PHYSICAL REVIEW C **89**, 035801 (2014)

Rapid spin deceleration of magnetized protoneutron stars via asymmetric neutrino emission

Tomoyuki Maruyama,[1,2] Jun Hidaka,[2] Toshitaka Kajino,[2,3] Nobutoshi Yasutake,[4] Takami Kuroda,[2] Tomoya Takiwaki,[2] Myung-Ki Cheoun,[2,5] Chung-Yeol Ryu,[6] and Grant J. Mathews[7]

[1]*College of Bioresource Sciences, Nihon University, Fujisawa 252-8510, Japan*

[2]*National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan*

[3]*Department of Astronomy, Graduate School of Science, University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113-0033, Japan*

[4]*Department of Physics, Chiba Institute of Technology, 2-1-1 Shibazono, Narashino, Chiba 275-0023, Japan*

[5]*Department of Physics, Soongsil University, Seoul 156-743, Korea*

[6]*General Education Curriculum Center, Hanyang University, Seoul 133-791, Korea*

[7]*Department of Physics, Center of Astrophysics, University of Notre Dame, Notre Dame, Indiana 46556, USA*

(Received 25 January 2013; revised manuscript received 9 August 2013; published 3 March 2014)

We estimate the maximum possible contribution to the early spin deceleration of protoneutron stars because of asymmetric neutrino absorption. We calculate the neutrino scattering in the context of a fully relativistic mean field theory and estimate for the first time the spin deceleration of neutron stars because of asymmetric neutrino absorption in a toroidal magnetic field configuration. We find that the deceleration can be much larger for asymmetric neutrino absorption in a toroidal magnetic field than the braking due to magnetic dipole radiation. Nevertheless, the effect is estimated to be less than the angular momentum loss due to the transport of magnetically locked material in the neutrino energized wind.

DOI: 10.1103/PhysRevC.89.035801 PACS number(s): 25.30.Pt, 26.60.−c, 24.10.Jv, 97.60.Jd

I. INTRODUCTION

Magnetic fields play an important role in many astrophysical phenomena. The observed asymmetry in supernova (SN) remnants, pulsar kick velocities [1], and the existence of magnetars [2,3] all suggest that strong magnetic fields affect the dynamics of core-collapse SN explosions and the velocity [4] that protoneutron stars (PNSs) receive at birth.

There are at least two major SN explosion scenarios leading to asymmetric morphologies in observed SN remnants. One of them is the standing accretion shock instability (SASI)-aided neutrino driven explosion [5,6]. The other is the magnetorotational explosion (MRE) [7,8]. Both mechanisms may also be a source for pulsar kick velocities [9,10]. The MRE takes place through the extraction of the rotational energy of the PNS via strongly amplified magnetic fields $\sim 10^{15}$ G. In this case a few processes can be candidates for the amplification mechanisms such as the winding effect or the magnetorotational instability [11]. Thus, the MRE is expected to leave behind a magnetar remnant. However, there are many unknown aspects of these scenarios, such as the progenitor and final core rotation and magnetic field profile. Hence, there is not yet a definitive understanding of the observed asymmetry and remnant kick velocities in core-collapse supernovae.

Moreover, it has been pointed out [12] that the characteristic spin-down ages ($P/2\dot{P}$) of magnetars appear to be systematically overestimated compared to ages of the associated supernova remnants. This suggests that there may be additional loss of angular momentum, perhaps due [12] to the dissipation of rotational energy into magnetic dipole radiation. It has also been proposed [13] that magnetic protoneutron stars can lose angular momentum from the ejection of magnetically coupled material via neutrinos. However, there are alternative explanations for the spin down of magnetic neutron stars as summarized in Refs. [14,15]. Even in the nonmagnetized case a rotating neutron star will lose considerable amounts of angular momentum by neutrino emission as first pointed out in Refs. [16,17] and discussed in more detail in Ref. [18], and applied to a discussion of neutron star birth properties by Refs. [14,15,19].

Nevertheless, in this work we point out that there is yet another source of angular momentum loss via neutrino emission in magnetic stars. In this case it is due to asymmetric neutrino scattering in strong toroidal magnetic fields. In this work we estimate the maximum possible effect from this asymmetric scattering and compare it specifically with the spin down calculated by the mechanism of Ref. [13]. We find that even in the best case, this contribution to spin down is less than that of other mechanisms. Nevertheless, it does contribute as an independent possible process and one should consider this effect in models for the early spin down of protoneutron stars.

Although we will approach obtaining estimates in a simple best case model, one should keep in mind that this effect should be studied in the context of more complex neutron star models (e.g., Refs. [20,21] and references therein). Both static and dynamic properties of neutron-star matter have been studied (e.g., Refs. [22–24]) at high temperature and density in the context of spherical nonmagnetic neutron star models. Such aspects as an exotic phase of strangeness condensation (e.g., Refs. [25–27]), nucleon superfluidity (cf. [28]), rotation-powered thermal evolution (e.g., Ref. [29]), a quark-hadron phase transition (e.g., [30]), etc., have been considered. Neutrino propagation has also been studied for PNS matter including hyperons (cf. [31]). These theoretical treatments of high-density hadronic matter, however, have not yet considered the effects of strong magnetic fields.

Although previous work (e.g., Refs. [32,33]) has studied the effects of magnetic fields on the asymmetry of neutrino emission, the neutrino-nucleon scattering processes were calculated in a nonrelativistic framework [32] and only a uniform dipole field configuration was considered.

Our studies of neutrino scattering and absorption cross sections in hot and dense magnetized neutron-star matter (including hyperons) [34,35] are based upon the fully relativistic mean-field (RMF) theory [36]. Our previous papers demonstrated that poloidal magnetic fields enhance the scattering cross sections for neutrinos in the direction parallel to the magnetic field, while also reducing the absorption cross-sections in the same direction. When the direction is antiparallel, the opposite occurs.

It was shown in Ref. [35] that for interior magnetic field strengths near the equipartition limit, where by equipartition we mean gas pressure $\approx$ magnetic pressure. This occurs for field strengths of order 10^{16-18} G. For such field strengths the enhancement of the scattering cross-sections is $\sim 1\%$ at a baryon density of $\rho_B = 3\rho_0$, while the reduction in the absorption cross section is $\sim 2\%$. This enhancement and reduction were shown to increase the neutrino momentum flux emitted along the direction of the dipole magnetic field and to decrease the emitted momentum flux emitted antiparallel to the magnetic field. This asymmetry was then applied to a calculation of pulsar-kick velocities in the context of a one-dimensional Boltzmann equation including only the dominant effect of neutrino absorption. PNS kick velocities of ~ 550 km s^{-1} were estimated.

Of relevance to the present work, however, are recent magnetohydrodynamic (MHD) PNS simulations (e.g., Refs. [37–39]), which demonstrate that the magnetic field inside a neutron star can obtain a toroidal configuration. It was also demonstrated [38] that the field strength of toroidal magnetic field is $\sim$100 times stronger than that of a poloidal magnetic field due to winding effects on the original dipole field lines for rapidly rotating the protoneutron stars.

Here, we show that the early spin deceleration of a PNS could result from an asymmetry in the neutrino emission that arises from parity violation in weak interactions [40,41] and/or an asymmetric distribution of the magnetic field [42] in strongly magnetized PNSs. (However, for an alternative scenario see Ref. [13].) Theoretical calculations [32,33] have suggested that as little $\sim 1\%$ asymmetry in the neutrino emission out of a total neutrino luminosity of $\sim 10^{53}$ ergs is enough to explain the observed pulsar kick velocities. We here study the asymmetric neutrino absorption in the case of a toroidal magnetic field inside a protoneutron star. If neutrinos are preferentially emitted along a direction opposite to that of the rotation. This could enhance the spin down rate of PNSs. In this article, we present for the first time a study of the effect of asymmetric neutrino absorption on the spin deceleration of PNSs.

II. MODEL

A. Neutrino transport in relativistic mean-field theory

Even a strong magnetic field has less mass energy than the baryonic chemical potential in degenerate neutron-star matter, i.e., $\sqrt{eB} \ll \varepsilon_b - M_b$, where ε_b and M_b are the chemical potential and rest mass of the baryon b, respectively. Hence, we can treat the magnetic field as a perturbation. We then ignore the contribution from convection currents and consider only the spin interaction. We also assume that $|\mu_b B| \ll E_b^*(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^2 + M_b^{*2}}$, and treat the single-particle energies and the wave function in a perturbative way.

In this framework we then obtain the wave function in a magnetic field by solving the Dirac equation:

$$[\gamma_\mu p^\mu - M_b^* - U_0(b)\gamma_0 - \mu_b B \sigma_z] u_b(p,s) = 0, \quad (1)$$

where $M_b^* = M_b - U_s(b)$, while $U_s(b)$ and $U_0(b)$ are respectively the scalar mean field and the time component of the vector mean field for the baryons b. These scalar and vector fields are calculated in the context of RMF theory.

In Refs. [34,35] we calculated the neutrino absorption cross section σ_A in PNS matter for an interior magnetic field strength near equipartition and a temperature of $T = 20$ MeV. Those results demonstrated that the absorption cross sections are suppressed in the direction parallel to the magnetic field $\boldsymbol{B}$ by about 2–4 % in the density region of $\rho_B = (1-3)\rho_0$. The opposite effect occurs in the antiparallel direction. The net effect of these changes in the absorption cross sections leads to an increase in the emitted neutrino momentum flux along the direction of the magnetic field and a decrease of the momentum flux emitted in the antiparallel direction.

However, it is quite likely [37–39] that the magnetic field exhibits a toroidal configuration within the PNS. Hence, we now consider the implications of a toroidal field configuration on neutrino transport in a strongly magnetized PNS. For this purpose we solve for the neutrino phase-space distribution function $f_\nu(\boldsymbol{r},\boldsymbol{k})$ using a Boltzmann equation as described below and in Ref. [35].

We assume that the system is static and nearly in local thermodynamic equilibrium. Under these assumptions the phase-space distribution function satisfies $\partial f_\nu/\partial t = 0$ and can be expanded as $f_\nu(\boldsymbol{r},\boldsymbol{k}) = f_0(\boldsymbol{r},\boldsymbol{k}) + \Delta f(\boldsymbol{r},\boldsymbol{k})$, where the first term is the local equilibrium part, and the second term is its deviation.

Furthermore, we assume that only the dominant effect of absorption contributes to the neutrino transport, and that the neutrinos travel along a straight line. It is common (e.g., Ref. [7]) to utilize a one-dimensional (1D) Boltzmann equation in simulations of PNS formation, and hence, the straight line approximation is adequate for our purpose. The 1D Boltzmann equation for f_ν in our simulation can then be written:

$$\hat{k} \cdot \frac{\partial}{\partial \boldsymbol{r}} f_\nu(\boldsymbol{r},\boldsymbol{k}) = \hat{k} \cdot \frac{\partial \varepsilon_\nu(\boldsymbol{r})}{\partial \boldsymbol{r}} \frac{\partial f_0}{\partial \varepsilon_\nu} + \hat{k} \cdot \frac{\partial \Delta f}{\partial \boldsymbol{r}} \approx -\frac{\sigma_A(\boldsymbol{r},\boldsymbol{k})}{V} \Delta f(\boldsymbol{r},\boldsymbol{k}), \quad (2)$$

where $\varepsilon_\nu(\boldsymbol{r})$ is the neutrino chemical potential at coordinate $\boldsymbol{r}$. Here, we define the variables $x_L \equiv (\boldsymbol{r}\cdot\boldsymbol{k})/|\boldsymbol{k}|$ and $\boldsymbol{R}_T \equiv \boldsymbol{r} - (\boldsymbol{r}\cdot\boldsymbol{k})\boldsymbol{k}/k^2$, and then solve Eq. (2) analytically

$$\Delta f(x_L, R_T, \boldsymbol{k}) = \int_0^{x_L} dy \left[-\frac{\partial \varepsilon_\nu}{\partial y} \frac{\partial f_0}{\partial \varepsilon_\nu} \right] \times \exp\left[-\int_y^{x_L} dz \frac{\sigma_A(z, R_T, \boldsymbol{k})}{V} \right], \quad (3)$$

where the center of the neutron star is at $\boldsymbol{r} = (0,0,0)$, and all of the integrations are performed along a straight line.

This simplified Boltzmann equation is adequate for our purpose, which is to estimate the relative difference between scattering aligned with the magnetic field vs antialigned. When the neutron star is rotating, however, the neutrino transport should to be treated in the comoving frame of the fluid. This causes additional angular momentum loss as we discuss below.

In this work we utilize an equation of state (EOS) at a fixed temperature and lepton fraction by using the parameter set PM1-L1 [43] for the RMF as in previous work [34,35]. When Λ particles are not included, the PM1-L1 EOS is sufficiently stiff [27] to give a maximum neutron star mass with about 2.2 solar mass, which is larger than the value observed for PSR J1614-2230 [44]. When the Λ particles are included, however, the EOS becomes softer and gives about 1.7 solar mass as a maximum neutron-star mass. This could be resolved, however, if we were to introduce additional repulsive force between the Λs [45] consistent with hypernuclear data. Another possibility would be introducing a repulsive three-body nuclear force.

We show the baryon density in Fig. 1(a) and the particle fractions in Fig. 1(b) as a function of the neutron-star radius. For this figure we assume a neutron-star baryonic mass of $M_{NS} = 1.68 M_\odot$, a temperature of $T = 20$ MeV, and a lepton fraction of $Y_L = 0.4$. The moment of inertia of the neutron star becomes $I_{NS} = 1.54 \times 10^{45}$ g cm^2 or 1.36×10^{45} g cm^2 with or without Λ particles, respectively. We note, however, that magnetic fields of this strength will also slightly increase the neutron star radius due to the additional magnetic pressure. The associated increase in the moment of inertia, would therefore decrease slightly the spin-down rate estimated here [see Eq. (6) below]. Nevertheless, we ignore this effect as our purpose is to estimate the maximum possible spin-down rate from asymmetric neutrino scattering.

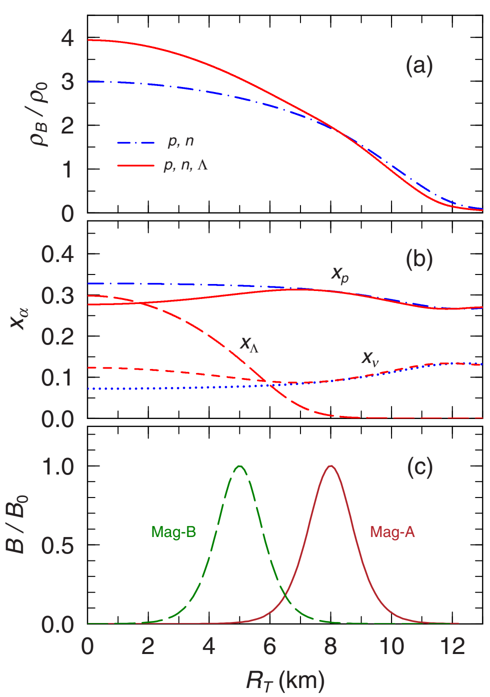

FIG. 1. (Color online) (a) shows the baryon density distribution for a PNS with $T = 20$ MeV and $Y_L = 0.4$. The solid and long-dashed lines show results with (red solid line) and without (blue dot-dashed line) Λs, respectively. (b) shows the number fractions for protons, Λs and neutrinos in a PNS. The (red) solid and (blue) dot-dashed lines show the proton fraction in systems with and without Λs. The (red) long-dashed line indicates the Λ fraction. The (red) dashed and (blue) dotted lines denote the neutrino fraction in systems with and without Λs, respectively. (c) shows the field strength distribution at $z = 0$ for the toroidal magnetic fields considered here. The (wine red) solid and (dark green) dashed lines represent those for $r_0 = 8$ km (Mag-A) and 5 km (Mag-B), respectively.

One can see in Figs. 1(a) and 1(b) the appearance of Λs for a baryon density greater than about twice the saturation density of nuclear matter, i.e., $\rho_B \gtrsim 2\rho_0$, where $\rho_0 \approx 2.7 \times 10^{14}$ g cm^{-3}. This softens the EOS and leads to an increase in the baryon density and neutrino fraction for $r \lesssim 8$ km relative to hadronic matter without Λs.

B. Toroidal magnetic field

The ratio of the total rate of angular momentum loss to the total power radiated by neutrinos at a given spherical surface S_N is

$$\left(\frac{c dL_z/dt}{dE_T/dt}\right) = \frac{\int_{S_N} d\Omega_r \int \frac{d^3k}{(2\pi)^3} \Delta f(\boldsymbol{r},\boldsymbol{k})(\boldsymbol{r} \times \boldsymbol{k}) \cdot \boldsymbol{n}}{\int_{S_N} d\Omega_r \int \frac{d^3k}{(2\pi)^3} \Delta f(\boldsymbol{r},\boldsymbol{k}) \boldsymbol{k} \cdot \boldsymbol{n}}, \quad (4)$$

where $\boldsymbol{n}$ is the unit vector normal to S_N. For illustration we will consider surfaces for which $\rho_B = \rho_0$ and $\rho_B = \rho_0/10$. We also adopt the speed of light for the neutrino propagation velocity. We can then obtain the angular acceleration from the neutrino luminosity, $\mathcal{L}_\nu = (dE_T/dt)$,

$$\dot{\omega} = -\frac{1}{cI_{NS}} \left(\frac{c dL_z/dt}{dE_T/dt}\right) \mathcal{L}_\nu. \quad (5)$$

For a PNS with spin period P, the angular velocity is $\omega = 2\pi/P$, and the angular acceleration is defined by $\dot{\omega} = -2\pi \dot{P}/P^2$. Thus, we obtain

$$\frac{\dot{P}}{P} = \frac{P}{2\pi c I_{NS}} \left(\frac{c dL_z/dt}{dE_T/dt}\right) \mathcal{L}_\nu. \quad (6)$$

We adopt the following parametrization for the toroidal magnetic field configuration in cylindrical coordinates (r_T, ϕ, z),

$$\vec{B} = B_\phi G_L(z) G_T(r_T) \hat{e}_\phi, \quad (7)$$

where $\hat{e}_\phi = (-\sin\phi, \cos\phi, 0)$ in terms of the azimuthal angle ϕ, and

$$G_L(z) = \frac{4e^{z/a_0}}{[1 + e^{z/a_0}]^2}, \quad G_T(r_T) = \frac{4e^{(r_T - r_0)/a_0}}{[1 + e^{(r_T - r_0)/a_0}]^2}. \quad (8)$$

This functional form was chosen to approximate the results of numerical simulations [38,39] of toroidal magnetic field amplification. For purposes of estimating the maximum possible effect we assume a toroidal magnetic field that is aligned along the direction of the spin rotation. Admittedly, this is an oversimplification, but it is adequate for our purposes of estimating the maximum possible effect. Nature, however, could be more complicated. Toroidal fields could be oppositely oriented and can even invert with time. This would imply that neutrino emission could also accelerate the stellar rotation. Another plausible case is an (antiparallel) poloidal torus configuration of the magnetic field. In this case a more complicated scenario could be possible. Assuming that in the northern hemisphere the direction of the magnetic field and the spin rotation are the same, then the effect described here would would operate to decelerate the rotation. In the southern hemisphere, however, the magnetic field and spin could be antiparallel. In this case the asymmetry in neutrino absorption may even accelerate the rotation. This might lead to a complicated twisting mode.

In Fig. 1(c) we illustrate the magnetic field strength $|\boldsymbol{B}/B_\phi|$ for different field configurations, with $a_0 = 0.5$ km and $r_0 = 8.0$ km (Mag-A) or $r_0 = 5.0$ km (Mag-B). These parameters are chosen to represent a best case and a typical case. As such, this should bracket the cases for which the effect studied here may be of interest. The fact that the spin down is still significant in both limits supports the robustness of these results. We here take $\mathcal{L}_\nu \approx 3 \times 10^{52}$ erg s^{-1} [33] as a typical value of the neutrino luminosity from the protoneutron star, and the spin period is chosen to be $P = 10$ ms, while the observed spin period of magnetars is about 10 s [46,47]. We discuss below more details regarding this choice of spin period.

III. RESULT AND DISCUSSIONS

Numerical simulations [38,39] have shown that the strength of the toroidal magnetic field can easily amplify to $B_\phi = 10^{16}$ G or more from an initial value of $\sim 10^{14}$ G due to the winding of the magnetic field lines in rapidly rotating of PNSs. We therefore adopt these typical values for both components $B_{\rm pol} = 10^{14}$ G and $B_\phi = 10^{16}$ G, respectively.

We summarize the calculated results in Table I. It includes two cases by taking the PNS surface S_N at different locations, one at $\rho_B = \rho_0$ and the other at $\rho_B = \rho_0/10$, to illustrate the robustness of this braking mechanism. We obtain the results that $\dot{P}/P \sim 10^{-6}$ in Mag-A and $\dot{P}/P \sim 10^{-7}$ in Mag-B when $P = 10$ ms.

To compare with the rate of spin down due [12] to dissipating rotational energy into magnetic dipole radiation the sixth column shows $\dot{P}/P$ calculated with the magnetic dipole radiation (MDR) formula [48].

$$P\dot{P} = B_{\rm pol}^2 \left(\frac{3I_{NS}c^3}{8\pi^2 R^6}\right)^{-1} = B_{\rm pol}^2 \left(\frac{3M_{NS}^3 c^3}{125\pi^2 I_{NS}^2}\right)^{-1}, \quad (9)$$

where R and $I_{NS} = 2M_{NS}R^2/5$ are the NS radius and the moment of inertia. These quantities are determined from the EOS as discussed above. For these particular parameters we see that the spin deceleration from asymmetric neutrino emission can be more effective than that of MDR when the neutrino luminosity is high.

If we consider the case $P \approx 1$ ms, these two mechanisms give comparative results. This is because $\dot{P}/P$ is proportional to P in our model, while it is proportional to P^{-2} in the MDR according to Eqs. (6) and (9). If we consider the alternative case of a stronger poloidal magnetic field $B_{\rm pol} = 10^{15}$ G while keeping $P = 10$ ms, the two mechanisms also give comparable strength because $\dot{P}/P$ is proportional to B in our model, but to B^2 in MDR. Either conditions of a longer spin period or a weaker field strength would thus lead to a dominance of our new mechanism over MDR.

However, other means to spin down magnetic neutron stars by neutrino emission have been proposed [13–15]. Even nonmagnetized rotating neutron star will lose angular momentum by neutrino emission [14–19].

For illustration, therefore, we also compare with the spin down of the protoneutron star as was proposed by Thompson *et al.* [13]. This mechanism utilizes the neutrino-driven winds to push magnetically locked matter away from PNS and slow down the rotation. In this mechanism spin-down rate in the dipole magnetic field is given by [13],

$$(\dot{P}/P)_{\rm Poloidal} = 4.14 \times 10^{-3}[{\rm s}^{-1}] \left(\frac{M_{NS}}{1.4M_\odot}\right)^{-1} \times \left(\frac{\dot{M}}{10^{-3}M_\odot}\right)^{+3/5} \left(\frac{R}{10\,[{\rm km}]}\right)^{+2/5} \times \left(\frac{B_{\rm pol}}{10^{14}\,[{\rm G}]}\right)^{+4/5} \left(\frac{P}{10\,[{\rm ms}]}\right)^{+2/5}, \quad (10)$$

where $\dot{M}$ is the wind mass loss rate. A comparison between our rate Eq. (6) from asymmetric neutrino emission and the neutrino-driven wind Eq. (10) is shown in the seventh column

TABLE I. The first column shows the presumed composition of nuclear matter, i.e., p, n for nucleonic and p, n, Λ for hyperonic matter. The second column gives the model for the toroidal magnetic field configuration (see text). The third column denotes results from Eq. (4), the fourth and fifth columns are results obtained using Eq. (6) at the indicated baryon density. The sixth column shows the spin-down rate from magnetic dipole radiation, Eq. (9). The seventh column shows the spin-down rates from the model of Thompson *et al.* [13] for the neutrino-driven winds coupled with the poloidal magnetic field, Eq. (10). The spin period is taken to be $P = 10$ ms, and magnetic field strengths of $B_{\rm pol} = 10^{14}$ G and $B_\phi = 10^{16}$ G are used in these calculations.

Comp.	Mag.	$\frac{cdL_z/dt}{dE_T/dt}$	$\dot{P}/P$ (s^{-1})			
			$\rho_B = \rho_0$	$\rho_B = \rho_0/10$	MDR	Thompson
p, n	Mag-A	3.34	3.45×10^{-6}	7.25×10^{-7}	9.86×10^{-8}	3.56×10^{-3}
	Mag-B	0.482	4.97×10^{-7}	3.16×10^{-7}		
p, n, Λ	Mag-A	5.45	6.39×10^{-6}	1.02×10^{-6}	7.76×10^{-8}	3.50×10^{-3}
	Mag-B	0.390	4.57×10^{-7}	2.01×10^{-7}		

of Table I. For this comparison we use the standard parameter values of $M_{NS} = 1.68 M_\odot$, $\dot{M} = 10^{-3} M_\odot$, $B_{\rm pol} = 10^{14}$ G, $P = 10$ ms, and $R = 10.1$ km (with Λs) and 10.8 km (without Λs), which are obtained from I_{NS}.

One can also estimate the effect from nonmagnetic neutrino transport. This effect arises [18] when the neutrino transport is treated in the comoving frame of the fluid. In this corotating frame the neutrino distribution will be isotropic in equilibrium in the absence of strong magnetic fields. In the laboratory frame, however, the rotation of the neutron star creates an emission asymmetry by which the neutrinos are able to carry away angular momentum [18]. Based upon Eq. (15) in Ref. [18], one can estimate that $\dot{P}/P \leqslant 2.5(\dot{M}/M) < 3 \times 10^{-3}$ s^{-1}, for $\dot{M}/M \sim 10^{-3}$ s^{-1} as in the wind model above [18]. Hence, this mechanism may also exceed or be comparable to the effect from asymmetric neutrino scattering described in the present work.

Nevertheless, the spin-down mechanism described in the present work can be an additional effect that works together with the other neutrino-emission mechanisms. It may cause further enhancement of the spin-down rate, and therefore warrants consideration in models for the spin down of the PNS.

We note, however, that it may be difficult to directly confirm by observations the asymmetric neutrino scattering mechanism described herein. In principle one might eventually confirm this effect via a detection of neutrinos aligned or antialigned with a magnetic field. In this regard, there is another consequence of asymmetric neutrino scattering that is more directly observable, i.e., the observed pulsar kick velocities. In our previous paper [34,35], we showed that the neutrino asymmetric emission can lead to pulsar kick velocities of $v_{\rm kick} = 500 \sim 600$ km/s, that are comparable to the observed values of 400–1500 km/s. Hence, asymmetric neutrino emission may affect a variety of observed dynamical processes associated with SN explosions.

We note that in the present calculation we have ignored the neutrino scattering and production processes. The neutrino scattering process enhances the asymmetry of the emission, although its contribution to the mean-free path is much smaller than that from absorption in the density region of interest, $\rho_0 \lesssim \rho_B \lesssim 3\rho_0$ [35].

Neutrino production in a magnetic field is known to cause asymmetry in the neutrino emission [49,50]. The cross section for the neutrino production reaction, $e^- + p \to n, \Lambda + \nu_e$, is qualitatively the same as that for the absorption reaction, $\nu_e + n, \Lambda \to p + e^-$. The only difference is the small contribution from the magnetic part of the initial and final electron states. Hence, this production process would tend to enhance the asymmetry and also contribute to the spin deceleration.

Our goal in this work has been to estimate the maximum possible effect of asymmetric neutrino scattering on the spin-down rate of PNSs. Even so, there are a number of uncertainties in our estimate of $\dot{P}/P$. These include the interior strength and configuration of the magnetic field, along with the spin period of the NS core, etc. This process may or may not contribute, but should at least be considered in a more realistic calculation. Since our value of $\dot{P}/P$ is at least 10^2 times larger than that for the MDR spin-down mechanism, asymmetric neutrino emission could be significant at some point during the early stages of SN explosion. Moreover, as discussed above, other processes such as neutrino scattering and production tend to increase the asymmetry in neutrino emission and lead to additional spin deceleration. Thus, we can conclude that asymmetric neutrino emission from PNSs may play a role in the spin deceleration of a magnetic PNS and should be considered.

IV. SUMMARY

We have estimated a best case scenario for the possible spin down of a PNS due to asymmetric neutrino absorption. We consider the optimum case of a toroidal magnetic field configuration aligned with the neutron-star spin direction. We calculated the cross sections for asymmetric neutrino absorption and scattering in the context of RMF theory. We then solved the Boltzmann equation using a one-dimensional attenuation method, assuming that the neutrinos propagate along an approximately straight line, and that the system is in quasiequilibrium. We only included neutrino absorption, which dominates [35] over scattering in producing asymmetric momentum transfer to the PNS.

In this simplified model we found that asymmetric neutrino emission can have an effect on the early spin deceleration of a PNS. Indeed, this effect can initially be larger than the braking from a magnetic dipole field configuration, but is probably smaller than that due to the magnetized neutrino wind breaking mechanism of Ref. [13]. Finally, we caution that definitive conclusions should involve a fully dynamical MHD simulation of the evolution of a PNS with asymmetric neutrino scattering and production as well as absorption in a strong magnetic field. Nevertheless, the results presented here suggest a possible influence of asymmetric neutrino absorption on the early formation process of magnetars and therefore warrant further investigation.

ACKNOWLEDGMENTS

This work was supported in part by the Grants-in-Aid for the Scientific Research from the Ministry of Education, Science and Culture of Japan (Grants No. 20105004, No. 21105512, No. 2324036, No. 2354032, No. 24340060, and No. 25105510) and Nihon University College of Bioresource Sciences Research Grant for 2013. Work (M.K.C.) is supported by the National Research Foundation of Korea (2012R1A1A3009733, 2012M7A1A2055605, 2011-0015467). Work at the University of Notre Dame (GJM) is supported by the US Department of Energy under Nuclear Theory Grant No. DE-FG02-95-ER40934.

[1] A. G. Lyne and D. R. Lorimer, Nature (London) **369**, 127 (1994).
[2] B. Paczyński, Acta. Astron. **41**, 145 (1992).
[3] For a review, G. Chanmugam, Annu. Rev. Astron. Astrophys. **30**, 143 (1992).
[4] R. E. Rothchild, S. R. Kulkarni, and R. E. Lingenfelter, Nature (London) **368**, 432 (1994).
[5] A. Marek and H.-T. Janka, Astrophys. J. **694**, 664 (2009).

[6] T. Takiwaki, K. Kotake, and Y. Suwa, Astrophys. J. **749**, 98 (2012).
[7] A. Burrows, L. Dessart, E. Livne, C. D. Ott, and J. Murphy, Astrophys. J. **664**, 416 (2007).
[8] K. Kotake, H. Sawai, S. Yamada, and K. Sato, Astrophys. J. **608**, 391 (2004).
[9] H. Sawai, K. Kotake, and S. Yamada, Astrophys. J. **672**, 465 (2008).
[10] J. Nordhaus, T. D. Brandt, A. Burrows, E. Livne, and C. D. Ott, Phys. Rev. D **82**, 103016 (2010).
[11] S. Akiyama and C. Wheeler, Astrophys. J. **584**, 954 (2003).
[12] T. Nakano, K. Makishima, K. Nakazawa, H. Uchiyama, and T. Enotoet, AIP Conf. Proc. **1427**, 126 (2012).
[13] T. A. Thompson, P. Chang, and E. Quataert, Astrophys. J. **611**, 380 (2004).
[14] A. Heger, S. E. Woosley, and H. C. Spruit, Astrophys. J. **626**, 350 (2005).
[15] C. A. Ott *et al.*, Astrophys. J. Suppl. Ser. **164**, 130 (2006).
[16] D. Kazanas, Nature (London) **267**, 501 (1977).
[17] R. Epstein, Astrophys. J. **219**, L39 (1978).
[18] T. W. Baumgarte and S. L. Shapiro, ApJ **504**, 431 (1998).
[19] H.-T. Janka, Proc. IAU Symposium on Young Neutron Stars and their Environments, **218**, 3 (2004).
[20] J. M. Lattimer and M. Prakash, Science **304**, 536 (2004).
[21] J. M. Lattimer, Ann. Rev. Nucl. Part. Sci. **62**, 485 (2012).
[22] G. E. Brown and H. A. Bethe, Astrophys. J. **423**, 659 (1994).
[23] M. Prakash *et al.*, Phys. Rep. **280**, 1 (1997).
[24] N. K. Glendenning, Phys. Rep. **342**, 393 (2001).
[25] D. B. Kaplan and A. E. Nelson, Phys. Lett. B **175**, 57 (1986); **179**, 409(E) (1986).
[26] G. E. Brown, K. Kubodera, D. Page, and P. Pizzochero, Phys. Rev. D **37**, 2042 (1988).
[27] T. Maruyama, H. Fujii, T. Muto, and T. Tatsumi, Phys. Lett. B **337**, 19 (1994).
[28] D. Page and E. Baron, Astrophys. J. **354**, L17 (1990).
[29] S. Tsuruta, Phys. Rep. **292**, 1 (1998).
[30] N. Yasutake and K. Kashiwa, Phys. Rev. D **79**, 043012 (2009); N. Yasutake, T. Noda, H. Sotani, T. Maruyama, and T. Tatsumi, *Recent Advances in Quarks Research*, Nova, (2013), Chap. 4, p. 63, arXiv:1208.0427 [astro-ph].
[31] S. Reddy, M. Prakash, and J. M. Lattimer, Phys. Rev. D **58**, 013009 (1998).
[32] P. Arras and D. Lai, Phys. Rev. D **60**, 043001 (1999).
[33] D. Lai and Y.-Z. Qian, Astrophys. J. **501**, L155 (1998); **505**, 844 (1998).
[34] T. Maruyama, T. Kajino, N. Yasutake, M. K. Cheoun, and C. Y. Ryu, Phys. Rev. D **83**, 081303(R) (2011).
[35] T. Maruyama, N. Yasutake, M. K. Cheoun, J. Hidaka, T. Kajino, G. J. Mathews, and C. Y. Ryu, Phys. Rev. D **86**, 123003 (2012).
[36] B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E **6**, 515 (1997).
[37] J. Braithwaite and H. C. Spruit, Nature (London) **431**, 819 (2004).
[38] T. Takiwaki, K. Katake, and K. Sato, Astrophys. J. **691**, 1360 (2009).
[39] T. Kuroda and H. Umeda, Astrophys. J. Suppl. **191**, 439 (2010).
[40] A. Vilenkin, Astrophys. J. **451**, 700 (1995).
[41] C. J. Horowitz and G. Li, Phys. Rev. Lett. **80**, 3694 (1998).
[42] G. S. Bisnovatyi-Kogan, Astron. Astrophys. Trans. **3**, 287 (1993).
[43] T. Maruyama, H. Shin, H. Fujii, and T. Tatsumi, Prog. Theo. Phys. **102**, 809 (1999).
[44] P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts, and J. W. T. Hessels, Nature (London) **467**, 1081 (2012).
[45] I. Bednarek, P. Haensel, J. L. Zdunik, M. Bejger, and R. Manka, A & A **543**, A157 (2012).
[46] Radio pulsar data courtesy of the ATNF Oulsar Group, http://www.atnf.csiro.au/research/pulsar/psrcat/; D. R. Lorimer and M. Kramer, *Handbook of Pulsar Astronomy* (Cambridge University Press, Cambridge, 2004).
[47] P. M. Woods and C. Thompson, arXiv:astro-ph/0406133.
[48] J. E. Gunn and J. P. Ostriker, Nature (London) **221**, 454 (1968); A. G. Lyne and F. G. Smith, Pulsar Astronomy CUP (1990).
[49] N. N. Chugai, Sov. Astron. Lett. **10**, 87 (1984).
[50] E. M. Henley, M. B. Johnson, and L. S. Kisslinger, Phys. Rev. D **76**, 125007 (2007).