}. \quad (6)$$

Examples of correlation functions transformed in this way are shown in Fig. 2. The $p\bar{\Lambda}$ function for a given source size, calculated according to Eq. (5), is given for comparison. It has positive correlation at very low k^* coming from the positive real part of the scattering length f_0 and a wide anticorrelation coming from the positive imaginary part of f_0. This anticorrelation is wide, extending beyond 0.4 GeV/c, so its width is larger than any combination of decay momenta given in Table I. The residual correlations are calculated for the same source size and radius parameters, so in their respective k^* variables they look identical to $C^{p\bar{\Lambda}}(k^*_{p\bar{\Lambda}})$. After the transformation given by Eq. (6) the correlation is diluted at low k^*. However, at higher values the shape of the function changes very little and is almost the same for the parent and

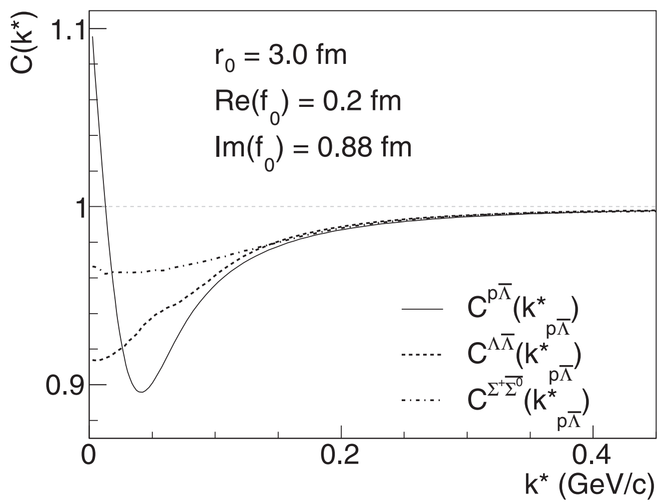

FIG. 2. Theoretical correlation function for a given source size for $p\bar{\Lambda}$ and two examples of residual correlation functions for $\Lambda\bar{\Lambda}$ and $\Sigma^+\bar{\Sigma}^0$ pairs.

residual correlation. The spike at $k^* = 0$ is transformed with the matrix in Fig. 1 to a slight bump in $k^*_{p\bar{\Lambda}}$ where W touches the x axis, that is, around 50 MeV/c, half of the decay momentum of Λ into proton. The same function is diluted twice as strong at low k^* for the pair where both particles decay ($\Sigma^+\bar{\Sigma}^0$). This difference persists up to around 100 MeV/c; above this value both functions are similar to each other and to the original correlation.

In terms of the physics picture the contribution of the $p\bar{p}$ correlation to the $p\bar{\Lambda}$ one is not RC; instead it comes from fake association of primary proton to a Λ particle. However, the formalism to deal with such situation is exactly the same as in the case of RC, and Eq. (6) can be used. The difference is that the $p\bar{p}$ correlation function has a Coulomb FSI component in addition to the strong FSI, which must be taken into account when calculating $C^{p\bar{p}}$. The W matrix for $p\Lambda$ to pp pair transformation can be used.

Once each of the RC components is determined, the complete correlation function for the $p\bar{\Lambda}$ system can be written:

$$C(k^*_{p\bar{\Lambda}}) = 1 + \lambda_{p\Lambda}(C^{p\bar{\Lambda}}(k^*_{p\bar{\Lambda}}) - 1) + \sum_{X\bar{Y}} \lambda_{XY}(C^{X\bar{Y}}(k^*_{p\bar{\Lambda}}) - 1), \quad (7)$$

where the λ values are equivalent to the pair fractions given in Table I. It is an additional factor that decreases the correlation for the RC; however, for some pairs it is almost as large as λ for true $p\bar{\Lambda}$ pairs. Equation (7) is the final formula that can be fitted directly to experimental data. In principle each $C^{X\bar{Y}}$ depends on four independent parameters: the source size r_0, real and imaginary parts of f_0, and the value of d_0, giving 37 independent parameters (f_0 and d_0 for the $p\bar{p}$ pair is known). Some assumptions are obviously needed to reduce this number; we propose several options in Sec. III.

III. THEORETICAL SCENARIOS

Following the procedure employed by STAR in Ref. [13] we put the effective range $d_0 = 0$ fm for all calculations. Radius for the various systems in central Au + Au collisions at RHIC energies is expected to follow hydrodynamic predictions, which give $r_0 \sim 1/\sqrt{\langle m_T \rangle}$, where m_T is the transverse mass of the pair. For baryons m_T is large, and the decrease is not expected to be steep (see Fig. 5 in Ref. [13] for illustration). $\langle m_T \rangle$ for a given pair depends on the momentum spectra of particles taken for this analysis, which is not specified in Ref. [13]. We expect that $\langle m_T \rangle$ for the pairs considered here will be within 20% of each other, giving little variation of the scaling factor. Therefore, we make a simplifying assumption that system size r_0 for each pair is the same.

With these assumptions 18 components of f_0 remain for the nine pairs. Little theoretical guidance is given for those values. An approach adopted in Ref. [11] equates all annihilation cross sections for the $B\bar{B}$ pairs and assumes they are equal to the one for $p\bar{p}$. In Ref. [7] the value of $\Im f_0 = 0.88 \pm 0.09$ fm is given. This value is used to calculate the $p\bar{p}$ correlation functions. No such assumption is made for $\Re f_0$, which can vary significantly between various $B\bar{B}$ pairs. Therefore, we make two assumptions. $\Im f_0$ is assumed to be the same for all $B\bar{B}$ pairs, but it is not fixed to the $p\bar{p}$ value—it is treated as free in the fit. Similarly $\Re f_0$ is also assumed to be the same for all pairs and is free in the fit.

In Ref. [11] an alternative scenario for annihilation cross sections is given; namely, that they are the same as in $p\bar{p}$, but at the same $\sqrt{s}$ of the pair, not relative momentum. In UrQMD these assumptions differ little; the majority of hadronic rescatterings happen at large relative momentum, where the difference between cross sections scaled with k^* and $\sqrt{s}$ is small. However, in the case of femtoscopic correlations, which by definition are concentrated at low relative momentum, the two scenarios differ strongly. For example, a $\Sigma^+\Xi^0$ pair at $k^* = 10$ MeV/c taken at the same $\sqrt{s}$ corresponds to a pp pair at $k^* = 831.6$ MeV/c. This assumption would then significantly decrease the correlation for higher mass pairs, including $p\bar{\Lambda}$. For the unknown baryon-baryon interactions in UrQMD an additive quark model (AQM) is used to provide the cross sections [11]. The predicted values decrease with the increase of the number of strange quarks in the pair. We use this as another plausible scenario for the scaling of $B\bar{B}$ annihilation cross section, where the real and imaginary parts of f_0 for a given pair is scaled with a factor deduced from Table V in Ref. [11]. We notice immediately that replacing an up or down quark in one of the baryons with a strange quark results in the increase of the pair mass. Therefore, in both cases the proposed scalings result in the same ordering of pairs listed in Table I: from the lightest pp pair with no strange quarks to the heaviest $\Lambda^+\Xi^0$ and $\Sigma^+\Xi^0$ with three strange quarks each. The cross sections for those pairs should therefore change smoothly from the maximum value for the pp pair to the smallest value for $\Sigma^+\Xi^0$. Any nonmonotonic jump of the cross section with increasing pair mass or strangeness content will signal an internal inconsistency of the scenario. The exact rate of the decrease of the cross sections in the AQM scenario is smaller than in the $\sqrt{s}$ scenario, but their general behavior is similar. The AQM can then be treated as an intermediate scenario between k^* and $\sqrt{s}$ cases. We test if the accuracy of STAR data allows us to distinguish between them. We consider AQM and $\sqrt{s}$ cases together as one scenario and test whether it is consistent with data.

The next scenario comes from the fact that we have just shown that both $p\bar{p}$ and $\Lambda\bar{\Lambda}$ RC contribute to the $p\bar{\Lambda}$ correlation function. One can then ask if it is possible that the observed correlation is explained by annihilation of particle-antiparticle pairs only, not all $B\bar{B}$ pairs. In such scenario the imaginary part of the scattering length should be put to zero for all pairs except the ones in which the two particles have exactly opposite quark content.

The last scenario is the repetition of the STAR procedure, where no RC is included and correlation is present for $p\bar{\Lambda}$ pairs only.

In each of these four scenarios, all $C^{X\bar{Y}}$ functions can be calculated from Eq. (5). The last function remaining to be calculated is then $C^{p\bar{p}}$. A dedicated procedure is used. First the relative momenta distributions are taken from Therminator, from collisions simulated with parameters corresponding to central Au + Au collision at $\sqrt{s_{NN}} = 200$ GeV. Then a source size is assumed, equal to the one used for all other pairs. This allows the generation of r^* for each pair, according to the probability distribution from Eq. (4). This gives pairs, each with its k^* and r^*, which enables the calculation of Ψ. It is performed with a dedicated code from Lednicky [15], where the known $p\bar{p}$ interaction parameters are used. The resulting correlation function is then calculated according to Eq. (1), corresponding to the value of the source size r_0. As mentioned earlier, the W functions are also calculated from Therminator, for all parent pair types.

IV. ANALYSIS OF STAR DATA

The STAR data on $p\bar{\Lambda}$ correlation function [13] has been corrected for several effects, most of them experimental in nature. Two of those corrections must be re-examined for this analysis. The correlation was normalized above 0.35 GeV/c [13]. As can be seen in Fig. 2 the femtoscopic correlation is small but non-negligible in this region. However, the upper range for the normalization is not given. The number of pairs increases with k^*, so if the upper normalization range is large, pairs with negligible correlation will dominate the normalization factor. We assume this is the case, which means that the experimental correlation is properly normalized.

The data was also corrected for "purity," that is, the fraction of true $p\bar{\Lambda}$ pairs, given in Table I. The procedure used by STAR is correct only if all other pairs are not correlated. In Eq. (7) it would correspond to the scenario where all $C^{X\bar{Y}}$ are at 1.0 in the full k^* range. We have just shown that this assumption is explicitly violated by the RC effect, which is expected to be significant for $p\bar{\Lambda}$ correlations measured by STAR. Therefore, the experimental correlation function analyzed later is uncorrected for purity, with the purity factor equal to 0.15, taken from Table I, so that a fit according to Eq. (7) can be properly applied.

The fitting range was set to 0.45 GeV/c, the maximum range for which experimental data are available.

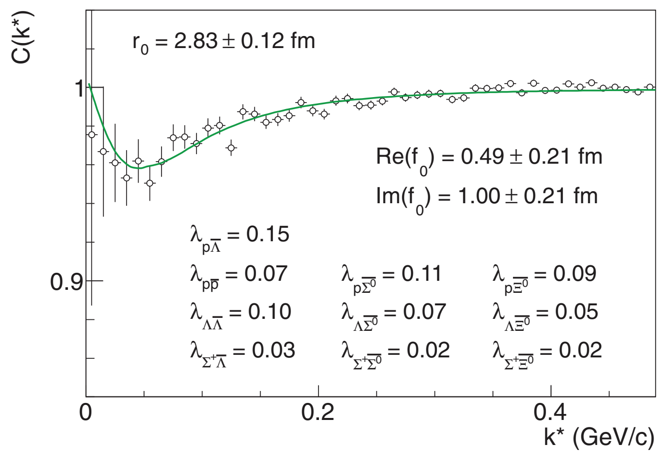

FIG. 3. (Color online) Fit to the STAR $p\bar{\Lambda}$ correlation function with Eq. (7), all residual correlation components included. χ^2/n dof = 1.5.

V. FITTING THE EXPERIMENTAL CORRELATION FUNCTION

Formula (7) is fitted to the STAR experimental data, with the theoretical assumptions mentioned above. Standard χ^2 minimization procedure is used. The result of the fit is shown in Fig. 3; the χ^2 values are on the order of 1.5 per dof and are similar for all fits discussed below. The fit gives the value of the source size $r_0 = 2.83 \pm 0.12$ fm, and the scattering length $f_0 = 0.49 \pm 0.21 + i(1.00 \pm 0.21)$ fm. The value of r_0 is significantly larger than given in Ref. [13], indicating that the RC plays a critical role in the extraction of physical quantities. The value extracted here is in good agreement with the values obtained for the $p\Lambda$ system. This consistency is naturally expected in practically all realistic models of heavy-ion collisions, while the previous STAR result was violating this consistency without providing any viable explanation. It is also consistent with expectation from hydrodynamical models, which are in good agreement with all other femtoscopic measurements at RHIC. Taking all those arguments into account we claim that the result presented in this work is the correct one, and that the result for $p\bar{\Lambda}$ from Ref. [13] should be considered obsolete.

The extracted imaginary part of the scattering length is significant and in agreement with the value given for the $p\bar{p}$ system. This means that the assumption that the annihilation process for any $B\bar{B}$ system is similar to that process for $p\bar{p}$, taken at the same relative momentum, is consistent with data.

In Fig. 4 all the residual correlation components of the fit are shown. The absolute value of the correlation effect $1 - C$ is plotted, and the logarithmic scale is needed to distinguish the small contributions. No single component is dominating the function; all 10 components are needed to describe the correlation. The largest ones are, as expected, the ones which have large pair fractions and small decay momenta, that is, $p\bar{\Lambda}$, $p\bar{\Sigma}^0$, $\Lambda\bar{\Lambda}$, and $p\bar{\Xi}^0$. The systems where both particles decay and the systems where the fraction is small contribute less. All the RC contributions are relevant through the whole k^* range.

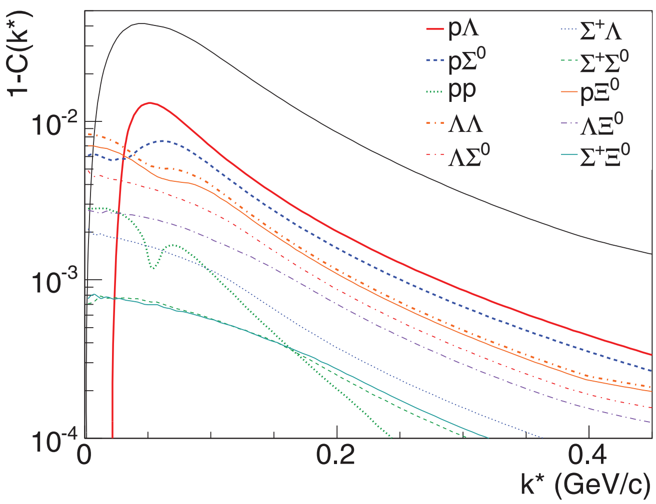

FIG. 4. (Color online) Comparison of all residual correlation components for the $p\bar{\Lambda}$ correlation function (thin black line). For better illustration unity minus the correlation effect is plotted in logarithmic scale.

In order to validate the procedure and the important result, several scenarios, described in Sec. III, have been tested. In Fig. 5 the fit was performed where no residual correlations were included. This is equivalent to the STAR procedure. The result from Ref. [13] is reproduced, and the resulting radius is small. $\Re f_0$ changes sign with respect to the default case, but interestingly $\Im f_0$ is consistent with the full RC fit.

The next scenario assumes annihilation for particle-antiparticle pairs only. By testing it we check if the annihilation is really necessary for all $B\bar{B}$ pairs, or if it is enough that it happens only with baryons having exactly the opposite quark content. A fit is performed, where only $p\bar{p}$, $\Lambda\bar{\Lambda}$, and $\Lambda\bar{\Sigma}^0$ RC is included, while for all other $B\bar{B}$ pairs (including $p\bar{\Lambda}$) there is no correlation. Results similar to the previous test are obtained—the radius is 1.5 ± 0.1 fm, while $\Im f_0$ for the strange pairs is twice the $p\bar{p}$ value. Even though the results cannot be rejected based on the quality of the fit, the resulting radius parameter is so small that it could be considered unphysical. Both scenarios are therefore unlikely. In other words the analysis gives a strong argument for the statement that the annihilation happens between all $B\bar{B}$ pairs, not just the ones with exactly opposite quark content and that this effect must be taken into account, via the RC formalism in any analysis of $B\bar{B}$ femtoscopic correlations.

In the last scenario, following the idea from Ref. [11] it was proposed that the annihilation cross section for $B\bar{B}$ pairs is the same for all pairs, but taken at the same $\sqrt{s}$ instead of the relative momentum. In femtoscopy such scaling would be reflected in Eq. (2) by taking f^S at a different k^*. In this work we treat the imaginary and real parts of f_0 for the $p\bar{\Lambda}$ system as fit parameters and scale the f^S for all other pairs, by taking the same f_0 parameters, but calculating f^S at

$$k^* = \left(\frac{s^2 + m_p^4 + m_\Lambda^4 - 2sm_p^2 - 2sm_\Lambda^2 - 2m_p^2 m_\Lambda^2}{4s} \right)^{1/2} \quad (8)$$

according to Eq. (3). s is the square of the total energy in PRF for the pair $X\bar{Y}$. f^S is a function rapidly decreasing with k^*. By taking s for the baryon pair, where one or both baryons have a mass higher than the proton or the Λ, one gets from Eq. (8) k^* higher than for the original pair, so f^S will be smaller. In Fig. 6 the result of such calculation is shown for a pair with smallest and largest mass difference to the $p\bar{\Lambda}$ pair. The strength of the correlation is visibly decreased. However, the shape is only slightly affected. In fact the functions can be described by Eq. (5), with altered values of f_0. The $\Re f_0$ is scaled to approximately 20% of the original value, whereas $\Im f_0$ is scaled to 60% (32%) of the original value for the pair with smallest (largest) mass difference, that is, $p\bar{\Sigma}^0$ ($\Sigma^+\bar{\Xi}^0$). These scaling factors provide the needed constraints on the fit parameters, and the fit can be performed as in the previous cases.

Figure 7 shows the result of the fit, with the scaling of f^S with $\sqrt{s}$ of the pair. The resulting source size is comparable to the default case. $\Im f_0$ is significantly larger than for the

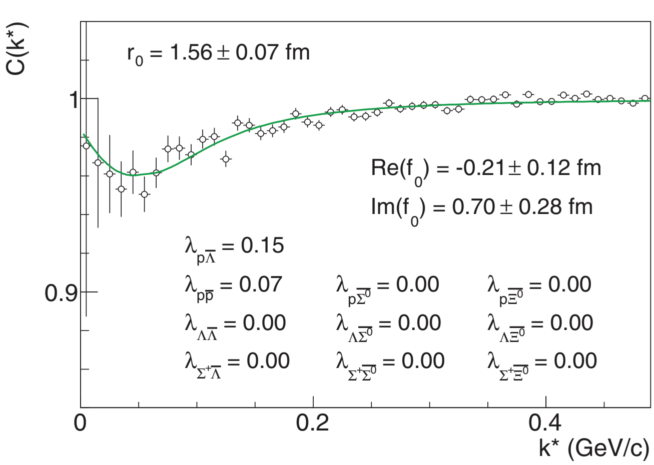

FIG. 5. (Color online) Fit to the STAR $p\bar{\Lambda}$ correlation function with Eq. (7), no residual correlation components included. χ^2/n dof $= 1.5$.

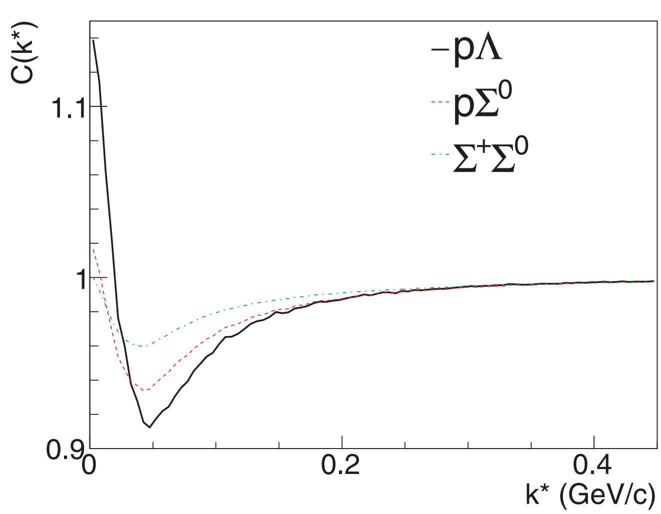

FIG. 6. (Color online) Calculation of correlation function, with f^S taken the same as the f^S for $p\bar{\Lambda}$ pair at the corresponding $\sqrt{s}$.

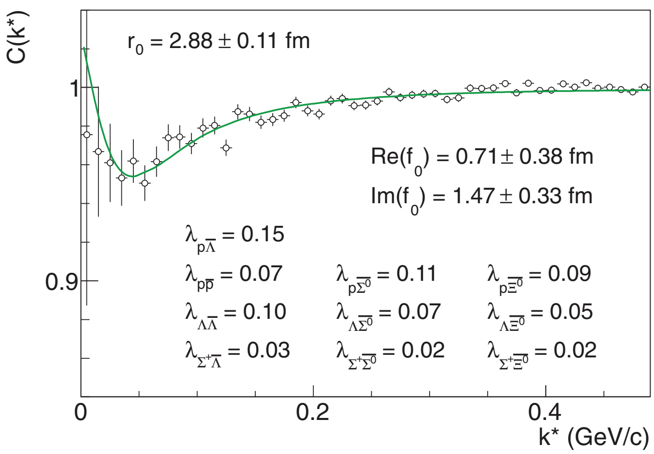

FIG. 7. (Color online) Fit the the STAR $p\bar{\Lambda}$ correlation function with Eq. (7), all residual correlations included, and strength of the interaction scaled according to the $\sqrt{s}$ of the pair (see text for details). χ^2/n dof $= 1.5$.

default fit and larger than the measured $p\bar{p}$ value. While this scenario is not ruled out by the data, it is internally inconsistent. It would mean that by moving from the lightest $p\bar{p}$ pair with no strangeness content to the heavier $p\bar{\Lambda}$ pair, the cross section increases sharply, but if one moves to an even heavier pair, it drops instead of increasing further. Such nonmonotonic behavior explicitly violates the smooth scaling of cross sections with $\sqrt{s}$ assumed in this scenario. If one takes the $p\bar{p}$ f_0 as the starting point, instead of $p\bar{\Lambda}$ (which would be a more literate implementation of the scenario proposed in Ref. [11]), then f_0 cannot be a free parameter. A fit gives $r_0 = 2.23 \pm 0.09$ fm, which is lower than the expected value. Such scenario cannot be ruled out, but is less likely, due to the disagreement of this value with r_0 for $p\Lambda$ pairs. In another version of the scenario cross sections are scaled with predictions from AQM, giving a similar value of the radius $(2.83 \pm 0.12$ fm) and $f_0 = 0.51 \pm 0.22 + i(1.04 \pm 0.22)$ fm. This also gives a nonmonotonic behavior of cross section with the increase of the strange quark content; however, the results are consistent with data within the current precision. So, as expected, the AQM scenario is an intermediate case, lying between k^* and $\sqrt{s}$ scalings.

The results discussed in this work rely on the comparison between results for BB and $B\bar{B}$ pairs. We show that RC are important for $B\bar{B}$ pairs and significantly influence the extracted physics results. It is therefore natural to ask whether RC have a similar effect for BB pairs. There are at least two reasons to expect that the influence of RC on BB pairs, and in particular $p\Lambda$ correlations, will be small, and therefore the comparisons drawn in this work are valid. In Sec. II A three conditions are given that need to be met simultaneously for RC to be important: Condition (a) requires that the interaction for parent particles is strong. For BB pairs only the real part of the scattering length is finite, and it is found to vary strongly between pairs. AQM [11] predicts that it decreases with the strange quark content of the pair. This is in contrast to $B\bar{B}$ pairs, where the $\Im f_0$ potentially varies less between pairs and may even be similar for all $B\bar{B}$ pairs, as this work suggests. In Eq. (5) the width of the correlation effect coming from $\Re f_0$ is limited, approximately 50 MeV$/c$ for values of r_0 expected in heavy-ion collision. Therefore, in contrast to the $B\bar{B}$ case where a wide effect coming from $\Im f_0$ gives significant contribution at large q, the expected width of the BB correlation effect is small with respect to the decay momenta given in Table I. For illustration, see Ref. [20]. This directly violates condition (c) given in Sec. II A. Therefore it is reasonable to assume that the RC will have a smaller influence on the $p\Lambda$ results. The quantitative estimate of this influence is beyond the scope of this paper and is planned for future works.

A. Systematic uncertainty discussion

All the values given above were obtained with certain assumptions, spelled out above, both related to the STAR data treatment as well as the methodology itself and the unknown strong interaction parameters. By varying those assumptions in a reasonable range one can estimate the systematic uncertainty on the extracted parameters coming from the application of the RC method and the assumptions made.

Restricting the fitting range to 0.35 GeV$/c$ (beginning of the normalization range) gives 5% variation in radius, while $\Im f_0$ decreases to 0.6 ± 0.2 fm and $\Re f_0$ is positive but consistent with zero. Performing the fit separately for $p\bar{\Lambda}$ and $\bar{p}\Lambda$ pairs gives r_0 statistically consistent with the default fit. $\Im f_0$ varies by up to 20%, and $\Re f_0$ by up to 50%. That is expected: $\Re f_0$ affects the function mostly at low k^*, where data are less precise, whereas $\Im f_0$ produces the wide anticorrelation, which is better constrained by the data. With the statistical power of the STAR data we were unable to test the influence of the d_0 parameter variation or independent variation of f_0 parameters for heavier $B\bar{B}$ pair types. In conclusion the source size r_0 is well constrained and comparable to r_0 measured by STAR [13] for $p\Lambda$ and $\bar{p}\bar{\Lambda}$ within the statistical and systematic uncertainty of this work. $\Im f_0$ is determined to be finite and positive, consistent with the hypothesis that its value for all $B\bar{B}$ pairs considered is similar to the value for $p\bar{p}$. The systematic uncertainty of the method is at least 20%. $\Re f_0$ is consistent with being finite and positive, although the systematic uncertainty of the method is at least 50%. There is also no theoretical expectation that $\Re f_0$ is similar for different $B\bar{B}$ pairs, so this measurement can be interpreted as "average effective" $\Re f_0$ for the considered $B\bar{B}$ pairs.

Certain other systematic uncertainties depend on the details of the experimental treatment. These include, among others, the variation of the normalization range, variation of the pair fractions, and the variation of the DCA cuts. Their estimation is beyond the scope of this work, as it requires direct access to experimental raw data and procedures.

VI. SUMMARY

We have presented the theoretical formalism for dealing with residual correlations in baryon-antibaryon femtoscopic correlations. We have shown that for realistic scenario of heavy-ion collision at $\sqrt{s_{NN}} = 200$ GeV such correlations are critical for the correct interpretation of data. The formalism has been applied to $p\bar{\Lambda}$ and $\bar{p}\Lambda$ femtoscopic correlations measured by STAR [13]. Different estimates for system size r_0 as well as real and imaginary parts of the scattering length f_0 have been obtained. This system size is consistent with results for $p\Lambda$ and $\bar{p}\bar{\Lambda}$ pairs and model expectations. Therefore, the puzzle of unexpectedly small $p\bar{\Lambda}$ system size reported by STAR in [13] is solved. In addition, more robust estimates for f_0 parameter are obtained, not only for the $p\bar{\Lambda}$ system, but also for a number of heavier $B\bar{B}$ pairs. A scenario where all $B\bar{B}$ pairs have similar annihilation cross section (expressed as a function of pair relative momentum) is judged to be most likely, as it gives the expected source size and is internally consistent. Other scenarios have been explored, but were judged to be less likely.

With this methodology it is possible to measure strong interaction potential for a number of $B\bar{B}$ pair types, including Λ and Ξ baryons. More precise data, differential in centrality and pair momentum and obtained for other pair types (e.g. $p\Xi^0$, $\Lambda\Lambda$, $\Lambda\Xi^0$), would help constrain these interesting, unknown quantities. In particular high statistics runs of Au + Au collisions at RHIC, as well as Pb–Pb collisions at the LHC, promise better quality data and give hope for more precise measurement in the near future.

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