Disappearance of Mott oscillations in sub-barrier elastic scattering of identical heavy ions, and the nuclear interaction

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Background: The sub-barrier elastic scattering of heavy ions is discussed.

Purpose: We investigate the sensitivity of the Mott cross section to the short-range nuclear interaction.

Method: The conventional scattering theory of identical particles is used to discuss the disappearance of Mott oscillations at a certain critical value of the Sommerfeld parameter. This, universal, phenomenon is called "transverse isotropy".

Results: It is found that a significant modification of the transverse isotropy arises from the inclusion of the short-range nuclear interaction even at the sub-barrier energies considered. Comparison with existing data on the $\alpha + \alpha$ system is made.

Conclusions: We suggest that the study of this lack of Mott oscillation, namely the "transverse isotropy" is a potentially useful mean to study the nuclear interaction.

I. INTRODUCTION

Deviations from pure Mott scattering in the case of heavy-ion systems have been the subject of investigation over a long period. In particular, it has been suggested [1,2] that heavy-ion systems, such as ^{12}C on ^{12}C and ^{16}O on ^{16}O at sub-barrier energies, may exhibit deviation from a pure Mott scattering of identical bosons, owing to the underlying Fermi nature of the constituent nucleons. These authors invoke the idea of parastatistics to quantify their suggestion. The concept of parastatistics is advanced to describe systems obeying neither the Bose, nor the Fermi statistics, but somewhere in between. Using a parameter that interpolates between the two major statistics, one can test possible deviations of composite bosonic systems such as even-even nuclei from the Bose statistics. Experiments at Yale of ^{12}C on ^{12}C and ^{16}O on ^{16}O elastic scattering at deep sub-barrier energies seem to exhibit such deviation, albeit small [3].

The quest for information about the short-range nuclear interaction from elastic scattering data has been going on for a long time. This is even more challenging in the case of elastic scattering of heavy ions, where the long-range Coulomb interaction is very important, especially at low energies. This fact prompted researchers to measure the cross section at higher energies, where the Coulomb effects are concentrated in the very small angular region around $\theta = 0$. Useful information was obtained about the nuclear interaction at these higher energies, especially in systems where nuclear rainbow dominates [4–6]. One may still wonder if low energy scattering could be used to obtain such information. In fact, it has been shown that information about several useful nuclear properties can be obtained when the energy is below the Coulomb barrier and the cross section is predominantly Coulomb [7–9]. Further, any deviation from the Coulomb interaction, even if very small,

may lead to measurable change in the characteristics of the Mott oscillations in the scattering of identical nuclei. This fact leads, among other things, to a test of the existence of color Van der Walls force in the Mott scattering of ^{208}Pb + ^{208}Pb [10,11]. In this paper we propose to study a special feature of the Mott scattering to obtain information about the short range nuclear interaction, which would, in principle, complement the information obtained at high energies. This special feature is the apparent disappearance of the oscillations at a certain critical value of the Sommerfeld parameter. Preliminary work on this has been done in [12], where the effect was coined transverse isotropy (TI). A recent experiment [13] on $\alpha + \alpha$ Mott scattering seems to show this TI. Here we go further, and demonstrate and this "transverse isotropy" is quite sensitive to the presence of the nuclear interaction, making it an attractive venue to look for the latter.

II. IDENTICAL PARTICLE SCATTERING

In the scattering of identical bosons the wave function is symmetric with respect to the exchange of the projectile and the target. In the simple case of spin 0, or collisions of polarized particles with spin aligned, the spacial part of the wave function must be symmetric. The angular distribution must then be given by the expression

$$\sigma(\theta) = |f(\theta) + f(180^\circ - \theta)|^2, \qquad (1)$$

where we use the short-hand notation: $\sigma(\theta) \equiv d\sigma/d\Omega$. Equation (1) can be written in the form

$$\sigma(\theta) = \sigma_{\text{inc}}(\theta) + \Xi_{\text{int}}(\theta), \qquad (2)$$

where $\sigma_{\mathrm{inc}}(\theta)$ is the incoherent sum of contributions from the two amplitudes,

$$\sigma_{\mathrm{inc}}(\theta) = |f(\theta)|^2 + |f(180^\mathrm{o} - \theta)|^2, \qquad (3)$$

and $\Xi_{\mathrm{int}}(\theta)$ is the interference term,

$$\Xi_{\mathrm{int}}(\theta) = 2\,\mathrm{Re}\{f^*(\theta) \times f(180^\mathrm{o} - \theta)\}. \qquad (4)$$

Note that the incoherent part of the cross section is positive-definite, whereas the interference term may assume positive or negative values.

Since the scattering amplitude must have a continuous derivative with respect to θ and the cross section is symmetric with respect to $\theta = 90^\mathrm{o}$, $\sigma(\theta)$ must have a vanishing slope at this angle. This poses a few interesting questions such as:

(i) What are the conditions for the angular distribution to have a maximum or a minimum at $\theta = 90^\mathrm{o}$?

(ii) Can a system present maxima and minima for different collision energies?

(iii) If the answer to the previous question is 'yes', how does the cross section behaves near the transition energy?

The aim of the present paper is to answer these questions.

III. MOTT SCATTERING

We begin by considering a simple scattering problem: the collision of structureless particles, interacting only through point-charge Coulomb forces. In this case, the problem has an analytical solution (see, e.g., Ref. [14]),

$$f_\mathrm{c}(\theta) = -\frac{a}{2}\, e^{2i\sigma_0}\, \frac{e^{-i\eta\ln(\sin^2\theta/2)}}{\sin^2(\theta/2)}, \qquad (5)$$

where η and a are, respectively, the Sommerfeld parameter and half the distance of closest approach in a head-on collision given by

$$\eta = \frac{q^2}{\hbar v}, \quad a = \frac{q^2}{2E}. \qquad (6)$$

Above, q is the charge of the identical particles (the projectile and the target), v is the relative velocity, and σ_0 is the s-wave Coulomb phase shift

$$\sigma_0 = \arg\{\Gamma(1 + i\eta)\}, \qquad (7)$$

with Γ standing for the usual Γ function.

Using these results and normalizing all functions with respect to the Mott cross section at $\theta = 90^\mathrm{o}$,

$$\sigma_{\mathrm{M}}(\theta) \rightarrow \overline{\sigma}_{\mathrm{M}}(\theta) = \frac{\sigma_{\mathrm{M}}(\theta)}{\sigma_{\mathrm{M}}(90^\mathrm{o})},$$

$$\sigma_{\mathrm{inc}}(\theta) \rightarrow \overline{\sigma}_{\mathrm{inc}}(\theta) = \frac{\sigma_{\mathrm{inc}}(\theta)}{\sigma_{\mathrm{M}}(90^\mathrm{o})},$$

$$\Xi_{\mathrm{int}}(\theta) \rightarrow \overline{\Xi}_{\mathrm{int}}(\theta) = \frac{\Xi_{\mathrm{int}}(\theta)}{\sigma_{\mathrm{M}}(90^\mathrm{o})},$$

we can write

$$\overline{\sigma}_{\mathrm{M}}(\theta) = \overline{\sigma}_{\mathrm{inc}}(\theta) + \overline{\Xi}_{\mathrm{int}}(\theta), \qquad (8)$$
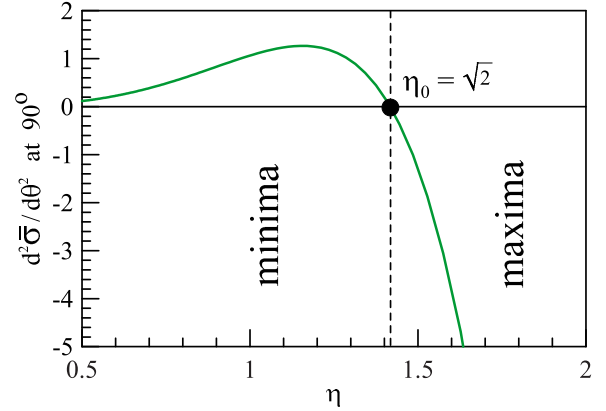

FIG. 1. (Color online) Second derivative of the renormalized Mott cross section at $\theta = 90^\mathrm{o}$.

with

$$\overline{\sigma}_{\mathrm{inc}}(\theta) = \frac{1}{16} \left[\frac{1}{\sin^4(\theta/2)} + \frac{1}{\cos^4(\theta/2)} \right] \qquad (9)$$

and

$$\overline{\Xi}_{\mathrm{int}}(\theta) = \frac{1}{16} \left[2\, \frac{\cos[2\eta\ln(\tan(\theta/2))]}{\sin^2(\theta/2)\,\cos^2(\theta/2)} \right].$$

Note that here the cross sections are normalized with respect to the Mott cross section at $\theta = 90^\mathrm{o}$, whereas in Ref. [12] the normalization was with respect to the Rutherford cross section. These normalizations differ by a factor 4.

The sign of the second derivative of the angular distribution at $\theta = 90^\mathrm{o}$ would indicate wether the cross section has a maximum or a minimum at this angle. Note that the normalized cross section depends exclusively on the value of the Sommerfeld parameter. In Fig. 1 we show the second derivative of the angular distribution at $\theta = 90^\mathrm{o}$, as a function of the Sommerfeld parameter. We see that the second derivative is positive for small values of η and is negative at large values. This means that the angular distribution has a minima for $\eta < \eta_0$ and a maximum above η_0. The transition value of the Sommerfeld parameter can be obtained analytically. After a lengthy calculation [12], one obtains $\eta_0 = \sqrt{2}$.

To illustrate this behavior we show in Fig. 2 the normalized Mott cross sections for a value of the Sommerfeld parameter below η_0 ($\eta = 0.2$) and one value above ($\eta = 4.0$). As expected the former has a minimum at $\theta = 90^\mathrm{o}$, whereas the latter has a maximum. However, the most interesting feature of this figure is the cross section at the critical value of the Sommerfeld parameter, $\eta = \sqrt{2}$. In this case, the cross section is remarkably flat around 90°. This phenomenon was called *transverse isotropy* in Ref. [12]. The important question at this stage is: Can this behavior be observed in some physical system?

IV. TRANSVERSE ISOTROPY IN NUCLEAR PHYSICS

In principle, nuclear systems can be good candidates to exhibit the flat cross sections discussed in the previous section. However, nuclei interact through both Coulomb and nuclear forces. Thus, the prediction of a flat cross section at the critical value of the Sommerfeld parameter will only be valid if the
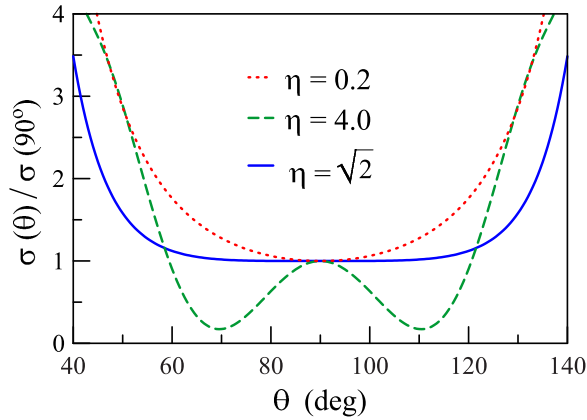
FIG. 2. (Color online) Mott cross sections for three values of the Sommerfeld parameter, normalized with respect to their value at $\theta = 90^\circ$.

corresponding collision energy is below the height of the Coulomb barrier. We should then check if there are nuclear systems satisfying this condition. As qualitative approach to the problem, we set $Z_P = Z_T = Z$ and $A_P = A_T = 2Z$, evaluate the collision energy corresponding to η_0,

$$E_0 = \frac{Z^2 e^2}{\hbar \eta_0} \sqrt{\frac{m_0 Z}{2}},$$

and estimate the barrier height by the approximate expression,

$$V_B \simeq \frac{Z_P Z_T e^2}{r_0 \left(A_P^{1/3} + A_T^{1/3}\right)} = \frac{e^2}{r_0} Z^{5/3} 2^{-4/3}.$$

In Fig. 3 we plot the ratio E_0/V_B against the atomic number. We see that only two systems satisfy the condition that the transition energy lies below the Coulomb Barrier: $^2H + ^2H$ and $^4He + ^4He$. For a third system, $^6Li + ^6Li$, this condition is nearly satisfied and for heavier systems the transition energy lies well above the barrier. In the cases of $^2H + ^2H$ and $^6Li + ^6Li$ the problems is more complicated because these nuclei do not have spin zero. In this way, the above discussion of the cross section only applies for
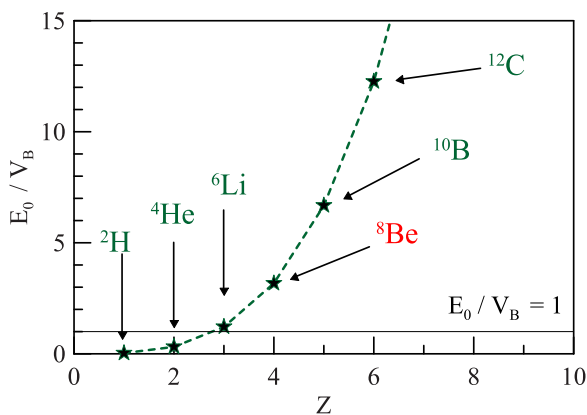

FIG. 3. (Color online) The ratio of the energy E_0 to the Coulomb barrier, V_B, as a function of the atomic number of the identical collision partners. For details see the text.
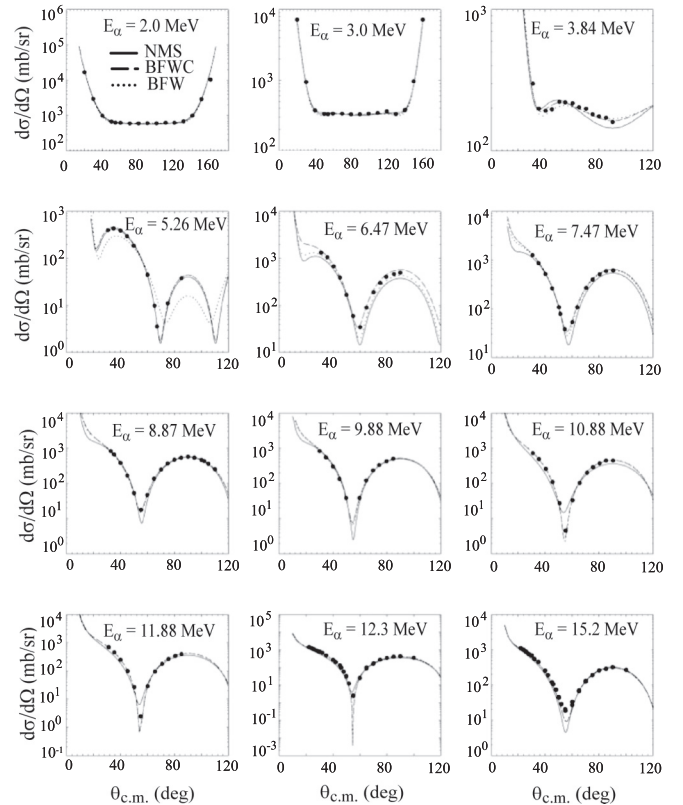

FIG. 4. The experimental angular distributions for the $^4He + ^4He$ system at several collision energies. The figure was taken from Abdullah *et al.* [13]. For details see the text.

polarized projectile and target with spin aligned. Therefore, we concentrate the discussion to the $^4He + ^4He$ collision.

Figure 4 shows experimental angular distributions for the $^4He + ^4He$ system at several collision energies. The figure was taken from Abdullah *et al.* [13] (note that the collision energies are given in the laboratory frame). The lowest energy where data were taken in [13] is $E_{lab} = 2.0$ MeV ($E_{c.m.} = 1.0$ MeV), which is higher than the collision energy where the cross section is expected to be flat. For this system, $\eta = \sqrt{2}$ corresponds to $E_{c.m.} = 0.397$ MeV. However, the data show the flat behavior for the two lowest energies, which correspond to $E_{c.m.} = 1.0$ MeV and $E_{c.m.} = 1.5$ MeV. This is surprising and one can only assume that the nuclear interaction is producing the flat behavior at higher energies. To check this point we include a nuclear potential in the Hamiltonian and re-evaluate the second derivative of the cross section. We first try a nuclear potential frequently used to describe nuclear collisions. We consider the Akÿuz-Winther potential [14,15]. This potential is an approximation to the double folding interaction, parametrized by the Woods-Saxon shape,

$$V(r) = \frac{V_0}{1 + \exp\left[(r - R_0)/a\right]}.$$

The parameters V_0, R_0, and a are functions of the mass numbers of the projectile and the target and for the $^4He + ^4He$ system they have the values: $V_0 = -22.21$ MeV, $R_0 = 3.63$ fm, and $a = 0.5152$ fm. Since the ^4He nuclei do
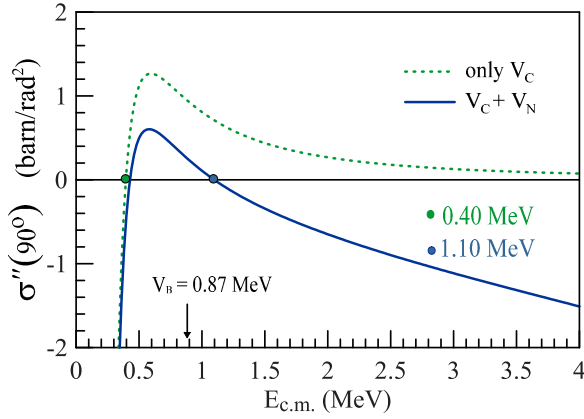
FIG. 5. (Color online) Second derivative of the Mott cross section at $\theta = 90^\circ$ for the Akÿus-Winther potential. The dotted line is basically the same line of Fig. 1, except that here the plot is against the collision energy. The solid line takes into account both the Coulomb and the nuclear potentials.

not have excited states at low energies, we do not include an imaginary part in the nuclear interaction.

In Fig. 5 we show the second derivative of the cross section,

$$\sigma''(\theta) \equiv \frac{d^2\sigma(\theta)}{d\theta^2},$$

at $\theta = 90^\circ$, both including (solid line) and not including (dotted line) nuclear forces. Using the Akÿuz-Winther potential, the Coulomb barrier for the $^4\text{He} + {}^4\text{He}$ system takes the value $V_B = 0.87$ MeV, as indicated within the figure. We see that the two curves are very close below V_B and the transition energy E_0 is not significantly changed by the nuclear interaction. However, at higher energies the two curves become progressively different. A very interesting effect of the nuclear force is that it leads to a second transition energy $E_0' = 1.10$ MeV. In this way, we can distinguish three energy regimes:

reg. 1 : $E < E_0 \;\rightarrow\; \sigma''(90^\circ) < 0$ (maximum),

reg. 2 : $E_0 < E < E_0' \;\rightarrow\; \sigma''(90^\circ) > 0$ (minimum),

reg. 3 : $E_0 > E_0' \;\rightarrow\; \sigma''(90^\circ) < 0$ (maximum).

In regions 1 and 3 the second derivative is negative, hence $\sigma(90^\circ)$ is a maximum. On the other hand, in region 2 the second derivative is positive and $\sigma(90^\circ)$ is a minimum.

Figure 5 explains some features of the experimental cross sections of Fig. 4. First, it predicts a second region where the cross sections is flat. This regions is around $E_{\text{c.m.}} \sim 1$ MeV, as in the data of Abdullah et al. [13]. Above this region, Fig. 5 predicts that cross section will have a maximum at $\theta = 90^\circ$. This is correct, with one exception: the data at $E_{\text{lab}} = 3.84$ MeV. At this energy the experimental cross section has a minimum, whereas following Fig. 6, a maximum is predicted. This point should be studied more carefully.

The Akÿuz-Winther interaction is quite successful for the description of heavy ion collision. However, it is not suitable for ^4He. The data in the α energy region $E_\alpha = 2\text{--}34.2$ MeV were accounted for by a real nuclear interaction given by the sum of two gaussians, one attractive and one repulsive, plus the Coulomb interaction. This potential, which in the work of
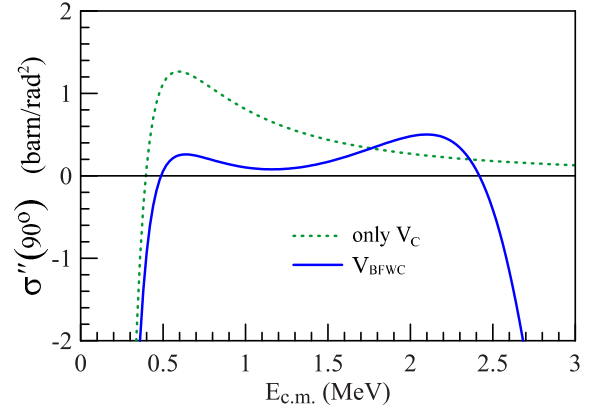

FIG. 6. (Color online) Same as Fig. 5 but now the nuclear potential is given by Eqs. (10)–(12), with the parameters given in the text.

Abdullah et al. [13], was called BFWC, is given by

$$V_{\text{BFWC}}(r) = -V_A\, e^{-r^2/R_A^2} + V_R\, e^{-r^2/R_R^2} + V_C(r). \quad (10)$$

Above, $V_C(r)$ is the Coulomb interaction,

$$V_C(r) = \frac{4e^2}{2R_C}\left(3 - \frac{r^2}{R_C^2}\right), \qquad \text{for } r < R_C \quad (11)$$

$$= \frac{4e^2}{r} \quad \text{for } r \geqslant R_C, \quad (12)$$

with $R_C = 5.8$ fm, and the parameters of the nuclear potential are $V_A = 122.62$ MeV, $R_A = 2.132$ fm, $V_R = 3.0$ MeV, and $R_R = 2.0$ fm.

We then adopt this potential and evaluate the second derivative of the cross section at $\theta = 90^\circ$, as a function of the collision energy. The result is given in Fig. 6. The lowest transition energy shifts slightly, taking the value $E_0 = 0.47$ MeV and the second one moves to $E_0' = 2.41$ MeV. Between these two energies the second derivative remains very small, getting very close to zero around 2.2 MeV. The energy $E_{\text{lab}} = 3.84$, where the experimental cross section shows a minimum at 90° is in the region where the second derivative grows before dropping as the energy approaches E_0'.

The results shown in Fig. 6 imply that the cross sections must be very flat at the energies $E_{\text{c.m.}} = 0.47$ MeV and $E_{\text{c.m.}} = 2.41$ MeV and between these values the cross section has a slight minimum. Above 2.41 MeV, the cross sections present pronounced maxima at 90°. This is effectively the experimental behavior [13]. No experimental results are available for the other predicted transition point at $E_{\text{c.m.}} = 0.47$ MeV.

V. CONCLUSIONS

We have discussed the effect of the nuclear interaction on the transverse isotropy, namely, the angular region where the Mott cross section becomes flat. Application was made for the $\alpha + \alpha$ system, where data exist at the near-barrier energies considered [13]. We have found an important sensitivity to the nuclear interaction. Our finding should be helpful to investigate the nuclear interaction in the Mott scattering of heavy ions. In particular the transition region with transverse isotropy

predicted at $E_{\text{c.m.}} = 0.47$ MeV in the ^{4}He $+$ ^{4}He system should give a precise determination of the long range part of the nuclear potential.

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