Examination of directed flow as a signal for a phase transition in relativistic nuclear collisions

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The sign change of the slope of the directed flow of baryons has been predicted as a signal for a first order phase transition within fluid dynamical calculations. Recently, the directed flow of identified particles was measured by the STAR Collaboration in the beam energy scan program. In this article, we examine the collision energy dependence of directed flow v_1 in fluid dynamical model descriptions of heavy ion collisions for $\sqrt{s_{NN}} = 3$–20 GeV. The first step is to reproduce the existing predictions within pure fluid dynamical calculations. As a second step we investigate the influence of the order of the phase transition on the anisotropic flow within a state-of-the-art hybrid approach that describes other global observables reasonably well. We find that, in the hybrid approach, there seems to be no sensitivity of the directed flow on the equation of state and in particular on the existence of a first order phase transition. In addition, we explore more subtle sensitivities such as the Cooper-Frye transition criterion and discuss how momentum conservation and the definition of the event plane affects the results. At this point, none of our calculations matches qualitatively the behavior of the STAR data; the values of the slopes are always larger than in the data.

I. INTRODUCTION

The anisotropic flow of particles has been an interesting observable since data from the first heavy ion collisions became available at the Bevalac. The deflection of the produced particles in the reaction plane (defined as the plane between the impact parameter and the beam direction) can be quantified by the so-called directed flow, v_1. At very low beam energies of $E_{lab} < 1$ GeV per nucleon, the rotation of the system leads to a strong overall directed flow coefficient, which has been observed and understood within fluid dynamical calculations [1–3].

At very high beam energies, as they are achieved at the Large Hadron Collider (LHC) and the Relativistic Heavy Ion Collider (RHIC), the slope of the traditional directed flow is close to zero at midrapidity due to the almost perfect transparency of the colliding nuclei. The small negative slope of charged particles (mostly pions) at top RHIC energy can be explained within a fluid dynamical model and a slightly tilted initial state [4] as well as within a hadronic transport model [5]. Recently in Ref. [6] the possibility of rotation was discussed, due to the increasing initial angular momentum at very high (e.g., LHC) collision energies in peripheral collisions.

In the past three years more studies were focused on odd flow coefficients related to initial state fluctuations. The so-called rapidity-even v_1 or directed flow was defined to quantify the dipole moment generated by fluctuations in the initial transverse density profile. In the present study, we are solely interested in the traditional rapidity-odd directed flow that forms independently of initial fluctuations.

At intermediate colliding energies, studied at the beam energy scan program at RHIC, the future Facility for Antiproton and Ion Research (FAIR) and the former Alternating Gradient Synchrotron (AGS)-Super Proton Synchrotron (SPS) experiments, the systematic study of directed flow is thought to be more interesting. Within fluid dynamical calculations, it has been predicted that the slope of the directed flow of baryons will turn negative and then positive again as a function of energy if a first order phase transition is present. This means that more protons (most of the baryons at lower beam energies are protons) are emitted in a direction opposite to the spectators than aligned with them. This effect, called "antiflow" or "collapse of flow," has been attributed to a softening of the equation of state (EOS), during the expansion, due to a first order phase transition [7–10], leading to a tilt of the fireball in the reaction plane [8,9]. In these theoretical models the pressure gradient in the initial state was strongly reduced due to the transition. The shock compression and heating resulted in a highly compressed intermediate state, which reversed the direction of the directed (v_1) flow.

The corresponding measurements of the NA49 Collaboration [11] for the directed flow of protons had insufficient statistics to draw definite conclusions. Recently, the STAR Collaboration measured the predicted qualitative behavior of the slope of the net-proton directed flow as a function of beam energy which turns negative and then positive again [12] (antiflow). Since the early predictions were made with exclusively fluid dynamical models, which overpredicted all other flow components, the goal of our study is to understand the EOS dependence of directed flow within more modern transport approaches.

First, we validate the qualitative predictions within a pure fluid dynamical calculation and confirm that with a first order phase transition the proton v_1 slope has the expected qualitative behavior, including a dip below zero. As in the previous studies, this sign change happens at much lower beam energies than what STAR measured. In Sec. III we explore the influence of the freeze-out criterion on this result (isochronous compared to isoenergy density) and show the relation to the time evolution of the directed flow. Then we

perform the calculation within the ultrarelativistic quantum molecular dynamics (UrQMD) hybrid approach with a more realistic treatment of the initial state and final stages employing nonequilibrium hadron-string transport. In this calculation the sensitivity of the directed flow to the equation of state is less obvious. Finally, in Sec. V we point out additional issues that need to be addressed before a clear conclusion can be drawn.

II. THE EQUATIONS OF STATE

In the following we study the effect of the equation of state of hot and dense nuclear matter on the directed flow measures in relativistic nuclear collisions. In particular we want to know whether the slope of the directed flow, as a function of rapidity, is sensitive to the order of the QCD phase transition. We therefore have to compare two different scenarios: one where the QCD transition is of first order and one where it is a crossover.

For the first order transition scenario we employ a well known Maxwell construction which has been used in several investigations on the effect of the EOS [13]. The Maxwell construction is used to connect a mean field type $SU(2)_f$ hadronic model (HM) and a bag model EOS (BM) that consists of free quarks and gluons. The conditions for a Maxwell construction are the equality of the thermodynamic variables temperature $T_{BM} = T_{HM}$, baryochemical potential $\mu_{BM} = \mu_{HM}$, and pressure $p_{BM} = p_{HM}$. As a result of the construction one obtains a single phase system inside the coexistence region of the transition. For simplicity, in the following we refer to the constructed EOS only as the bag model EOS (BM).

Because of the Maxwell construction the isothermal speed of sound, c_s^{IT}, essentially vanishes, and also the isentropic speed of sound, c_s^{IE}, drops considerably inside the coexistence region. Note that the Maxwell construction only accounts for the so-called "softening" of the equation of state, due to the phase transition, and it lacks important features associated with a first order phase transition, e.g., a region of mechanical instability or the surface tension [14,15]. However, because we are only interested in the effect of the softening on the bulk dynamics, the Maxwell constructed EOS will suffice for our current investigations.

Alternatively we use an equation of state which follows from the combination of a chiral hadronic model with a constituent quark model [16], later referred to as the χ-over equation of state. This EOS gives a reasonable description of lattice QCD results at vanishing net baryon density, including a smooth crossover from a confined hadronic phase to a deconfined quark phase. This crossover continues into the finite density region of the phase diagram for essentially all densities relevant for the present investigations.

We therefore are able to compare the fluid dynamics of systems that always evolve through a first order transition to those that always evolve through a smooth crossover.

III. COLD NUCLEAR MATTER INITIALIZATION

Early studies on the directed flow in relativistic nuclear collisions suggested the "collapse of flow" to be a possible

signal for a first order phase transition in dense nuclear matter. In particular one extracted the net x-momentum per nucleon, p_x^{dir}/N, defined in the direction of the impact parameter, in a given rapidity window from fluid dynamical simulations as

$$p_x^{dir}/N = \frac{\int \rho_B(r) m_N v_x(r) dr}{\int \rho_B(r) dr}, \qquad (1)$$

where $\rho_B(r)$ and $v_x(r)$ are the local net baryon density and fluid velocity and m_N is the nucleon mass. It was found that, when the bag model equation state, with a strong first order phase transition, was used, $dp_x^{dir}/dy|_{y=0}$, i.e., the slope of the directed net momentum with rapidity, would be negative for collisions where the system remains in the mixed phase for a considerable time [7–9].

As a first step we want to reproduce these results with the above described bag model EOS that includes a Maxwell construction from a hadronic to a quark gluon plasma phase. Furthermore, we also use the χ-over EOS that only shows a crossover and is consistent with lattice data at $\mu_B = 0$ to see if the observed "antiflow" is unique for a first order transition, i.e., a very strong softening.

We use the one-fluid SHASTA algorithm, which is an ideal (3+1)-dimensional fluid dynamic code described in Ref. [17] for all calculations.

In early investigations [7] the full collision was simulated within ideal fluid dynamics. As a consequence the two colliding nuclei have to be described as two homogeneous density distributions colliding head on. In this so-called "cold nuclear matter initialization" no distinct nucleons exist, but rather two distributions of cold, locally equilibrated, nuclear matter. We therefore initialize two energy- and baryon-density distributions, according to boosted Woods-Saxon profiles with a central density of saturated nuclear matter $\rho_0 \approx 0.16$ fm^{-3}, corresponding to the two Au nuclei with a given center-of-mass energy, at impact parameter $b = 8$ fm. The simulation is started at a point in time just before the two nuclei first make contact. In the early stage of the collision the kinetic energy of the nuclei is then rapidly stopped and transformed into large local densities. From the consecutive fluid dynamical simulation we can extract $\langle p_x^{dir}/N \rangle$, according to Eq. (1) as a function of time and at fixed rapidity, in the center-of-mass frame. Figure 1 shows a comparison of the time dependent directed momentum per nucleon as extracted from the pure one-fluid simulation at two different beam energies. A noticeable nonmonotonic dependence of the directed flow on time can be observed, due to the angular momentum of the fireball, and we expect the final result to depend considerably on the decoupling time of the evolution.

A typical transition point, from the fluid dynamical phase to the final hadronic decoupling, used in most recent simulations [18,19] is roughly four times the nuclear ground state energy density ϵ_0. The slope of p_x^{dir}/N is obtained by a linear fit to the rapidity dependent $p_x^{dir}/N(y)$ in the range $-0.5 < y < 0.5$, at the time when all cells of the calculation are below that criterion. In Fig. 2 we compare the beam energy dependence of the slopes from both possible equations of state, the first order transition, and crossover scenarios.
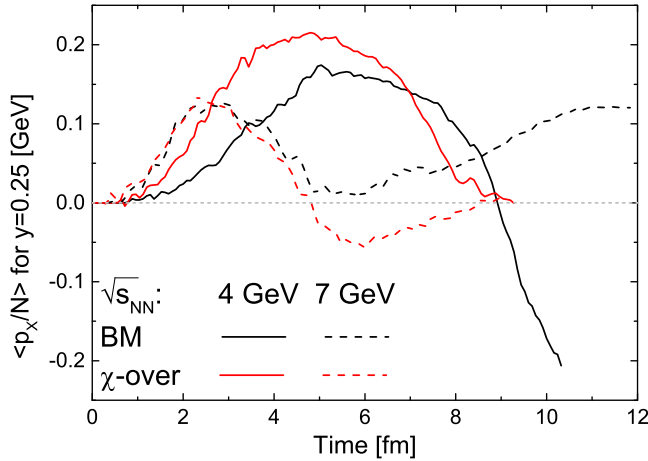
FIG. 1. (Color online) The time dependence of $\langle p_x^{\mathrm{dir}}/N \rangle$ at fixed rapidity window $y = 0.25 \pm 0.05$ from the ideal one-fluid calculations with a bag model and crossover EOS. We show two different beam energies, $\sqrt{s_{NN}} = 4$ and 7 GeV (solid and dashed lines, respectively). There is an evident nonmonotonic time dependence of $\langle p_x^{\mathrm{dir}}/N \rangle$.

As can be seen, we reproduce the predicted negative slope of the directed flow around $\sqrt{s_{NN}} = 4$ GeV when a first order transition is present [7]. The crossover EOS also shows a minimum over a range of $\sqrt{s_{NN}} = 4$–10 GeV; however, the slope always remains positive.

Already in the early studies it was noted that the quantity extracted with Eq. (1) is not directly comparable to the experimentally measured, identified particle v_1, defined as

$$v_1 = \langle \cos(\phi - \Psi_{\mathrm{RP}}) \rangle, \qquad (2)$$

where Ψ_{RP} is the reaction plane angle and ϕ the transverse angle of a particular particle. The average is usually performed
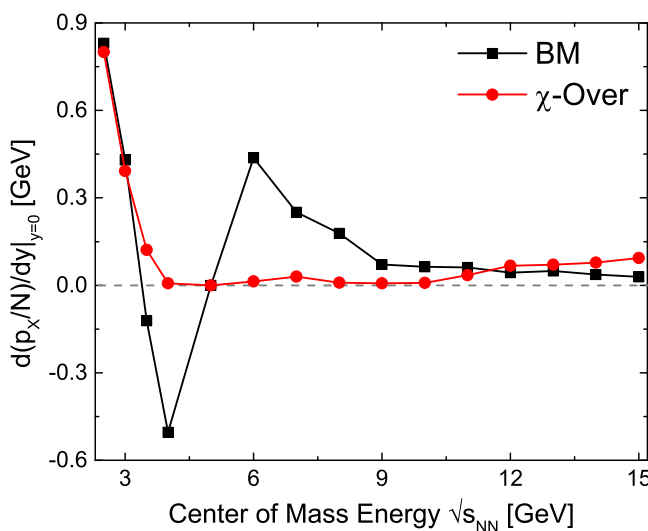

FIG. 2. (Color online) Beam energy dependence of the directed flow slope around midrapidity. Extracted from the ideal one-fluid calculations with a bag model (black) and χ-over EOS (red) for Au+Au collisions, with an impact parameter of $b = 8$ fm.
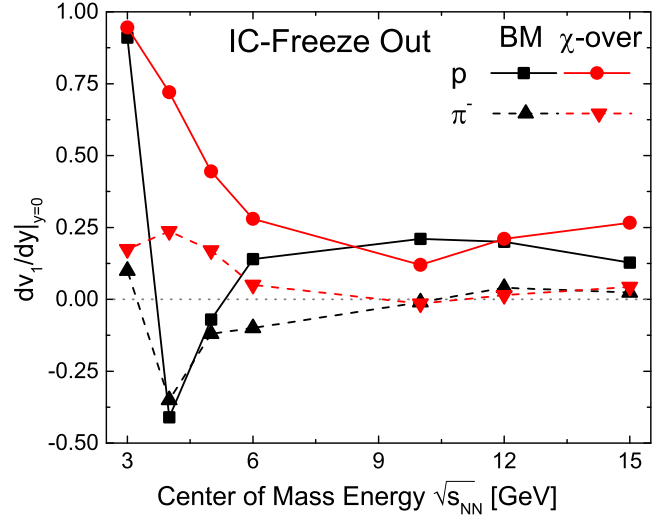

FIG. 3. (Color online) Beam energy dependence of the v_1 slope of protons and negatively charged pions around midrapidity extracted from the ideal one-fluid calculations with a bag model (black) and χ-over EOS (red). For particle production we applied a Cooper-Frye prescription on an isochronous hypersurface.

over all particles in all events, in a given rapidity bin. In order to transform the fluid dynamical fields into real particles we use the Cooper-Frye prescription [20] on a predefined hypersurface. The hypersurface, extracted from the unique fluid dynamical final state, is then used to sample a large number of hadronic final states which are independently evolved in the UrQMD transport model. Because the slope of the directed x-momentum was extracted from the fluid dynamical simulation at a fixed time we first use an isochronous hypersurface for our particle production.

The resulting slopes of the directed flow around midrapidity (fitted for $-0.5 < y < 0.5$), for different particle species, are shown in Fig. 3. Again, the negative slope is observed in the first order transition scenario around $\sqrt{s_{NN}} = 4$ GeV for protons and pions. The calculation with the χ-over EOS shows only a broad minimum in the dv_1/dy slope for protons and pions. However, it always remains positive. The softening of the two EOS therefore leads to a minimum of the directed flow slope, but not always to a negative "antiflow." Also the position of the minimum in beam energy varies with the EOS, as the crossover leads to a softening also at larger densities, resulting in a very shallow minimum at larger beam energy.

In Fig. 4 we show the same quantity as in Fig. 3, but this time we use an isoenergy density hypersurface for the transition in the subsequent hadronic afterburner. To construct this hypersurface we employ the Cornelius hypersurface finder [21], which was already successfully used in previous studies [22]. The minimum in the extracted dv_1/dy slopes occurs again at the same beam energies as with the isochronous freeze-out; however, the proton v_1 slope remains now always positive, even when we use the EOS with the large softening due to the phase transition. As shown in Fig. 1 the directed flow, at a given rapidity, shows a nonmonotonic time dependence. The positive proton v_1 slope in the isoenergy density freeze-out scenario can therefore be regarded as the
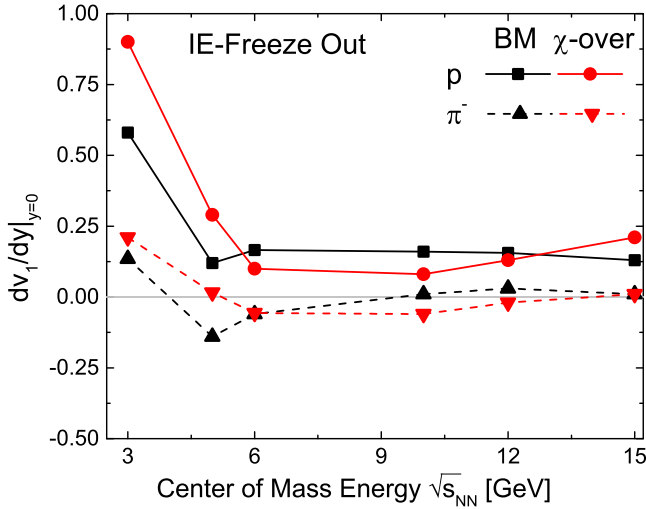
FIG. 4. (Color online) Slope of v_1 of protons and pions around midrapidity extracted from the ideal one-fluid calculations with a bag model (black) and χ-over EOS (red). For particle production we applied a Cooper-Frye prescription on an isoenergy density hypersurface.

result of an effective shortening of the fluid dynamical "low viscosity" phase as compared to the isochronous freeze-out.

It is noteworthy that in both discussed cases the slope of the proton v_1 always turns positive again once the beam energy is increased above the "softest point" of the EOS and that the pion directed flow shows always the same qualitative behavior as the proton flow.

IV. HYBRID MODEL

Until now we have assumed that the colliding systems are in local equilibrium from the beginning of the collision in order to apply ideal fluid dynamics also for the initial interpenetration phase. This had the advantage that we could use different equations of state also for the initial phase, which leads to different compression dynamics and subsequently has an impact on the directed flow. However, the assumption of local equilibrium is certainly not justified for the very early stage of a nucleus-nucleus collision. In this stage a nonequilibrium approach is better suited to describe the early time dynamics. One example of such an approach is studied in this section.

The UrQMD hybrid approach, described in detail in Ref. [18], was introduced to combine the advantages of a Boltzmann transport approach with fluid dynamics. Because the UrQMD model is used for the initial interpenetration stage of the collision, the effective equation of state during that stage is defined by the microscopic properties of the model, i.e., a purely hadronic phase. As stated in Sec. III, the fluid dynamical evolution is realized using the SHASTA algorithm, while the initial state before equilibrium and the final state with hadronic rescatterings and decays are computed using UrQMD.

In the hybrid simulations, the transition from initial transport to fluid dynamics happens when the two colliding nuclei have passed through each other: $t_{\text{start}} = \frac{2R}{\sqrt{\gamma_{\text{c.m.}}^2 - 1}}$, where R
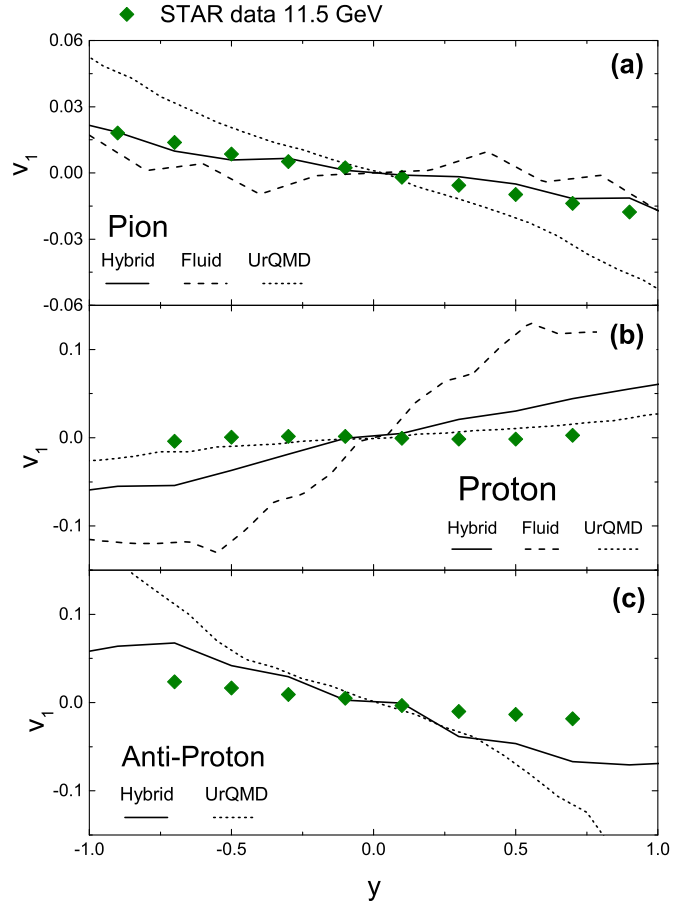

FIG. 5. (Color online) Comparison of (a) pion, (b) proton, and (c) antiproton $v_1(y)$ from the various models, for a beam energy of $\sqrt{s_{NN}} = 11.5$ GeV, compared with data [12]. Here we always used the χ-over EOS in the fluid dynamical phase.

represents the nuclear radius and $\gamma_{\text{c.m.}}$ is the Lorentz factor in the center-of-mass frame of the colliding nuclei. The transition from fluid dynamics back to transport happens on an isoenergy density $\epsilon = 4\epsilon_0 \approx 0.6 \, \text{GeV/fm}^3$ surface, which is constructed using the Cornelius hypersurface finder.

The directed flow was calculated using events with impact parameter $b = 4.6–9.4$ fm, to approximate the 10–40 % centrality range of the STAR data. As seen in Figs. 5(b) and 5(c), the hybrid model overestimates the directed flow as a function of rapidity for protons and antiprotons at $\sqrt{s_{NN}} = 11.5$ GeV, in comparison to the experimental data and the standard UrQMD result.[1] However, for charged pion v_1 at the same collision energy the hybrid model results agree with experimental data better than standard UrQMD or the pure fluid dynamical simulation [Fig. 5(a)]. All the hybrid model calculations, up to $\sqrt{s_{NN}} \approx 16$ GeV, reproduce the qualitative feature observed at lower SPS energy that the proton v_1 has the opposite sign of the pion v_1.

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[1]Note that we do not show the antiproton $v_1(y)$ from the pure fluid dynamical simulation due to limited statistics.
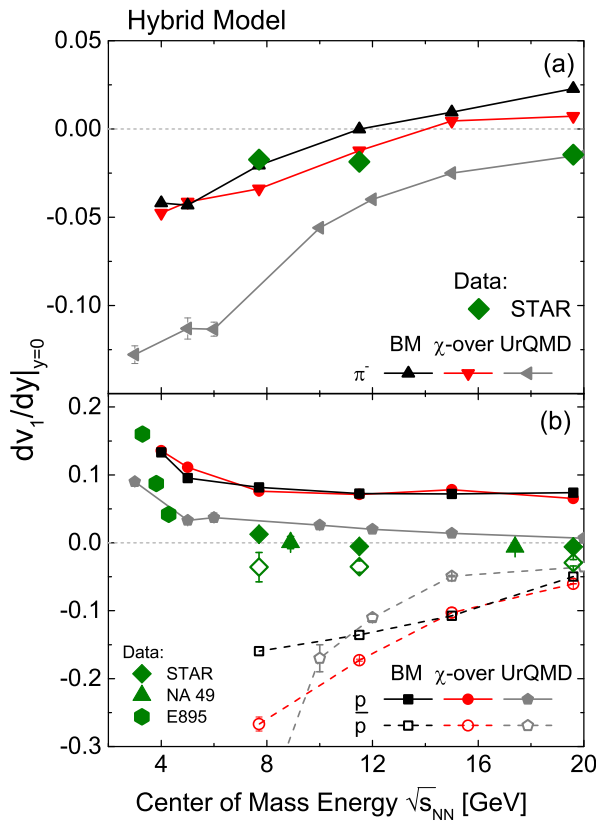
FIG. 6. (Color online) Slope of v_1 of (a) negatively charged pions and (b) protons and antiprotons around midrapidity extracted from the hybrid model calculations with a bag model and crossover EOS. We compare with standard UrQMD and experimental data [11,12,23].

The full beam energy dependence for the hybrid model results of the midrapidity v_1 slopes for negatively charged pions and protons and antiprotons are shown in Figs. 6(a) and 6(b), respectively. Both proton and antiproton slopes are overestimated for the whole examined collision energy range, while $dv_1/dy|_{y=0}$ for pions agrees with the data at lower collision energies but changes sign at $\sqrt{s_{NN}} \approx 10$–15 GeV, which is not supported by the STAR data. While the difference between the investigated equations of state was already rather small in the pure fluid results (Fig. 4), the two EOS are completely indistinguishable in the hybrid simulations.

For comparison we also present the standard UrQMD results as grey lines. The qualitative behavior is very similar to the hybrid model results. The standard UrQMD appears to better describe the experimental proton data, however.

Figure 7 shows the comparison of the hybrid and the pure hydro model results for the midrapidity dv_1/dy for protons and antiprotons, where the bag model equation of state is used. Both approaches give significantly too large slopes compared to the STAR data. As noted already in Sec. III, the proton dv_1/dy changes sign only for the model with isochronous Cooper-Frye hypersurface.

No model calculation seems to capture the qualitative experimental trend showing the directed flow slope for all particles turning negative at some point and approaching zero
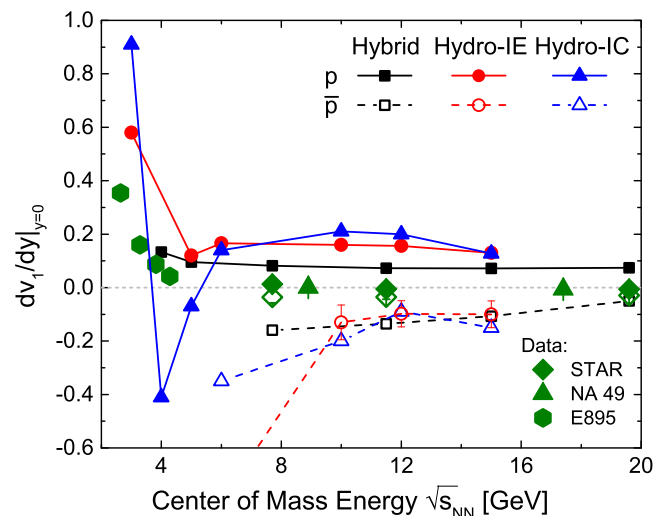

FIG. 7. (Color online) Comparison of results for pure hydro and hybrid model calculations with a first order EOS. Shown is the slope of v_1 of protons and antiprotons around midrapidity compared to experimental data.

from below. Also the overall magnitude seems to be strongly overestimated.

V. DISCUSSION

As observed in previous studies [21,24–26], the hybrid model (and to some extent also standard UrQMD) usually shows a reasonable agreement with experimental particle spectra, as well as the second and third order flow coefficients, at the energies investigated in this article. Furthermore, it has been shown [6,27–29] that fluid dynamical simulations can quite successfully account for the rapidity-even v_1 moment, which is caused by initial state fluctuations.

The strong deviation of the directed flow of all models, as compared to data, noted in the previous section is therefore surprising and requires a further discussion about the possible sources of differences in the v_1 extracted from the model as compared to the experiment.

One particular difference, for example, lies in the determination of the reaction plane angles Φ_{RP} used in Eq. (2). In our study the reaction plane (RP) angle is always defined to be zero along the impact parameter axis (x axis). Experiments determine a v_1 event plane (EP) using certain assumptions. Usually the EP for the directed flow is defined along the vector of the projectile and target spectator transverse momentum motion (defined also by measurement) [30,31]. For the Beam Energy Scan (BES) STAR data presented in this article a different method is used. Here the participant charged particles, detected by the inner tiles of the beam-beam counters (BBCs), with a pseudorapidity coverage of $3 < |\eta| < 5$ were used to determine the v_1 event plane. In an ideal scenario a model study would also define the EP in such a way.

Furthermore, it was pointed out in Ref. [32] and references therein that random initial state fluctuations, especially in the longitudinal direction, may lead to significant c.m. rapidity fluctuations, which diminish the v_1 signal. This is due to

the superposition of several rapidity-odd functions, with fluctuating zero points (which are supposed to be at the c.m. of the system). In Refs. [32] and [33,34] it is argued that the identification of the event-by-event (EbE) c.m. is important to determine the correct amplitude of v_1.

However, STAR compared the v_1 from the UrQMD event generator, which should naturally include longitudinal and transverse fluctuations, using a detector simulation to find the EP and the centrality selection, versus v_1 just straight from the model (as used by our study) with the same centrality selection. Normal analysis cuts were applied in both cases and the difference is very minor at all beam energies [12,35]. If such a comparison holds true also for our hybrid model (which uses UrQMD for the initial state on an event-by-event basis), the contributions for v_1 from fluctuations in the initial state, fireball and the spectator momentum, cannot explain the strong deviation of the fluid dynamics results from the experimental data.

Of course we do not know whether the real data contain additional correlations between the spectators and the fireball as well as unknown sources of fluctuations in the longitudinal and transverse directions. A more realistic scenario should not only consider the momentum transfer from the spectator to the fireball (and vice versa) from the random Fermi momenta but also from likely correlations and binding of the cold nuclei [36]. We know experiment measures a finite p_x for the spectators, but not its origin. This momentum must be balanced, so the fireball should have a momentum contributing opposite to the "naive" v_1 by definition, which is lacking in our model study.

Future quantitative investigations on the correlations between the spectators as well as their average transverse momentum might help to constrain model uncertainties arising from the incomplete description of spectator-fireball momentum transfer.

VI. CONCLUSIONS

We presented model simulation results on the directed flow of identified particles in nuclear collisions of beam energies in the range $\sqrt{s_{NN}} = 3$–20 GeV. To describe the strongly interacting systems created in these collisions we used different approaches, combining hadronic transport and ideal fluid dynamics.

We find that the pure fluid dynamical approach can reproduce older findings [7,9], which predicted a negative slope of the proton directed flow if a strong first order phase transition is present in the EOS. However, we also find that this "antiflow" is only observed if, and only if, the full dynamics, expansion, and initial compression are treated fully in the ideal fluid dynamics model. In the idealized one-fluid case the slope of the directed flow becomes positive at beam energies above the softest point of the EOS, just as observed in the three-fluid simulations [9] and hybrid model.

When we apply a more realistic freeze-out procedure and a hadronic transport model for the initial state, we observe a positive slope of the proton directed flow for all beam energies under investigation. Comparing our results to experimental data we find that essentially all models, including the standard hadronic transport UrQMD, cannot even describe the qualitative behavior, observed by experiment, of the proton directed flow. All models severely overestimate the data, even though other observables, like the radial or elliptic flow, are usually well described within these models.

Therefore, the measured negative slope of the directed flow is not explained. The calculated directed flow is very sensitive to details in the description of the initial state, the freeze-out prescription (and freeze-out time) as well as, potentially, the method of determining the event plane. The momentum transfer between the spectators and the fireball is not properly treated in the present model calculations. These issues need to be addressed before definite conclusions about the relation between the slope of directed flow and the EOS (including a phase transition) can be made.

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