Shell-model study of the 4th- and 6th-forbidden β-decay branches of ^{48}Ca

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The highly forbidden β^- decay of ^{48}Ca is reexamined by performing shell-model calculations with the GXPF1A effective interaction. We examine the three available decay branches to the lowest 6^+, 5^+, and 4^+ states of ^{48}Sc, and extract a theoretical half-life of $T_{1/2}^{\beta} = 5.2^{+1.7}_{-1.3} \times 10^{20}\, g_A^{-2}$ yr for the β^- decay, where g_A is the value of the axial-vector coupling constant. The current half-life estimate suggests stronger competition between the single-β-decay and double-β-decay branches of ^{48}Ca than previously expected on theoretical grounds.

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I. INTRODUCTION

Aside from being doubly magic, the nucleus ^{48}Ca features another interesting property: The double-β-decay channel competes with the ordinary single-β-decay channels (see Fig. 1). Studies of the possible double-β-decay branches of ^{48}Ca have been carried out both theoretically and experimentally [1–5]. The two-neutrino-emitting mode ($2\nu\beta\beta$ decay) of this decay channel is dominated by the ground-state-to-ground-state transition $0^+ \rightarrow 0^+$ with a energy release of 4.274 ± 0.004 MeV [6]. The current most up-to-date half-life for this branch is derived to be $T_{1/2}^{2\nu} = 4.4^{+0.6}_{-0.5} \times 10^{19}$ yr [4].

The theoretical formalism for the nuclear single-β-decay process is well established [7]. The predictions of the theory have extensively been verified against the experiment at least for the most typical types of β-decay transitions. Recently the theoretical formalism has been extended to studies of more extreme types of decays. These involve β-decay branches suppressed by high level of forbiddenness and/or low Q value (see, e.g., [8–14]). For highly forbidden transitions the theoretical analysis is complicated by the many nuclear matrix elements involved. On the experimental side, careful preparation and efficient detection techniques are needed to detect the tiny decay rates.

The retardation of the ordinary β decay of ^{48}Ca results from two unfavorable features of the decay. There is a large angular momentum change ($\Delta J = 4$, 5, or 6) between the initial and final nuclear states, and the energy release is small. The experimentally measured Q value for the 6th-forbidden ground-state-to-ground-state transition is 278 ± 5 keV [6]. Using this value, the Q values for the 4th-forbidden branches are deduced to be 147 keV (5^+) and 26 keV (4^+) with an uncertainty of 5 keV for each. Since the transition to the 5^+ state is unique, the phase-space considerations suggest this to be strongly favored over the other two branches. The ordinary β-decay branches, not yet experimentally verified, can be associated with partial half-life lower limits of 0.71×10^{20} yr (6^+), 1.1×10^{20} yr (5^+), and 0.82×10^{20} yr (4^+) [5].

An earlier study for ordinary β-decay branches of ^{48}Ca was carried out by Aunola, Suhonen, and Siiskonen [8]. In that work the one-body transition densities for the initial and final nuclear states were computed using the shell model with the FPBP and FPKB3 interactions. Based on these results a half-life of $T_{1/2}^{\beta} = 1.1^{+0.8}_{-0.6} \times 10^{21}$ yr was derived for the ordinary β decay. As remarked by the authors this value is 25 times longer than that of the double-β decay. This suggests that the double-β-decay branch is only very weakly challenged by the ordinary β decay.

In this work we are revisiting the study of Ref. [8]. The structure of the ^{48}Ca ground state and that of the low-lying states in ^{48}Sc were obtained from shell-model calculations performed in the pf shell using the GXPF1A interaction [15,16]. This interaction was extensively checked and validated for pf-shell nuclei, and it was successfully used to describe β decays [17] and Gamow-Teller strengths extracted from charge-exchange reactions [18]. It was also used to describe the β decay of the ^{48}Sc ground state to the low-lying states of ^{48}Ti (see, e.g., Table I of Ref. [19]) and the cumulative $2\nu\beta\beta$ matrix element of ^{48}Ca [19]. This last quantity was later confirmed by experimental data [20]. The same effective interaction was also recently used to describe the ^{48}Ca neutrinoless double-β ($0\nu\beta\beta$) decay nuclear matrix elements for the light Majorana neutrino exchange mechanism [21], and for other mechanisms that could potentially contribute to the $0\nu\beta\beta$-decay process [22].

This article is organized as follows. In Sec. II we give a brief overview on the general β-decay theory. The theoretical formalism of the double-β decay is not discussed in this work, but the general aspects of that theory can be found, e.g., in Ref. [23]. In Sec. III we summarize the computed results for the partial half-lives of the β^- decay branches and compare them with the former study [8]. In Sec. IV we draw conclusions. An earlier theoretical study for the unique 5th-forbidden β-decay branch of ^{48}Ca by Warburton can be found in Ref. [9]. A *pn*-quasiparticle random-phase approximation study for a very similar case of ^{97}Zr by Heiskanen *et al.* can be found in Ref. [10].

II. OVERVIEW OF THE β-DECAY THEORY

The general theoretical formalism for β decay is discussed in detail, e.g., in the book by Behrens and Bühring [7]. A streamlined overview of that discussion concentrating on the practical application of the theory can, however, be found from Refs. [24,25]. In these articles both the unique and nonunique transitions are discussed together with the allowed transitions.

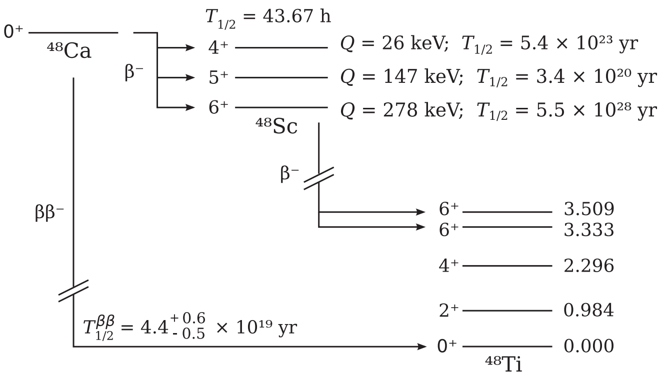

FIG. 1. β-decay scheme of ^{48}Ca. In addition to the three ordinary β-decay branches of ^{48}Ca, the ground-state-to-ground-state double-β-decay branch $0^+ \rightarrow 0^+$ is also shown. The values for the partial half-lives of the ordinary β-decay branches are taken from Table I with $g_A = 1.25$. For ^{48}Sc, only the dominant decay branches are indicated.

In the β-decay theory the partial half-life, denoted by $t_{1/2}$, of a given decay branch can be expressed as

$$t_{1/2} = \kappa/\tilde{C}. \tag{1}$$

The constant κ is

$$\kappa = \frac{2\pi^3\hbar \ln 2}{(m_e c^2)^5 G_{\mathrm{F}}^2/(\hbar c)^6}, \tag{2}$$

where m_e is the electron rest mass, and G_{F} is the effective Fermi coupling constant. The adopted choice for the numerator of Eq. (1) leaves the integrated shape factor $\tilde{C}$ dimensionless, that is,

$$\tilde{C} = \int_1^{w_0} C(w_e) p w_e (w_0 - w_e)^2 F_0(Z, w_e) \mathrm{d}w_e, \tag{3}$$

where $w_e = W_e/(m_e c^2)$ and $p = p_e c/(m_e c^2)$. The quantity W_e (p_e) is the energy (momentum) of the electron and Z is the proton number of the daughter nucleus. The upper limit of the integral (3), w_0, is the end-point energy of the β spectrum in units of $m_e c^2$. The function $F_0(Z, w_e)$ is the Fermi function for β^- transitions [see Eq. (32) in Ref. [24]].

The shape factor $C(w_e)$ of Eq. (3) contains the nuclear structure information in the nuclear matrix elements of the form

$$^{V/A}\mathcal{M}_{KLS} = \frac{1}{\sqrt{2J_i + 1}} \sum_{pn} {^{V/A}m_{KLS}(pn)} (\psi_f || [c_p^\dagger \tilde{c}_n]_K || \psi_i). \tag{4}$$

These elements are composed of two parts: the single-particle matrix elements $^{V/A}m_{KLS}(pn)$ and the reduced one-body transitions densities $(\psi_f || [c_p^\dagger \tilde{c}_n]_K || \psi_i)$ between the initial (ψ_i) and final (ψ_f) nuclear states. The single-particle matrix elements are universal for all nuclear models since they only characterize the properties of the transition operators. The one-body transition densities (OBTDs), on the other hand, are model and case specific, and in the context of shell-model studies, expressions for the OBTD can be found in Ref. [26].

The number of involved matrix elements for the nonunique transitions is determined by the level of forbiddenness K. For the 4th-forbidden nonunique decay to the 4^+ state the number of needed matrix elements is 12 and for the 6th-forbidden nonunique decay to the 6^+ state, 16. In the former case 8 and in the latter case 12 of these matrix elements involve radial integrals of the single-particle wave functions that involve Coulomb factors (see Ref. [24]). An important special case to the general β-decay theory is the class of unique transitions. For these transitions the angular momentum change between the initial and final nuclear states is maximal for a given forbiddenness of the transition, i.e., $\Delta J = |J_f - J_i| = K + 1$. In this case the shape factor is drastically simplified as only one matrix element contributes to it.

In the present work the single-particle matrix elements are calculated using harmonic oscillator wave functions (see, e.g., [24]). A more sophisticated way would be to use the Woods-Saxon wave functions (this approach was adopted, e.g., in [8]). The choice for the first is usually well justified by the argument that the low-energy harmonic oscillator functions are sufficient to describe the well-bound nuclear states [27].

TABLE I. Theoretical predictions for the partial half-lives (in years) of the β^- decay branches of ^{48}Ca. For each transition the upper values are computed with $g_A = 1.00$ and the lower values with $g_A = 1.25$.

Transition	Forbiddenness	$Q_{\beta^-}(0^+_{\mathrm{g.s.}} \rightarrow 6^+_{\mathrm{g.s.}})$ (keV)		
		278 − 5	278	278 + 5
$0^+_{\mathrm{g.s.}} \rightarrow 6^+_{\mathrm{g.s.}}$	6	6.29×10^{28}	5.18×10^{28}	4.27×10^{28}
		6.64×10^{28}	5.46×10^{28}	4.50×10^{28}
$0^+_{\mathrm{g.s.}} \rightarrow 5^+_1$	4u	6.95×10^{20}	5.25×10^{20}	4.01×10^{20}
		4.44×10^{20}	3.36×10^{20}	2.57×10^{20}
$0^+_{\mathrm{g.s.}} \rightarrow 4^+_1$	4	1.54×10^{24}	4.50×10^{23}	1.59×10^{23}
		1.84×10^{24}	5.37×10^{23}	1.90×10^{23}

III. CALCULATIONS AND DISCUSSION

The calculations of the partial half-lives for the three β^- decay branches of ^{48}Ca were performed following the procedures outlined in Refs. [24,25], when applicable. This involved, first, the construction of the nuclear matrix elements from the shell-model one-body transition densities, and second, the evaluation of the integrals of the form (3). The one-body transition densities were calculated for the full pf shell using the shell-model code OXBASH [28] with the GXFP1A interaction [15,16].

In β-decay theory the axial-vector current coupling strength of weak interactions is represented by the constant g_A. For nonunique transitions the dependence of partial half-lives on this constant is complicated as can be seen from the explicit expressions of Ref. [24]. In the case of unique transitions, however, the dependence is simple, namely, $t_{1/2} \propto 1/g_A^2$, as the shape factor is composed of only a single nuclear matrix element. Although we are using the quenched shell-model type of value $g_A = 1.00$ alongside the bare nucleon value $g_A = 1.25$ in our calculations, it should be strongly emphasized that the justification for the adopted choice in the case of highly forbidden transitions is not at all abiding. The value $g_A = 1.00$

as such is appropriate only for allowed Gamow-Teller type of transitions in untruncated single major-shell shell-model calculations. The quenching of g_A arises from the truncation of the model space to a major shell, and the failure to include two-meson exchange operators. Since the calculation of these effects is very multipole dependent, the purpose for adopting the quenched value in the current work is mainly to inspect the dependence of the transition half-lives on the axial vector coupling strength.

The computed partial half-lives are summarized in Table I. The Q values of the decay branches are deduced from that of the ground-state-to-ground-state transition. These values are taken to be the ones mentioned in Sec. I of this work. The associated experimental uncertainty of 5 keV yields corresponding uncertainties for the partial half-lives. As it is to be expected by way of phase-space considerations, the uncertainties for half-lives increase when approaching the low-Q-value limit. In the case of the dominant unique decay branch, the reduction (increase) of transition Q value by 5 keV yields approximately a 32% longer (24% shorter) partial half-life.

The increase of the coupling strength g_A from the value 1.00 to 1.25 has the largest effect for the unique decay branch, decreasing the partial half-lives by 36%. The corresponding change in the coupling strength is less important for the nonunique decay branches, and it is seen to work in the opposite direction. Hence, the increase in g_A also increases the dominance of the unique decay branch.

Comparison of the results of Table I with those of the former study (Table I in Ref. [8]) shows a drastic increase in the decay probability. The partial half-lives of that work are over two and three orders of magnitude longer than those of ours for the transitions to the 6^+ and 4^+ states, respectively. In the case of the unique decay branch the deviations are less, but still the current values are half of those of [8]. It should be noted that the most recent mass evaluation [6] yields exactly the same Q values for the three β-decay branches as the ones used in [8]. Thus the comparison of the results is straightforward in terms of energetics of the decays.

A possible source of uncertainty in our calculations is the use of harmonic oscillator wave functions in the evaluation of the single-particle matrix elements. The effects that arise from differences in the proton and neutron single-particle wave functions were not examined in this work. In the work [8] it was demonstrated that the more accurate wave functions can significantly affect the decay rates of the nonunique decay branches. For these branches the deduced partial half-lives with harmonic oscillator wave functions were more than 60 times ($J_f = 6^+$) and 500 times ($J_f = 4^+$) longer than those with the Woods-Saxon ones. The observed strong correspondence between the decay rate and the type of single-particle wave function was directly associated by Aunola *et al.* [8] with the behavior of the recoil nuclear matrix elements: When the harmonic oscillator wave functions were used in the pf shell, the recoil matrix elements completely vanished, while Woods-Saxon wave functions induced center-of-mass spurious effects.

For the unique decay branch the deduced partial half-lives of Ref. [8] were much more consistent. The use of Woods-Saxon wave functions resulted in an increase of the partial half-life by only some 10%. This indicates that the only surviving matrix element for unique decays is only modestly affected by the choice of single-particle wave functions. Because of that we do not expect the decay rate of the unique branch to change drastically by the choice of single-particle wave functions.

In the present work the screening of the atomic electrons was taken into account by following the discussion by Brown in Ref. [29]. This involves a modification of the Fermi function of the integrand of Eq. (3) by adding an exponential factor [see Eq. (46) in Ref. [29]] to it. The purpose of this extra factor for β^- decays is to effectively reduce the size of the transition phase space at low electron energies. At large energies the phase space is normalized to coincide with that of the unmodified case. Based on our calculations performed with the screening-corrected Fermi function, we conclude that screening results in a less than 1% effect on the partial half-lives. Therefore, for the purposes of this study we do not consider the screening corrections to be significant.

It is worth noting that when dealing with the screening corrections, Aunola *et al.* [8] referred to the study by Warburton in Ref. [9]. According to those calculations the increase in the partial half-life of the unique decay branch is expected to be as much as 11%. A screening effect of this magnitude is significantly stronger than that suggested by the present calculations.

Interestingly the current partial half-lives (Table I) for the nonunique decay branches are closer to the ones obtained by using the Woods-Saxon wave functions in the former study (Table 2 in Ref. [8]). Although some enhancement of the decay rates of nonunique decay branches is to be expected from the use of the Woods-Saxon wave functions, the effect on the total β^--decay half-life is damped by the dominance of the unique branch that is insensitive to the choice of the single-particle wave functions. Neglecting the screening corrections as discussed above, we present in this work a half-life of $T^{\beta}_{1/2} = 5.2^{+1.7}_{-1.3} \times 10^{20}\, g_{\mathrm{A}}^{-2}$ yr for the β decay of ^{48}Ca, where we can present the half-life in terms of the axial-vector coupling constant g_A since the dominant transition is unique and solely determines the value of the half-life. The error limits of the result stem entirely from those of the transition Q values.

The current prediction for the β-decay half-life is a factor 5–7 higher than the experimental lower limits for the individual β-decay branches. On the other hand, the current half-life is 53% shorter than the value $T^{\beta}_{1/2} = 1.1^{+0.8}_{-0.6} \times 10^{21}\, g_{\mathrm{A}}^{-2}$ yr presented by Aunola *et al.* in Ref. [8]. Hence, our present result suggests a stronger competition between the ordinary β-decay and double-β decay branches than formerly expected. The current value is only 12 times longer than the measured half-life for the two-neutrino-emitting mode of double-β decay.

IV. CONCLUSIONS

In this work the shell-model study of Ref. [8] for the ordinary β decay of ^{48}Ca was reexamined by using the GXPF1A interaction in the full pf shell. We considered all three possible β^--decay branches to the 6^+, 5^+, and 4^+ states of the daughter nucleus ^{48}Sc, and deduced the corresponding theoretical partial half-lives.

The computed values for the partial half-lives (Table I) are well compatible with and not far from the experimental lower limits of Ref. [5]. The present calculations confirm the dominance of the unique 4th-forbidden branch over the nonunique ones. According to the study of Ref. [8] the decay rates of the nonunique decay branches can be considerably enhanced by the use of the Woods-Saxon single-particle wave functions. However, this choice does not significantly change the total half-life due to the dominance of the unique decay branch, which is only slightly affected by the choice of single-particle wave functions.

Based on our calculations we propose a theoretical half-life of $T^{\beta}_{1/2} = 5.2^{+1.7}_{-1.3} \times 10^{20}\, g_{\rm A}^{-2}$ yr for the ordinary β decay of ^{48}Ca. This value is only about 12 times longer than the experimentally measured half-life $T^{2\nu}_{1/2} = 4.4^{+0.6}_{-0.5} \times 10^{19}$ yr for the two-neutrino-emitting mode of the double-β decay. Although the double-β-decay branch is still relatively weakly challenged by the ordinary β decay, we note that the competition between the decay channels is expected to be stronger than suggested by Aunola *et al.* [8].

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