Properties of low-lying levels of ^{14}Be

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(Received 12 January 2014; revised manuscript received 16 March 2014; published 9 April 2014)

In a simple model, I have calculated energy splittings and wave functions of low-lying states in ^{14}Be. Data from $2n$ decays of 2^+ states allow estimates of the $(sd)^4$ admixtures in the predominantly $(sd)^2$ states. I demonstrate that the first 2^+ state is predominantly of ds structure, not dd as recently claimed.

DOI: 10.1103/PhysRevC.89.044312 PACS number(s): 21.10.Jx, 21.60.Cs, 27.20.+n

I. INTRODUCTION

A limited amount of information is available concerning levels of ^{14}Be, for which only the ground state (g.s.) is bound (by 1.27(13) MeV to ^{12}Be + $2n$ [1]).The g.s. mass excess was first measured in a pion-induced double-charge-exchange reaction, ^{14}C(π^-, π^+)^{14}Be [2]. The first-excited 2^+ state, at E_x = 1.54 MeV, E_{2n} = 0.28 MeV, was first observed in a heavy-ion reaction [3] and later confirmed by other experiments [4–6]. One experiment [6] reported a state at 4.1 MeV.

A very recent experiment [7] used the inelastic scattering of ^{14}Be from hydrogen to investigate low-lying resonances, by detecting ^{12}Be + n + n in coincidence. They observed the extreme tail of the first 2^+ state and reported two other resonances at E_{2n} = 2.28(9) and 3.99(14) MeV, with widths of 1.5 and 1.0 MeV, respectively. These widths were not obtained by fitting, but rather were held fixed in the analysis. They assigned 2^+ to the first of these and suggested (3^-) for the second. They were able to fit their data without any continuum background. Here, I use a simple model to examine the expected structure of these and other states.

II. CALCULATIONS

All the low-lying positive-parity states of ^{14}Be are expected to be well described in terms of two components—a p-shell ^{12}Be(g.s.) coupled to two sd-shell neutrons and a p-shell ^{10}Be(g.s.) plus $\nu(sd)^4$, with the former dominating in the lowest states. The expectation that the bulk of the $(sd)^4$ strength will lie considerably higher than that for $(sd)^2$ is assured by the large difference of energies for s and d orbitals, with s being lower, plus the fact that the s orbital can contain only two neutrons.

A simple model has proven quite successful in describing the properties of $(sd)^2$ states in light nuclei [8–10]. For nucleus $A+2$, with nucleus A fully within the p shell, the model uses the energies of the $1/2^+$ and $5/2^+$ states in nucleus A + 1 as single-particle energies (SPE's) of s and d neutrons, respectively. Two-body matrix elements (2BME's) (Table I) are taken from earlier work on ^{18}O [10] and are assumed to be the same for all the nuclei considered. Reference [10] described ^{18}O in a shell model plus a collective model. For nine positive-parity states of ^{18}O, they varied wave-function components to reproduce a vast array of experimental information—primarily *1n* and *2n* transfer and electromagnetic strengths. The main accomplishment was the separation of the two-neutron structure from other structures involving core excitation. The resulting wave functions enabled construction of the relevant Hamiltonian matrices for the $(sd)^2$ space. These Hamiltonians were subsequently used successfully in $(sd)^2$ shell-model calculations for ^{14,16}C [8] and ^{17}N [9], where matrix diagonalizations produced energies and wave functions. I have applied this model to the $(sd)^2$ states of ^{14}Be.

It might appear that the simplest approach would be to obtain ^{14}Be by adding two sd-shell neutrons to the g.s. of ^{12}Be. However, two related problems exist for this calculation. (i) The configuration I desire has two sd-shell neutrons coupled to a p-shell ^{12}Be(g.s.), not to the physical ^{12}Be(g.s.), which is known to have a significant (about 68% [11–13]) $(sd)^2$ component. Thus, the simple procedure would run afoul of the Pauli principle. (ii) The $1/2^+$ and $5/2^+$ SPE's are not well determined in ^{13}Be. Even if they were, they would be relative to the physical ^{12}Be$+n$, whereas I need them relative to ^{12}Be$_{1p}+n$.

The standard representation of the ^{12}Be(g.s.) wave function is

$$^{12}\mathrm{Be(g.s.)} = {}^{10}\mathrm{Be}_{1p} \otimes (\alpha s^2 + \beta d^2) + \gamma\, {}^{12}\mathrm{Be}_{1p}\left[+\delta\, {}^{10}\mathrm{Be}_{1p}(2^+) \otimes (sd)^2_{2+}\right],$$

where s and d stand for $s_{1/2}$ and $d_{5/2}$, respectively, and the subscript *1p* denotes pure p-shell structures. My favorite wave function [11] has $\alpha^2 = 0.53(3)$, $\beta^2 = 0.15$, and $\gamma^2 = 0.32$. The term in parentheses must be present at some level, but it is usually omitted in most treatments. I list it because of its possible relevance later below.

Luckily, the wave functions of the $(sd)^2$ states in ^{14}Be will depend only on the energy difference E_d–E_s, and not on the actual energies. I previously estimated this difference (Table I) to be about 2.3 MeV [14]. The absolute energies of the calculated states would depend on the actual d and s energies, and not just the difference. Thus, I am not attempting a calculation of absolute energies in ^{14}Be. The low-energy positive-parity spectrum will contain two 0^+ states, two 2^+ states, and one 3^+ state. A 4^+ state will lie somewhat (1–2 MeV) higher. (In my space, the 4^+ state is pure dd and the 3^+ state is pure ds. In ^{16}C [8], where the ds energy splitting is only 0.74 MeV, the 4^+ state is just above the 3^+ state. Thus, in ^{14}Be, where the ds splitting is 2.3 MeV, the 4^+ state should be about 1.6 MeV above the 3^+ state.) The energy splitting between the two 0^+ states and between

TABLE I. Energies (MeV) relevant to calculations of ^{12}Be x $(sd)^2$ statesin ^{14}Be.

Quantity	Value[a]
$E_d - E_s$	2.3[b]
$\langle s^2, Vs^2\rangle_0$	−1.54
$\langle s^2, Vd^2\rangle_0$	−1.72
$\langle d^2, Vd^2\rangle_0$	−2.78
$\langle ds, Vds\rangle_2$	−0.62
$\langle ds, Vd^2\rangle_2$	−0.61
$\langle d^2, Vd^2\rangle_2$	−1.04

[a]Reference [10], unless otherwise noted.
[b]Reference [14].

the two 2^+ states will also depend only on E_d–E_s. Results of the calculation are listed in Table II. The wave functions are those that result from simple diagonalization of the relevant Hamiltonian matrices of Table I. The energy splittings δE are just the differences between the eigenvalues that result from those diagonalizations. We note that the configuration of the lower 0^+ state is predominantly (85 %) s^2. This is in agreement with conclusions from experiments [15]. The second 0^+ $(sd)^2$ state is predicted to be reasonably far away (4.8 MeV) from the g.s. I expect the $(sd)^4$ configuration to be only a minor component of the physical g.s., but it could be important for the second 0^+ state. The first 2^+ state is dominated (92%) by the configuration *ds*, and the predicted 2^+ states are reasonably close together, certainly close enough (2.2 MeV) that the first $(sd)^4$ 2^+ state is probably above both.

III. DECAYS

Aksyutina *et al.* [7] detected ^{12}Be + n + n in coincidence following inelastic scattering of ^{14}Be from hydrogen. They analyzed fractional energy distributions as functions of ε_{fn} and ε_{nn}. For the first two 2^+ states, they assumed "democratic" decay (no ^{13}Be intermediate state) and did the analysis in terms of the ℓ values of the two neutrons, allowing combinations of *dd* and *ds*. They did not include *pp* decays because of their assumption that the $p_{3/2}$ orbital was filled in the g.s. of ^{10}Be. However, in the wave functions of Cohen and Kurath [16], for example, of the two *p*-shell neutron vacancies in ^{10}Be(g.s.), only 1.2 are $p_{1/2}$, with the other 0.8 being $p_{3/2}$. Furthermore, the δ term in ^{12}Be(g.s.) (if present) will also allow

TABLE II. Energy splittings and wave functions of the lowest $(sd)^2$ states in ^{14}Be.

J^π	Wave function		δE (MeV)
0^+	0.85 s^2	0.15 d^2	4.81
	0.15 s^2	0.85 d^2	
2^+	0.92 *ds*	0.08 d^2	2.20
	0.08 *ds*	0.92 d^2	

pp decays through an amplitude that is δ times the amplitude for $^{12}\mathrm{Be}_{1p}$(g.s.) → $^{10}\mathrm{Be}_{1p}(2^+)$. Refitting the $2n$ decay data with *pp* decays included might provide an estimate of the magnitude of this δ term. Nevertheless, I use here the results of Ref. [7].

For the analysis of the first 2^+ state, Aksyutina *et al.* [7] obtained a *dd* contribution of 41(7)%, and they obtained a *dd* contribution of 9(4)% for the second 2^+ state. They concluded that they had confirmed the $(0d_{5/2})^2$ nature of the first 2^+ state and had determined $(0d_{5/2})(1s_{1/2})$ for the second. (They credit Refs. [3,4] for the claim that the first 2^+ state is d^2, but I do not find that statement in either reference.) These results might appear to be in conflict with the wave functions of Table II, but I show here that the numerical results do not conflict. It is important to realize that, if decay is forbidden from the major component of the wave function, decay can take place through even a quite small component, because $2n$ decay will still overwhelm γ decay. In the present case, $2n$ decay from the first $(sd)^2$ ^{14}Be 2^+ state cannot proceed to the major $(sd)^2$ component of ^{12}Be(g.s.), but only to the $^{12}\mathrm{Be}_{1p}$ component. With the wave function from Table II, the ratio would be $dd/ds = 0.08/0.92$. But, I show that a small amount of the $(sd)^4$ configuration remedies the problem. Let the wave function of the first 2^+ state be

$$^{14}\mathrm{Be}(2_1^+) = A\ ^{12}\mathrm{Be}_{1p} \otimes (sd)^2_{2+} + \mathrm{B}\ ^{10}\mathrm{Be}_{1p} \otimes (sd)^4_{2+},$$

with $A^2 + B^2 = 1$.

[Another term that could be present is $^{10}\mathrm{Be}_{1p}(2^+) \otimes (sd)^4{}_0$. However, as we shall see, the second term above turns out to be small, so that this possible third component should be quite small. Additionally, it cannot decay to the first three terms of the g.s. of ^{12}Be via $2n$ emission.] The dd/ds ratio for the first term of ^{14}Be(2^+) is as in Table II. The $(sd)^4$ component has

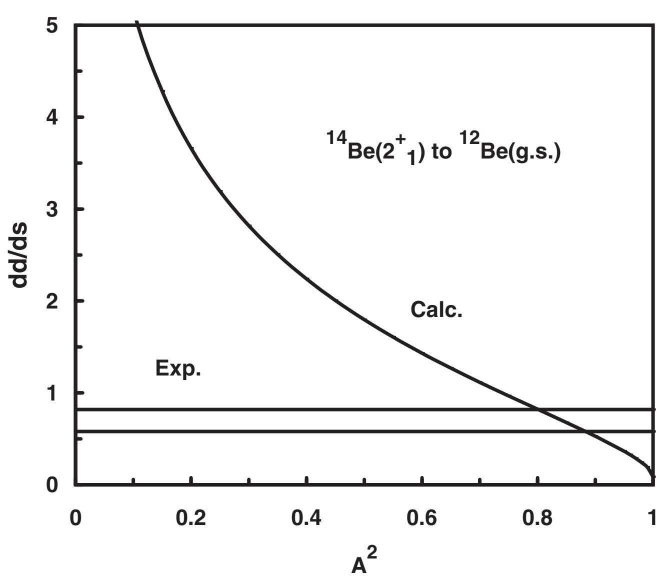

FIG. 1. For $2n$ decay of the first 2^+ state of ^{14}Be, the calculated dd/ds ratio is plotted vs A^2, the amount of $(sd)^2$ in the decaying state. The horizontal lines represent the experimental result [7]. Calculation and experiment agree for $A^2 = 0.84(4)$.

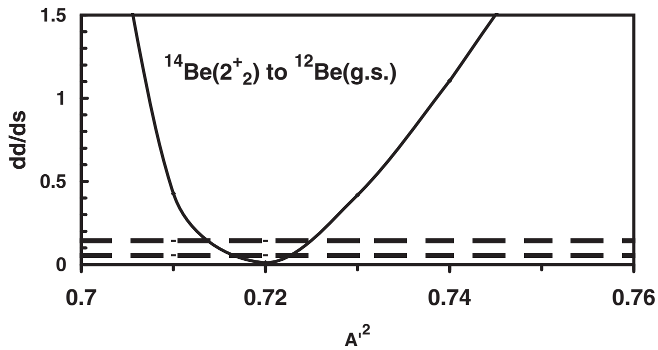

FIG. 2. Same as Fig. 1, but for the second 2^+ state of ^{14}Be. The calculated dd/ds ratio is plotted (solid curve) vs A'^2, the amount of $(sd)^2$ in the decaying state. The dashed horizontal lines represent the experimental ratio, implying $A'^2 \sim 0.72$.

configurations s^2d^2, sd^3, and d^4. The first and third can decay via dd to the s^2 and d^2 terms, respectively, in ^{12}Be(g.s.). The second term will decay via ds to the d^2 term. With a simple $(sd)^4$ wave function, the calculated dd/ds ratio varies with A^2 as plotted in Fig. 1. We thus see that a very small [16(4)%] admixture of the $(sd)^4$ component in the dominantly $(sd)^2$ 2^+ state reproduces agreement with the measured ratio. The decay data are thus in agreement with a wave function for the first 2^+ state that is predominantly of ds character.

For the second 2^+ state, the very small amount of dd decay requires destructive interference between two major amplitudes. As above, let the wave function of the second 2^+ state be

$$^{14}\mathrm{Be}(2_2^+) = A'\,^{12}\mathrm{Be}_{1p} \otimes (sd)^2_{2^+2} + B'\,^{10}\mathrm{Be}_{1p} \otimes (sd)^4_{2^+},$$

with $A'^2 + B'^2 = 1$.

With destructive interference, the dd amplitude will vanish for some value of A', as depicted in Fig. 2. Thus, to fit the measured dd intensity of 9(4)% (dashed horizontal lines in Fig. 2) would require A'^2 in the vicinity of $A'^2 \sim 0.72$.

The experiment identified another resonance, at E_x = 5.25(19) MeV. A 3^+ $(sd)^2$ state is expected near the position of the second 2^+ state, but it should be somewhat weak in inelastic scattering. The authors suggest that one possibility is 3^-. If so, it would be outside the present model.

IV. SUMMARY

In a simple model, I have computed the energy splittings and wave functions of the 0^+ and 2^+ $(sd)^2$ states in ^{14}Be. I have used information from $2n$ decays of the 2^+ states [7] to estimate the amount of the $(sd)^4$ configuration in the predominantly $(sd)^2$ states. Results are 16(4)% for the first 2^+ state and ∼28% for the second. If the third resonance observed in Ref. [7] is neither 3^+ nor 0^+, it is outside the present model. The first 2^+ state is found to be predominantly of ds structure, not dd as claimed in Ref. [7].

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