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Higher-order symmetry energy of nuclear matter and the inner edge of neutron star crusts

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The parabolic approximation to the equation of state of the isospin asymmetric nuclear matter (ANM) is widely used in the literature to make predictions for the nuclear structure and the neutron star properties. Based on the realistic M3Y-Paris and M3Y-Reid nucleon-nucleon interactions, we investigate the effects of the higher-order symmetry energy on the proton fraction in neutron stars and the location of the inner edge of their crusts and their core-crust transition density and pressure, thermodynamically. Analytical expressions for different-order symmetry energy coefficients of ANM are derived using the realistic interactions mentioned above. It is found that the higher-order terms of the symmetry-energy coefficients up to its eighth order (E_{sym8}) contributes substantially to the proton fraction in β-stable neutron star matter at different nuclear matter densities, the core-crust transition density and pressure. Even by considering the symmetry-energy coefficients up to E_{sym8}, we obtain a significant change of about 40% in the transition pressure value from the one based on the exact equation of state. The slope parameters of the symmetry energies for the M3Y-Paris (Reid) interaction, at the saturation density, are $L = 47.51$ (50.98), $L_4 = -0.47$ (-1.43), $L_6 = 0.58$ (0.67), and $L_8 = 0.126$ (0.133) MeV. Using equations of state based on both Paris and Reid effective interactions which provide saturation incompressibility of symmetric nuclear matter in the range of $220 \leqslant K_0 \leqslant 270$ MeV, we estimate the ranges $0.090 \leqslant \rho_t \leqslant 0.095$ fm^{-3} and $0.49 \leqslant P_t \leqslant 0.59$ MeV fm^{-3} for the liquid–core-solid-crust transition density and pressure, respectively. The corresponding range of the proton fraction obtained at this ρ_t range is $0.029 \leqslant x_{p(t)} \leqslant 0.032$.

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To understand many astrophysical phenomena, we need to know accurate information about the density and isospin dependencies of the equation of state (EOS) of the isospin-asymmetric nuclear matter (ANM), the AEOS, which are still largely unknown [1]. The AEOS plays a significant role in determining the different properties of neutron stars (NSs) such as the proton fraction in their matter and the critical density for the direct URCA process, and consequently the cooling rate of NSs. Also, the location of the inner edge of the NSs crusts, their core-crust transition density and pressure, the crustal fraction of their moment of inertia, and the critical frequency of a rotating NS are examples of such properties. However, the expansion of the AEOS with respect to its density ρ and isospin asymmetry I is commonly used to study the nuclear matter (NM) [2,3], nuclear structure [1,4,5], and NS properties [6,7,8]. For example, based on the M3Y-Paris [9] and M3Y-Reid [10] interactions, it is found that the fourth-order symmetry energy $E_{\text{sym4}}(\rho)$ is needed to express the energy of pure neutron matter (PNM) at $\rho \geqslant 4\rho_0$ [4]. Also, $E_{\text{sym4}}(\rho)$ enhances the calculated proton fraction in β-stable $npe\mu$ matter at high densities and reduces the core-crust transition density and pressure in NS [6]. Furthermore, the constraints on the symmetry incompressibility [11,12,13] upon neglecting the higher-order symmetry energies give some discrepancies among the different studies [4,5,14,15]. The conclusion drawn is that the widely used empirical parabolic approximation of the AEOS may produce significant errors in the calculated ANM properties.

In the framework of a nonrelativistic Hartree-Fock scheme [16], the ANM energy per nucleon based on the density-dependent M3Y-Paris (Reid) NN interaction reads [17]

$$E_A(\rho,I) = \frac{3\hbar^2 k_F^2[(1+I)]^{5/3} + (1-I)^{5/3}}{20m} + f(\rho)\frac{\rho}{2}\Big\{C_0 J_{00}^D + I^2 C_1 J_{01}^D + \frac{1}{4}\int \big[C_0 v_{00}^{Ex} B_0^2 + C_1 v_{01}^{Ex} B_1^2\big] d\vec{r}\Big\},$$

$$B_{\substack{0\\(1)}}(I,r) = (1+I)\hat{j}_1(k_{Fn}r)^{+}_{(-)}(1-I)\hat{j}_1(k_{Fp}r). \quad (1)$$

Here, $I = (\rho_n - \rho_p)/\rho$ and m is the nucleonic mass. k_{Fn},k_{Fp}, and k_F denote the neutron, proton, and total Fermi momenta, respectively. $v_{00}^{D(Ex)}$ and $v_{01}^{D(Ex)}$ are the central isoscalar and isovector direct (exchange) components [4,16,18] of the M3Y interactions, respectively, $J_{00(01)}^D = \int v_{00(01)}^D(r) d\vec{r}$. In terms of the first-order spherical Bessel function, $\hat{J}_1(x) = 3 j_1(x)/x$. The CDM3Y density-dependent form of the M3Y effective interaction is given as [16,19,20]

$$v_{00(01)}^{D(Ex)}(\rho,r) = C_{0(1)} f(\rho) v_{00(01)}^{D(Ex)}(r) = C_{0(1)}(1 + \alpha e^{-\beta\rho} - \gamma\rho) v_{00(01)}^{D(Ex)}(r).$$

Around $I = 0$, we can expand the AEOS as

$$E_A(\rho,I) = E_A(\rho,0) + E_{\text{sym}}(\rho) I^2 + E_{\text{sym4}}(\rho) I^4 + E_{\text{sym6}}(\rho) I^6 + E_{\text{sym8}}(\rho) I^8 + \cdots, \quad (2)$$

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$E_{\rm sym}(\rho) \equiv E_{\rm sym2}$. According to Eqs. (1) and (2), the symmetry energy coefficients read [4],

$$E_{\rm sym}(\rho) = \frac{\hbar^2 k_F^2}{6m} + \frac{f(\rho)\rho}{2}\Big\{C_1 J_{01}^D + \frac{1}{4}\int \big[C_0 v_{00}^{Ex} B_{00} B_{00}^{(2)} + C_1 v_{01}^{Ex}\big(B_{10}^{(1)}\big)^2\big] d\vec{r}\Big\},$$

$$E_{\rm sym4}(\rho) = \frac{\hbar^2 k_F^2}{162m} + \frac{f(\rho)\rho}{96} \times \int \big[C_0 v_{00}^{Ex} M_0 + C_1 v_{01}^{Ex} M_1\big] d\vec{r},$$

$$E_{\rm sym6}(\rho) = \frac{77\hbar^2 k_F^2}{20\,(3^7)m} + \frac{f(\rho)\rho}{6!4} \times \int \big[C_0 v_{00}^{Ex} M_2 + C_1 v_{01}^{Ex} M_3\big] d\vec{r},$$

$$E_{\rm sym8}(\rho) = \frac{1309\hbar^2 k_F^2}{80\,(3^9)m} + \frac{f(\rho)\rho}{8!4} \times \int \big[C_0 v_{00}^{Ex} M_4 + C_1 v_{01}^{Ex} M_5\big] d\vec{r}, \tag{3}$$

where $M_0 = 3(B_{00}^{(2)})^2 + B_{00}B_{00}^{(4)}$, $M_1 = 4B_{10}^{(1)}B_{10}^{(3)}$, $M_2 = 15B_{00}^{(2)}B_{00}^{(4)} + B_{00}B_{00}^{(6)}$, $M_3 = 10(B_{10}^{(3)})^2 + 6B_{10}^{(1)}B_{10}^{(5)}$, $M_4 = 35(B_{00}^{(4)})^2 + 28B_{00}^{(2)}B_{00}^{(6)} + B_{00}B_{00}^{(8)}$, $M_5 = 56B_{10}^{(3)}B_{10}^{(5)} + 8B_{10}^{(1)}B_{10}^{(7)}$, $B_{00(10)} \equiv B_{0(1)}(I=0)$, and $B_{00(10)}^{(n)} \equiv \frac{\partial^n B_{0(1)}}{\partial I^n}|_{I=0}$.

Thermodynamically, the chemical equilibrium of the direct URCA reactions, $n \to p + e^- + \tilde{\nu}_e$ and $p + e^- \to n + \nu_e$, in β-stable matter ($npe\mu$) of a NS yields [7]

$$\mu_e = \mu_n - \mu_p = 2\frac{\partial E_A(\rho,I)}{\partial I}. \tag{4}$$

$\mu_i (i = n,p,e)$ are the chemical potentials for the neutrons, protons, and electrons, respectively. Because they require high electronic chemical potential [21], muons start to appear at $\rho \geqslant \rho_0$ [22] providing very little contribution to the chemical equilibrium. The charge neutrality of NSs implies $\rho_e = \rho_p = \rho x (k_{Fe} = k_{Fp})$. $x = \rho_p/\rho$ is the proton fraction of ANM, $I = 1 - 2x$. The chemical potential of the relativistic electrons becomes

$$\mu_e = \sqrt{k_{Fe}^2 c^2 + m_e^2 c^4} \approx k_{Fe} c = \hbar c (3\pi^2 \rho x)^{1/3}. \tag{5}$$

According to Eqs. (1), (4), and (5), the proton fraction of β-stable matter (x_p) is determined by

$$x_p = \frac{1}{3\pi^2\rho}\Bigg(\frac{\hbar k_F^2}{2mc}\big[(2-2x_p)^{\frac{2}{3}} - (2x_p)^{\frac{2}{3}}\big] + \frac{2f(\rho)\rho}{\hbar c}\Big\{(1-2x_p)c_1 J_{01}^D + \frac{1}{4}\int \big[c_0 v_{00}^{Ex} B_0 B_0^{(1)} + c_1 v_{01}^{Ex} B_1 B_1^{(1)}\big] d\vec{r}\Big\}\Bigg)^3. \tag{6}$$

One can approximate x_p using the AEOS expansion, Eq. (2), in addition to Eqs. (4) and (5), as

$$x_p = \frac{1}{2}\left[1 - \frac{\hbar c (3\pi^2 \rho x_p)^{\frac{1}{3}}}{4[E_{\rm sym} + 2E_{\rm sym4}(1-2x_p)^2 + 3E_{\rm sym6}(1-2x_p)^4 + 4E_{\rm sym8}(1-2x_p)^6]}\right]. \tag{7}$$

In terms of the pressures of baryons [17] $P_N(\rho,x)$ and electrons $P_e(\rho,x)$, the total pressure of the npe matter, using Eq. (1), becomes

$$P(\rho,x) = P_N(\rho,x) + P_e(\rho,x) = \frac{\hbar^2 k_F^2\big[(2-2x)^{\frac{5}{3}} + (2x)^{\frac{5}{3}}\big]\rho}{10m} + \frac{1}{2}(\rho^3 f'(\rho) + \rho^2 f(\rho))\Big\{C_0 J_{00}^D + (1-2x)^2 C_1 J_{01}^D + \frac{1}{4}\int \big(C_0 v_{00}^{Ex} B_0^2 + C_1 v_{01}^{Ex} B_1^2\big) d\vec{r}\Big\} - \frac{\rho^2}{4} f(\rho)\int \big(C_0 v_{00}^{Ex} B_0 B_2 + C_1 v_{01}^{Ex} B_1 B_3\big) d\vec{r} + \frac{\hbar c}{12\pi^2}\big(3\pi^2\rho x\big)^{4/3} \tag{8}$$

where $B_{\underset{(3)}{2}}(I,r) = (1+I)\, j_2(k_{Fn} r) \overset{+}{\underset{(-)}{}} (1-I) j_2(k_{Fp} r)$ and $f'(\rho) \equiv \frac{\partial f(\rho)}{\partial \rho}$. Correspondingly, based on Eq. (2), an approximate total pressure becomes

$$P(\rho,x) = \rho^2\big[E_A^{[1]}(\rho, x=0.5) + E_{\rm sym}^{[1]}(1-2x)^2 + E_{\rm sym4}^{[1]}(1-2x)^4 + E_{\rm sym6}^{[1]}(1-2x)^6 + E_{\rm sym8}^{[1]}(1-2x)^8\big] + \frac{\hbar c}{12\pi^2}(3\pi^2\rho x)^{4/3}, \tag{9}$$

where $E_{{\rm sym}j}^{[n]} (j = 0,2,4,6,8;\ n = 1,2,\ldots) = \frac{d^n E_{{\rm sym}j}(\rho)}{d\rho^n}$.

The intrinsic stability condition of a single phase for locally neutral matter under β equilibrium is determined, thermodynamically, by the positivity of the compressibility of matter K_μ, under constant chemical potential [23],

$$K_\mu = \left(\frac{\partial P}{\partial \rho}\right)_\mu = \frac{K(\rho,I)}{9} - \frac{\left(\frac{\partial^2 E_A(\rho,I)}{\partial\rho\partial I}\rho\right)^2}{\frac{\partial^2 E_A(\rho,I)}{\partial I^2}} > 0, \tag{10}$$

$K(\rho,I) = 9[2\rho \frac{\partial E_A(\rho,I)}{\partial \rho} + \rho^2 \frac{\partial^2 E_A(\rho,I)}{\partial \rho^2}]$ is the known ANM incompressibility [17]. The last term in Eq. (10) arises from the leptonic pressure. Another stability condition regarding the electrical capacitance of matter $[\chi_v = -(\partial q/\partial \mu)_v > 0]$ is usually valid in our case [21,23]. However, the limiting density that breaks these conditions will correspond to the core-crust (liquid-solid) phase transition. Using Eq. (1), we obtain

$$\begin{aligned} K(\rho,I) = {} & \frac{3\hbar^2 k_F^2\left[(1+I)^{\frac{5}{3}} + (1-I)^{\frac{5}{3}}\right]}{2m} \\ & + 9\left(\frac{\rho^3}{2} f''(\rho) + 2\rho^2 f'(\rho) + \rho f(\rho)\right)\left\{C_0 J_{00}^D + I^2 C_1 J_{01}^D + \frac{1}{4}\int \left(C_0 v_{00}^{Ex} B_0^2 + C_1 v_{01}^{Ex} B_1^2\right) d\vec{r}\right\} \\ & - \frac{9}{4}[2\rho^2 f'(\rho) + 3\rho f(\rho)]\int \left(C_0 v_{00}^{Ex} B_0 B_2 + C_1 v_{01}^{Ex} B_1 B_3\right) d\vec{r} \\ & - \frac{9\rho}{4} f(\rho)\int \left[C_0 v_{00}^{Ex}\left(B_0 B_4 - B_2^2\right) + C_1 v_{01}^{Ex}\left(B_1 B_5 - B_3^2\right)\right] d\vec{r}, \end{aligned} \tag{11}$$

$$\begin{aligned} \frac{\partial^2 E_A(\rho,I)}{\partial \rho \partial I} = {} & \frac{\hbar^2 k_F^2}{6m\rho}\left[(1+I)^{\frac{2}{3}} - (1-I)^{\frac{2}{3}}\right] + [f'(\rho)\rho + f(\rho)]\left\{I C_1 J_{01}^D + \frac{1}{4}\int \left[C_0 v_{00}^{Ex} B_0 B_0^{(1)} + C_1 v_{01}^{Ex} B_1 B_1^{(1)}\right] d\vec{r}\right\} \\ & + \frac{1}{4} f(\rho)\rho \int \left[C_0 v_{00}^{Ex}\left(B_0^{[1]} B_0^{(1)} + B_0 B_0^{(11)}\right) + C_1 v_{01}^{Ex}\left(B_1^{[1]} B_1^{(1)} + B_1 B_1^{(11)}\right)\right] d\vec{r}, \end{aligned} \tag{12}$$

and

$$\begin{aligned} \frac{\partial^2 E_A(\rho,I)}{\partial I^2} = {} & \frac{\hbar^2 k_F^2}{6m}\left[(1+I)^{-\frac{1}{3}} + (1-I)^{-\frac{1}{3}}\right] \\ & + f(\rho)\rho\left\{C_1 J_{01}^D + \frac{1}{4}\int \left[C_0 v_{00}^{Ex}\left(\left(B_0^{(1)}\right)^2 + B_0 B_0^{(2)}\right) + C_1 v_{01}^{Ex}\left(\left(B_1^{(1)}\right)^2 + B_1 B_1^{(2)}\right)\right] d\vec{r}\right\}, \end{aligned}$$

$$B_{\underset{(5)}{4}}(I,r) = \{(1+I)\,[2j_2(k_{Fn}r) - (k_{Fn}r)\,j_3(k_{Fn}r)] \underset{(-)}{+} (1-I)\,[2j_2(k_{Fp}r) - (k_{Fp}r) j_3(k_{Fp}r)]\}/3, \tag{13}$$

where $B_i^{(n)}(i = 0,1,\ldots;\ n = 1,2,\ldots) \equiv \frac{\partial^n B_i}{\partial I^n}, B_i^{[1]} = \frac{\partial B_i}{\partial \rho}$, and $B_i^{(11)} = \frac{\partial^2 B_i}{\partial \rho \partial I}$.

Employing Eq. (2), we can express the incompressibility condition, Eq. (10), as

$$\begin{aligned} K_\mu = {} & 2\rho\left[E_A^{[1]}(\rho,0) + E_{\text{sym}}^{[1]} I^2 + E_{\text{sym4}}^{[1]} I^4 + E_{\text{sym6}}^{[1]} I^6 + E_{\text{sym8}}^{[1]} I^8\right] + \rho^2\left[E_A^{[2]}(\rho,0) + E_{\text{sym}}^{[2]} I^2 + E_{\text{sym4}}^{[2]} I^4 + E_{\text{sym6}}^{[2]} I^6 + E_{\text{sym8}}^{[2]} I^8\right] \\ & - \frac{2I^2\rho^2\left[E_{\text{sym}}^{[1]} + 2E_{\text{sym4}}^{[1]} I^2 + 3E_{\text{sym6}}^{[1]} I^4 + 4E_{\text{sym8}}^{[1]} I^6\right]^2}{E_{\text{sym}} + 6E_{\text{sym4}} I^2 + 15E_{\text{sym6}} I^4 + 28E_{\text{sym8}} I^6} > 0. \end{aligned} \tag{14}$$

Shown in Fig. 1 is the density dependence of the proton fraction (x_p) in β-stable $npe\mu$ (npe) matter based on the M3Y-Paris, Figs. 1(a) and 1(c), and M3Y-Reid, Fig. 1(b), interactions with different parametrizations [17] of their CDM3Y-K density-dependent form. These parametrizations generate equations of state characterized by saturation incompressibility values in the range of $220 \leqslant K_0 \leqslant 270$ MeV. In Figs. 1(a) and 1(b), the calculations based on the different symmetry energies, Eq. (7), are compared with those based on the full AEOS, Eq. (6). The predicted proton fraction from the different equations of state based on the Paris interaction are displayed in Fig. 1(c). As can be seen, the proton fraction based on both the M3Y-Paris and M3Y-Reid interactions show almost the same behavior with density. For the CDM3Y-240 ($K_0 = 240$ MeV) form of the Paris (Reid) interaction, the proton fraction increases with ρ in the low density region, reaching $x_p^{\max} = 0.049$ at $\rho = 0.28$ (0.27) fm^{-3}. It then starts to decrease with ρ in the region of $\rho \geqslant 0.28$ fm^{-3}. The β-stable NM becomes proton-free, and consequently electron-free, matter at $\rho = 1.08$ (0.95) fm^{-3}. Even by considering the higher-order symmetry energies, up to E_{sym8}, the exact $x_p^{\max} = 0.049$ is not obtained. The AEOS expansion up to E_{sym}, E_{sym4}, E_{sym6}, and E_{sym8} yields $x_p^{\max} = 0.043$ (0.044), 0.043, 0.045, and 0.045, respectively. The different equations of state give a similar behavior for x_p but with a slight shift in $x_p^{\max}$, $0.050 \geqslant x_p^{\max} \geqslant 0.048$ in the range of $0.30 \geqslant \rho \geqslant 0.25$ fm^{-3}, Fig. 1(c). The AEOS does not affect x_p up to a density of $\rho \approx 0.25$ fm^{-3}. Its main effect appears in the density limit at which the β-stable matter becomes PNM. This limiting density decreases as the stiffness of the EOS increases. It extends to $\rho = 1.54$, 1.25, 1.08, 0.96, and 0.79 fm^{-3} for an AEOS of $K_0 = 220$, 230, 240, 250, and 270 MeV, respectively. Because $x_p^{\max} = 0.049$, the direct URCA process in NS would be then forbidden. This process is permitted only for $x_p \geqslant 1/9$ [24,25]. This confirms the suggested relatively slow cooling process of NS $[n + (n,p) \to p + (n,p) + e^- + \tilde{\upsilon}_e$ and $p + (n,p) \to n + (n,p) + e^+ + \upsilon_e]$ [24,26,27]. It was also concluded theoretically that an acceptable AEOS shall

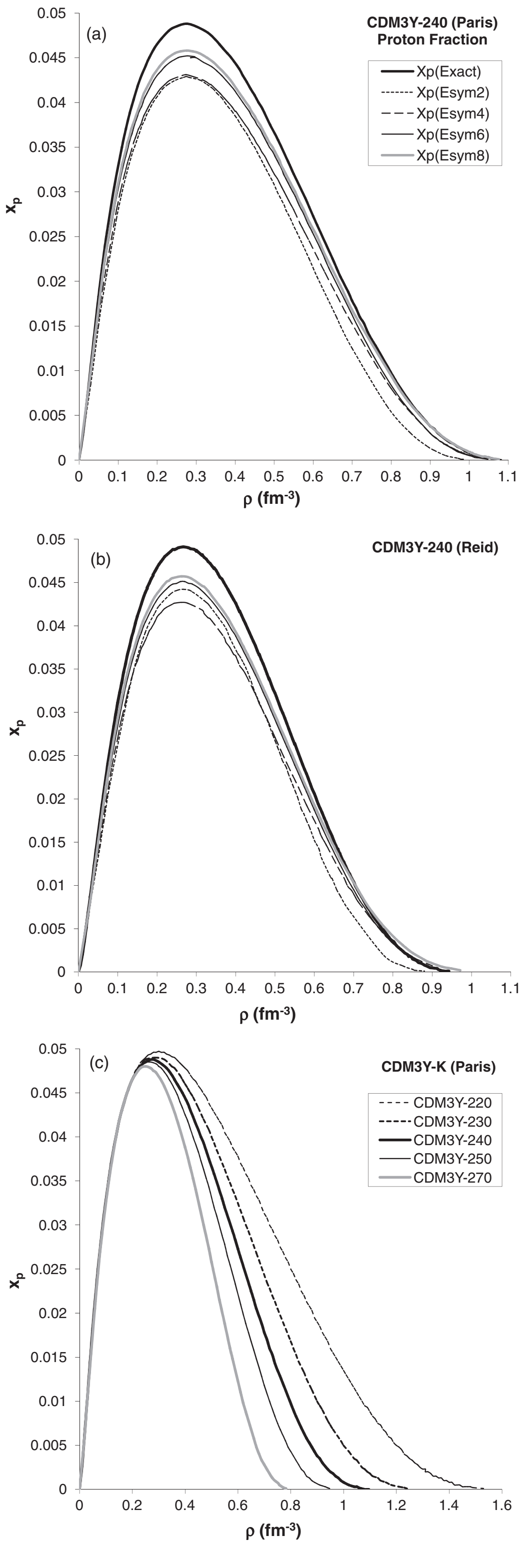

FIG. 1. Density dependence of the proton fraction in the β-stable *npe* matter based on the M3Y (a) Paris and (b) Reid *NN* interactions in their CDM3Y-240 density-dependent form in terms of the full AEOS, Eq. (6). The approximate calculations based on the expansion of the EOS up to different-order symmetry energies, $E_{\mathrm{sym}\boldsymbol{n}}(n = 2,4,6,8)$, are presented for comparison. (c) Exact calculations based on different EOSs characterized by incompressibility values of $220 \leqslant K_0 \leqslant 270$ MeV.

not allow the direct URCA process to occur in NSs with masses below 1.5 solar masses [28]. Even recent experimental observations that suggest high heat conductivity and an enhanced core cooling process indicating the enhanced level of neutrino emission were not attributed to the direct URCA process but were proposed to be due to the breaking and formation of neutron Cooper pairs [29–32].

Table I presents the core-crust transition density in NSs as extracted from the exact calculations of the incompressibility condition, Eqs. (10)–(13), based on equations of state of incompressibility range $K_0 = 220$–250 MeV. This range is estimated for the EOSs in various studies on NM [17], finite nuclei [5,13,33,34,35], and nuclear reactions [19,20,36,37,38]. Also, the exact transition pressure, Eq. (8), and the corresponding proton fraction, Eq. (6), are presented in Table I. We may question now the degree of accuracy of calculating the NS properties based on the isospin-asymmetry expansion of the AEOS. To this aim the approximate calculations of the transition density, Eq. (14), and pressure, Eq. (9), and the corresponding equilibrium proton fraction, Eq. (7), using the different symmetry-energy coefficients, are presented in Table I. As seen, the transition density and pressure and the proton fraction are estimated exactly to be within the ranges of $0.090 \leqslant \rho_t \leqslant 0.095$ fm^{-3}, $0.49 \leqslant P_t \leqslant 0.59$ MeV fm^{-3}, and $0.029 \leqslant x_{p(t)} \leqslant 0.032$, respectively. Using the Gogny MDI and 51 Skyrme interactions, the limits of $0.040 \leqslant \rho_t < 0.065$ fm^{-3} and $0.01 < P_t \leqslant 0.26$ MeV fm^{-3} are imposed [21]. The calculations based on the FSUGold and IU-FSU interactions yielded $0.051 \leqslant \rho_t \leqslant 0.077$ fm^{-3} and $0.24 \leqslant P_t \leqslant 0.53$ MeV fm^{-3} [6]. Further, constraints of $0.086 \leqslant \rho_t \leqslant 0.090$ fm^{-3} and $0.30 \leqslant P_t \leqslant 0.76$ MeV fm^{-3} are obtained through the relativistic energy density functional [8]. As shown in Table I, disregarding the higher-order symmetry energies increases ρ_t and P_t and reduces slightly $x_{p(t)}$. A similar increase in ρ_t and P_t due the parabolic approximation of the EOS is demonstrated based on nonrelativistic [21] and relativistic mean field models [6]. The approximate calculations of ρ_t in terms of the symmetry energies up to E_{sym}, E_{sym4}, E_{sym6}, and E_{sym8} led to errors of about $14 \pm 2\%$, $11 \pm 2\%$, $7 \pm 2\%$, and $5 \pm 1\%$, respectively, compared with the exact calculations. The errors in the corresponding P_t ($x_{p(t)}$) values are $64 \pm 7\% (7 \pm 3\%)$, $53 \pm 5\%$ $(6 \pm 3\%)$, $44 \pm 3\%$ $(3 \pm 3\%)$ and $38 \pm 2\%$ $(3 \pm 3\%)$, respectively. However, we need to consider up to E_{sym8} to get the transition density and proton fraction with the inevitable small error ($\leqslant 6\%$). Even so, the errors in the approximate transition pressure are always large ($\geqslant 40\%$). The obtained errors are generally smaller in the case of the M3Y-Paris interaction than in the Reid one. We can relate this to the symmetry energies and their slope parameters [8,21]. The symmetry-energy coefficients and their slope parameters for the M3Y-Paris (M3Y-Reid) interaction, at the saturation density, are $E_{\mathrm{sym}}(\rho_0) = 30.85$ (31.11), $E_{\mathrm{sym4}}(\rho_0) = 0.10$ (-0.13), $E_{\mathrm{sym6}}(\rho_0) = 0.28$ (0.31), and $E_{\mathrm{sym8}}(\rho_0) = 0.098$ (0.102) MeV, and $L(\rho_0) = 47.51$ (50.98), $L_4(\rho_0) = -0.47$ (-1.43), $L_6(\rho_0) = 0.58$ (0.67), and $L_8(\rho_0) = 0.126$ (0.133) MeV. These values are independent of the density-dependence form and consequently independent of the saturation incompressibility value [4]. However, the Paris interaction, which yields smaller symmetry energies and slope

TABLE I. The exact calculations of the core-crust transition density ρ_t [Eqs. (10)–(13)] and pressure P_t [Eq. (8)], and the corresponding proton fraction $x_{p(t)}$ [Eq. (6)], in NSs using full EOSs ($K_0 = 220$-250 MeV) based on the CDM3Y-Paris and CDM3Y-Reid interactions. $\rho_t^{E_{\mathrm{sym}n}}$ [Eq. (14)],$P_t^{E_{\mathrm{sym}n}}$ [Eq. (9)], and $x_{p(t)}^{E_{\mathrm{sym}n}}$ [Eq. (7)] are the approximate values based on the isospin-asymmetry expansion of the EOS, up to different-order symmetry energies, $E_{\mathrm{sym}\boldsymbol{n}}$.

K_0 (MeV)	M3Y-Paris				M3Y-Reid			
	220	230	240	250	220	230	240	250
ρ_t^{Exact} (fm^{-3})	0.091	0.093	0.094	0.095	0.090	0.092	0.093	0.094
$\rho_t^{E_{\mathrm{sym}}}$ (fm^{-3})	0.102	0.103	0.104	0.105	0.104	0.106	0.107	0.108
$\rho_t^{E_{\mathrm{sym4}}}$ (fm^{-3})	0.100	0.101	0.102	0.103	0.102	0.103	0.104	0.105
$\rho_t^{E_{\mathrm{sym6}}}$ (fm^{-3})	0.097	0.098	0.099	0.100	0.098	0.099	0.100	0.102
$\rho_t^{E_{\mathrm{sym8}}}$ (fm^{-3})	0.095	0.096	0.097	0.098	0.095	0.097	0.098	0.099
P_t^{Exact} (MeV fm^{-3})	0.485	0.505	0.511	0.513	0.551	0.573	0.581	0.589
$P_t^{E_{\mathrm{sym}}}$ (MeV fm^{-3})	0.785	0.796	0.807	0.818	0.940	0.975	0.988	1.001
$P_t^{E_{\mathrm{sym4}}}$ (MeV fm^{-3})	0.743	0.753	0.763	0.774	0.872	0.884	0.896	0.907
$P_t^{E_{\mathrm{sym6}}}$ (MeV fm^{-3})	0.707	0.717	0.728	0.738	0.812	0.824	0.856	0.867
$P_t^{E_{\mathrm{sym8}}}$ (MeV fm^{-3})	0.678	0.688	0.698	0.709	0.761	0.791	0.803	0.814
$x_{p(t)}^{\mathrm{Exact}}$	0.031	0.031	0.032	0.032	0.029	0.029	0.029	0.029
$x_{p(t)}^{E_{\mathrm{sym}}}$	0.028	0.029	0.029	0.029	0.028	0.028	0.028	0.028
$x_{p(t)}^{E_{\mathrm{sym4}}}$	0.029	0.029	0.029	0.029	0.028	0.028	0.028	0.028
$x_{p(t)}^{E_{\mathrm{sym6}}}$	0.030	0.030	0.030	0.030	0.028	0.029	0.029	0.029
$x_{p(t)}^{E_{\mathrm{sym8}}}$	0.030	0.030	0.030	0.030	0.028	0.029	0.029	0.029

parameters than those of the Reid one, achieves a slight improvement in the calculations based the EOS expansion. We also observe a slight decrease in the transition pressure upon decreasing the symmetry energies. Actually, the criteria of accepting or rejecting a definite degree of accuracy for the approximate calculations of x_p, ρ_t and P_t depends on how much the other physical predictions related to NS are sensitive to them.

In conclusion, the higher-order symmetry-energy coefficients up to E_{sym8} are needed to describe reasonably well the proton fraction of the β stable (npe) matter at high nuclear densities, and the core-crust transition density. The parabolic approximation of the EOS does not affect seriously the proton fraction at the transition density. On the contrary, the calculations of the core-crust transition pressure upon the symmetry energies up to E_{sym8} show a deviation of as high as 40% from the exact calculations. The slope parameters of the symmetry-energy coefficients for the M3Y-Paris (Reid) interaction are $L(\rho_0) = 47.51$ (50.98), $L_4(\rho_0) = -0.47$ (-1.43), $L_6(\rho_0) = 0.58$ (0.67), and $L_8(\rho_0) = 0.126$ (0.133) MeV. Based on the CDM3Y-Paris and CDM3Y-Reid interactions ($K_0 = 220$-250 MeV), we estimate the constraints of $0.090 \leqslant \rho_t \leqslant 0.095$ fm^{-3}, $0.485 \leqslant P_t \leqslant 0.589$ MeV fm^{-3}, and $0.029 \leqslant x_{p(t)} \leqslant 0.032$ on the core-crust transition density and pressure of a NS, and the corresponding proton fraction, respectively.

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