Pionic and hidden-color, six-quark contributions to the deuteron b_1 structure function

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The b_1 structure function is an observable feature of a spin-1 system sensitive to non-nucleonic components of the target nuclear wave function. The contributions of exchanged pions in the deuteron are estimated and found to be of measurable size for values of x of about 0.1. A simple model for a hidden-color, six-quark configuration (with $\sim 0.15\%$ probability to exist in the deuteron) is proposed and found to give substantial contributions for values of $x > 0.2$. Good agreement with Hermes data is obtained. Predictions are made for an upcoming JLab experiment. The Close and Kumano sum rule is investigated and found to be a useful guide to understanding various possible effects that may contribute.

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I. INTRODUCTION

Deep inelastic scattering from a spin-one target has features, residing in the leading-twist b_1 structure function, that are not present for a spin-1/2 target [1,2]. In the quark-parton model

$$b_1 = \sum_q e_q^2 \left[q_\uparrow^2 - \frac{1}{2}(q_\uparrow^1 + q_\uparrow^1) \right] \equiv \sum_i e_q^2 \delta q_i, \qquad (1)$$

where $q_\uparrow^m(q_\downarrow^m)$ is the number density of quarks with spin up(down) along the z axis in a target hadron with helicity m. The function b_1 is called the tensor structure function of the deuteron because it has been observed by the Hermes collaboration using a tensor polarized deuteron target [3] for values of Bjorken $0.01 < x < 0.45$. The function b_1 takes on its largest value of about 10^{-2} at the lowest measured value of x (0.012), decreases with increasing x through zero and takes on a minimum value of roughly -4×10^{-3}.

The function b_1 nearly vanishes if the spin-one target is made of constituents in a relative s state, and is very small for a target of spin 1/2 particles moving nonrelativistically in higher angular momentum states [1,4–6]. Thus one expects [1] that a nuclear b_1 may be dominated by non-nucleonic components of the target nuclear wave function. Consequently, a Jefferson Laboratory experiment [7] is planned to measure b_1 for values of x in the range $0.16 < x < 0.49$ and $1 < Q^2 < 5\ \text{GeV}^2$ with the aim of reducing the error bars.

At very small values of x effects of shadowing (double scattering) are expected to be important [8–10]. Our focus here is on the kinematic region of higher values of x that are available to the JLab experiment. It is therefore natural to think of the nuclear Sullivan mechanism [11], Fig. 1, in which an exchanged pion is struck by a virtual photon produced by an incoming lepton. That the one-pion exchange potential OPEP gives a tensor force of paramount importance in deuteron physics is a nuclear physics textbook item [12]. Indeed, realistic deuteron wave functions can be constructed using only the OPEP along with a suitable cutoff at short distances [13–16]. Therefore it is reasonable to estimate the size of such pionic effects. The present author did this in 1989 conference proceeding [4], finding that the effects are small, see also [8]. However, as experimental techniques have improved dramatically, the meaning of small has changed. Therefore, considering the planned JLab experiment, it is worthwhile to reassess the size and uncertainties of the pionic effects.

In particular, the Hermes experimental result [3] presents an interesting puzzle because it observed a significant negative value of b_1 for $x = 0.45$. At such a value of x, any sea quark effect such as arising from double-scattering or virtual pions is completely negligible. Furthermore, the nucleonic contributions are computed to be very small [4–6], so one must consider other possibilities. We therefore take up the possibility that the deuteron has a six-quark component that is orthogonal to two nucleons. Such configurations are known to be dominated by the effects of so-called hidden-color states in which two color-octet baryons combine to form a color singlet [17]. Such configurations can be generated, for example, if two nucleons exchange a single gluon leading to a quantum fluctuation involving an color octet and color anti-octet baryon.

In particular, a component of the deuteron in which all six quarks are in the same spatial wave function ($|6q\rangle$) can be expressed in terms on nucleon-nucleon NN, Delta-Delta $\Delta\Delta$, and hidden color components CC as [17]

$$|6q\rangle = \sqrt{1/9}|N^2\rangle + \sqrt{4/45}|\Delta^2\rangle + \sqrt{4/5}|CC\rangle. \qquad (2)$$

This particular state has an 80% probability of hidden color and only an 11% probability to be a nucleon-nucleon configuration. The 80% cited here is a purely algebraic number that applies only for completely overlapping nucleons. The real question is the probability that the deuteron consists of 6 quarks are in the same spatial wave function, which is denoted here as P_{6q}. Early pioneering work [18] claims a probability for the $|CC\rangle$ admixture in the range of 1% to 1.5% is needed to describe nucleon-nucleon scattering and deuteron properties. However, high accuracy conventional nuclear theory calculations of nucleon-nucleon scattering [19–21] and deuteron properties [22] provide essentially perfect descriptions of the data without making explicit reference to hidden color states. Given the success of those calculation a value of P_{6q} in the range of 1% to 1.5% can only be taken as an upper limit. We shall show that a much, much smaller value of P_{6q} can have a big impact on the computed value of b_1. A recent review of hidden color phenomena is presented in [23]. In the following, the term $|6q\rangle$ is referred to interchangeably as either a six-quark or hidden color state.

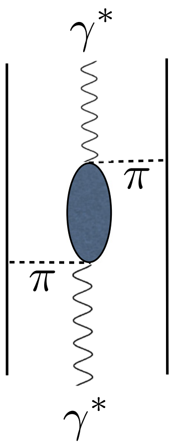

FIG. 1. (Color online) Forward Compton scattering diagram for the Sullivan process. The virtual photon γ^* encounters an exchanged pion (dashed line), breaking it up forming a complicated state (blob) which then emits the pion which is absorbed by another nucleon. The imaginary part of this graph is related to the deep inelastic structure functions of the deuteron.

The discovery of the EMC effect [24] caused researchers to consider the effects of such six-quark states [25] in a variety of nuclear phenomena [26–28]. Furthermore, the possible discovery of such a state as a di-baryon resonance has drawn recent interest [29]. Therefore we propose a model of a hidden-color six-quark components of the s and d states of the deuteron. We also note that including a six-quark hidden color component of the deuteron does not lead to a conflict with the measured asymptotic d to s ratio of the deuteron [30]. The EMC effect remains the only nuclear effect that has not been explained using conventional (nonquark) dynamics [31–33].

Section II provides a brief discussion of the formalism required to compute deep inelastic scattering cross sections for nuclei. Section III applies this formalism to compute pionic contributions to b_1. Section IV presents our simple model for the hidden color s and d states of the deuteron. Our aim here is only to provide an exploratory calculation to show the relevance of hidden color to the $b_!$ structure function. Section V compares the effects of pions and hidden color with the existing Hermes data and makes predictions for the upcoming JLab experiment. The sum rule of Close and Kumano [34] that $\int dx\, b_1(x) = 0$ is discussed in Sec. VI and summary remarks are presented in Sec. VII.

II. NUCLEAR DEEP INELASTIC SCATTERING

The nuclear dependence of nucleon quark distributions, known as the EMC effect, has been reviewed many times. See for example, [31–33]. The techniques for including the effects of bound nucleons and nuclear virtual pions are well established. Structure functions measured in deep-inelastic lepton scattering are determined by the squares of frame independent light front (LF) wave functions of the target. We also note that structure functions, expressed in terms of quark light-cone correlation functions in which quarks are evaluated at light-like separations (the quarks are at equal LF times), can be computed using the target rest frame, see, for example, [35–37]. These quark light-cone correlations are matrix elements taken within the target-rest frame wave function. This formulation is used by all calculations of the EMC effect. Even though the momentum transfer is high, the observable structure function is determined by a ground-state matrix element.

III. ONE PION EXCHANGE EFFECTS

A nuclear pion excess as a possible explanation of the EMC effect was introduced early on by Ericson and Thomas [38]. The motivation for this is that the contribution is expected to occur at values of Bjorken $x \sim m_\pi/M_N \approx 1/7$ which were relevant for the EMC experiment. This very same range of x is also relevant for the HERMES and proposed JLab experiments.

The pionic contribution to the nuclear quark distribution for a spin 1 target of $J_z = m$ and atomic number A is given by [4,39]

$$\Delta_\pi q^{(m)}(x) = \int_x^A \frac{dy}{y} q^\pi(x/y) f_\pi^{(m)}(y), \tag{3}$$

where $q^\pi(x)$ is the charged-weighted quark structure function of the pion (assumed to be the same for nuclear pions as for free pions):

$$q^\pi(x) = \tfrac{5}{9}u_v^\pi(x) + \tfrac{10}{9}\bar{u}^\pi + \tfrac{2}{9}s^\pi(x), \tag{4}$$

where u_v^π is the valence u quark distribution of the π^+ and s^π is the sea quark distribution of a flavor symmetric pion sea, and the probability to find a pion residing in a deuteron D of $S_z = m$ is given by

$$f_\pi^{(m)}(y_A) = \int \frac{d\xi^-}{2\pi} e^{-iy_A P_D^+ \xi^-} \langle D,m|\phi_\pi(\xi^-)\phi_\pi(0)|D,m\rangle_c, \tag{5}$$

where the subscript c stands for connected terms. The matrix element in Eq. (5) is a light-cone correlation function evaluated in the laboratory frame, so that $P_D^+ = M_D$. We suppress the notation for the Q^2 dependence of the pion structure function, but include its effects in calculations discussed below. The support of $f_\pi^{(m)}(y)$ extends to $y = A(=2)$ because it is possible, though highly unlikely, that a single pion could carry the entire plus-component of the nuclear momentum.

The resulting contribution to b_1 is given by

$$b_1^\pi(x) = \tfrac{1}{2}(\Delta_\pi q^{(0)}(x) - \Delta_\pi q^{(1)}(x)). \tag{6}$$

The expression, Eq. (5), for $f_\pi^{(m)}$ is evaluated by saturating the intermediate states with two nucleon, one pion states. We use the nucleon variable y with $yM = y_A M_D$ (M is the nucleon mass). Evaluation using nonrelativistic dynamics and neglecting retardation effects in the pion propagator (so that $q^+ = q^0 + q_z \approx q_z$) leads to

$$f_\pi^{(m)}(y) = \frac{-3yg^2}{(2\pi)^3} \int \frac{d^3q}{(\mathbf{q}^2 + m_\pi^2)^2} \frac{G_A^2(\mathbf{q}^2)}{G_A^2(0)} \times \delta(My - q_z) F_m(\mathbf{q}), \tag{7}$$

with

$$F_m(\mathbf{q}) \equiv \int d^3r \langle D,m|e^{-i\mathbf{q}\cdot\mathbf{r}} \boldsymbol{\sigma}_1 \cdot \mathbf{q}\, \boldsymbol{\sigma}_2 \cdot \mathbf{q}|D,m\rangle, \tag{8}$$

where $\mathbf{r}$ is the displacement between the neutron and proton, g is the pion-nucleon coupling constant (we use 13.5) and M is the nucleon mass. The nucleons involved in nonrelativistic nuclear wave functions are on their mass shell. This means that one may use the generalized Goldberger-Treiman relation [37] to relate the pion-nucleon form factor $G_{\pi N}(t)$ to the axial form factor:

$$G_{\pi N}(t) = \frac{M}{f_\pi} G_A(t), \tag{9}$$

where t is the square of the four-momentum transferred to the nucleons, $G_A(t)$ is the axial vector form factor, f_π is the pion decay constant, and $G_{\pi N}(0) \approx g$. Using Eq. (9) has obvious practical value because it relates an essentially unmeasurable quantity $G_{\pi N}$ with one G_A that is constrained by experiments, and has been used in obtaining Eq. (7). We use the dipole form: $G_A(Q^2)/G_A(0) = 1/(1+(Q^2/M_A^2))^2$, with M_A as the so-called axial mass. The values of M_A are given by $M_A = 1.03 \pm 0.04$ GeV as reviewed in [37]. This range is consistent with the one reported in a later review [40].

To proceed it is convenient to use the following representation [12] of the deuteron wave function:

$$\langle \mathbf{r}|D,m\rangle = \frac{1}{\sqrt{4\pi}}\left[\frac{u(r)}{r}|1,m\rangle + \frac{w(r)}{r}\frac{(3\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}}\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}}-1)}{\sqrt{8}}|1,m\rangle\right], \tag{10}$$

where $|1,m\rangle$ represents the triplet spin wave function. One might think that it is necessary to use a relativistic deuteron wave function computed using light front variables. However, as shown by several authors including [41–44] relativistic effects in the deuteron are very small, consistent with the very small binding energy. For a system that is inherently nonrelativistic one can make the transition to light-front variables using the simple translation $p^+ = M + p_z$, where M is the constituent mass [45], but there is no need to do that here.

Using the wave function of Eq. (10) to evaluate the expression appearing in Eq. (7) yields the results

$$F_m(\mathbf{q}) = F_m^{uu}(\mathbf{q}) + F_m^{uw}(\mathbf{q}) + F_m^{ww}(\mathbf{q}), \tag{11}$$

where

$$F_{\pm1}^{uu} = q_z^2 I_{uu0}(q), \quad F_0^{uu} = \left(q_\perp^2 - q_z^2\right) I_{uu0}(q), \tag{12}$$

$$F_{\pm1}^{uw} = -\frac{1}{\sqrt{2}}\left(3\mathbf{q}^2 - q_z^2\right) I_{uw2}(q),$$

$$F_0^{uw} = -\frac{1}{\sqrt{2}}\left(3\mathbf{q}^2 - \left(q_\perp^2 - q_z^2\right)\right) I_{uw2}(q), \tag{13}$$

$$F_{\pm1}^{ww} = q_z^2 I_{ww0}(q) + \frac{1}{4}\left(3\mathbf{q}^2 - q_z^2\right) I_{ww2}(q), \tag{14}$$

$$F_0^{ww} = \left(q_\perp^2 - q_z^2\right) I_{ww2}(q) + \frac{1}{4}\left(3\mathbf{q}^2 - \left(q_\perp^2 - q_z^2\right)\right) I_{ww2}(q), \tag{15}$$

$$I_{abL}(q) \equiv \int_0^\infty dr\, a(r) b(r) j_L(qr), \tag{16}$$

where $a,b = u,w$, $L = 0,2$, and $\mathbf{q}^2 = q^2 = q_z^2 + q_\perp^2$. We need the combinations $F_0^{ab}(\mathbf{q}) - F_1^{ab}(\mathbf{q}) \propto (\mathbf{q}^2 - 3q_z^2)$ to compute b_1. Therefore it is useful to define the integral which gives the individual terms of $f_\pi^{(0)}(y) - f_\pi^{(1)}(y)$:

$$\begin{aligned} f_{abL}(y) &\equiv -\frac{3yg^2}{(2\pi)^2}\int \frac{d^3q}{\left(\mathbf{q}^2+m_\pi^2\right)^2}\frac{\delta(My-q_z)}{\left(1+\frac{\mathbf{q}^2}{M_A^2}\right)^4} \\ &\quad\times\left(\mathbf{q}^2 - 3q_z^2\right) I_{abL}(q), \qquad (17)\\ &= -\frac{3yg^2}{8\pi^2}\int_0^\infty \frac{dq_\perp^2}{\left(q_\perp^2+M^2y^2+m_\pi^2\right)^2}\frac{1}{\left(1+\frac{q_\perp^2+M^2y^2}{M_A^2}\right)^4} \\ &\quad\times(q_\perp^2 - 2M^2y^2) I_{abL}(\sqrt{q_\perp^2+M^2y^2}). \qquad (18)\end{aligned}$$

Then

$$\begin{aligned}\delta f_\pi(y) &\equiv f_\pi^{(0)}(y) - f_\pi^{(1)}(y) \\ &= f_{uu0}(y) + \frac{\sqrt{2}}{2} f_{uw2}(y) + f_{ww0}(y) - \frac{1}{4} f_{ww2}(y)\end{aligned} \tag{19}$$

and

$$b_1^\pi(x) = \frac{1}{2}\int_x^2 \frac{dy}{y} q^\pi(x/y)\,\delta f_\pi(y). \tag{20}$$

The key output of the present section is the function $\delta f_\pi(y)$, which is displayed in Fig. 2. The Argonne V18 deuteron wave function [20] is used here, but virtually identical results are obtained with the Reid '93 potential [47]. Note the double node structure, a consequence of the tensor nature of the operator, that can be understood by examining Eq. (17) and the functions $I_{abL}(q)$. For small values of y, $f_{abL}(y) \propto (-y)$, but for larger values of $y = q_z$, the integrand changes sign. A node in the functions $I_{abL}(q)$ causes another sign change at still larger values of y. Indeed, we may use Eq. (17) to obtain a sum rule:

$$\int_0^2 dy \frac{f_{abL}(y)}{y} = 0, \tag{21}$$

with the 0 resulting from the feature $\int d^3q f(\mathbf{q}^2)(\mathbf{q}^2 - 3q_z^2) = 0$. This sum rule has been used as a numerical check on the integrals. No general result for $\int dy\, f(y)$ can be obtained because of the factor y appearing in front of the integral in Eq. (17).

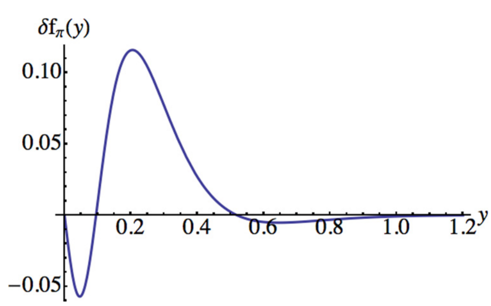

FIG. 2. (Color online) $\delta f_\pi(y)$ of Eq. (19). The results obtained with the Argonne V18 deuteron wave function [20] overlap with those of the Reid '93 potential [47].

IV. HIDDEN-COLOR SIX-QUARK STATES

We investigate the possible relevance of hidden-color six-quark states. There have been many models and attempts to provide definitive evidence for the existence of such states. However, no model has been unambiguously and uniquely verified by experiment. This is because conventional nuclear theory makes no reference to such states and yet successfully reproduces all known nuclear phenomena except for the EMC effect. In this paper, we take the view that the HERMES observation at $x = 0.452$ (their largest value of x) may provide another example requiring unconventional nuclear wave functions, and therefore may offer an opportunity to finally learn something definite. The key point is that nucleonic (and mesonic) effects, as presently computed, offer much smaller (in magnitude) values of b_1 than found by HERMES. A value of $x = 0.452$ involves valence quarks. Since nucleons do not provide a mechanism, one is naturally encouraged to look at six quark hidden color components, which should have support at such values of x. Since no definitive model exists, it is sufficient to use the simplest of many possible models for the present exploratory calculation.

We proceed by assuming the existence of a deuteron component consisting of six nonrelativistic quarks in an S state. As stated in the Introduction, such a state has only a probability of 1/9 to be a nucleon-nucleon component, and is to a reasonable approximation a hidden color state, so we use the terminology six-quark, hidden color state. Then we obtain the corresponding d state component by promoting any one of the quarks to a $d_{3/2}$ state. We define these states by combining five s-state quarks into a spin 1/2 component, which couples with the either the $s_{1/2}$ of $d_{3/2}$ single-quark state to make a total angular momentum of 1. We therefore write the wave functions of these states for a deuteron of $J_z = H$ as

$$\psi_{j,l,H}(\mathbf{p}) = \sqrt{N_l}\, f_l(p) \sum_{m_s,m_j} \mathcal{Y}_{jlm_j}\left\langle jm_j, \frac{1}{2}m_s|1H\right\rangle, \quad (22)$$

where $l,j = s_{1/2}$ or $d_{3/2}$, N_l is a normalization constant chosen so that $\int d^3p\bar{\psi}_{j,l,H}(\mathbf{p})\gamma^+\psi_{j,l,H}(\mathbf{p}) = 1$ and $\mathcal{Y}_{jlm_j}$ is a spinor spherical harmonic. The matrix element for transition between the $l = 0$ and $l = 2$ states is given by the light-cone distribution:

$$F_H(x_{6q}) = \frac{1}{2}\int d^3p\bar{\psi}_{1/2,0,H}(\mathbf{p})\gamma^+\psi_{3/2,2,H}(\mathbf{p}) \times \delta\left(\frac{p\cos\theta + E(p)}{M_{6q}} - x_{6q}\right), \quad (23)$$

where $E(p) = \sqrt{p^2 + m^2}$ with m as the quark mass, and M_{6q} is the mass of the six-quark bag, x_{6q} is the momentum fraction of the six-quark bag carried by a single quark and $x_{6q}M_{6q} = xM$ [25]. Note that $p\cos\theta$ is the third (z) component of the momentum, so that the plus component of the quark momentum is $E(p) + p\cos\theta$. We take $M_{6q} = 2M$ (its lowest possible value) to make a conservative estimate.

The term of interest $b_1(x)$ is given by

$$b_1^{6q}(x) = \tfrac{1}{2}(2)(F_0(x) - F_1(x))P_{6q}, \quad (24)$$

where P_{6q} is the product of the probability amplitudes for the six-quark states to exist in the deuteron, and the factor of 2 enters because either state can be in the d wave. Evaluation of F_H using Eq. (22) leads to the result

$$b_1^{6q}(x) = -\sqrt{\frac{N_0N_2}{2}}\frac{3}{4\pi}\int d^3p f_0 f_2(3\cos^2\theta - 1) \times \delta\left(\frac{p\cos\theta + E(p)}{M} - x\right)P_{6q}. \quad (25)$$

To proceed further we specify the wave functions to be harmonic oscillator wave functions. We take $f_2(p) = -p^2R^2e^{-p^2R^2/2}$, $f_0(p) = e^{-p^2R^2/2}$, where R is the radius parameter. Within the present framework, the model is specified by only three parameters—R, the quark mass m, and P_{6q}. The key question is whether such a model can reproduce the HERMES data point at $x = 0.452$ without using a value of P_{6q} large enough to conflict with conventional nuclear physics calculations that do not require a nonzero value [19–22]. In other words, we ask if the hidden color states provide a substantial mechanism to make b_1 nonzero at large values of x. We note that the HERMES data have a large error bar, and our numerical results are chosen to reproduce the central value.

Our procedure is to adjust the value of P_{6q} to reproduce the data at that point and see how large a value is needed. To proceed further, we use a quark mass of 338 MeV [48]. We expect that the six-quark state should be somewhat larger than that of a nucleon, and therefore choose R to be 1.2 fm. We shall see that the calculations are not very sensitive to the exact value of R. The dependence on all three parameters is examined below.

The evaluation of $b_1^{6q}(x)$ proceeds by using $d^3p = 2\pi p^2dpd\cos\theta$, integrating over $\cos\theta$, and changing variables to $u \equiv p^2R^2$. The result is

$$b_1^{6q}(x) = \frac{6MR}{\sqrt{30\pi}}\int_{u_{\min(x)}}^{\infty} du\, e^{-u}[3((x^2M^2 + m^2)R^2 + u - 2xMR\sqrt{u + m^2R^2}) - u]P_{6q}, \quad (26)$$

where

$$u_{\min(x)} \equiv \frac{(x^2M^2 - m^2)^2R^2}{4x^2M^2}. \quad (27)$$

V. RESULTS

We may now start examining the resulting phenomenology, considering first the pionic contributions. The quark distribution function of the pion, q^π is needed to evaluate b_1^π as shown in Eq. (20). Evaluation requires knowledge of this function over a wide range of its argument and x/y can be very small. However, knowledge of q^π comes from fixed-target Drell-Yan data at values of $x \geqslant 0.3$ [49,50]. We display the sensitivity to different versions of q^π in Fig. 3. We display results for the full and valence distributions of [46], and for the full and valence distributions of [51] at $Q^2 = 1.17$ GeV^2. The sea is important for values of x less than about 0.1. This is unfortunate because the Drell-Yan data at large x embody little sensitivity to the sea. However, the computed values of b_1^π are not very different for

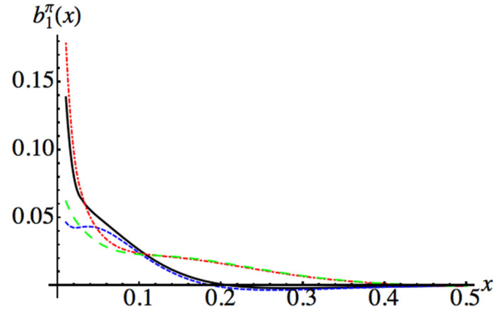

FIG. 3. (Color online) Computed values of b_1^π, for different pion structure function at $Q^2 = 1.17$ GeV^2. Solid lines—full structure function [46], short-dashed lines (blue)—valence [46], dot dashed lines (red)—full structure function (mode 3) [51], long dashed lines (green) (mode 3) [51].

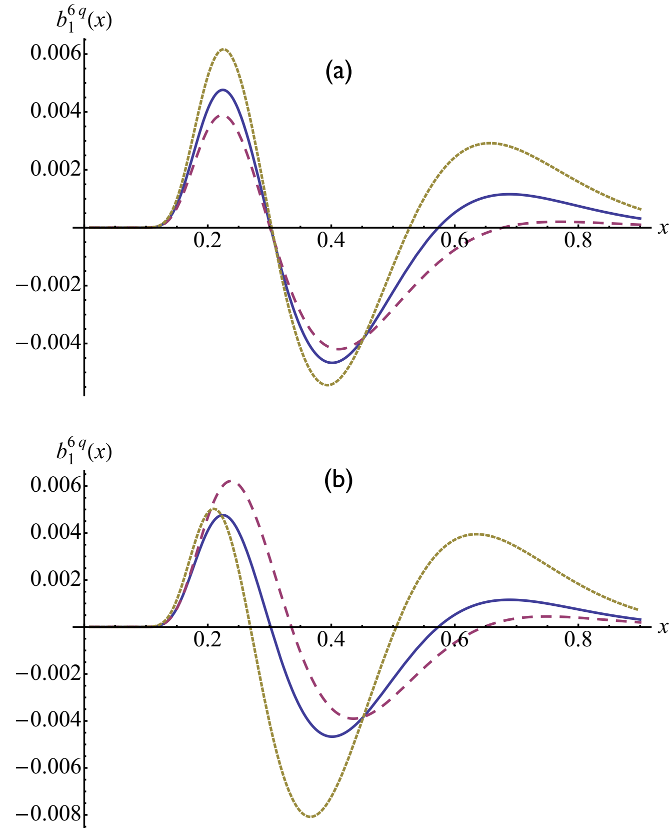

FIG. 4. (Color online) Computed values of b_1^{6q} from Eq. (26). Sensitivity to parameters is displayed. (a) Solid lines (blue) uses $R = 1.2$ fm, $m = 338$ MeV, long dashed lines (red) R is decreased by 10%, dotted lines (green) R is increased by 10%. (b) Solid lines (blue) uses $R = 1.2$ fm, $m = 338$ MeV, long dashed (red) m is increased by 10%, dotted lines (green), m is decreased by 10%.

the two parametrizations, except for very small values of x. The solid and dashed curves show the result of using two different valence quark distributions of [46] at $Q^2 = 0.4$ GeV^2. For Fit 3 (solid) $q^\pi(x) \sim x^{-0.3}$ while for Fit 4 (dashed) $q^\pi(x) \sim x^{0.06}$.

We now turn to the determine the contributions due to hidden color, b_1^{6q} provided by Eq. (26). The value of $b_1^{6q}(x = 0.452)$ is relevant because the pionic contribution is negligible, and the measured value, $b_1 = -3.8 \pm 0.16 \times 10^{-3}$, differs from zero. We choose $P_{6q} = 0.0015$ to reproduce the central value using $R = 1.2$ fm and $m = 338$ MeV. The small value of P_{6q} shows that the six-quark configuration has great impact on the computed value of b_1. It is noteworthy that such a very, very small value cannot be ruled out by any observations. Our value of P_{6q} is very small: small enough to evade any limits imposed by high accuracy conventional nuclear theory calculations of nucleon-nucleon scattering [19–21] and deuteron properties [22] that make no explicit reference to hidden color states. Our value of 0.0015 is too small to conflict with conventional nuclear calculations.

The results for b_1^{6q} are shown in Fig. 4. Results using the model parameters $R = 1.2$ fm, $m = 338$ MeV are shown as the solid curves in Figs. 4 a and b, which also displays the sensitivity to the values of the parameters. There is relatively little sensitivity to the value of R, but more sensitivity to the value of m. However, there is wider dependence upon the choice of models. For example, if the HERMES point at $x = 0.452$ is not reproduced, this particular model will be ruled out. The exponential appearing in Eq. (26) renders the contribution very small for small values of x. This is seen in the figure. However, there is a large negative contribution at values of $x \approx 0.4$, as well as a double-node structure. The latter arises from the factor $3\cos^2\theta - 1$ appearing in the integrand of Eq. (25). The contributions of the hidden-color configurations are generally much smaller than those of exchanged pions except for values of x larger than about 0.35. We also predict that, for even larger values of x, b_1 changes sign and may have another maximum. This mechanism allows contributions at large values of x. A quark in a hidden color, six quark configuration can have up to two units of x. The parameter dependence of the model is also explored. Fig. 4 a shows the dependence on the value of R and Fig. 4 a shows the dependence on the value of m. For each of the curves P_{6q} is chosen so that the value at $x = 0.452$ is the same. Shifting the value of R while keeping $b_1^{6q}(0.452)$ fixed requires less than 4% changes in the value of P_{6q}. Increasing the value of the quark mass produces larger effects. Keeping $b_1^{6q}(0.452)$ fixed requires that the value of P_{6q} needs to be decreased by 20% if the value of the quark mass is increased by 10%, and the value of P_{6q} needs to be increased by about a factor of 1.8 if the value of the quark mass is decreased by 10%. In the remainder of this paper, we use the central values $R = 1.2$ fm, $m = 338$ MeV.

At this stage we can assess the size of our computed b_1^π and b_1^{6q} versus the only existing data [3]. These data is given in Table I along with our computed values of b_1^π using the pion structure functions of Ref. [46] and the three modes of [51]. These modes differ in the fraction of momentum carried by the sea: 10%, 15%, and 20% for modes 1 and 3, respectively. The differences obtained by using different structure functions are generally not larger than the experimental error bars. For values of x less than about 0.2, there is qualitative agreement between the measurements and the calculations of b_1^π (which are much larger than those of b_1^{6q}), given the stated experimental uncertainties and the unquantifiable uncertainty caused by lack of knowledge of the sea. However, the large-magnitude negative central value measured at $x = 0.452$ is two standard

TABLE I. Measured values (in 10^{-2} units) of the tensor structure function b_1. Both the statistical and systematic uncertainties are listed. The numbers in parenthesis refer to the structure function modes of Ref. [51].

	$\langle Q^2\rangle$	$b_1 \pm \delta b_1^{\rm stat} \pm \delta b_1^{\rm sys}$			b_1^π [46] b_1^π [51] (1) b_1^π [51] (3)			b_1^{6q}
$\langle x\rangle$	[GeV^2]	[10^{-2}]	[10^{-2}]	[10^{-2}]	[10^{-2}]	[10^{-2}]	[10^{-2}]	[10^{-2}]
0.012	0.51	11.20	5.51	2.77	10.5	15.5	24.1	0.00
0.032	1.06	5.50	2.53	1.84	5.6	6.8	8.9	0.00
0.063	1.65	3.82	1.11	0.60	4.2	3.7	4.1	0.00
0.128	2.33	0.29	0.53	0.44	1.6	1.3	1.3	0.01
0.248	3.11	0.29	0.28	0.24	−0.55	0.13	0.12	0.41
0.452	4.69	−0.38	0.16	0.03	−0.02	−0.02	−0.022	−0.38

deviations away from the value provided by b_1^π but in accord with the value provided by b_1^{6q}. Thus our result is that one can reproduce the Hermes measurements by using pion exchange contributions at low values of x and hidden-color configurations at larger values of x. This is also shown in Fig. 5, where very good agreement between data and our model can be observed. The contributions of double scattering [10] are far smaller than the measurements for the values of x displayed in the table and in Fig. 5, and are therefore neglected here.

The next step is to make predictions for the JLab experiment. Our results for $b_1 = b_1^\pi + b_1^{6q}$ are shown in Fig. 6. For values of x less than about 0.2 the computed values of b_1 are dominated by those of b_1^π. For larger values of x the computed results are not significantly different from 0. This result combined with the very small large x results for nucleonic [1,5,6] and double-scattering contributions [8–10], makes the case that an observation of a value of b_1 significantly different than zero for values of x greater than about 0.3 would represent a discovery of some sort of exotic nuclear physics. Our model Eq. (26) leads to an effect that does contribute at larger values of x. One may roughly think of the prediction for the JLab experiment as arising from b_1^π for $x < 0.2$ and from b_1^{6q} for $x > 0.2$.

VI. SUM RULE OF CLOSE AND KUMANO

Close and Kumano [34] found a sum rule that the integral of $b_1(x)$ vanishes:

$$\int dx b_1(x) = 0, \tag{28}$$

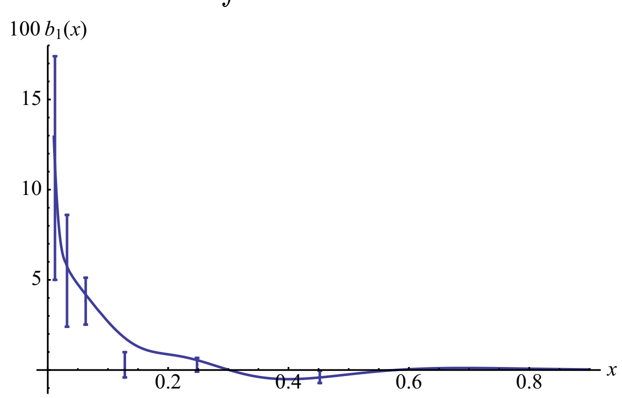

FIG. 5. (Color online) Computed values of $b_1 = b_1^\pi + b_1^{6q}$ from Eq. (20) and Eq. (26). The pion structure function is that of [46], model 1.

provided that the sea is unpolarized, as is the case for the pion contribution discussed here. This sum rule is interesting because it shows that if $b_1(x)$ is significantly different from 0 at one value of x, it must take on significant values of the opposing sign for other values of x.

A visual inspection of the Fig. 3 shows immediately that the pionic contribution does not obey this sum rule. This result can be seen analytically by integrating Eq. (20) over x:

$$\int_0^1 dx b_1^\pi(x) = \frac{1}{2}\int_0^1 dx \int_x^2 \frac{dy}{y} q^\pi(x/y)\delta f_\pi(y) \tag{29}$$

$$= \frac{1}{2}\int_0^2 dy \delta f_\pi(y) \int_0^1 du q^\pi(u). \tag{30}$$

The above result is obtained by interchanging the order of the integration over x and y and changing variables from x to $u = x/y$. There are two reasons why the product of integrals on the right does not vanish. The first is displayed above in Eq. (21); the integral of $\delta f_\pi(y)/y$ vanishes, so the integral of $\delta f_\pi(y)$ can not vanish. The second is that the integral of the quark distribution function of the pion is infinite for any of the published structure functions. Thus the value of the sum rule is infinity.

Given this violation of the sum rule of Close and Kumano, it is interesting to see if the extant calculations of other mechanisms are consistent with the sum rule. Consider first the nucleonic contribution [1,5,6]. In particular we examine

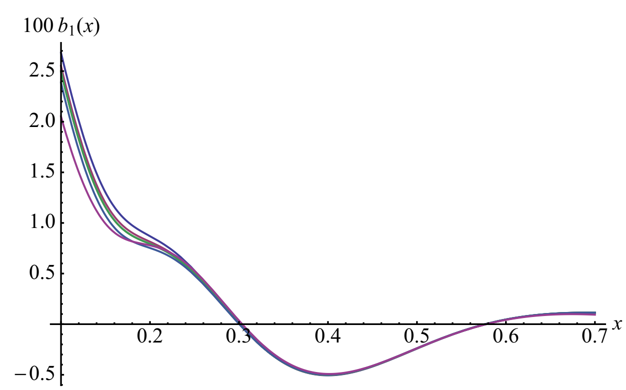

FIG. 6. (Color online) Computed values of $100\,(b_1^\pi + b_1^{6q})$, for values of $Q^2 = 1.17$, 1.76, 2.12, and 3.25 GeV^2 [46] distributions and for [51] (lowest curve at $x = 0.15$). For the other curves, b_1^π increases as Q^2 increases for small values of x.

Eq. (15,16) of [5]:

$$b_1^N(x) = \int_x^2 \frac{dy}{y}\Delta b(y) F_{1N}(x/y), \tag{31}$$

$$\Delta b(y) = \int d^3p F_d(p)(3\cos^2\theta - 1)\left(1 + \frac{p\cos\theta}{M} + \frac{p^2}{4M^2}\right) \times \delta\left(y - \frac{p\cos\theta + E(p)}{M}\right), \tag{32}$$

$$F(p) \equiv -\frac{3}{4\pi\sqrt{2}}\sin\alpha\cos\alpha u_s(p)u_d(p) + \frac{3}{16\pi}\sin^2\alpha u_d^2(p), \tag{33}$$

where M is the nucleon mass, $E(p) = \sqrt{p^2 + M^2}$, and u_s, u_d are the s and d state components of the deuteron wave function. The angle θ is the polar angle for the vector $\mathbf{p}$. Integration over x and repeating the above manipulations leads to the result

$$\int_0^1 dx b_1^N(x) = \int_0^2 dy \Delta b(y) \int_0^1 du F_{1N}(u). \tag{34}$$

There is again a product of integrals, with the one involving F_{1N} being infinite for any of the published structure functions. Close and Kumano state that the integral over y vanishes. If that were correct the value of the sum rule would be zero times infinity, or indefinite. However, while the integral over y is very small, we can argue generally that it is nonzero.

First note that

$$\int_0^2 dy\, \delta\left(y - \frac{(E + p\cos\theta)}{M}\right) = \theta(2M - E - p\cos\theta), \tag{35}$$

which does not vanish if $p > 3/4M$. This nonvanishing is sufficient to show that value of the sum rule takes on the value infinity because the integral over d^3p would not vanish. It is possible that the integral over d^3p could vanish accidentally, but if such a cancellation occurs for one given deuteron wave function, it would not occur for another.

We explain this in more detail. The integral over $\cos\theta$ appearing in Eq. (32) can be evaluated in closed form with the result

$$\int_0^2 dy\, \Delta b(y) = \frac{\pi}{2}M^3 \int_{\frac{3}{4}}^\infty \frac{dl}{l} F_d(p = lM)(4l^4 + (98 - 29\sqrt{l^2+1})l^2 - 172\sqrt{l^2+1} + 179). \tag{36}$$

The expression within the parenthesis is negative for $l = p/M < 2.63$ and the integrand varies as $l^3 F_d(l)$ for large values of l. The wave functions $u_{s,d}(p)$ each fall at least as rapidly as $1/p^4$ for large p, so the integrand falls at least as fast as $1/l^5$ for large values of l. Thus the value of integral is noninfinite and very small. This is because all known deuteron wave functions are nearly vanishing for $p/M > 3/4$, and can be regarded as vanishing for $p/M > 2.63$. The net result is that the Close-Kumano sum rule evaluates to the product of a number of very small magnitude times infinity, which is infinite.

Another known mechanism is double scattering, $b_2^{(2)}(x, Q^2)$. Here we use the result of [10], obtained after integrating over $k_\perp$ in their Eq. (20):

$$b_2^{(2)}(x, Q^2) = \frac{-3}{(\pi)^4}\frac{Q^2}{16\sqrt{2}\alpha}\,\mathrm{Im}\, i\int d^2b \times \int dz\, u_0(r)u_2(r)\frac{2z^2 - b^2}{(z^2+b^2)^2}\frac{\pi}{a}e^{-\frac{b^2}{4a}}\sum_V e^{iz/\lambda_V} \times \frac{M_V^4}{(M_V^2 + Q^2)^2}\frac{d\sigma}{dt}\bigg|_{\gamma N \to VN, t=0}, \tag{37}$$

where

$$\lambda_V = \frac{2\nu}{M_V^2 + Q^2} = \frac{Q^2}{Mx(M_V^2 + Q^2)} \equiv \frac{\Lambda_V}{x}. \tag{38}$$

The x dependence enters through the dependence of λ_V on x.

To test the sum rule we integrate over x.

$$\int_0^1 dx b_2^{(2)}(x, Q^2) = \frac{-3}{(\pi)^4}\frac{Q^2}{16\sqrt{2}\alpha}\int d^2b \int dz\, u_0(r)u_2(r) \times \frac{2z^2 - b^2}{(z^2+b^2)^2}\frac{\pi}{a}e^{-\frac{b^2}{4a}}\sum_V \frac{\Lambda_V}{z}\sin\frac{z}{\Lambda_V} \times \frac{M_V^4}{(M_V^2 + Q^2)^2}\frac{d\sigma}{dt}\bigg|_{\gamma N \to VN, t=0}. \tag{39}$$

The three-dimensional spatial integral involving $(2z^2 - b^2) = 3z^2 - r^2$ would vanish if multiplied by a function that depended only on r. However, the integrand contains the exponential involving b^2 and the sin involving z. Therefore the integral does not vanish and the double scattering term violates the sum rule. The nonvanishing of the sum rule integral for the pionic and double scattering mechanisms is in agreement with an earlier finding by [8].

Thus three published mechanisms that contribute to b_1, and all violate the sum rule of Close and Kumano. However, one can see from Eq. (25) that

$$\int dx b_1^{6q}(x) = 0. \tag{40}$$

The integral over all values of x leads to a three-dimensional integral involving $3\cos^2\theta - 1$ which must vanish. Moreover, a glance at Fig. 4 leads to the expectation that the integral of $b_1^{6q}(x)$ over the values of x displayed vanishes. Indeed, numerical integration leads to a zero within one part in 10^8. Furthermore, the model of [1] involving massless relativistic quarks with $j = 3/2$ moving in a central potential also satisfies the sum rule for the same reasons.

Given the different possible values that the integral of b_1 may take, it seems reasonable to re-examine the derivation and meaning of the sum rule of Eq. (28). The key equations of Ref. [34] are their Eqs. (15,16):

$$\Gamma_{H,H} = \langle p, H|J_0(0)|p, H\rangle, \tag{41}$$

$$\frac{1}{2}(\Gamma_{0,0} - \Gamma_{1,1}) = \sum_i e_i \int dx \delta q_{i,v}^A(x), \tag{42}$$

where the sum is over quarks of flavor i and charges e_i of a hadron of z-projection H. The term δq_i is defined in Eq. (1), and involves only valence quarks,which have nonzero integrals over all x that are related to baryon number and charge.

That the sum rule does not hold for mechanisms in which b_1 is generated by nonvalence quark contributions, such as the double-scattering and pion exchange mechanisms, is consistent with the derivation based on valence quark dominance. The sum rule does not hold for the contributions of the d state nucleons, but those contributions to b_1 are nearly vanishing for nonzero values of x. The sum rule does hold for the hidden-color six-quark configurations, in which a valence quark contributes. Thus the sum rule is a useful guide to the physics relevant for b_1. An observation of its failure means that sea effects are important. Measuring significant positive and negative values of b_1 at large x could signify the importance of an exotic valence quark effect.

VII. SUMMARY

This paper contains an evaluation of the pion exchange and six-quark, hidden-color contribution to the b_1 structure function of the deuteron. The pion-nucleon form factor is constrained phenomenologically to reduce a possible uncertainty. There is some numerical sensitivity to using different pionic structure functions. The pionic mechanism is sizable for small values of x, and can reproduce Hermes data [3] for values of x less than 0.2. A postulated model involving hidden-color components of the deuteron is shown to complement the effects of pion exchange in reproducing the Hermes data for all measured values of x. This model is based on the accuracy of the Hermes data for its largest value of $x = 0.452$, and is chosen for simplicity. Many other models possible and we welcome further work to improve such models. Nevertheless, the availability of the Hermes data enables us to make predictions for an upcoming JLab experiment [7]. The sum-rule of Close and Kumano, Eq. (28) is shown to be violated for the three previously published mechanisms that contribute to b_1. However, the sum rule holds when the mechanism involves valence quarks, such as in the present hidden color model. This means that such contributions (if nonzero) must yield negative and positive contributions to b_1. Finding such an up-down pattern is an interesting and significant problem for experimentalists. A clear observation of such a pattern would provide significant evidence for the existence of hidden-color components of the deuteron.

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