PHYSICAL REVIEW C **89**, 024904 (2014)

Coulomb corrections to photon and dilepton production in high-energy pA collisions

Kirill Tuchin
Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA
(Received 18 November 2013; published 10 February 2014)

I consider particle production in high-energy pA collisions. In addition to the coherent interactions with the nuclear color field, I take into account coherent interactions with the nuclear electromagnetic Coulomb field. Employing the dipole model, I sum up the leading multiple color and electromagnetic interactions and derive inclusive cross sections for photon and dilepton production. I found that the Coulomb corrections are up to 10% at $\sqrt{s} = 200$ GeV per nucleon.

DOI: 10.1103/PhysRevC.89.024904 PACS number(s): 25.75.Cj, 12.20.−m, 24.85.+p

I. INTRODUCTION

A pivotal feature of high-energy pA and AA collisions at the Brookhaven Relativistic Heavy Ion Collider (RHIC) and CERN Large Hadron Collider (LHC) is large longitudinal coherence length that by far exceeds radii of heavy nuclei. In QCD, color fields of nucleons in a heavy nucleus fuse to create an intense coherent color field, which has fundamental theoretical and phenomenological importance. Since nuclear force is short-range, only nucleons along the same impact parameter add up to form a coherent field. Because the QCD contribution to the scattering amplitude at high energy is imaginary, it is proportional to α_s^2. Thus, the parameter that characterizes the color-coherent field is $\alpha_s^2 A^{1/3} \sim 1$, where A is atomic weight. The longitudinal coherence length increases with the collision energy, but decreases with momentum transfer, so that at low energies color coherence is a nonperturbative phenomenon. At RHIC the longitudinal coherence length is large even for semihard transverse momenta (a few GeV's), authorizing application of the perturbation theory to color-coherent processes [1–3].

Along with strong color field, heavy ions also possess strong electromagnetic Coulomb field. The electromagnetic force is long range, so that all Z protons of an ion contribute to the field. Also, the QED contribution to the scattering amplitude is approximately real. As a result, the parameter that characterizes the coherent electromagnetic field is $\alpha Z \sim 1$. Since both parameters $\alpha_s^2 A^{1/3}$ and αZ are of the same order of magnitude in heavy ions, electromagnetic force must be taken into account along with the color one. This observation is a direct consequence of coherence which enhances the electromagnetic contribution by a large factor Z. Not all particle production channels in high-energy pA and AA collisions are equally affected by the nuclear Coulomb field(s). Our main observation is that gluon emission off a fast quark is completely unaffected in the eikonal approximation, whereas photon and dilepton production are moderately modified. The central goal of this article is to evaluate the magnitude of the Coulomb corrections to these processes.

The article is structured as follows. In Sec. II I develop a formalism, inspired by the Glauber-Mueller model [4] that takes into account both color and electromagnetic coherence by means of multiple scattering resummation. This formalism is applied in Sec. III to calculate the scattering amplitude of color-electric dipole of size r on heavy nucleus. The dipole-nucleus amplitude is employed in Secs. IV and V to compute inclusive photon and dilepton cross sections correspondingly. I conclude in Sec. VI with a discussion of our results and their ramifications on pA and AA phenomenology.

Strong electromagnetic interactions in pA and AA collisions were investigated before by many authors [5–11] who where concerned with pure QED contributions. In this paper I am more interested to study an interplay between the QCD and QED dynamics.

II. GLAUBER MODEL

Let the nucleus quantum state be described by the wave function ψ_A that depends on positions $\{\boldsymbol{b}_a, z_a\}_{a=1}^A$ of all A nucleons, where $\boldsymbol{b}_a$ and z_a are the transverse and the longitudinal positions of a nucleon a correspondingly. (In our notation, transverse vectors are in bold face). If the proton-nucleus scattering amplitude $i\Gamma^{pA}$ is known for a certain distribution of nucleons, then the average scattering amplitude is

$$\langle \Gamma^{pA}(\boldsymbol{b},s)\rangle = \int \prod_{a=1}^{Z} d^2\boldsymbol{b}_a\, dz_a\, |\psi_A(\boldsymbol{b}_1,z_1,\boldsymbol{b}_2,z_2,\ldots)|^2 \times \Gamma^{pA}(\boldsymbol{b}-\boldsymbol{b}_1,z_1,\boldsymbol{b}-\boldsymbol{b}_2,z_2,\ldots,s). \quad (1)$$

The scattering amplitude is simply related to the scattering matrix element S as $\Gamma(\boldsymbol{b},s) = 1 - S(\boldsymbol{b},s)$. The later can in turn be represented in terms of the phase shift χ so that in our case

$$\Gamma^{pA}(\boldsymbol{b}-\boldsymbol{b}_1,z_1,\boldsymbol{b}-\boldsymbol{b}_2,z_2,\ldots,s) = 1 - \exp\{-i\chi^{pA}(\boldsymbol{b}-\boldsymbol{b}_1,z_1,\boldsymbol{b}-\boldsymbol{b}_2,z_2,\ldots,s)\}. \quad (2)$$

At high energies, interaction of the projectile proton with different nucleons is independent inasmuch as the nucleons do not overlap in the longitudinal direction. This assumption is tantamount to taking into account only two-body interactions, while neglecting the many-body ones [12]. In this approximation the phase shift $\chi^{l\bar{l}Z}$ in the proton-nucleus interaction is just a sum of the phase shifts $\chi^{l\bar{l}p}$ in the proton-nucleon interactions and correlations between nucleons in the impact parameter space are neglected. I have

$$\langle \Gamma^{pA}(\boldsymbol{b},s)\rangle = \left\langle 1 - e^{-i\chi^{pA}}\right\rangle = \left\langle 1 - e^{-i\sum_a \chi^{pN}}\right\rangle = 1 - e^{-i\sum_a \langle\chi^{pN}\rangle}, \quad (3)$$

where in the last term $\langle\cdots\rangle$ stands for an average over a single nucleon position in the nucleus, defined below in Eq. (7). To the leading order in coupling α_s, the phase shift χ^{pN} can be expanded as $-i\chi^{pN} = \ln(1-\Gamma^{pN}) \approx -\Gamma^{pN}$. Therefore, I can write

$$\langle\Gamma^{pA}(\boldsymbol{b},s)\rangle = 1 - \exp\left\{-\sum_a \langle\Gamma^{pN}(\boldsymbol{b},s)\rangle\right\}. \tag{4}$$

Strong and electromagnetic contributions decouple in the elastic scattering amplitude at the leading order in respective couplings:

$$\Gamma^{pN} = \Gamma_s^{pN} + \Gamma_{\rm em}^{pN}. \tag{5}$$

Indeed, as I discuss below $i\Gamma_{\rm em}^{pN}$ is real, while $i\Gamma_s^{pN}$ is imaginary, which is a consequence of the fact that SU(3) generators are traceless. Owing to Eq. (5) I can cast Eq. (4) in the form

$$\langle\Gamma^{pA}(\boldsymbol{b},s)\rangle = 1 - \exp\left\{-A\left\langle\Gamma_s^{pN}(\boldsymbol{b},s)\right\rangle - Z\left\langle\Gamma_{\rm em}^{pN}(\boldsymbol{b},s)\right\rangle\right\}, \tag{6}$$

where Z is the number of protons. In the Glauber model I average over the nucleus using the nuclear density ρ as follows:

$$\left\langle\Gamma_s^{pN}(\boldsymbol{b},s)\right\rangle = \frac{1}{A}\int_{-\infty}^{\infty} dz_a \int d^2b_a\, \rho(\boldsymbol{b}_a,z_a)\Gamma_s^{pN}(\boldsymbol{b}-\boldsymbol{b}_a,s). \tag{7}$$

Neglecting the diffusion region, nuclear density is approximately constant $\rho = A/(\frac{4}{3}\pi R_A^3)$ for points inside the nucleus and zero otherwise. The range of the nuclear force is about a fm, which is much smaller than the radius R_A of a heavy nucleus. Therefore, $\boldsymbol{b} \approx \boldsymbol{b}_a$ and

$$\left\langle\Gamma_s^{pN}(\boldsymbol{b},s)\right\rangle = \frac{1}{A}\, 2\sqrt{R_A^2 - b^2}\,\pi R_A^2\,\rho\Gamma_s^{pN}(0,s). \tag{8}$$

In this approximation the total proton-nucleon cross section is $\sigma^{pN}(s) = 2\pi R_p^2 \Gamma_s^{pN}(0,s)$, with R_p being proton's radius, so that

$$\left\langle\Gamma_s^{pN}(\boldsymbol{b},s)\right\rangle = \frac{1}{A}\rho T(b)\frac{1}{2}\sigma^{pN}(s), \tag{9}$$

where $T(b) = 2\sqrt{R_A^2 - b^2}$ is the thickness function. It follows from Eq. (9) that $A\langle\Gamma_s^{pN}\rangle \sim \alpha_s^2 A^{1/3}$, which implies that Eq. (6) sums up terms of order $\alpha_s^2 A^{1/3} \sim 1$ at $\alpha_s \ll 1$. Indeed, the leading strong-interaction contribution to the pN elastic scattering amplitude corresponds to double-gluon exchange. Note also, that the corresponding $\langle i\Gamma_s^{pN}\rangle$ is purely imaginary.

Proton density in the nucleus is $Z\rho/A$, hence

$$\left\langle\Gamma_{\rm em}^{pN}(\boldsymbol{b},s)\right\rangle = \frac{1}{Z}\int_{-\infty}^{\infty} dz_a \int d^2b_a\, \frac{Z}{A}\rho(\boldsymbol{b}_a,z_a)\Gamma_{\rm em}^{pN}(\boldsymbol{b}-\boldsymbol{b}_a,s) \tag{10}$$

$$= \frac{1}{A}\rho\int d^2b_a\, T(b_a)\Gamma_{\rm em}^{pN}(\boldsymbol{b}-\boldsymbol{b}_a,s). \tag{11}$$

Electromagnetic interaction is long-range, therefore all values of impact parameter b contribute to the total cross section. Moreover, the leading logarithmic contribution comes from impact parameters far away from the nucleus $b \gg b_a \sim R_A$. In this case,

$$\left\langle\Gamma_{\rm em}^{pN}(\boldsymbol{b},s)\right\rangle = \frac{1}{A}\rho\,\Gamma_{\rm em}^{pN}(\boldsymbol{b},s)\int d^2b_a 2\sqrt{R_A^2 - b_a^2}$$
$$= \Gamma_{\rm em}^{pN}(\boldsymbol{b},s), \quad b \gg R_A. \tag{12}$$

However, if I am interested in differential cross section at impact parameters $b \sim R_A$ no such approximation is possible. The leading electromagnetic contribution to elastic pN scattering amplitude arises from one photon exchange; the corresponding $\langle i\Gamma_{\rm em}^{pN}\rangle$ is purely real. I note, that Eq. (7) sums up terms of order $\alpha Z \sim 1$ at $\alpha \ll 1$.

The total pA cross section can be computed using the optical theorem as follows:

$$\sigma_{\rm tot}^{pA}(s) = 2\int d^2b\, {\rm Im}\,[i\Gamma^{pA}(\boldsymbol{b},s)]$$
$$= 2\int d^2b\left\{1 - \exp\left[-A\left\langle\Gamma_s^{pN}(\boldsymbol{b},s)\right\rangle\right]\right.$$
$$\left.\times \cos\left[Z\left\langle i\Gamma_{\rm em}^{pN}(\boldsymbol{b},s)\right\rangle\right]\right\}. \tag{13}$$

III. DIPOLE-NUCLEUS SCATTERING

Similarly to the proton-nucleus scattering, one can consider scattering of color *and* electric singlet $q\bar{q}$ pair (dipole) of size $\boldsymbol{r}$ off a heavy nucleus. Since a single gluon exchange is an inelastic process, the leading in α_s contribution to the elastic scattering amplitude comes from the double gluon exchange given by

$$A\left\langle\Gamma_s^{q\bar{q}N}(\boldsymbol{b},s;\boldsymbol{r})\right\rangle = \frac{2C_F}{N_c}\rho T(b)\frac{1}{2}\pi r^2\alpha_s^2 \ln\frac{1}{r\mu}, \tag{14}$$

where μ is an infrared scale and s the center-of-mass energy squared, while the leading in α term arises from a singe photon exchange given by

$$Z\left\langle i\Gamma_{\rm em}^{q\bar{q}N}(\boldsymbol{b},s;\boldsymbol{r})\right\rangle = \frac{Z}{A}\rho\, 2\alpha\int d^2b_a\, T(b_a)$$
$$\times \ln\frac{|\boldsymbol{b}-\boldsymbol{b}_a-\boldsymbol{r}/2|}{|\boldsymbol{b}-\boldsymbol{b}_a+\boldsymbol{r}/2|}. \tag{15}$$

In this paper I employ a simple but quite accurate "cylindrical nucleus" model (see, e.g., [13,14]). Namely, I set $T(b) = 2R_A$ if $b < R_A$ and zero otherwise. The impact parameter integrals in Eqs. (13) and (15) can now be taken exactly. In particular, integration over $\boldsymbol{b}_a$ is described in the Appendix. Since in QCD $r \ll R_A$ I can neglect a very narrow region $|\boldsymbol{b}-\boldsymbol{r}/2| < R_A < |\boldsymbol{b}+\boldsymbol{r}/2|$ in which case Eq. (A3) yields for the electromagnetic term in the elastic dipole-nucleon scattering amplitude

$$\left\langle i\Gamma_{\rm em}^{q\bar{q}N}(\boldsymbol{b},s;\boldsymbol{r})\right\rangle$$
$$= 2\alpha\left[-\frac{\boldsymbol{b}\cdot\boldsymbol{r}}{R_A^2}\theta(R_A - b) + \ln\frac{|\boldsymbol{b}-\boldsymbol{r}/2|}{|\boldsymbol{b}+\boldsymbol{r}/2|}\theta(b - R_A)\right]. \tag{16}$$

The total cross section for dipole-nucleus scattering has the same form as Eq. (13) and can now be written as

$$\sigma_{\rm tot}^{q\bar{q}A}(s;r) = 2\int d^2b\left\{1-\exp\left[-A\langle\Gamma_{\rm s}^{q\bar{q}N}(\mathbf{0},s;\boldsymbol{r})\rangle\right]\cos\left(2\alpha Z\frac{\boldsymbol{b}\cdot\boldsymbol{r}}{R_A^2}\right)\right\}\theta(R_A-b) + 2\int d^2b\left\{1-\cos\left(2\alpha Z\ln\frac{|\boldsymbol{b}-\boldsymbol{r}/2|}{|\boldsymbol{b}+\boldsymbol{r}/2|}\right)\right\}\theta(b-R_A)\,. \tag{17}$$

In the first line of Eq. (17) I can replace the cosine by one, because $r \ll R_A$, $\alpha Z \sim 1$. The corresponding contribution to the cross section is

$$\sigma_{\rm s}^{q\bar{q}A}(s;r) = 2\pi R_A^2\left\{1-\exp\left[-A\langle\Gamma_{\rm s}^{q\bar{q}N}(0,s;\boldsymbol{r})\rangle\right]\right\}, \tag{18}$$

which is a purely QCD term, hence the subscript "s" for the "strong" interaction. Integral in the second line of Eq. (17) can be taken exactly and yields the QED contribution [15,16]

$$\sigma_{\rm em}^{q\bar{q}A}(s;r) \equiv 2\int_{R_A}^{b_{\max}} db\, b\int_0^{2\pi}d\phi\left\{1-\cos\left(\alpha Z\ln\frac{b^2+r^2/4-br\cos\phi}{b^2+r^2/4+br\cos\phi}\right)\right\} = 4\pi r^2(\alpha Z)^2\ln\frac{b_{\max}}{R_A} = 4\pi r^2(\alpha Z)^2\ln\frac{s}{4m_q^2 m_N R_A}\,, \tag{19}$$

where m_q and m_N are quark and nucleon masses correspondingly. Terms of order r^2/R_A^2 are neglected in Eq. (19).[1] Energy dependence arises from the long distance cutoff $b_{\max} = s/(4m_N m_q^2)$ of the b integral.

The total cross section is thus simply a sum of the QCD and QED terms

$$\sigma_{\rm tot}^{q\bar{q}A}(s;r) = \sigma_{\rm s}^{q\bar{q}A}(s;r) + \sigma_{\rm em}^{q\bar{q}A}(s;r)\,. \tag{20}$$

From a comparison of Eqs. (18) and (19) it is clear that the QED contribution to the total cross section is suppressed relative to the QCD term by $(r\alpha Z/R_A)^2\log s$, hence the Coulomb correction is largest for soft processes with larger r. Since the largest dipole size is of order R_p, the smallest suppression factor is of order $(\alpha Z)^2/A^{2/3}\log s$, which for gold nucleus is about 0.1 at $\sqrt{s} = 200$ GeV. Because, $Z \sim A$, the relative contribution of the Coulomb correction increases with A.

At high energies, dipole-nucleon scattering amplitude acquires energy dependence $\Gamma^{q\bar{q}N} \sim s^{1+\Delta}$, where to the leading order in QCD $\Delta_{\rm s} = 4\ln 2(\alpha_s N_c/\pi)$ [19,20] and in QED $\Delta_{\rm em} = (11/32)\pi\alpha^2$ [21,22]. Since $\Delta_{\rm em} \ll \Delta_{\rm s}$ I can neglect the effect of QED evolution. A phenomenological way to take QCD evolution into account is to parametrize the scattering amplitude in terms of quark saturation momentum $\tilde{Q}_s$ and anomalous dimension γ as follows:

$$A\langle\Gamma_{\rm s}^{q\bar{q}N}(0,s;\boldsymbol{r})\rangle = \tfrac{1}{4}\left(r^2\tilde{Q}_s^2\right)^\gamma, \tag{21}$$

where $\tilde{Q}_s^2 \approx 0.16A^{1/3}$ GeV^2 and $\gamma \approx 0.63$ [23]. Numerical value of the saturation momentum is known from DIS and heavy-ion phenomenology (see, e.g., [24]).

IV. PHOTON PRODUCTION

In this and the next section I discuss photon and dilepton production in high-energy pA collisions. Photon production without electromagnetic corrections was calculated in [25]. If I assume the validity of the collinear factorization on the proton side, the problem reduces to computing the photon and dilepton production in qA collisions.[2] I adopt the following notations: four-momenta of incoming quark, photon, and outgoing quark are q, k_1, and k_2, correspondingly; bold face denotes their respective transverse components; $z = k_{1+}/q_+$. Transverse coordinates of incoming quark, photon and outgoing quark in the amplitude are $\boldsymbol{u}$, $\boldsymbol{x}_1$, and $\boldsymbol{x}_2$; those in the complex conjugated amplitude are distinguished by a prime: $\boldsymbol{r} = \boldsymbol{x}_1 - \boldsymbol{x}_2$, $\boldsymbol{r}' = \boldsymbol{x}_1' - \boldsymbol{x}_2'$, $\boldsymbol{b} = (\boldsymbol{x}_1+\boldsymbol{x}_2)/2$, $\boldsymbol{b}' = (\boldsymbol{x}_1'+\boldsymbol{x}_2')/2$. I also define the following scattering matrix element:

$$\mathcal{S}(\boldsymbol{b},\boldsymbol{r}) = 1 - \,{\rm Im}\,[i\Gamma^{q\bar{q}A}(\boldsymbol{b},s;\boldsymbol{r})] = \exp\left[-A\langle\Gamma_{\rm s}^{q\bar{q}N}(\boldsymbol{b},s;\boldsymbol{r})\rangle\right]\cos\left[Z\langle i\Gamma_{\rm em}^{q\bar{q}N}(\boldsymbol{b},s;\boldsymbol{r})\rangle\right]. \tag{22}$$

[1] I would like to stress that approximation $r \ll R_A$ holds only if the dipole size r is determined by a QCD scale. For example, in photon emission $r\sim 1/k$, where k is photon's momentum, in dilepton production $r \sim 1/M$, where M is dilepton's invariant mass. Therefore, only if k and M are at or above $\sim 1/R_p \sim 200$ MeV can this approximation be used. The original calculation of [17,18] was done in QED in the opposite limit of a point-like nucleus $r \gg R_A$.

[2] One should be cautious with the collinear factorization of dilute projectiles at high energies since it is not valid in exclusive processes, see, e.g., [26], and is violated even in some inclusive processes [27,28].

With these notations I can write down the double-inclusive cross section as follows [29–31]:

$$\frac{d\sigma^{qA\to\gamma qX}}{d^2k_1 d^2k_2 dz} = \frac{1}{2(2\pi)^5}\int d^2u\, d^2u'\, d^2x_1\, d^2x_2\, d^2x_1'\, d^2x_2'\, e^{-\boldsymbol{k}_1\cdot(\boldsymbol{x}_1-\boldsymbol{x}_1')-i\boldsymbol{k}_2\cdot(\boldsymbol{x}_2-\boldsymbol{x}_2')}\phi^{q\to q\gamma}(\boldsymbol{r},\boldsymbol{r}',z)$$
$$\times [-\mathcal{S}((\boldsymbol{x}_2'+\boldsymbol{u})/2,\boldsymbol{x}_2'-\boldsymbol{u}) - \mathcal{S}((\boldsymbol{x}_2+\boldsymbol{u}')/2,\boldsymbol{x}_2-\boldsymbol{u}') + \mathcal{S}((\boldsymbol{x}_2'+\boldsymbol{x}_2)/2,\boldsymbol{x}_2'-\boldsymbol{x}_2) + \mathcal{S}((\boldsymbol{u}+\boldsymbol{u}')/2,\boldsymbol{u}-\boldsymbol{u}')], \tag{23}$$

where the square of the light-cone wave function

$$\phi^{q\to q\gamma}(\boldsymbol{r},\boldsymbol{r}',z) = \frac{2e_f^2}{(2\pi)^2}\frac{\boldsymbol{r}\cdot\boldsymbol{r}'}{r^2r'^2}\frac{1+(1-z)^2}{z}\delta(\boldsymbol{u}-z\boldsymbol{x}_1-(1-z)\boldsymbol{x}_2)\delta(\boldsymbol{u}'-z\boldsymbol{x}_1'-(1-z)\boldsymbol{x}_2') \tag{24}$$

describes photon emission off quark in the chiral limit. According to Eqs. (9), (10), and (22) I have

$$\mathcal{S}(\boldsymbol{b},\boldsymbol{r}) = \exp\left\{-\frac{2C_F}{N_c}\rho T(b)\frac{1}{2}\pi r^2\alpha_s^2\ln\frac{1}{r\mu}\right\}\cos\left\{\frac{Z}{A}\rho\, 2\alpha\int d^2b_a\, T(b_a)\ln\frac{|\boldsymbol{b}-\boldsymbol{r}/2-\boldsymbol{b}_a|}{|\boldsymbol{b}+\boldsymbol{r}/2-\boldsymbol{b}_a|}\right\}. \tag{25}$$

Integration over the final quark transverse momentum $\boldsymbol{k}_2$ gives the single-inclusive cross section

$$\frac{d\sigma^{qA\to\gamma qX}}{d^2k_1 dz} = 2\alpha e_f^2\frac{1+(1-z)^2}{z}\int d^2\tilde{b}\int\frac{d^2r}{(2\pi)^2}\int\frac{d^2r'}{(2\pi)^2}e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{r}-\boldsymbol{r}')}\frac{\boldsymbol{r}\cdot\boldsymbol{r}'}{r^2r'^2}\phi^{q\to q\gamma}(\boldsymbol{r},\boldsymbol{r}',z)$$
$$\times[-\mathcal{S}(\tilde{\boldsymbol{b}}+z\boldsymbol{r}/2,z\boldsymbol{r}) - \mathcal{S}(\tilde{\boldsymbol{b}}+z\boldsymbol{r}'/2,z\boldsymbol{r}') + 1 + \mathcal{S}(\tilde{\boldsymbol{b}}+z(\boldsymbol{r}+\boldsymbol{r}')/2,z(\boldsymbol{r}-\boldsymbol{r}'))], \tag{26}$$

where $\tilde{\boldsymbol{b}} = \boldsymbol{b}-\boldsymbol{r}/2$.

As in the previous section I utilize the "cylindrical nucleus" model to take the impact parameter integrals. In particular, taking integral over $\boldsymbol{b}_a$ in Eq. (25) yields

$$\mathcal{S}(\boldsymbol{b},\boldsymbol{r}) = e^{-\frac{1}{4}(\tilde{Q}_s^2r^2)^\gamma}\theta(R_A-b) + \cos\left(2\alpha Z\ln\frac{|\boldsymbol{b}-\boldsymbol{r}/2|}{|\boldsymbol{b}+\boldsymbol{r}/2|}\right)\theta(b-R_A) \tag{27}$$

up to terms of order r^2/R_A^2. With the same accuracy, integration over $\boldsymbol{b}$ in Eq. (26) can now be done explicitly using Eqs. (13), (18), (19), and (22):

$$\frac{d\sigma^{qA\to\gamma qX}}{d^2k_1 dz} = \alpha e_f^2\frac{1+(1-z)^2}{z}\int\frac{d^2r}{(2\pi)^2}\int\frac{d^2r'}{(2\pi)^2}e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{r}-\boldsymbol{r}')}\frac{\boldsymbol{r}\cdot\boldsymbol{r}'}{r^2r'^2}\phi^{q\to q\gamma}(\boldsymbol{r},\boldsymbol{r}',z)$$
$$\times\left[\sigma_{\rm tot}^{q\bar{q}A}(s;z\boldsymbol{r}) + \sigma_{\rm tot}^{q\bar{q}A}(s;z\boldsymbol{r}') - \sigma_{\rm tot}^{q\bar{q}A}(s;z(\boldsymbol{r}-\boldsymbol{r}'))\right]. \tag{28}$$

Equation (28) can be cast into a factorized form by employing the following identities [13,32]:

$$\int d^2x\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\frac{\boldsymbol{x}}{x^2} = -2\pi i\frac{\boldsymbol{k}}{k^2}, \tag{29}$$

$$\int d^2x'\frac{\boldsymbol{x}'\cdot(\boldsymbol{x}+\boldsymbol{x}')}{\boldsymbol{x}'^2(\boldsymbol{x}+\boldsymbol{x}')^2} = \pi\ln\frac{1}{x^2}. \tag{30}$$

The result reads

$$\frac{d\sigma^{qA\to\gamma qX}}{d^2k_1 dz} = \frac{\alpha}{(2\pi)^3}e_f^2\frac{1}{k_1^2}\frac{1+(1-z)^2}{z}\int d^2x\, e^{-i\boldsymbol{k}_1\cdot\boldsymbol{x}}\ln\frac{1}{x\mu}\nabla_{\boldsymbol{x}}^2\sigma_{\rm tot}^{q\bar{q}A}(s;z\boldsymbol{x}). \tag{31}$$

The electromagnetic contribution can be calculated exactly:

$$\frac{d\sigma_{\rm em}^{qA\to\gamma qX}}{d^2k_1 dz} = \frac{\alpha}{(2\pi)^2}e_f^2\frac{8\pi}{k_1^4}\frac{1+(1-z)^2}{z}4\pi z^2(\alpha Z)^2\ln\frac{s}{4m_q^2m_NR_A}, \tag{32}$$

where I used

$$\int d^2x\,\ln\frac{1}{x}\,e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} = \frac{2\pi}{k^2}. \tag{33}$$

To obtain a qualitative estimate of the QCD contribution to the inclusive cross section (31), note that unless $x<2/k_1$ the exponent is rapidly oscillating. Furthermore, the integrand is exponentially suppressed at $zx>2/\tilde{Q}_s$. I thus obtain

$$\frac{d\sigma_{\rm s}^{qA\to\gamma qX}}{d^2k_1 dz} \approx \frac{\alpha}{(2\pi)^2}e_f^2\frac{1}{k_1^2}\frac{1+(1-z)^2}{z}\int_0^{x_0}dx\,\ln\frac{1}{x}\,\partial_x\left[x\partial_x\left(\sigma_{\rm tot}^{q\bar{q}A}(s;zx)\right)\right], \tag{34}$$

where x_0 is the smallest of three scales $2/k_1$, $2/(z\tilde{Q}_s)$, and $1/\mu$. Expanding Eq. (18) with Eq. (21) at small zx I find

$$\sigma_s^{q\bar{q}A}(s;zx) \approx 2\pi R_A^2 \tfrac{1}{4}\left(\tilde{Q}_s^2 z^2 x^2\right)^\gamma , \tag{35}$$

which upon substitution into Eq. (34) and combining with Eq. (32) produces

$$\frac{d\sigma^{qA\to\gamma qX}}{d^2k_1 dz} \approx \frac{\alpha}{\pi} e_f^2 \frac{1}{k_1^4} \frac{1+(1-z)^2}{z} \left[8\pi z^2 (\alpha Z)^2 \ln \frac{s}{4m_q^2 m_N R_A} + \frac{1}{4}\gamma k_1^2 R_A^2 \tilde{Q}_s^{2\gamma}(x_0 z)^{2\gamma} \ln \frac{1}{x_0} \right] . \tag{36}$$

I see that the ratio of the QCD and the QED terms is of order $(R_A\tilde{Q}_s)^2(k_1^2/\tilde{Q}_s^2)^\eta$, with $\eta = 1$, if $k_1 \ll \tilde{Q}_s$ and $\eta = 1-\gamma$, if $k_1 \gg \tilde{Q}_s$. Thus, the QED interactions have the largest relative impact at small photon transverse momenta and in more peripheral events. Note also that the role of QED interactions diminishes with energy because the saturation momentum increases as a power of energy, whereas the QED contribution is only logarithmic. The distinct feature of z dependence of inclusive photon production cross section is that it vanishes in the eikonal limit $z \to 0$, which is evident from Eq. (26).

The relative magnitude of the Coulomb correction to the photon spectrum can be expressed in terms of the ratio

$$\mathcal{R}_\gamma = \frac{d\sigma_{\rm em}^{qA\to\gamma qX}}{d^2k_1 dz} \Bigg/ \frac{d\sigma^{qA\to\gamma qX}}{d^2k_1 dz} , \tag{37}$$

which is plotted in Fig. 1. As expected, the Coulomb correction is largest at small k_1 and for heavier nucleus. For p-Au collisions at $\sqrt{s} = 200$ GeV it constitutes about 7% at small k_1. At larger energies it slowly increases as $\log s$.

V. DILEPTON PRODUCTION

Dilepton production by an incident quark is quite complicated because both the quark and the produced dileptons interact with the electromagnetic field of the target nucleus, and quark also interacts with the nuclear color field. At large invariant mass M of produced dilepton pair, an intermediate process of photon splitting $\gamma^* \to \ell^+\ell^-$ can be factored out, which leads to significant simplifications. This will be our assumption throughout this section. A detailed analysis of this approximation can be found in [27].

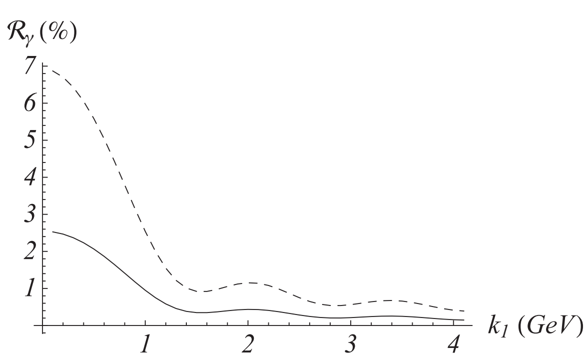

FIG. 1. Fraction of the QED contribution in the total differential photon production cross section. Solid line: Cu; dashed line: Au, $\sqrt{s} = 200$ GeV, $\mu = 1/\text{fm}$, quark mass $m_q = 150$ MeV.

Our notation scheme in this section follows the same pattern as in the previous one. Momenta of incident photon and outgoing leptons are q, k_1, and k_2 correspondingly; the lepton's light-cone momentum fraction is $z = k_{1+}/q_+$. Transverse coordinates of leptons are $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$; $\boldsymbol{r} = \boldsymbol{x}_1 - \boldsymbol{x}_2$ is dipole size, $\boldsymbol{b} = (\boldsymbol{x}_1 + \boldsymbol{x}_2)/2$ its impact parameter. Prime indicates coordinates in the complex conjugated amplitude. With these notations the double inclusive cross section for dilepton production reads

$$\begin{aligned}\frac{d\sigma^{\gamma^* A\to\ell^+\ell^-}}{d^2k_1 d^2k_2} &= \frac{\pi}{(2\pi)^6}\int dz \int d^2x_1 d^2x_2 d^2x_1' d^2x_2' e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{x}_1-\boldsymbol{x}_1')} \\ &\quad\times e^{-i\boldsymbol{k}_2\cdot(\boldsymbol{x}_2-\boldsymbol{x}_2')}\phi^{\gamma^*\to\ell^+\ell^-}(\boldsymbol{r},\boldsymbol{r}',z) \\ &\quad\times [1 + \mathcal{Q}_{\rm em}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_1',\boldsymbol{x}_2') - \mathcal{S}_{\rm em}(\boldsymbol{b},\boldsymbol{r}) - \mathcal{S}_{\rm em}(\boldsymbol{b}',\boldsymbol{r}')] ,\end{aligned} \tag{38}$$

where the squared light-cone wave-function describing photon splitting into dilepton pair is given by

$$\begin{aligned}\phi^{\gamma^*\to\ell^+\ell^-}(\boldsymbol{r},\boldsymbol{r}',z) = \frac{2\alpha}{\pi} m^2 \Bigg\{ &\frac{\boldsymbol{r}\cdot\boldsymbol{r}'}{rr'} K_1(rm_\ell)K_1(r'm_\ell) \\ &\times [z^2 + (1-z)^2] + K_0(rm_\ell)K_0(r'm_\ell) \Bigg\} .\end{aligned} \tag{39}$$

The scattering matrix elements of electric dipole is [cf. Eq. (25)]

$$\begin{aligned}\mathcal{S}_{\rm em}(\boldsymbol{b},\boldsymbol{r}) &= \cos\left\{ Z\langle i\Gamma_{\rm em}^{q\bar{q}N}(\boldsymbol{b},s;\boldsymbol{r})\rangle \right\} \\ &= \cos\left\{ \frac{Z}{A}\rho\, 2\alpha \int d^2b_a\, T(b_a) \ln \frac{|\boldsymbol{b}-\boldsymbol{b}_a-\boldsymbol{r}/2|}{|\boldsymbol{b}-\boldsymbol{b}_a+\boldsymbol{r}/2|} \right\} ,\end{aligned} \tag{40}$$

and that of electric quadrupole is $\mathcal{Q}_{\rm em}$. The later is a complicated function of its coordinates. Explicit form of its QCD analog can be found in [31]; it significantly simplifies in the large N_c approximation [33]. If either $\boldsymbol{x}_1 = \boldsymbol{x}_1'$ or $\boldsymbol{x}_2 = \boldsymbol{x}_2'$, the quadrupole reduce to a dipole, e.g.,

$$\mathcal{Q}_{\rm em}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_1',\boldsymbol{x}_2')|_{x_2=x_2'} = \mathcal{S}_{\rm em}((\boldsymbol{x}_1+\boldsymbol{x}_1')/2, \boldsymbol{x}_1 - \boldsymbol{x}_1') . \tag{41}$$

Upon integration over $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, Eq. (38) gives the total inclusive cross section that agrees with results of [5].

Since I am interested in invariant mass distribution, it is convenient to introduce another pair of independent momenta, photon transverse momentum $\boldsymbol{q}$, and the relative momentum of the pair $\boldsymbol{\ell}$, as follows:

$$\boldsymbol{q} = \boldsymbol{k}_1 + \boldsymbol{k}_2, \quad \boldsymbol{\ell} = (1-z)\boldsymbol{k}_1 - z\boldsymbol{k}_2 . \tag{42}$$

The invariant mass of dilepton can be expressed as

$$M^2 = (k_1+k_2)^2 = q_+(k_{1-}+k_{2-}) - (\boldsymbol{k}_1+\boldsymbol{k}_2)^2 = \frac{m^2+\boldsymbol{\ell}^2}{z(1-z)}. \tag{43}$$

I took into account that in the light-cone perturbation theory $q_- \neq k_{1-} + k_{2-}$ because photon splitting is only an intermediate step in dilepton production. Using these notations, the phase in Eq. (38) can be written as

$$-i\boldsymbol{k}_1\cdot(\boldsymbol{x}_1-\boldsymbol{x}_1') - i\boldsymbol{k}_2\cdot(\boldsymbol{x}_2-\boldsymbol{x}_2') = -i\boldsymbol{\ell}\cdot(\boldsymbol{r}-\boldsymbol{r}') - i\boldsymbol{q}\cdot(\boldsymbol{b}-\boldsymbol{b}') - i\boldsymbol{q}\cdot(\boldsymbol{r}-\boldsymbol{r}')(z-1/2). \tag{44}$$

Factorization of photon decay assumes that $\ell \sim 1/M$ and $q < 2m_\ell$ [27]. Therefore, I can neglect the last term in Eq. (44):

$$-i\boldsymbol{k}_1\cdot(\boldsymbol{x}_1-\boldsymbol{x}_1') - i\boldsymbol{k}_2\cdot(\boldsymbol{x}_2-\boldsymbol{x}_2') \approx -i\boldsymbol{\ell}\cdot(\boldsymbol{r}-\boldsymbol{r}') - i\boldsymbol{q}\cdot(\boldsymbol{b}-\boldsymbol{b}'). \tag{45}$$

For an almost on-mass-shell photon, the transverse polarization is dominant. Expanding Eq. (39) at small m_ℓ and keeping only the term dominant at small dipole sizes, I get

$$\phi^{\gamma^*\to\ell^+\ell^-}(\boldsymbol{r},\boldsymbol{r}',z) \approx \frac{2\alpha}{\pi}\frac{\boldsymbol{r}\cdot\boldsymbol{r}'}{r^2r'^2}[z^2+(1-z)^2]. \tag{46}$$

Since Eq. (46) as well as the scattering factors are q independent, I can integrate in Eq. (38) over $\boldsymbol{q}$, which in view of Eq. (45) yields $(2\pi)^2\delta(\boldsymbol{b}-\boldsymbol{b}')$. Moreover, since M is larger than the typical momentum transfer $\Delta \sim \sqrt{\alpha Z}/b$ by a t-channel photon, i.e., $\Delta \ll M$, I can expand the quadrupole amplitude at small difference $|\boldsymbol{r}-\boldsymbol{r}'| \ll |\boldsymbol{r}+\boldsymbol{r}'|/2$, which yields $\mathcal{Q}_{\rm em} \approx \mathcal{S}_{\rm em}(\boldsymbol{b},\boldsymbol{r}-\boldsymbol{r}')$. [Other scattering factors in Eq. (38) do not depend on this difference.] With these assumptions and approximations I derive at large M

$$\frac{d\sigma^{\gamma^*A\to\ell^+\ell^-}}{d^2\ell d^2b} = \frac{\pi}{(2\pi)^4}\int dz[z^2+(1-z)^2]\int d^2r d^2r'\, e^{-i\boldsymbol{\ell}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \times \frac{2\alpha}{\pi}\frac{\boldsymbol{r}\cdot\boldsymbol{r}'}{r^2r'^2}[1+\mathcal{S}_{\rm em}(\boldsymbol{b},\boldsymbol{r}-\boldsymbol{r}')-\mathcal{S}_{\rm em}(\boldsymbol{b},\boldsymbol{r}) - \mathcal{S}_{\rm em}(\boldsymbol{b},\boldsymbol{r}')]. \tag{47}$$

I can take one of the two-dimensional integrals using Eqs. (29) and (30). This gives

$$\frac{d\sigma^{\gamma^*A\to\ell^+\ell^-}}{d^2\ell d^2b} = \frac{\pi}{(2\pi)^4}\frac{2\alpha}{\pi}\int dz[z^2+(1-z)^2]\frac{2\pi}{\ell^2}\int d^2r e^{-i\boldsymbol{\ell}\cdot\boldsymbol{r}} \times \ln\frac{1}{r}\nabla_{\boldsymbol{r}}^2[1-\mathcal{S}_{\rm em}(\boldsymbol{b},\boldsymbol{r})]. \tag{48}$$

To calculate the Laplacian appearing in the right-hand side of Eq. (48) I use the expression for the scattering amplitude in the integrand of Eq. (17) (with $\Gamma_s = 0$):

$$\nabla_{\boldsymbol{r}}^2[1-\mathcal{S}_{\rm em}(\boldsymbol{b},\boldsymbol{r})] = (2\alpha Z)^2\frac{b^2}{R_A^4}\cos\left(2\alpha Z\frac{\boldsymbol{b}\cdot\boldsymbol{r}}{R_A^2}\right)\theta(R_A-b) + \frac{b^2}{(\boldsymbol{b}-\boldsymbol{r}/2)^2(\boldsymbol{b}+\boldsymbol{r}/2)^2}(2\alpha Z)^2 \times\cos\left(2\alpha Z\ln\frac{|\boldsymbol{b}-\boldsymbol{r}/2|}{|\boldsymbol{b}+\boldsymbol{r}/2|}\right)\theta(R_A-b). \tag{49}$$

As mentioned before, at $b < R_A$ I can expand this expression in powers of r^2/R_A^2, while at $b > R_A$ in powers r^2/b^2. I have

$$\nabla_{\boldsymbol{r}}^2[1-\mathcal{S}_{\rm em}(\boldsymbol{b},\boldsymbol{r})] \approx (2\alpha Z)^2\left[\frac{b^2}{R_A^4}\theta(R_A-b)+\frac{1}{b^2}\theta(R_A-b)\right]. \tag{50}$$

Plugging this into Eq. (48) and employing Eq. (33) yields

$$\frac{d\sigma^{\gamma^*A\to\ell^+\ell^-}}{d^2\ell d^2b} = \frac{4}{3\pi^2}\frac{\alpha}{\ell^4}(\alpha Z)^2\left[\frac{b^2}{R_A^4}\theta(R_A-b)+\frac{1}{b^2}\theta(R_A-b)\right]. \tag{51}$$

Notice that the dilepton spectrum at a given impact parameter is energy-independent. This a consequence of the quasiclassical approximation. Integration over impact parameter can be done directly in Eq. (48) using Eqs. (19) and (33) if I neglect a small contribution at $b < R_A$. The result is

$$\frac{d\sigma^{\gamma^*A\to\ell^+\ell^-}}{d^2\ell} = \frac{\pi}{(2\pi)^4}\frac{2\alpha}{\pi}\int_0^1 dz[z^2+(1-z)^2]\frac{2\pi}{\ell^2} \times\int d^2re^{-i\boldsymbol{\ell}\cdot\boldsymbol{r}}\ln\frac{1}{r}8\pi(\alpha Z)^2\ln\frac{s}{4m_\ell^2m_NR_A} = \frac{8\alpha}{3\pi}\frac{1}{\ell^4}(\alpha Z)^2\ln\frac{s}{4m_\ell^2m_NR_A}. \tag{52}$$

The same formula is obtained by integration of an approximate formula (51) over b. This is because Eq. (19) assumes that $b \gg R_A$. Note that a b-integrated cross section exhibits logarithmic dependence on energy, which enters through the cutoff $b_{\rm max}$ [see Eq. (19)].

If there were no QED interactions of dilepton with the nucleus I would have, instead of Eq. (47),

$$\frac{d\sigma_0^{\gamma^*\to\ell^+\ell^-}}{d^2\ell d^2b} = \frac{\pi}{(2\pi)^4}\int dz[z^2+(1-z)^2]\int d^2rd^2r'\,e^{-i\boldsymbol{\ell}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\frac{2\alpha}{\pi}\frac{\boldsymbol{r}\cdot\boldsymbol{r}'}{r^2r'^2} = \frac{\pi}{(2\pi)^2}\int dz[z^2+(1-z)^2]\frac{2\alpha}{\pi}\frac{1}{\ell^2} = \frac{\alpha}{3\pi^2}\frac{1}{\ell^2}. \tag{53}$$

Changing the integration variable from ℓ to M I obtain the well-known QED result for virtual photon decay probability

$$\frac{d\sigma_0^{\gamma^*\to\ell^+\ell^-}}{d^2b} = \frac{2\alpha}{3\pi}\frac{dM}{M}. \tag{54}$$

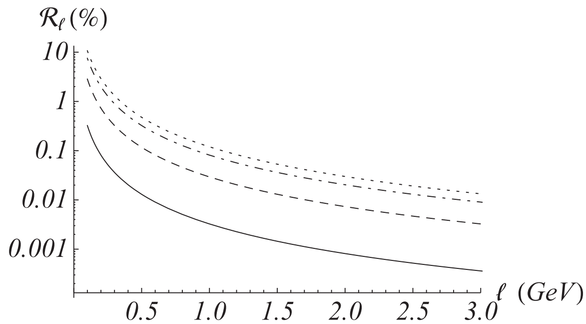

FIG. 2. Fraction of the QED contribution in the e^+e^- dilepton production cross section in the Coulomb field of gold nucleus, $A = 197$, $Z = 79$. Solid line: $b = 1$ fm, dashed line: $b = 3$ fm, dashed-dotted line: $b = 5$ fm, dotted line: $b = 7$ fm.

The difference between the dilepton production cross section in the Coulomb field and in vacuum can be expressed as the following ratio:

$$f(\ell,b) = \frac{d\sigma^{\gamma^* A\to \ell^+\ell^-}}{d^2b d^2\ell} \Big/ \frac{d\sigma_0^{\gamma^*\to \ell^+\ell^-}}{d^2\ell d^2 b}\,. \tag{55}$$

Using Eqs. (51) and (53) I derive that at large invariant masses

$$f(\ell,b) = \frac{4(\alpha Z)^2}{R_A^2\ell^2}\left[\frac{b^2}{R_A^2}\theta(R_A-b) + \frac{R_A^2}{b^2}\theta(R_A-b)\right]. \tag{56}$$

As in the previous section, I express the relative magnitude of the Coulomb correction to the dilepton spectrum as a ratio

$$\mathcal{R}_\ell = \frac{f_\ell}{1+f_\ell}\,, \tag{57}$$

which is plotted in Fig. 2 for electron-positron pair production by high energy virtual photon in a Coulomb field of gold nucleus. I observe that the relative contribution of the Coulomb corrections to dilepton production increases at smaller $M \sim 2\ell$ and toward the nucleus boundary and can reach 10% in semiperipheral and peripheral collisions.

VI. DISCUSSION AND SUMMARY

In this article I investigated the role of electromagnetic Coulomb interactions in photon and dilepton production in high-energy pA collisions. Among other important processes that receive electromagnetic corrections is gluon emission off a fast quark and $q\bar{q}$ production. Photon production vanishes in the eikonal approximation, i.e., when a valence quark moves strictly along the straight line, corresponding to $z \to 0$. In contrast, gluon production cross section diverges in this limit as $1/z$ giving the leading logarithmic term to the rapidity distribution. Therefore, QED contribution to gluon production appears only as a correction to a subleading order in α_s and can be safely neglected. In $q\bar{q}$ production via gluon splitting, Coulomb corrections come about already at the leading order because at least one fermion carries finite z.

QED corrections to photon production are largest at small transverse momentum of photon and increase with energy and nuclear weight. In p-Au collisions at $\sqrt{s} = 200$ GeV per nucleon, the Coulomb correction to photon production reaches 7%. Dilepton production receives QED contributions at two stages: at virtual photon emission, which is qualitatively similar to photon production, and at virtual photon splitting into a dilepton pair. The latter can proceed even in vacuum. I computed the Coulomb correction to this process and found that it is largest for small invariant masses M and increases with impact parameter. In p-Au collisions at $\sqrt{s} = 200$ GeV per nucleon, the Coulomb correction is up to 10% at $M \sim 200$ MeV. An upshot of this is that the prompt photon yield extracted from the dilepton spectrum using the equation $\frac{dN^{\ell^+\ell^-}}{dM} = \frac{2\alpha}{3\pi M}N^\gamma$ is overestimated by about 10%.

It is of a special interest to extend the analysis of this article to the initial stage of heavy-ion collisions. At a qualitative level, I expect that the main features that I observed in pA scattering are carried over to AA scattering. However, a quantitative estimate of the Coulomb corrections in heavy-ion collisions require further analytical investigation.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-87ER40371.

APPENDIX

The integral appearing in Eq. (15) can be written in the cylindrical nucleus model as

$$\begin{aligned} I &= \int d^2b_a \ln\frac{|\boldsymbol{b}-\boldsymbol{r}/2-\boldsymbol{b}_a|}{|\boldsymbol{b}+\boldsymbol{r}/2-\boldsymbol{b}_a|}\theta(R_A-b_a) \\ &= \int d^2b_a \ln\frac{|\boldsymbol{x}-\boldsymbol{b}_a|}{|\boldsymbol{y}-\boldsymbol{b}_a|}\theta(R_A-b_a)\,, \end{aligned} \tag{A1}$$

where θ is the step function and I denoted $\boldsymbol{x} = \boldsymbol{b}-\boldsymbol{r}/2$ and $\boldsymbol{y} = \boldsymbol{b}+\boldsymbol{r}/2$. Introducing a dimensionless variable $\xi = b_a/x$ I have

$$\begin{aligned} &\int d^2b_a \ln|\boldsymbol{x}-\boldsymbol{b}_a|\theta(R_A-b_a) \\ &= \frac{1}{2}x^2\int_0^{R_A/x} d\xi\,\xi\int_0^{2\pi} d\phi[\ln x^2 + \ln(1+\xi^2-2\xi\cos\phi)] \\ &= \pi x^2\int_0^{R_A/x} d\xi\,\xi\left[\ln x^2 + \ln\frac{2}{1+\xi^2+|\xi^2-1|}\right] \\ &= \frac{\pi}{2}\times\begin{cases} x^2 - R_A^2 + R_A^2\ln R_A^2\,, & x \geqslant R_A, \\ R_A^2\ln x^2\,, & x < R_A\,. \end{cases} \end{aligned} \tag{A2}$$

Suppose now for definitiveness that $x > y$. Then

$$I = \frac{\pi}{2}\times\begin{cases} 2R_A^2\ln\frac{x}{y}\,, & x,y > R_A, \\ R_A^2\ln\frac{x^2}{R_A^2} + R_A^2 - y^2\,, & x > R_A > y, \\ x^2-y^2, & x,y < R_A. \end{cases} \tag{A3}$$

[1] L. V. Gribov, E. M. Levin, and M. G. Ryskin, Phys. Rep. **100**, 1 (1983).
[2] L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 3352 (1994).
[3] Y. V. Kovchegov, Phys. Rev. D **54**, 5463 (1996).
[4] A. H. Mueller, Nucl. Phys. B **415**, 373 (1994); A. H. Mueller and B. Patel, *ibid.* **425**, 471 (1994); A. H. Mueller, *ibid.* **437**, 107 (1995).
[5] D. Ivanov and K. Melnikov, Phys. Rev. D **57**, 4025 (1998).
[6] D. Ivanov, E. A. Kuraev, A. Schiller, and V. G. Serbo, Phys. Lett. B **442**, 453 (1998).
[7] D. Y. Ivanov, A. Schiller, and V. G. Serbo, Phys. Lett. B **454**, 155 (1999).
[8] A. J. Baltz, F. Gelis, L. D. McLerran, and A. Peshier, Nucl. Phys. A **695**, 395 (2001).
[9] R. N. Lee, A. I. Milstein, and V. G. Serbo, Phys. Rev. A **65**, 022102 (2002).
[10] E. Bartos, S. R. Gevorkyan, E. A. Kuraev, and N. N. Nikolaev, Phys. Rev. A **66**, 042720 (2002).
[11] G. Baur, K. Hencken, and D. Trautmann, Phys. Rep. **453**, 1 (2007).
[12] R. J. Glauber, in *Geometrical Pictures in Hadronic Collisions*, edited by S. Y. Lo (World Scientific, Singapore, 1987), pp. 83–182.
[13] Y. V. Kovchegov and K. Tuchin, Phys. Rev. D **65**, 074026 (2002).
[14] D. Kharzeev, Y. V. Kovchegov, and K. Tuchin, Phys. Lett. B **599**, 23 (2004).
[15] B. Z. Kopeliovich, A. V. Tarasov, and O. O. Voskresenskaya, Eur. Phys. J. A **11**, 345 (2001).
[16] K. Tuchin, Phys. Rev. D **80**, 093006 (2009).
[17] H. Bethe and W. Heitler, Proc. Roy. Soc. Lond. A **146**, 83 (1934).
[18] H. A. Bethe and L. C. Maximon, Phys. Rev. **93**, 768 (1954).
[19] I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978) [Yad. Fiz. **28**, 1597 (1978)].
[20] E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Sov. Phys. JETP **45**, 199 (1977) [Zh. Eksp. Teor. Fiz. **72**, 377 (1977)].
[21] V. N. Gribov, L. N. Lipatov, and G. V. Frolov, Sov. J. Nucl. Phys. **12**, 543 (1971) [Yad. Fiz. **12**, 994 (1970)].
[22] A. H. Mueller, Nucl. Phys. B **317**, 573 (1989).
[23] A. H. Mueller and D. N. Triantafyllopoulos, Nucl. Phys. B **640**, 331 (2002).
[24] Y. V. Kovchegov and E. Levin, *Quantum Chromodynamics at High Energy* (Cambridge University Press, Cambridge, 2013).
[25] J. Jalilian-Marian, Nucl. Phys. A **753**, 307 (2005).
[26] Y. Li and K. Tuchin, Phys. Rev. C **78**, 024905 (2008).
[27] K. Tuchin, Nucl. Phys. A **899**, 44 (2013).
[28] T. Altinoluk and A. Kovner, Phys. Rev. D **83**, 105004 (2011).
[29] F. Gelis and J. Jalilian-Marian, Phys. Rev. D **66**, 014021 (2002).
[30] R. Baier, A. H. Mueller, and D. Schiff, Nucl. Phys. A **741**, 358 (2004).
[31] F. Dominguez, C. Marquet, B.-W. Xiao, and F. Yuan, Phys. Rev. D **83**, 105005 (2011).
[32] D. Kharzeev, Y. V. Kovchegov, and K. Tuchin, Phys. Rev. D **68**, 094013 (2003).
[33] J. Jalilian-Marian and Y. V. Kovchegov, Phys. Rev. D **70**, 114017 (2004); **71**, 079901(E) (2005).