Two-photon-exchange amplitude with πN intermediate states: P_{33} channel

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We consider two-photon exchange (TPE) in elastic electron-proton scattering and evaluate the effect of πN (pion + nucleon) intermediate hadronic states. Among different πN states, we concentrate on the P_{33} channel; thus we effectively include $\Delta(1232)$ resonance with realistic width and shape and corresponding background as well. In agreement with the previous result, obtained for the zero-width resonance, we observe that the TPE correction to the electric form factor is the largest one; it grows with Q^2 and at $Q^2 \gtrsim 2.5$ GeV^2 exceeds the corresponding elastic contribution.

I. INTRODUCTION

Elastic electron scattering is the primary tool for measuring the nucleon electromagnetic form factors which, in turn, reflect the internal structure of the nucleon and dynamics of the strong interaction inside it. There are two main methods for measuring the form factors: the standard Rosenbluth separation method and the modern polarization transfer method (used since 1998 [1]). It was realized about ten years ago that these two methods are in serious disagreement with respect to the proton form factor ratio G_E/G_M at high Q^2 [2]. While the Rosenbluth method indicated that $G_E/G_M \sim$ const, the polarization data suggested that the ratio decreases almost linearly with Q^2. This disagreement was further confirmed with increased precision experiments, using both Rosenbluth [3] and polarization [4] techniques. It was suggested that the discrepancy may be due to data analysis done in the Born approximation, thus leaving out nontrivial higher-order terms, such as two-photon exchange (TPE). Indeed, first calculations had shown [5] that the discrepancy is, at least partially, resolved by including TPE corrections.

Besides that, there is now much experimental activity aimed at the direct observation of TPE effects in the ep scattering. This includes experiments which are already completed [6], in progress [7], approved, or proposed [8]. A comprehensive review of experimental and theoretical studies of TPE in $ep \rightarrow ep$ and other processes, along with a bibliography, can be found in Ref. [9].

There are two mainstream approaches to the theoretical evaluation of the TPE amplitude: "quark" and "hadronic" ones. In the quark approach, as its name suggests, the nucleon is viewed as an ensemble of quarks (partons), interacting according to QCD [10–13]. Naturally, the applicability of this approach is limited to the high-Q^2 region. Despite all its advantages, the serious drawback is that it is hard to calculate the TPE correction to the electric form factor G_E in this approach, while this is surely needed for the correct interpretation of G_E/G_M measurements.

In the hadronic approach TPE is mediated by the production of virtual hadrons and/or hadronic resonances. The TPE amplitudes are broken into different contributions according to the intermediate state involved. The most important and well established one is the elastic contribution, which corresponds to pure nucleon intermediate state. In turn, all other contributions are called inelastic. Among them, the contributions of some prominent resonances [$\Delta(1232)$ and others] were studied in Refs. [14–16]. In Refs. [14,15] it was shown that their overall effect *on the cross section* is smaller than that of the elastic contribution, with $\Delta(1232)$ yielding its main part and the contributions of other resonances partially canceling each other.

Later, it was found [16] that $\Delta(1232)$ yields relatively large correction to the G_E/G_M form factor ratio at high Q^2 (far exceeding that of the elastic intermediate state), and that the correction grows with Q^2. This result suggests that the contributions of other inelastic states may also be important and at least should be estimated carefully. Unfortunately, all the above-mentioned papers use "zero-width" approximation; i.e., widths of resonances are assumed to be negligibly small. This approximation seems rather crude, especially for $\Delta(1232)$, since its width ($\Gamma_\Delta \sim 110$ MeV) is comparable to the distance from the threshold ($M_\Delta - M - m_\pi \sim 160$ MeV).

To overcome this issue, in the present paper we estimate the inelastic contribution to the TPE amplitude, arising from the πN (pion+nucleon) intermediate states. This may be viewed as a significant improvement of the previous "resonance" calculations, since most resonances have dominant πN content. Consequently, the advantages of our approach are

(i) automatically having correct resonance width,
(ii) automatically having correct resonance shape,
(iii) including not only resonances but background as well.

The πN contribution may further be split into the contributions of different partial waves of the πN system. Though, in principle, all partial waves may be taken into account in our method, it is particularly useful for the P_{33} channel, where Δ resides. The Δ resonance has almost 100% πN content, thus we will get pure improvement with respect to previous works. The situation is not so simple for other resonances, such as S_{11} and D_{13}, since they have significant $\pi\pi N$ branching ratios; the corresponding contribution will be missing in the present approach.

Only P_{33} channel will be considered in full detail further.

II. BACKGROUND

The idea of the present calculation is the following. The πN system is fully described by its isospin, spin-parity, and invariant mass. No other internal quantum numbers exist. Thus, with respect to the calculation of the TPE amplitudes, the πN system in the intermediate state is fully equivalent to the single particle with the same isospin, spin-parity, and mass (and properly defined transition amplitudes). If we are able to calculate the TPE contribution of the resonance with given quantum numbers, we can do precisely the same thing for the πN system of fixed invariant mass and then integrate over invariant masses.

Specifically, if, for the zero-width particle R (resonance) with mass M_R and $R \to \gamma^* N$ transition form factor $A^R(q^2)$ we have

$$
\begin{aligned}
\delta\mathcal{G}^R &= \delta\mathcal{G}[M_R, A^R(q^2)] \\
&\equiv \int A^R(q_1^2) A^R(q_2^2) \mathcal{K}(M_R, q_1^2, q_2^2) dq_1^2 dq_2^2
\end{aligned} \tag{1}
$$

(where $\delta\mathcal{G}$ stands for any TPE amplitude, and $\mathcal{K}$ is some kernel, irrelevant for the following discussion), then the full contribution of the πN partial wave with the same quantum numbers will be

$$
\delta\mathcal{G}^{\pi N} = \int \delta\mathcal{G}[W, A^{\pi N}(q^2, W)] dW^2, \tag{2}
$$

where the integration variable W is the invariant mass of the πN system and $A^{\pi N}$ is appropriately defined transition form factor. Note that q^2 stands for the square of virtual photon momentum and should not be confused with the total momentum transfer in the elastic process, Q^2.

For the P_{33} channel, all needed formulas [Eq. (1)] are already derived [16]. All we have to do is to establish a correspondence between the transition form factors, used in Ref. [16], and the multipole amplitudes for the production of the πN system.

The pion electroproduction is commonly described by the multipole amplitudes $E_{l\pm}$, $M_{l\pm}$, and $S_{l\pm}$, which are functions of q^2 and W; the subscript l is pion orbital quantum number and $\pm$ indicates the total angular momentum $j = l \pm 1/2$. One can also define helicity amplitudes as follows (see, e.g., [17,18]):

$$
\begin{aligned}
A_{1/2} &= -\frac{1}{2}\left[(j+1/2\pm 1)E_{l\pm} \pm (j+1/2\mp 1)M_{l\pm}\right], \\
A_{3/2} &= \frac{1}{2}\sqrt{(j-1/2)(j+3/2)}\left[\pm E_{l\pm} - M_{l\pm}\right], \\
S_{1/2} &= -\frac{1}{\sqrt{2}}(j+1/2)S_{l\pm}.
\end{aligned} \tag{3}
$$

Thus we have three "flavors" of the transition amplitude: A^R for the single narrow resonance, A for the πN system, and $A^{\pi N}$ for the "effective resonances" describing πN continuum. As one can guess, the amplitudes $A^{\pi N}$, which are to be put into Eq. (2), can differ from A [Eq. (3)] only in overall q^2-independent factor, which arises from different normalizations of resonance and πN states. Thus, the factor can easily be determined by considering forward kinematics, where the imaginary part of the TPE amplitudes is related to the cross

section via the optical theorem. Nevertheless, a straightforward calculation (in arbitrary kinematics) is certainly possible, and to be rigorous, we perform such calculation in the Appendix.

The sought relation is

$$
A_H^{\pi N}(q^2, W) = A_H(q^2, W)\sqrt{\frac{2Wr(2j+1)}{M(W^2-M^2)}}, \tag{4}
$$

where M is the nucleon mass, r is the pion momentum in the πN center-of-mass system, and we use the shorthand A_H for any of $A_{1/2}$, $A_{3/2}$, $S_{1/2}$. Now the argument from the beginning of this section applies, and the TPE amplitudes will be given by Eq. (2). The apparent dimension mismatch between $A_H^{\pi N}$ and A_H is not an error; recall that the dimension of A_H is GeV^{-1}, the dimension of A_H^R, meant in Eq. (1), is GeV$^{-1/2}$, and Eq. (2) contains additional integration over dW^2.

For actual calculation of the TPE amplitudes we wish to employ the TPECALC program [19]. However, both TPECALC and Ref. [16] use not A_H, but covariant form factors $F_{1,2,3}$ to describe nucleon-resonance transition. The latter are related to the transition current matrix element for $R \to \gamma^* N$ as[1]

$$
\begin{aligned}
\langle N|J^\mu|R\rangle = \frac{1}{4M^2\sqrt{MW}}(g^{\mu\alpha}q^\nu - g^{\mu\nu}q^\alpha) \\
\times \bar{U}\left[(\hat{p}\gamma_\nu - p_\nu)F_1 - p_\nu F_2 + q_\nu F_3\right]\gamma_5 V_\alpha, \tag{5}
\end{aligned}
$$

where p and q are resonance and photon momenta, U and V_α are nucleon and resonance spinors, the states $|N\rangle$ and $|R\rangle$ are normalized to unity, and the resonance mass is taken to be W.

The relationship between A_H and F_i can be obtained using the definitions of A_H (Ref. [17], Eqs. (31)–(33)) and reads

$$
\begin{aligned}
K F_1 &= [(W-M)^2 - q^2](A_{3/2} + \sqrt{3}A_{1/2}), \\
K F_2 &= [W^2 - M^2 + q^2](A_{3/2} - \sqrt{3}A_{1/2}) + 2q^2\frac{W\sqrt{6}}{|\vec{q}|}S_{1/2}, \\
K F_3 &= 2W^2(A_{3/2} - \sqrt{3}A_{1/2}) + [W^2 - M^2 + q^2]\frac{W\sqrt{6}}{|\vec{q}|}S_{1/2},
\end{aligned} \tag{6}
$$

where $|\vec{q}|$ is photon momentum in the resonance rest frame, and

$$
\begin{aligned}
K &= \frac{1}{2M^2}[(W+M)^2 - q^2][(W-M)^2 - q^2] \\
&\times \sqrt{\pi\alpha\frac{(W-M)^2 - q^2}{M(W^2-M^2)}}. \tag{7}
\end{aligned}
$$

In full analogy to A_H, the form factors F_i should be then "renormalized" according to Eq. (4),

$$
F_i^{\pi N}(q^2, W) = F_i(q^2, W)\sqrt{\frac{2Wr(2j+1)}{M(W^2-M^2)}}. \tag{8}
$$

Note that, though for a single narrow resonance F_i are purely real, the calculation of the TPE amplitudes is well possible

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[1] In arXiv:1209.2746v1, the sign of the $p_\nu F_2$ term is incorrect.

when they are complex; the latter is obviously the case here, since the amplitudes A_H are complex.

III. TECHNICAL DETAILS

The multipole amplitudes $E_{l\pm}$, $M_{l\pm}$, and $S_{l\pm}$ were taken from the unitary isobar model MAID2007 [18]; the numerical values were downloaded from the dedicated website [20] for q^2 from 0 to 3 GeV^2 in steps of 0.05 GeV^2 and W from 1082 to 1550 MeV in steps of 15 MeV. Note that the site gives the amplitudes for the isospin channels, named $A_p^{(1/2)}$, $A_n^{(1/2)}$, and $A^{(3/2)}$. The amplitudes for $\gamma^* p \to \pi N (I = 3/2)$, which we need here, are $\sqrt{2/3} A^{(3/2)}$. Then, for each discrete W value W_i, the multipole amplitudes were converted into the helicity amplitudes A_H [Eq. (3)], and then into the transition form factors $F_{1,2,3}^{\pi N}$ [Eqs. (6) and (8)]. Much like it was described in Refs. [16,19], the resulting form factors were fitted with the sum of poles:

$$F(q^2, W_i) = \sum_{j=1}^{7} \frac{c_{ij} q^2}{q^2 - m_{ij}^2}, \qquad (9)$$

where c_{ij} are complex and m_{ij} are real parameters, with the restrictions $m_{i1} = 0$, $m_{i2} = M + W_i$. The TPE contributions were then calculated: at first for each individual W value, following procedures described in Ref. [16]; and finally they were integrated over W (with the rectangle method), yielding the total $\pi N (P_{33})$ contribution.

To get an impression of the numerical integration errors, we choose several representative kinematical points, and for that points, tried to

(i) vary sampling step in W (between 5, 10 and 15 MeV),
(ii) use Simpson's rule instead of the rectangle method.

In all cases the TPE amplitudes changed by no more than 2%–3%, which is small enough (note that since we are anyway using an expansion in α, no more than $\sim 1\%$ accuracy is needed).

Another source of uncertainty is the choice of the upper integration limit in W. In principle, we should integrate up to infinity, but for a numerical calculation we have to choose some
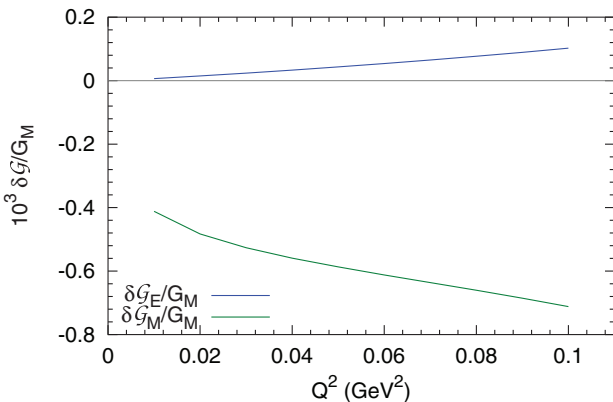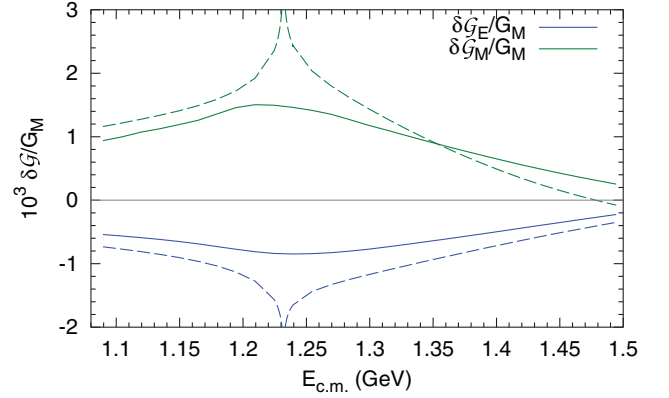

FIG. 2. (Color online) The TPE amplitudes near the Δ resonance, $\theta_{c.m.} = 90°$; πN contribution from this work (solid) and zero-width Δ [16] (dashed).

finite value, and 1550 MeV was used in most calculations. To probe the error resulting from cutting the integral off, we try to extend the integration limit from 1550 MeV to 1750 MeV or 2 GeV (note that the MAID multipoles only exist up to $W = 2$ GeV). Again, the change in the TPE amplitudes was no more than 3%, except in high-Q^2 region. If Q^2 is high (~ 5 GeV^2), then increasing upper integration limit changes the TPE amplitudes by about 5%–7%. Though such precision is still quite acceptable, this implies that the role of the intermediate states with higher masses increases with Q^2.

IV. RESULTS

As usual, we describe TPE by three invariant amplitudes (generalized form factors) $\delta \mathcal{G}_E$, $\delta \mathcal{G}_M$, and $\delta \mathcal{G}_3$. The corrections to the cross-section or polarization observables can be expressed in terms of these amplitudes; for all relevant formulas see Refs. [16,19].

There are three kinematical regions, where it is interesting to look at newly calculated $\pi N (P_{33})$ contribution:

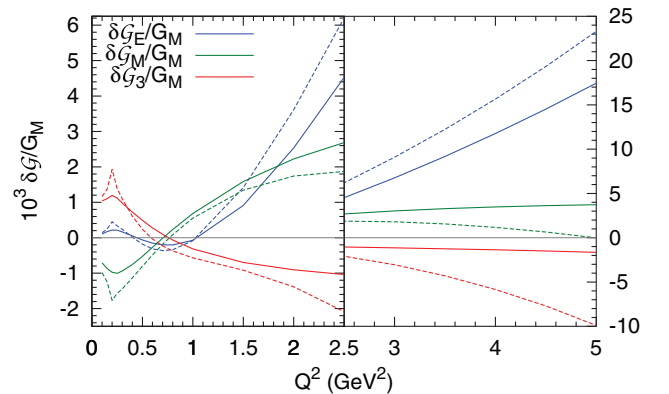(i) the low-Q^2 region, which might affect proton radius extraction,

FIG. 1. (Color online) The πN contribution to the TPE amplitudes at $\varepsilon = 0.25$ and low Q^2.

FIG. 3. (Color online) Q^2 dependence of the πN contribution to the TPE amplitudes, at fixed $\varepsilon = 0.25$. This work (solid) and Ref. [16] (dashed). Note the different y-axis scales in the left and right subplots.
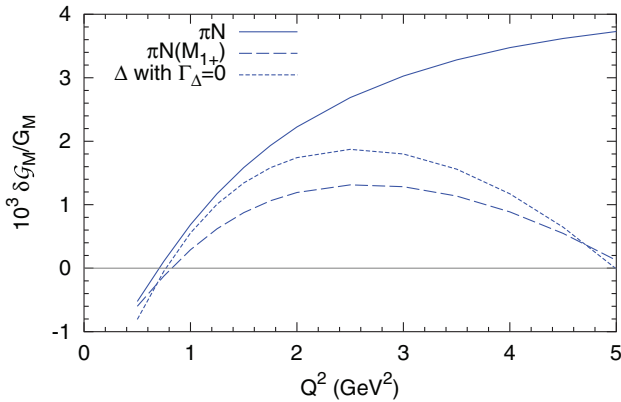
FIG. 4. (Color online) The TPE amplitude $\delta\mathcal{G}_M/G_M$, at fixed $\varepsilon = 0.25$, calculated in different approximations.

(ii) the vicinity of the resonance, where the zero-width approximation fails,

(iii) the high-Q^2 region, where previous works have revealed the substantial growth of the TPE corrections to the P_t/P_l polarization ratio.

At low Q^2 the πN contributions are small (with respect to the elastic contribution) and change moderately with Q^2 (Fig. 1), thus the proton radius extraction is not affected by the πN intermediate states.

The TPE amplitudes in the resonance region are shown in Fig. 2. Just as it was expected, there are smooth bumps at the resonance position, instead of the sharp peaks, which are seen in the zero-width approximation [16] (dashed lines).

Figure 3 shows Q^2 dependence of the calculated TPE amplitudes at fixed $\varepsilon = 0.25$ for Q^2 up to 5 GeV^2. The calculation of Ref. [16] is shown with dashed lines for comparison. We see that for Q^2 below ~ 2.5 GeV^2 the two approaches give very similar results. For higher Q^2, again in the agreement with Ref. [16], the amplitude $\delta\mathcal{G}_E$ dominates and grows with Q^2 (though its numerical value is somewhat smaller); in contrast, the amplitudes $\delta\mathcal{G}_M$ and $\delta\mathcal{G}_3$ have quite different values here and in Ref. [16]. When comparing these numerical values, one must keep in mind that present approach differs from that of Ref. [16] in several aspects:

(i) now we include E_{1+}, M_{1+}, and S_{1+} multipole amplitudes, whereas Ref. [16] effectively includes only M_{1+} amplitude (magnetic transition);

(ii) the resonance shape differs from pure Breit-Wigner;

(iii) the background contribution is included.

This is illustrated in Fig. 4, which shows the amplitude $\delta\mathcal{G}_M$, calculated in three ways:

(i) in the present approach,

(ii) in the present approach with magnetic transition only (F_2 and F_3 transition form factors set to zero),

(iii) in the approach of Ref. [16].

We see that the difference mainly results from neglecting electric transition in Ref. [16]. In a similar way we have found that the difference of the amplitude $\delta\mathcal{G}_E$ in the two approaches is mainly due to neglecting Δ resonance width. (Theoretically, the difference also could result from the contribution of states with higher W's, which are missing in "narrow Δ" calculation. However, actually this contribution is small; see the end of Sec. III).

Finally, in Fig. 5 we plot the TPE correction to the polarization ratio. At high Q^2 we see the same behavior which was found in Ref. [16], namely the correction grows rapidly with Q^2. Numerically the correction is $\sim 30\%$ smaller than that obtained in Ref. [16] (for the reasons discussed above).

V. CONCLUSIONS

We have considered the inelastic TPE contribution, originating from πN intermediate states, and, specifically, the P_{33} partial wave. Thus we effectively include $\Delta(1232)$ resonance and take into account its finite width. Numerically we obtain the following results:

(i) At small Q^2 this contribution is small (negligible w.r.t. the elastic one).

(ii) The TPE amplitudes have smooth maxima at the resonance position ($E_{\text{c.m.}} \approx M_\Delta$).

(iii) At high Q^2 we confirm the findings of Ref. [16], obtained with the zero-width Δ. The main correction comes to the generalized electric form factor. This correction (and, consequently, the correction to the
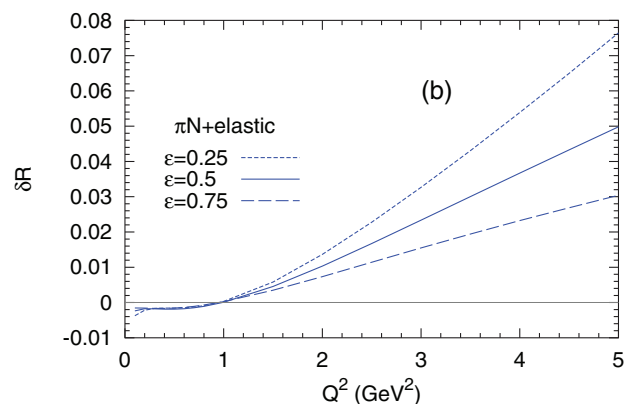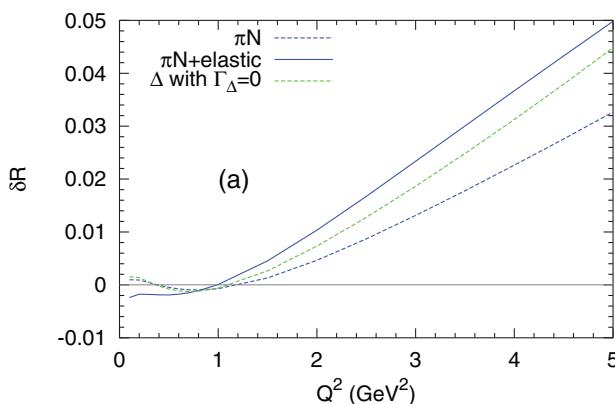

FIG. 5. (Color online) The TPE correction to the proton form factor ratio $R = \mu G_E/G_M$, as measured in polarization experiments; various contributions at fixed $\varepsilon = 0.5$ (a) and total at different values of ε (b).

polarization ratio) is relatively large and grows with Q^2. Its numerical value is somewhat smaller than in Ref. [16].

In summary, we see that (contrary to the common belief) the TPE corrections to the polarization ratio are not negligible at high Q^2. The questions which remains open are whether or not the contributions of partial waves other than P_{33} are small, how many of them should be taken into account, and how large is the error resulting from leaving out partial waves with higher spins. Surely, it is desirable to answer these questions before the TPE corrections are applied to experimental data. However, this is a separate task, and we plan to do it in further papers.

ACKNOWLEDGMENT

APPENDIX: STRAIGHTFORWARD CALCULATION OF THE πN CONTRIBUTION VIA MULTIPOLES

Let the process kinematics be

$$e(k) + p(p) \to e(k'') + \pi(r) + p(p'') \to e(k') + p(p'),$$
(A1)

and virtual photon momenta $q_1 = k - k''$ and $q_2 = k' - k''$. The imaginary part of the TPE amplitude is given by

$$\text{Im}\,\mathcal{M}_{fi} = \frac{1}{8\pi^2} \int \frac{(4\pi\alpha)^2}{q_1^2 q_2^2} L_{\alpha\beta} W_{\alpha\beta} d^4k'' \delta(k''^2 - m^2), \quad (A2)$$

where $L_{\alpha\beta} = \bar{u}'\gamma_\alpha(\hat{k}'' + m)\gamma_\beta u$ is leptonic tensor and

$$W_{\alpha\beta} = \sum_h \langle p'|J_\alpha|h\rangle\langle h|J_\beta|p\rangle(2\pi)^3\delta(p_h - p - q_1) \quad (A3)$$

is hadronic tensor. Here J_α is electromagnetic current operator, $|p\rangle$ and $|p'\rangle$ are initial and final protons, $|h\rangle$ is intermediate hadronic state (consisting of one or more particles), and p_h is its total momentum. The summation over h actually consists of

   (i) the summation over different particle types,
 (ii) the integration over their momenta (the integration element is $\frac{1}{(2\pi)^3}\frac{d\vec{p}}{2p_0}$),
(iii) the summation over their spins.

This allows us to distinguish between elastic and inelastic contributions, contributions of various resonances, etc., depending of the nature of the intermediate state h. The contribution of the πN intermediate states, which is under consideration now, has the form

$$W_{\alpha\beta} = \int \frac{d\vec{p}''}{2p_0''} \frac{d\vec{r}}{2r_0} \frac{1}{(2\pi)^3} \sum_{\text{spin}} \langle p'|J_\alpha|\pi N\rangle\langle \pi N|J_\beta|p\rangle$$
$$\times \delta(p'' + r - p - q_1), \quad (A4)$$

where p'' and r are the momenta of the nucleon and pion, respectively. The tensor $W_{\alpha\beta}$ is convenient to evaluate in

the πN center-of-mass system, where $\vec{p} + \vec{q}_1 = 0$. Assuming initial and final particles have definite helicities, we may write

$$W_{\lambda'\mu';\lambda\mu} \equiv \overset{*(\lambda')}{\varepsilon_\alpha}\varepsilon_\beta^{(\lambda)} W_{\alpha\beta} = \frac{(8\pi W)^2}{4\pi\alpha}\frac{|\vec{r}|}{(2\pi)^3 W}\int d\Omega$$
$$\times \sum_{\mu''}\langle \vec{q}_2\lambda'\mu'|f^+|\vec{r}\mu''\rangle\langle \vec{r}\mu''|f|\vec{q}_1\lambda\mu\rangle, \quad (A5)$$

where $\varepsilon_\alpha^{(\lambda)}$ is the polarization vector of virtual photon with helicity λ, $d\Omega$ is pion solid angle, μ, μ', and μ'' are helicities of the initial, final, and intermediate protons, and

$$\langle \vec{r}\mu''|f|\vec{q}_1\lambda\mu\rangle = \frac{\sqrt{4\pi\alpha}}{8\pi W}\varepsilon_\alpha^{(\lambda)}\langle \pi N|J_\alpha|p\rangle \quad (A6)$$

are the helicity amplitudes for the process $\gamma^* p \to \pi N$, defined according to [21]. The angular dependence of these amplitudes is determined by the general properties of space rotations; regardless of the interaction details, they have the following structure [21]:

$$\langle \vec{r}\mu''|f|\vec{q}_1\lambda\mu\rangle = \sum_j \frac{2j+1}{4\pi}\mathcal{D}^j_{-\mu'',\lambda-\mu}(\vec{r})\langle \mu''|f^j|\lambda\mu\rangle. \quad (A7)$$

Following [21], we use the notation

$$\mathcal{D}^j_{-\mu'',\lambda-\mu}(\vec{r}) \equiv \mathcal{D}^j_{-\mu'',\lambda-\mu}(\phi,\theta,0) \quad (A8)$$

for Wigner $\mathcal{D}$-functions (where ϕ, θ are spherical angles of the vector $\vec{r}$), and

$$\langle \mu''|f^j|\lambda\mu\rangle \equiv \langle jm\mu''|f|jm\lambda\mu\rangle, \quad (A9)$$

where $|jm\lambda\mu\rangle$ are states with definite angular momentum j and its projection m, and (here and below) $\vec{q}_1$ is directed along the z axis. For $\phi = 0$, these amplitudes are commonly denoted $H_{1..6}$ [17]:

$$
\begin{aligned}
H_1 &= H_{\frac{1}{2};-1\frac{1}{2}} = H_{-\frac{1}{2};1-\frac{1}{2}}, \\
H_2 &= H_{-\frac{1}{2};1\frac{1}{2}} = -H_{\frac{1}{2};-1-\frac{1}{2}}, \\
H_3 &= H_{\frac{1}{2};1-\frac{1}{2}} = -H_{-\frac{1}{2};-1\frac{1}{2}}, \\
H_4 &= H_{\frac{1}{2};1\frac{1}{2}} = H_{-\frac{1}{2};-1-\frac{1}{2}}, \\
H_5 &= -H_{\frac{1}{2};0\frac{1}{2}} = H_{-\frac{1}{2};0-\frac{1}{2}}, \\
H_6 &= H_{\frac{1}{2};0-\frac{1}{2}} = H_{-\frac{1}{2};0\frac{1}{2}},
\end{aligned}
\quad (A10)
$$

where

$$H_{\mu'';\lambda\mu} \equiv \langle \vec{r}\mu''|f|\vec{q}_1\lambda\mu\rangle. \quad (A11)$$

Comparing with Eqs. (7)–(12) of Ref. [17], we find

$$\langle \mu''|f^j|\lambda\mu\rangle = \frac{4\pi}{\sqrt{2}}\left[2\mu\mathcal{A}_{l+}^{(2\mu\lambda)} + 2\mu''\mathcal{A}_{l+1,-}^{(2\mu\lambda)}\right], \quad (A12)$$

where $l = j - 1/2$ and

$$\mathcal{A}_{l\pm}^{(1)} = -A_{l\pm},$$

$$(\mathcal{A}_{l+}^{(-1)}, \mathcal{A}_{l+1,-}^{(-1)}) = \frac{1}{2}\sqrt{l(l+2)}(-B_{l+}, B_{l+1,-}), \quad (A13)$$

$$(\mathcal{A}_{l+}^{(0)}, \mathcal{A}_{l+1,-}^{(0)}) = \frac{\sqrt{-q^2}}{|\vec{q}|}\frac{l+1}{\sqrt{2}}(S_{l+}, S_{l+1,-})$$

(where $A_{l\pm}$, $B_{l\pm}$, and $S_{l\pm}$ are usual multipole amplitudes [17]).

Now let us proceed with the calculation of $W_{\lambda'\mu';\lambda\mu}$. Switching again to the angular momentum eigenstates according to

$$\langle \vec{q}\lambda\mu|jm\lambda\mu\rangle = \sqrt{\frac{2j+1}{4\pi}}\,\mathcal{D}^j_{\lambda-\mu,m}(\vec{q}), \tag{A14}$$

we have

$$\int d\Omega \sum_{\mu''}\langle \vec{q}_2\lambda'\mu'|f^+|\vec{r}\mu''\rangle\langle \vec{r}\mu''|f|\vec{q}_1\lambda\mu\rangle = \sum_j \frac{2j+1}{4\pi}\mathcal{D}^j_{\lambda'-\mu',\lambda-\mu}(\vec{q}_2)\sum_{\mu''}\langle\lambda'\mu'|f^{j+}|\mu''\rangle\langle\mu''|f^j|\lambda\mu\rangle$$

$$= 4\pi \sum_j (2j+1)\mathcal{D}^j_{\lambda'-\mu',\lambda-\mu}(\vec{q}_2)\big[(2\mu)(2\mu')\overset{*}{\mathcal{A}}{}^{(2\mu'\lambda')}_{l+}\mathcal{A}^{(2\mu\lambda)}_{l+} + \overset{*}{\mathcal{A}}{}^{(2\mu'\lambda')}_{l+1,-}\mathcal{A}^{(2\mu\lambda)}_{l+1,-}\big], \tag{A15}$$

and finally

$$W_{\lambda'\mu';\lambda\mu} = \frac{(8\pi W)^2}{4\pi\alpha}\frac{|\vec{r}|}{2\pi W}\sum_j \frac{2j+1}{4\pi}\mathcal{D}^j_{\lambda'-\mu',\lambda-\mu}(\vec{q}_2)\big[(2\mu)(2\mu')\overset{*}{\mathcal{A}}{}^{(2\mu'\lambda')}_{l+}\mathcal{A}^{(2\mu\lambda)}_{l+} + \overset{*}{\mathcal{A}}{}^{(2\mu'\lambda')}_{l+1,-}\mathcal{A}^{(2\mu\lambda)}_{l+1,-}\big]. \tag{A16}$$

On the other hand, for the contribution of the infinitely narrow resonance with mass M_R, we will have instead of (A5), in full analogy with the above,

$$W^R_{\lambda'\mu';\lambda\mu} = \frac{(8\pi W)^2}{4\pi\alpha}\frac{\delta(W-M_R)}{2W}\sum_m \langle\vec{q}_2\lambda'\mu'|f^+|Rjm\rangle\langle Rjm|f|\vec{q}_1\lambda\mu\rangle$$

$$= \frac{(8\pi W)^2}{4\pi\alpha}\delta\big(W^2-M_R^2\big)\frac{2j+1}{4\pi}\mathcal{D}^j_{\lambda'-\mu',\lambda-\mu}(\vec{q}_2)\langle\lambda'\mu'|f^{j+}|R\rangle\langle R|f^j|\lambda\mu\rangle, \tag{A17}$$

where the photoproduction helicity amplitude is

$$\langle Rjm|f|\vec{q}_1\lambda\mu\rangle = \delta_{m,\lambda-\mu}\sqrt{\frac{2j+1}{4\pi}}\langle R|f^j|\lambda\mu\rangle = \frac{\sqrt{4\pi\alpha}}{8\pi W}\varepsilon^{(\lambda)}_\alpha\langle R|J_\alpha|p\rangle \tag{A18}$$

(remember $\vec{q}_1 \parallel \vec{e}_z$) and has the symmetry property

$$\langle R|f^j|\lambda\mu\rangle = \eta_R(-1)^{j-1/2}\langle R|f^j|-\lambda-\mu\rangle, \tag{A19}$$

where η_R is resonance parity. Carefully comparing (A18) with the definition of the standard resonance electroproduction amplitudes $A^R_{1/2}$, $A^R_{3/2}$, and $S^R_{1/2}$ [17], we find

$$\langle R|f^j|\lambda\mu\rangle = \frac{i}{8\pi W}\sqrt{\frac{4\pi}{2j+1}}\sqrt{4M(W^2-M^2)}\binom{2\mu}{1}\mathcal{A}^{(2\mu\lambda)}_R, \tag{A20}$$

where

$$\mathcal{A}^{(1)}_R = A^R_{1/2}, \quad \mathcal{A}^{(-1)}_R = \mp A^R_{3/2}, \quad \mathcal{A}^{(0)}_R = \mp\frac{\sqrt{-q^2}}{|\vec{q}|}S^R_{1/2}. \tag{A21}$$

Upper signs and symbols in the notation like $\binom{2\mu}{1}$ are taken for the resonance parity $\eta_R = (-1)^{j+1/2}$, and lower ones for $\eta_R = (-1)^{j-1/2}$. Note that overall phase factor is irrelevant. The corresponding hadronic tensor will be

$$W^R_{\lambda'\mu';\lambda\mu} = \frac{(8\pi W)^2}{4\pi\alpha}\delta\big(W^2-M_R^2\big)\frac{2j+1}{4\pi}\mathcal{D}^j_{\lambda'-\mu',\lambda-\mu}(\vec{q}_2)\frac{M(W^2-M^2)}{4\pi W^2(2j+1)}\binom{4\mu\mu'}{1}\overset{*}{\mathcal{A}}{}^{(2\mu'\lambda')}_R\mathcal{A}^{(2\mu\lambda)}_R. \tag{A22}$$

Comparing this with Eq. (A16), we easily deduce the relation (4). Indeed, putting in the last equation

$$\mathcal{A}_R = \mathcal{A}_{j\mp1/2,\pm}\sqrt{\frac{2W|\vec{r}|(2j+1)}{M(W^2-M^2)}}, \tag{A23}$$

and integrating it over dM_R^2, we obtain the term for corresponding spin and parity from Eq. (A16).

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