Magnetic properties of ^{177}Hf and ^{180}Hf in the strong-coupling deformed model

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This paper reports NMR measurements of the magnetic dipole moments of two high-K isomers, the $37/2^-$, 51.4 m, 2740 keV state in ^{177}Hf and the 8^-, 5.5 h, 1142 keV state in ^{180}Hf by the method of on-line nuclear orientation. Also included are results on the angular distributions of γ transitions in the decay of the ^{177}Hf isotope. These yield high precision $E2/M1$ multipole mixing ratios for transitions in bands built on the $23/2^+$, 1.1 s, isomer at 1315 keV and on the $9/2^+$, 0.663 ns, isomer at 321 keV. The new results are discussed in the light of the recently reported finding of systematic dependence of the behavior of the g_R parameter upon the quasiproton and quasineutron make up of high-K isomeric states in this region.

I. INTRODUCTION

This paper reports measurement of the magnetic dipole moments of the 51.4 m, 2740 keV, $37/2^-$ isomer in ^{177}Hf and of the 5.5 h, 1142 keV, 8^- isomer in ^{180}Hf, using the method of NMR on oriented nuclei, at the on-line NICOLE facility, ISOLDE, CERN. In addition, measurements of the angular distribution of γ transitions from the ^{177}Hf^{m2} isomer are analyzed to give precise values of the $E2/M1$ mixing ratios in transitions in bands built upon the $23/2^+$, 1.1 s, 1315 keV isomer and the $9/2^+$, 321 keV state of this nucleus.

Accurate measurements of the electromagnetic properties of ground states, isomers, and rotational band states in deformed nuclei yield parameters of the deformed nuclear model used to describe these nuclei. In the region between Yb and W ground states and isomers have close-to-constant deformation and the parameters of the model can be explored thoroughly. In this work the main focus is on isomeric state, band-head, magnetic dipole moments to yield values of the single particle g factor, g_K, with good precision. In addition, analysis of the angular distribution of mixed $M1/E2$ transitions in the decay of the isomers studied yields values of the mixing ratio δ which is related to the g factors g_K, and g_R, and intrinsic quadrupole moment Q_0, of the band head [see Eq. (4)].

Interest in obtaining the magnetic parameters g_K and g_R in these states derives from their involvement with ideas of superconductivity in nuclei. In the strong coupling model of

well-deformed nuclei the influence of the quasiparticle state upon the band properties arises through the dependence of the pairing strength, and hence the pairing gap Δ, upon the available orbitals close to the Fermi surface. In the deformed potential each state is doubly degenerate and is available for pair scattering if empty, however the occupation of a state by a single quasiparticle renders it unavailable to pair scattering. This is the process of blocking [1,2]. Since the pairing gap is determined by the number of pairs and the numbers of states between which they can scatter, increasing the number of quasiparticles increases blocking and reduces the gap Δ.

These considerations apply separately to protons and neutrons. Where the rotational energy levels of the bands are concerned, the relevant model parameter is the total moment of inertia $\mathcal{I}_{tot} = \mathcal{I}_p + \mathcal{I}_n$. The stronger the pairing associated with the state the more $\mathcal{I}_{tot}$ is reduced compared to the classical rigid body value. Blocking of either proton or neutron orbitals weakens the pairing and increases $\mathcal{I}_{tot}$. When electromagnetic properties of the band are considered, transition matrix elements, intensity ratios, multipole mixing ratios and the collective g factor g_R show different sensitivity to proton and neutron pairing, and the effects of blocking, through their differing effective charges, crudely 1 for protons and 0 for neutrons. In this limit g_R is given by the ratio $\mathcal{I}_p/\mathcal{I}_{tot}$, which yields the familiar simple result $g_R \sim Z/A$ for rigid rotation of the whole nucleus. It follows from these basic ideas that we can expect any increase in, for example, neutron blocking, by reducing the neutron pairing gap and increasing the neutron contribution to $\mathcal{I}_{tot}$, to reduce g_R relative to Z/A, while increased proton blocking will have the opposite result.

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*Deceased.

The new experimental results presented here, in combination with a much larger body of data from existing literature, have been used recently to explore these concepts [3]. It has been shown that, within the rather limited data presently available, the principle of additivity can be relied upon to give good estimates of the single particle g factor, g_K, in multi-quasi-particle isomers. The use of this principle to estimate g_K in many bands revealed a wide, systematic, dependence of the collective g factor, g_R, upon the neutron and proton quasiparticle make-up of the band-head states, presenting a new challenge to the theory of superconductivity, pairing and blocking in these deformed nuclei. This finding gives additional reason to extend, in particular, band-head magnetic moment measurements of good accuracy and to seek accurate mixing ratios to yield g_R, as are reported here.

The theoretical context of this work is given in Sec. II. Experimental details and new data on the angular properties of gamma transitions in the decay of the $37/2^-$ K isomer of ^{177}Hf are described in Secs. III and IV together with NMR results which yield the magnetic dipole moments of the $37/2^-$ K isomer and of the 8^- isomer in ^{180}Hf. Consistency of the new results with the findings of [3] is discussed in Sec. V.

II. THEORETICAL FRAMEWORK

Relevant expressions from the strong coupling model concerning electromagnetic properties of the nuclear states and transitions are given here [4]. For the static magnetic dipole moment of a band state of spin I based on a band head of spin K

$$\mu = g_R I + (g_K - g_R)\frac{K^2}{I+1} \tag{1}$$

which, for the band head $K = I$, gives

$$\mu = g_K[I^2/(I+1)] + g_R[I/(I+1)]. \tag{2}$$

Since the values of spin I in this paper are large, this expression makes it clear that the moment has only weak dependence upon the 'collective g_R factor and is largely determined by the quasiparticle g_K factor.

For a pure multi-quasi-particle state for which K is the simple sum of the constituent quasiparticle K_i's, the quasiparticle g factor g_K is given in terms of the constituent quasiparticle g_{K_i} factors by

$$g_K = \frac{1}{K}\sum_i K_i g_{K_i}. \tag{3}$$

Two spectroscopic variables, the $E2/M1$ ratio δ in transitions from a level of spin I to a lower level of spin $I-1$ and the branching ratio of transitions from a state of spin I to lower states of spin $I-1$ and $I-2$, depend upon the difference $(g_K - g_R)$ of the two (collective and quasiparticle) g factors. These variables can both be written in terms of δ given by

$$\delta = \frac{0.933 E_\gamma Q_0}{(g_K - g_R)\sqrt{(I^2-1)}}, \tag{4}$$

where E_γ is the transition energy in MeV and Q_0 is the intrinsic quadrupole moment in eb. From these expressions it is seen

that to separate the quasiparticle g factor g_K from the rotation g factor g_R it is necessary to have data on either transition branching ratios from states in the band to lower states of spin reduced by one and two units or $M1/E2$ mixing ratios for transitions between states of spin differing by one, combined with the static magnetic dipole moment of a state of the band. Of these, the static dipole moments are the rarest. In addition, knowledge of the intrinsic quadrupole moment Q_0 of the band states is required.

III. EXPERIMENTAL DETAILS

A. Nuclear orientation of the $37/2^-$, 51.4 m, K isomer in ^{177}Hf

At the ISOLDE isotope separator facility, CERN, new results have been obtained on the magnetic moment of the $37/2^-$ high-spin K isomer and on γ transitions in its decay to the ground state by the method of on-line nuclear orientation combined with NMR [5]. The 1.4 GeV proton beam from CERN's PSB synchrotrons was incident upon a mixed tantalum/tungsten foil target and separation of the Hf isotopes was achieved by introducing CF_4 gas into the plasma ion source and extracting a beam of HfF_3^+ ions [6]. The ions, accelerated through 60 keV, impinged upon a pure iron foil soldered to the cold finger of the NICOLE on-line nuclear orientation system [7] maintained at temperatures down to about 12 mK. The HfF_3^+ ions disintegrated at the foil surface and the Hf activity was implanted into the Fe lattice. By interrupting the implantation and allowing the dilution refrigerator to cool further, data were taken to about 6 mK as the activity decayed. The iron foil was magnetized by an applied magnetic field of 0.5 T.

At mass $A = 177$ the Hf beam contains both the isomer ^{177}Hf^{m2} ($I^\pi = 37/2^-$ $T_{1/2} = 51.4$ m) and stable ^{177}Hf ground state nuclei. The isomer activity made up about 1% of the beam. The decay scheme of ^{177}Hf^{m2} is shown in Fig 1. Angular distribution parameters for all transitions were obtained from measurements, as a function of the foil temperature, of the γ photopeak count rates in two pairs of high resolution Ge detectors placed at angles of 0, 90, 90, and 180 degrees to the axis of polarization. The counts were normalized to rates measured with the activity unoriented at a temperature close to 1 K. The foil temperature was obtained from measurements of the angular distributions of the 1333 keV and 1173 keV transitions in the decay of ^{60}Co activity in a Co single crystal soldered to the back of the cold finger. Since, for the ^{60}CoCo system, all orientation parameters are known, these measurements act as an accurate thermometer in the region between about 50 mK and 5 mK.

High degrees of polarization were achieved, produced by the large hyperfine field experienced by the Hf activity in the iron lattice. Examples of cold, oriented, gamma spectra compared with warm (1 K, unoriented) spectra in 0° and 90° detectors are shown in Fig. 2, in which the strongly differing anisotropies in transitions between levels in different bands of the decay are clearly seen.

In addition to γ transition angular distribution measurements, NMR of the implanted ^{177}Hf^{m2} parent state activity was sought by exposing the implanted nuclei to an RF field,
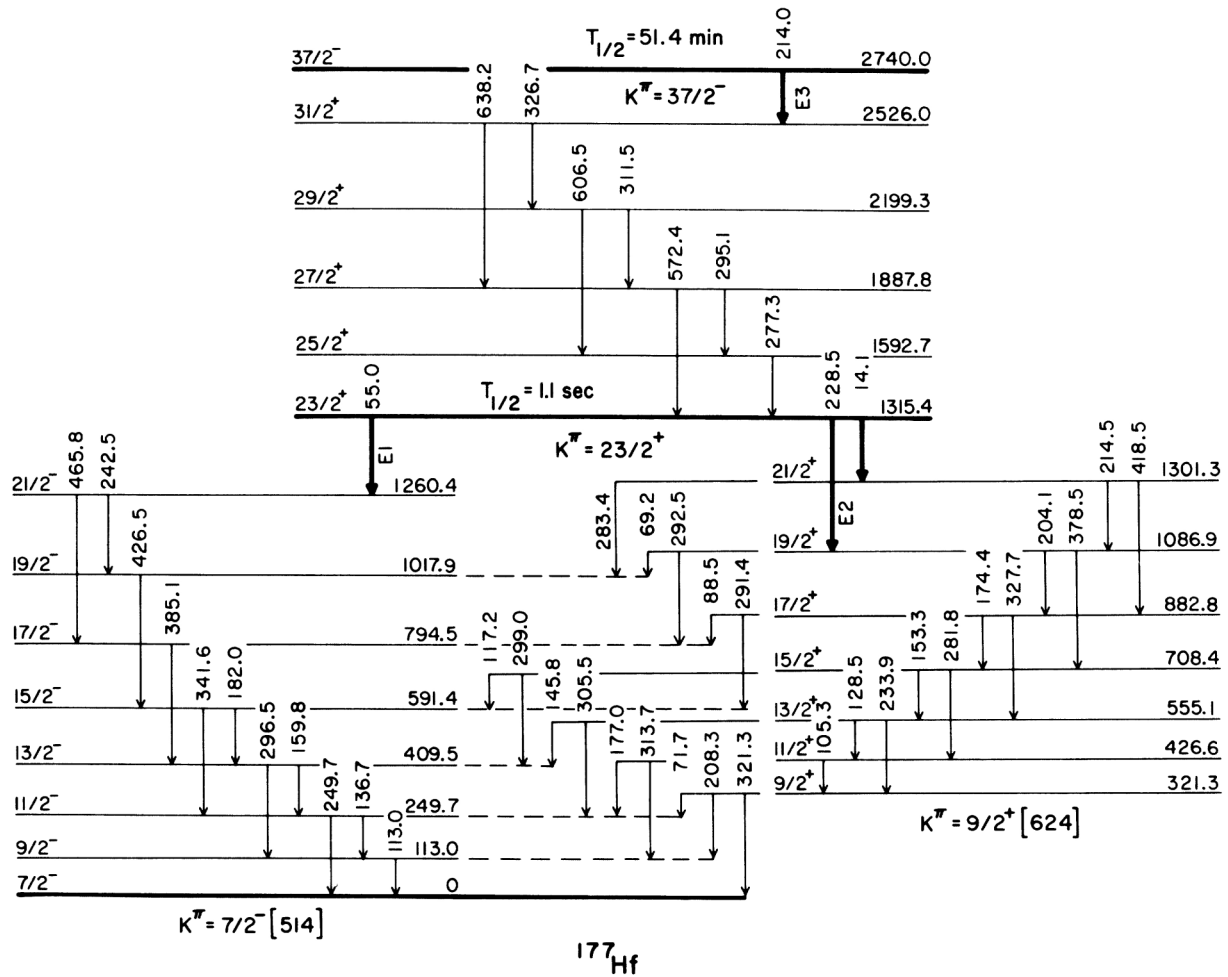
FIG. 1. ^{177}Hf^{m2} decay scheme. Taken from [19].

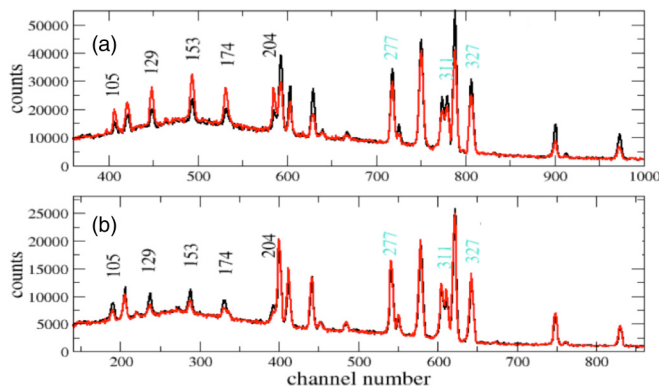transverse to the axis of polarization, produced by a simple two-turn coil.

FIG. 2. (Color online) Warm and cold spectra from $0°$ and $90°$ detectors for two series of $E2/M1$ transitions. Black/dark energy labels—in band built on $9/2^+$ [624] state—note increases at $0°$. Blue/pale energy labels—in band built on $23/2^+$ three-quasiparticle isomer—note decreases at $0°$.

B. Hyperfine interaction and nuclear spin-lattice relaxation time estimates

Implanted nuclei have a high probability of coming to rest in regular lattice sites and the well-defined hyperfine interaction allows observation of nuclear magnetic resonance by the disturbance caused to the gamma decay angular distribution by resonant absorption of RF energy. However even though the foil lattice may be at millikelvin temperature the nuclei are implanted unoriented and it takes a time, related to the nuclear spin-lattice relaxation time, T_1 for nuclear polarization to be established [8]. Both the resonant NMR frequency, ν_{res} and T_1 depend upon the strength of the hyperfine structure splitting of the state undergoing resonance and/or orientation, $h\nu_{res} = \mu B_{hf}/I$.

The nuclear moment of the $37/2^-$ isomer can be estimated to be $\sim 7\mu_N$ (see Sec. IV E below) and the hyperfine field at Hf in Fe is $B_{hf} = -67.4(9)$ T ([9] improved using ^{175}Hf moment 0.677(9) [10]). These values yield ν_{res} in the region of 200 MHz. There have been no spin lattice relaxation time measurements for Hf isotopes in iron, however an empirical relationship, $C_K T_{int}^2 = 1.4 \times 10^{-4}$ sK^3, where T$_{int}$ is the nuclear interaction strength, $T_{int} = \mu B_{hf}/kI$, can be used to give estimates of the Korringa constant C_K, usually

found reliable to better than a factor of two [8]. The resulting prediction is $T_1(37/2^-) \sim 19$ s at 10 mK which is much less than the isomer lifetime of 51.4 m, so full thermal equilibrium between the ^{177}Hf^{m2} nuclei and the lattice will be established prior to decay.

There remains the possibility that orientation may be perturbed by further interaction with the lattice during the lifetime of the 1.1 s $23/2^+$ isomeric state at 1315 keV. Any such re-orientation would affect observed anisotropies in transitions below the isomer. However, an estimate of T_1 for the $23/2^+$ isomer, based on an estimated $23/2^+$ moment and the above T_1 $(37/2^-)$ estimate gives the value $T_1 (25/2^+) \sim 8$ s at 10 mK. This is substantially longer than the lifetime of the isomer, indicating that any reorientation effect will be small.

C. Nuclear orientation of the 8^-, 5.5 h, 1142 keV, state in ^{180}Hf

At mass 180 the Hf beam has a strong isomeric ^{180}Hf component which decays by mixed $E1/M2/E3$ transitions to 8^+ and 6^+ states in the ground state band. These states decay by a sequence of pure $E2$ transitions which show strong anisotropy as has been reported previously [11]. Earlier measurements of the magnetic dipole moment of this isomer have not been of high accuracy. Körner *et al.* [12] reported 8.7(10) μ_N by a Mössbauer method and Krane *et al.* [13] 9.0(9) μ_N by an integral nuclear orientation experiment on sources of ^{180}Hf^m in HfZrFe$_2$ alloys. Since NMR on the implanted activity would provide a more precise result for the moment, resonance was sought with the activity implanted into an Fe foil. Unfortunately, at the high frequency at which resonance is expected for this isomer (in the region of 520 MHz) the RF line into the Nicole dilution refrigerator shows strong standing wave resonances which result in non-nuclear-resonance heating of the sample and render true resonance unobservable. In a second experiment the activity was implanted into a pure Ni foil in which the magnetic hyperfine field at Hf nuclei is much reduced. Resonance was observed at a frequency of 92.2(2) MHz in an applied field of 0.10 T. The result is discussed further below.

IV. ANALYSIS OF RESULTS

A. γ transition anisotropies

Photopeak counts for each transition in each spectrum for each γ detector were obtained using the fitting code DAMM. Counts taken with the sample at temperatures with appreciable orientation were normalized by taking ratios with counts from unoriented, warm (1 K), spectra. The gamma transition anisotropies, A, were obtained for each of the two pairs of detectors using the expressions

$$W(\theta, T) = N(\theta, T)/N(\theta, \text{warm}) \tag{5}$$

and

$$A = [W(0^0, T)/W(90^0, T) - 1]. \tag{6}$$

The anisotropies were analysed further using the standard nuclear orientation formalism [14]

$$W(\theta, T) = 1 + f \sum_\lambda B_\lambda U_\lambda A_\lambda Q_\lambda P_\lambda(\cos \theta). \tag{7}$$

Here f is the fraction in good sites. The B_λ factors describe orientation of the parent, isomeric state and depend upon the strength of the hyperfine interaction, the spin and the temperature. The U_λ factors are calculated for each state below the parent and require knowledge of the spins of any intervening states, the intensities of all transitions between them and their multipolarity, with admixtures treated without regard to phase. The A_λ parameters describe the observed emission and depend upon the initial and final state spins and the multipolarity, treated with sensitivity to phase of any multipole mixing ratio δ. Q_λ are correction factors which account for the finite solid angle subtended by the detectors and P_λ are the associated Legendre polynomials. Since γ emission conserves parity, the summation has only even terms (up to $\lambda = 2L_{\max}$, where $L_{\max}$ is the highest multipolarity in the observed emission).

In all on-line nuclear orientation experiments it has been found that the angular distributions can be well described as consisting of two components, the first from nuclei coming to rest in undisturbed (good) lattice sites and subject to the full hyperfine interaction and the second unoriented at all temperatures. The unoriented fraction describes nuclei which are stopped in the thin oxide layer which is always present on the implantation foil surface or are stopped in other very disturbed regions which are not magnetically ordered.

B. Analysis of the pure $E2$, $\Delta I = 2$, transitions: Determination of the fraction in good sites

The first transition emitted by the $37/2^-$ isomer is the 214 keV pure $E3$ transition. However this is an unresolved doublet with a mixed transition lower in the decay scheme so could not be used to evaluate the fraction f. The U_λ values for this pure multipole transition are known, however, so f can be obtained from the anisotropy of the pure $E2$, 638 keV, $(31/2^+-27/2^+)$ transition. As the other parameters necessary to calculate U_λ for lower states can all be extracted from the experimental data, values for f can also be found from data on the other $E2$ transitions between pairs of lower states built on the $23/2^+$ K isomer. The results are given in Table I. As an example, in Fig. 3 are shown the data for 606.5 keV transition

TABLE I. Fraction f of implanted Hf nuclei in good sites as extracted from fitting $E2$ transitions with known decay parameters.

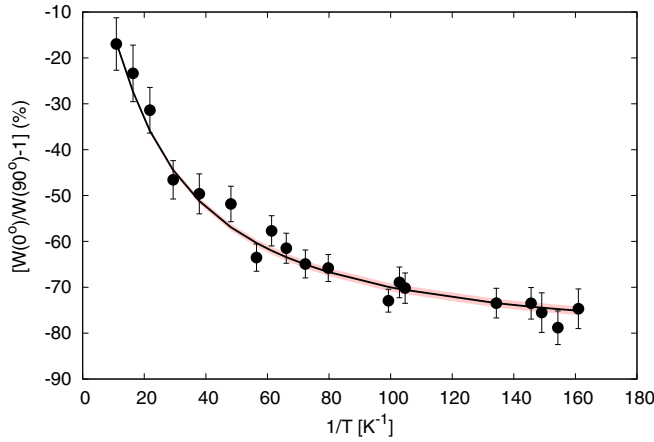| Transition		Energy	Fitted fraction f
I_1	I_2	(keV)	
Above the $23/2^+$ isomer			
31/2	27/2	638.2	0.755(7)
29/2	25/2	606.5	0.786(12)
27/2	23/2	572.4	0.768(12)
Below the $23/2^+$ isomer			
23/2	19/2	228.5	0.773(6)
21/2	17/2	418.5	0.773(13)
19/2	15/2	378.5	0.773(6)
15/2	11/2	281.8	0.779(9)
13/2	9/2	233.9	0.802(21)

FIG. 3. (Color online) Anisotropy of the 606.5 keV $29/2^+$–$25/2^+$ pure $E2$ transition in band built on $23/2^+$ isomer. Fitted curve with shaded band is for $f = 0.786(12)$.

between the $29/2^+$ and $25/2^+$ states and calculation using the fit f value, with its uncertainty.

C. Reorientation in the $23/2^+$ isomeric state

This state has lifetime 1.1 s and, although estimates given above suggest that the spin-lattice relaxation time T_1 of this state is considerably longer than the lifetime, this point requires further attention if anisotropies of γ transitions between states below the isomer are to be quantitatively analyzed. Data on anisotropies of the series of pure $E2$ transitions between states below the $23/2^+$ isomer have all be treated to yield values of f on the assumption that reorientation may be neglected. The results are given in Table I. The close agreement between values of f obtained from transitions above and below the isomer show clearly that the assumption is indeed valid. In Fig. 4 the data on the 228.5 keV transition between the $23/2^+$
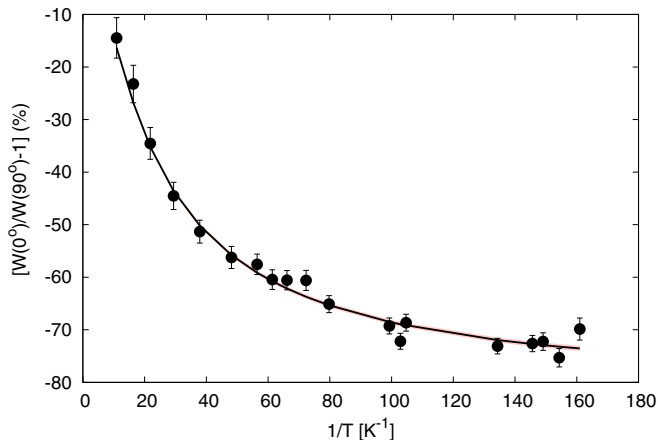

FIG. 4. (Color online) Anisotropy of the 228.5 keV $23/2^+$–$19/2^+$ pure $E2$ transition from the $23/2^+$ isomer to the band built on the $9/2^+$ [624] band. Fitted curve with shaded band is for $f = 0.773(6)$.

TABLE II. $E2/M1$ mixing ratios in transitions from levels of ^{177}Hf.

K	Level spin	E_γ (keV)	δ(In beam) [15]	$\delta(^{177}$Lu) [16]	$\delta(^{177}$Hf) this work	$(g_K - g_R)/Q_0$
37/2	41/2	420.9	0.74(23)			0.025(8)
	43/2	440.0	0.74(16)			0.025(5)
23/2	25/2	277.3			0.302(4)	0.069(1)
	27/2	295.1	0.31(2)			
	29/2	311.5	0.36(2)		0.285(5)	0.071(2)
	31/2	326.5	0.30(1)		0.278(5)	0.071(2)
	33/2	340.1	0.26(1)			
	35/2	351.9	0.21(1)			
	37/2	361.7	0.18(2)			
	39/2	369.4	0.20(2)			
9/2	11/2	105.3		−0.36(4)	−0.23(4)	−0.083(15)
	13/2	128.4	0.38(1)	−0.37(6)	−0.34(3)	−0.055(5)
	15/2	153.1	0.38(1)	−0.33(5)	−0.317(13)	−0.061(3)
	17/2	174.3	0.36(1)	−0.32(4)	−0.296(13)	−0.065(3)
	19/2	204.1	0.40(1)	−0.33(5)	−0.289(13)	−0.070(4)
	21/2	214.3	0.35(1)	−0.29(2)		
	23/2	260.5	0.50(2)			
	25/2	241.8	0.40(2)			
	27/2	325.4	0.61(7)			
	29/2	249.4	0.27(2)			

and $19/2^+$ states are shown, with calculation using the fit f value, with its uncertainty.

The data from both pairs of detectors were in very close agreement. An average value $f = 0.775(5)$ was adopted and used throughout the analysis.

D. Analysis of the mixed multipole $\Delta I = 1$ transitions: Determination of the $E2/M1$ mixing ratios

Each state in the bands built upon the $23/2^+$ multi-quasi-particle isomer and the $9/2^+$ and $7/2^-$ single quasiparticle states decays both by a direct $E2$, $\Delta I = 2$, transition and a mixed $E2/M1$, $\Delta I = 1$, transition. The $E2/M1$ multipole
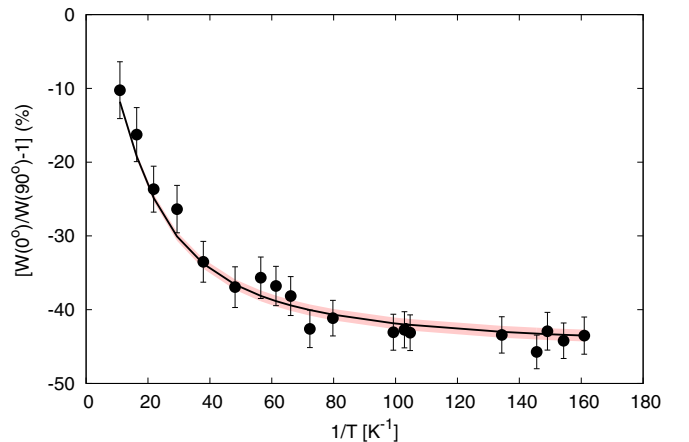

FIG. 5. (Color online) Anisotropy of the 277.3 keV $25/2^+$–$23/2^+$ mixed $E2/M1$ transition in band built on $23/2^+$ isomer. Fitted curve with shaded band is for $\delta [E2/M1] = +0.302(4)$.
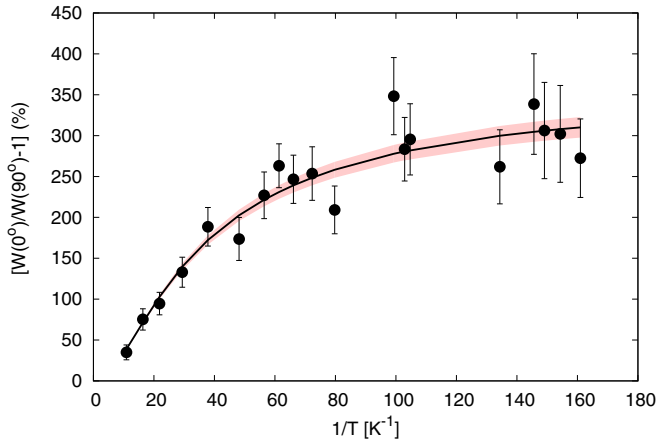
FIG. 6. (Color online) Anisotropy of the 153.1 keV $15/2^+$–$13/2^+$ mixed $E2/M1$ transition in band built on $9/2^+$ [624] band. Fitted curve with shaded band is for $\delta\,[E2/M1] = -0.317(13)$.

mixing ratio, δ, is a most useful quantity in establishing the parameters of the band, as described previously. For each of these transitions the anisotropies were analysed in a somewhat novel way to give the best fit values of δ. Each data point in the temperature dependence (19 in all) was regarded as an individual experiment and fitted with δ as the only unknown. The resulting set of δs was treated statistically to give the best fit value and the error on this value. Results are shown in Table II, compared to results from other investigations [15,16]. The overall agreement is excellent and the new results have significantly smaller experimental uncertainties. Examples of the fits are shown in Figs. 5 and 6. Figure 5 shows data for the 277.3 keV transition between the $25/2^+$ and $23/2^+$ states in the $23/2^+$ band. All mixed transitions in this band show similar negative anisotropies. Figure 6 shows data for the 153.1 keV transition between the $15/2^+$ and $13/2^+$ states in the $9/2^+$ band. All mixed transitions in this band show similar large positive anisotropies. The larger scatter and uncertainties for these transitions arise because they are measured on a large Compton-backscattered γ intensity so that the small $90°$ counts at lower temperatures have large uncertainties (see Fig. 2).
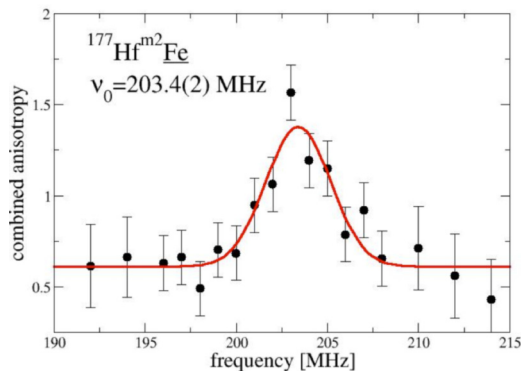

FIG. 7. (Color online) Resonance observed in a combined signal from the stretched $E2$ transitions in the decay of the $37/2^-$ isomer in ^{177}Hf.
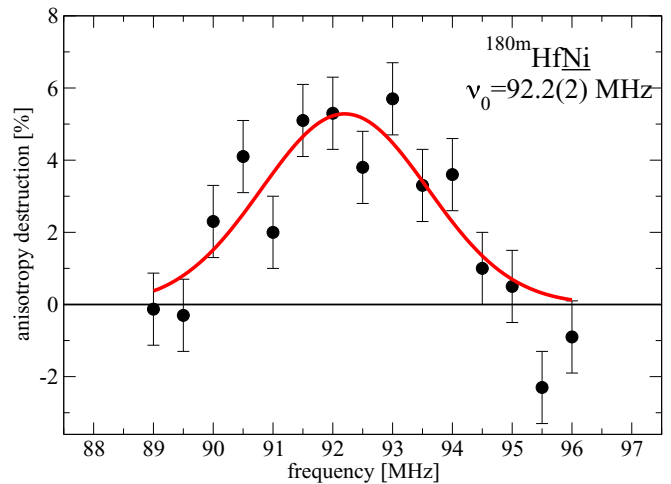

FIG. 8. (Color online) Resonance observed in percentage destruction of anisotropy of the 8^- isomer in ^{180}Hf.

E. The magnetic moment of the $37/2^-$ isomer in ^{177}Hf

The search for NMR of the $37/2^-$ isomer was carried out with RF swept between 180 and 230 MHz in 1 MHz steps with RF modulation of 1 MHz. A positive signal was found, as shown in Fig. 7, with center frequency

$$\nu_{\mathrm{res}}(^{177}\mathrm{Hf^{m2}}\underline{\mathrm{Fe}}) = 203.4(2)\,\mathrm{MHz}. \qquad (8)$$

This frequency, combined with the known hyperfine field $-67.4(9)$ T, adjusted for the external field of 0.100 T applied to the Fe foil during the NMR measurements, yields the magnitude of the magnetic moment of the $37/2^-$ isomer as

$$|\mu|(^{177}\mathrm{Hf^{m2}},37/2^-,51.4\mathrm{m}) = 7.33(9)\ \mu_N. \qquad (9)$$

Note that any hyperfine anomaly between the isotope ^{175}Hf, on which the hyperfine field measurement was made, and ^{177}Hf^{m2} is not expected to exceed 0.1% and has been neglected.
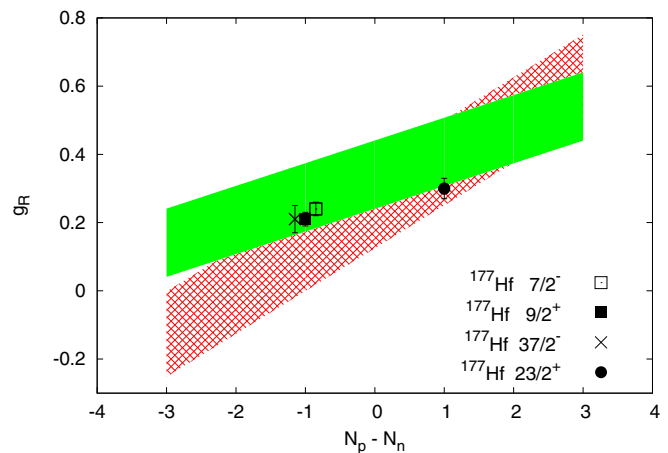

FIG. 9. (Color online) Plot of the collective g_R as a function of $\Delta = N_p - N_n$. For ease of viewing the three data points at -1 have been slightly off-set along the x axis. For discussion see text and Ref. [3].

TABLE III. Adopted g_K value for individual quasiparticle states. Experimental data taken from [18].

Quasiparticle state	Adopted g_K	Basis of adopted value
Protons		
$7/2^+$[404]	0.765(25)	moments of $7/2^+$ ground states in ^{175,177,179,181}Ta
$9/2^-$[514]	1.37(3)	moment of $9/2^-$ 6 keV state in ^{181}Ta
Neutrons		
$5/2^-$[512]	$-0.48(2)$	moment of $5/2^-$ ground state of ^{175}Hf
$7/2^-$[514]	0.206(14)	moment of $7/2^-$ ground state of ^{177}Hf
$9/2^+$[624]	$-0.239(11)$	moment of $9/2^+$ ground state of ^{179}Hf

F. The magnetic moment of the 8 isomer in ^{180}Hf

Extraction of the magnetic moment from the observed resonance (Fig. 8) requires knowledge of the hyperfine field acting on Hf nuclei in the Ni lattice. Several previous values for this field have been reported, none of them of the necessary accuracy. The best would appear to be the Mössbauer study by Aggarwal *et al.* [17] who gave the value $B_{hf}(Hf\underline{Ni}) = 11.8(2.6)$ T. Using this value the observed ^{180}Hf^m resonance yields the moment of the isomer to be $8.3(1.8)\mu_N$. Other, lower, reported Hf\underline{Ni} fields give unrealistically high values for the nuclear moment. While the hyperfine fields at some neighboring elements have better measurements in nickel they do not allow an improved estimate for the field at the Hf nucleus. Clearly as it stands the present value is no improvement on the earlier results and the observed NMR result awaits an improved field value. This can be obtained, for example, by the observation of NMR in ^{177}Hf^{m2} activity in Ni where the frequency should be in the region of 35 MHz.

V. DISCUSSION

The variation in g_R found in the analysis of this work (see below) prompted, in part, a comprehensive survey of nearly 100 high-K bands in isotopes in this region [3]. The results have been recently published and draw two significant conclusions: that the assumption of additivity of quasiparticle contributions is a sound basis for estimation of the quasiparticle g factor g_K in multi-quasi-particle isomers in the majority of cases and that, based on analysis of the mixing ratio and branching ratio data, there is a wide and systematic variation of the collective g factor g_R which depends upon the quasiparticle make-up of the band head isomer. Underlying ideas concerning pairing and blocking in these isomers are discussed in the paper. In this section we review the extent to which the experimental results found in this work are consistent with the two conclusions described.

A. Additivity of single quasiparticle g_K factors to form g_K for multi-quasi-particle states

In Table III we give the g_K factors adopted in the broad survey for those single-quasiparticle states which are relevant to the two isomers ^{177}Hf^{m2} and ^{180}Hf^m for which the magnetic moments have been measured in this work. The g_R factor makes a small contribution to the total magnetic moment since it is multiplied by the factor $I/(I + 1)$ as compared to $I^2/(I + 1)$ for g_K [see Eq. (2)] and the spins of the two isomers are high, 37/2 and 8, respectively. Taking g_R as 0.29(5) and assuming additivity, as described by Eq. (3) yields predicted moments of these isomers of $7.28(13)\mu_N$ and $8.12(15)\mu_N$ to be compared to the measured values, $7.33(9)\mu_N$ and $8.3(18)\mu_N$. Both results show agreement with the predictions based on additivity, consistent with the findings in [3].

B. The variation of g_R

Taking average values of the parameter $(g_K - g_R/Q_0)$ with $Q_0 = 7.2$ eb and g_K assuming additivity in the multi-quasi-particle states, values of g_R were extracted from this work for bands built on the $9/2^+$, $23/2^+$, and $37/2^-$ states in ^{177}Hf. Results are given in Table IV which also includes g_R for the band built on the $7/2^-$ ground state of ^{177}Hf. We reproduce in Fig. 9 the evidence of variation of g_R as a function of the difference, $N_p - N_n$, of the numbers of unpaired quasiprotons and quasineutrons in the isomer reported in Ref. [3], showing the broad band ranges which, it is suggested, are caused by variation in the contributions from breaking specific quasiparticle pairs and the individual results from Table IV.

VI. CONCLUSIONS

This paper reports new measurements of the magnetic properties of the isotopes ^{177}Hf and ^{180}Hf and their high-K isomers. The magnetic moments of the $37/2^-$ isomer in

TABLE IV. g_R values extracted from band-head moments and band spectroscopic data. Q_0 taken as 7.2 eb throughout analysis.

Isotope	State	Energy (keV)	N_p	N_n	$\Delta = N_p - N_n$	Moment Reference	Spectroscopy Reference	g_R
^{177}Hf	$37/2^-$	2740	2	3	-1	this work	[15]	0.21(4)
^{177}Hf	$7/2^-$	0	0	1	-1	[20]	[15]	0.24(2)
^{177}Hf	$9/2^+$	321	0	1	-1	[21]	this work	0.21(2)
^{177}Hf	$23/2^+$	1316	2	1	$+1$	additivity	this work	0.30(3)

^{177}Hf and the 8^- isomer in ^{180}Hf provide valuable additional evidence for the validity of additivity in estimating the quasiparticle g factor g_K in these deformed nuclei. Values of the collective g factor, g_R, obtained from detailed γ transition anisotropy measurements, are shown to be fully consistent with the recently revealed systematic dependence of this parameter upon the quasiparticle make-up of the bands involved.

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