

Adiabatic model of (d, p) reactions with explicitly energy-dependent nonlocal potentials

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We have developed an approximate way of dealing with explicit energy dependence of nonlocal nucleon optical potentials as used to predict the (d, p) cross sections within the adiabatic theory. Within this approximation, the nonlocal optical potentials have to be evaluated at an energy shifted from half the incident deuteron energy by the $n-p$ kinetic energy averaged over the range of the $n-p$ interaction and then treated as an energy-independent nonlocal potential. Thus, the evaluation of the distorting potential in the incident channel is reduced to a problem solved in our previous work [N. K. Timofeyuk and R. C. Johnson, *Phys. Rev. Lett.* **110**, 112501 (2013); *Phys. Rev. C* **87**, 064610 (2013)]. We have demonstrated how our new model works for the case of $^{16}\text{O}(d, p)^{17}\text{O}$, $^{36}\text{Ar}(d, p)^{37}\text{Ar}$, and $^{40}\text{Ca}(d, p)^{41}\text{Ca}$ reactions and highlighted the need for a detailed understanding of the energy dependence of nonlocal potentials. We have also suggested a simple way of correcting the d - A effective potentials for nonlocality when the underlying energy-dependent nonlocal nucleon potentials are unknown but energy-dependent local phenomenological nucleon potentials are available.

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I. INTRODUCTION

In our two previous publications [1,2] we proposed a method for calculating $A(d, p)B$ cross sections when the interaction of the neutron and proton in deuteron with the target A is energy independent but nonlocal. This development has been motivated by the growing number of (d, p) experiments performed at radioactive-beam facilities with the aim of extracting spectroscopic information beyond the limits of β stability, including cases important for astrophysical applications. The (d, p) reaction is known [3] to be dominated by those components of the total wave functions in which the separation between the neutron and proton in the incoming deuteron is less than the range of the $n-p$ interaction. Such components are often calculated in the adiabatic distorted-wave approximation (ADWA) where the effective deuteron potential is derived from neutron and proton optical potentials taken at some fixed energy [4,5]. Because the introduction of ADWA, the fixed neutron and proton energies used to calculate the adiabatic deuteron potential were always taken to be precisely half of the deuteron incident energy [4,6,7]. Such a prescription seemed to be reasonable from the intuitive point of view that appeals to the low binding energy of deuteron.

The first indications for a need to go beyond the approximation of fixed nucleon energies came from the Faddeev study of transfer reactions in Refs. [8,9]. The energy dependence of the nucleon optical potentials assumed there were either explicit [8] or a result of the nonlocality of energy-independent nucleon potential [9]. Given that the Faddeev formalism is too complicated for the routine analysis of deuteron stripping reactions, we have suggested [1,2] a practical way to account for the nonlocality of nucleon optical potentials in the ADWA developed in Refs. [4,6]. It turned out [2] that for energy-independent nonlocal potentials of the Perey-Buck type [10] a simple adiabatic deuteron potential can be constructed as a solution of a transcendental equation, similar to the one obtained for nucleon scattering in Ref. [10]. Moreover, for $Z = N$ nuclei and isospin-independent nucleon optical potentials this solution is related to the sum of nucleon potentials taken

at an energy shifted with respect to half the incident deuteron energy by about 40 MeV. We have shown that this large shift comes from the relative $n-p$ kinetic energy averaged over the short-range potential V_{np} [1,2]. The new effective deuteron potential is shallower than that traditionally used in the ADWA and this leads to a change in normalization of predicted (d, p) cross sections with consequences for their interpretation in terms of nuclear structural quantities.

The simple prescription of Refs. [1,2] to obtain the d - A potential for (d, p) reactions relies strongly on the assumption that the energy dependence of nucleon optical potentials is a consequence of the nonlocality of energy-independent potentials. However, the formal theory of optical potentials developed by Feshbach [11] shows that optical potentials are not only nonlocal but energy dependent as well. The explicit energy dependence of Feshbach potentials comes from the coupling of channels with the target, A , in its ground state to channels in which A is excited, i.e., the mechanism that gives rise to the imaginary part of the optical potential when the excited channels are open. The result is that global local energy-dependent potentials derived from analysis of experimental data may not be of the Perey-Buck type. We have checked this for optical potentials taken from frequently used global systematics CH89 [12]. We found that only their real parts satisfy the Perey-Buck form with the usual nonlocality range of 0.85 fm while the imaginary parts are definitely not of the Perey-Buck type and do not follow any immediately obvious systematic rule. Therefore, it is unclear how to treat explicit energy dependence of the popular CH89 global potential in the ADWA. This may be relevant to other energy-dependent global optical potentials available in the literature and used in (d, p) reaction calculations.

In this paper we suggest an approximate practical way of dealing with explicitly energy-dependent nonlocal optical potentials. For this purpose we first consider in Sec. II the connection between the three-body model and the underlying many-body problem that leads to the energy dependence of optical potentials. Then in Sec. III we give formal expressions

for optical potential operators in two- and three-body systems. We derive in Sec. IV the link between the adiabatic (d, p) models with energy-dependent and energy-independent optical potentials and make important comments in Sec. V. We apply in Sec. VI the new model to calculation of the cross sections of the same (d, p) reactions considered earlier by us in Ref. [2]. We discuss our model and the results obtained and make conclusions in Sec. VII. Important derivations are given in the Appendix.

II. CONNECTION BETWEEN THE THREE-BODY MODEL OF THE $n + p + A$ SYSTEM AND THE UNDERLYING MANY-BODY PROBLEM

Explicit energy dependence of the nucleon optical potential arises as the result of internal structure of the target and the consequential possibility that the target can be excited. However, it is customary to model the $A(d, p)B$ reaction in terms of scattering state solutions of a three-body Schrödinger equation in which only the coordinates of the n and p appear explicitly. We can formalize this procedure by regarding the three-body wave function of the model as the projection, $\Psi_{k_d}^{(+)}(n, p)\phi_A$, of the full $A + 2$ many-body wave function corresponding to a deuteron incident on a nucleus A in its ground state, ϕ_A , onto the ground state of A . This projection has outgoing waves that describe elastic deuteron scattering and elastic deuteron breakup exactly and outgoing waves in any open stripping channel in which B has a non-negligible component with A in its ground state. The complete many-body wave function has components in which A is not in its ground state. The existence of coupling to these components influences the projection $\Psi_{k_d}^{(+)}(n, p)\phi_A$, but we base our analysis on a model in which their explicit contribution to the $A(d, p)B$ transition matrix is neglected.

Within this framework an exact expression for $A(d, p)B$ transition matrix is

$$T_{dp} = \langle \Psi_{k_p, B}^{(-)}(p, B) | V_{np} | \Psi_{k_d}^{(+)}(n, p)\phi_A \rangle, \quad (1)$$

where V_{np} is the n - p interaction and $\Psi_{k_p, B}^{(-)}$ is a solution of the many-body Schrödinger equation corresponding to a proton with momentum k_p incident on B but with V_{np} deleted. This expression for the exact amplitude differs from the widely used alternative expression in which the transition operator is given by $V_{np} + \sum_{i \in A} V_{pi} - U_{pB}$ [13] and the bra-vector contains the product of the wave function of nucleus B and the distorted wave in the p - B channel generated by the (arbitrary) optical potential U_{pB} . The advantage of Eq. (1) is that it contains the short-ranged transition operator V_{np} only, which allows a simple approximation for the $\Psi_{k_d}^{(+)}(n, p)$ to be developed. However, the final-state wave function $\Psi_{k_p, B}^{(-)}$ in this expression becomes more complicated. To further discuss this issue we restrict ourselves to a three-body model, $n + p + A$, in which the internal degrees of freedom of A are not treated explicitly. In this case $\Psi_{k_p, B}^{(-)}$ is a solution of a three-body problem, but it is a very special one in which (i) two of the bodies, n and p , do not interact, and (ii) in practical applications to $A(d, p)B$ reactions A is frequently much more massive than a nucleon.

In fact, in the $A \rightarrow \infty$ limit excitation of B is impossible in this model and the exact solution for $\Psi_{k_p, B}^{(-)}$ is the product of $\chi_{k_p}^{(-)}(p)$, a proton scattering wave function distorted by the proton-target potential V_{pA} , and the wave function of the final nucleus B . The overlap of the latter with A gives the neutron overlap function I_{AB} . Derivations of these results are given in [14] where references to earlier work can be found. Corrections to this limit in the case of selected light nuclei A have been evaluated in [15] using the adiabatic approximation to handle the excitation and break-up of B by the recoil of A . It was shown in [15] how the recoil corrections modify the proton distorted wave, $\chi_{k_p}^{(-)}(p)$, by a factor that goes to unity for large A leaving a three-body wave function with a range limited in space by the neutron overlap function. We refer to [15] and [16] for further quantitative discussion of these results which show that recoil contributions to $\Psi_{k_p, B}^{(-)}$ can probably be ignored in the first instance except for the very lightest nuclei. The aim of the present paper is to clarify how a proper treatment of the wave function in the incident channel changes (d, p) predictions when the wave function in the final channel is fixed. Although to date a complete solution to the problem of calculating $\Psi_{k_p, B}^{(-)}$ for finite A has not yet been obtained the results of [15] and [16] give sufficient grounds for ignoring the finite A complications in $\Psi_{k_p, B}^{(-)}$ for our purposes.

The emphasis is now on the evaluation of $\Psi_{k_d}^{(+)}(n, p)$, which by definition is a function of the coordinates of n and p . We make heavy use of the fact that the evaluation of the amplitude (1) requires knowledge of this function within the range of V_{np} only and we make approximations that are inspired by this observation.

Using projection operator techniques originally given by Feshbach [11] it is possible to derive explicit expressions for the effective interactions that appear in the three-body Hamiltonian that drives $\Psi_{k_d}^{(+)}(n, p)$. This effective Hamiltonian is

$$H_{\text{eff}} = T_3 + V_{np} + U_n^0(n) + U_p^0(p) + \langle \phi_A | U | \phi_A \rangle, \quad (2)$$

where T_3 is the three-body kinetic energy operator for $n + p + A$ in the center-of-mass system and the bra-ket notation here implies integration over the target nucleus coordinates to leave an operator in n and p coordinates only. The interactions $U_N^0(N)$ that appear in Eq. (2) depend only on the n - A and p - A coordinates, respectively, being diagonal in states of the target A but, otherwise, are arbitrary at this point. They have no influence on the ground-state matrix elements of $U + U_n^0 + U_p^0$ or on the three-body wave function. The importance of the introduction of the U_N^0 's in the present context is that their arbitrariness can be used to decouple the neutron and proton contributions to the complex many-body operator U , as least as far as the lowest multiple scattering contributions are concerned. How this is done is discussed in Sec. IV.

The many-body operator U in Eq. (2) accounts for all the effects of the target nucleus degrees of freedom. It satisfies the

integral equation [see the Appendix (Sec. 1)]

$$U = \Delta v_{nA} + \Delta v_{pA} + (\Delta v_{nA} + \Delta v_{pA}) \frac{Q_A}{e} U, \quad (3)$$

$$\Delta v_{NA} = v_{NA} - U_N^0, \quad v_{NA} = \sum_{i=1}^A v_{Ni}, \quad N = n, p.$$

The operator Q_A projects onto excited states of the target and the energy denominator e is given by

$$e = E_3 + i0 - T_3 - V_{np} - U_n^0(n) - U_p^0(p) - (H_A - E_A), \quad (4)$$

where H_A and E_A are the internal Hamiltonian and the ground-state energy of the target A , respectively. E_3 is the three-body energy related to the incident center-of-mass kinetic energy E_d and deuteron binding energy ϵ_0 by $E_3 = E_d - \epsilon_0$.

A formal solution of Eq. (3) is

$$U = \Delta v_{nA} + \Delta v_{pA} + (\Delta v_{nA} + \Delta v_{pA}) \times Q_A \frac{1}{e - Q_A(\Delta v_{nA} + \Delta v_{pA})Q_A} Q_A(\Delta v_{nA} + \Delta v_{pA}). \quad (5)$$

$\langle \phi_A | U | \phi_A \rangle$ sums up all processes via excited target states and the deuteron ground and breakup states that are coupled to the target ground state by Δv_{NA} and which begin and end on the target ground state. The expansion that appears on iteration of Eq. (3) by repeated substitution for U on the right-hand side makes this clear:

$$U = \Delta v_{nA} + \Delta v_{pA} + (\Delta v_{nA} + \Delta v_{pA}) \frac{Q_A}{e} (\Delta v_{nA} + \Delta v_{pA}) + (\Delta v_{nA} + \Delta v_{pA}) \frac{Q_A}{e} (\Delta v_{nA} + \Delta v_{pA}) \times \frac{Q_A}{e} (\Delta v_{nA} + \Delta v_{pA}) + \dots. \quad (6)$$

When averaged over the target ground state and added to $U_n^0(n) + U_p^0(p)$, the first term on the right-hand side of Eq. (6) will be recognized as providing an expression for the sum of the real nucleon optical potentials calculated in a folding model using the free-space interactions between n and p with the target nucleons. The second- and higher-order terms include the effect of target excitation. Because of the propagator $\frac{1}{e}$ they are complex and nonlocal in neutron and proton coordinates even when the nucleon-nucleon interactions v_{Ni} are local. In addition, there is an explicit dependence on the energy E_3 through the energy denominator e . The purpose of this E_3 dependence is to link the three-body energy of the system to the thresholds of all relevant channels correctly. For example, if E_3 is less than the energy of a coupled intermediate excited state, then the corresponding contribution to U will be real (Hermitian); otherwise, a complex (non-Hermitian) contribution will result. This is the physical reason for expecting the effective interaction to be explicitly energy dependent.

Our aim is to make a connection between $U_n^0(n) + U_p^0(p) + \langle \phi_A | U | \phi_A \rangle$ and the neutron and proton optical potentials and, in particular, to determine the energies at which phenomeno-

logical optical models should be taken to reproduce the main properties of $U_n^0(n) + U_p^0(p) + \langle \phi_A | U | \phi_A \rangle$ for application to evaluation of the (d, p) transition amplitude (1). We see that this can only be done approximately.

III. FORMAL EXPRESSIONS FOR THE NUCLEON OPTICAL POTENTIAL IN TWO- AND THREE-BODY CHANNELS

A. Multiple scattering expansion of operator U

The operator U , which is a complicated operator in the coordinates of n , p , and A , can be partially separated into its p - A and n - A contributions by using manipulations from multiple scattering theory [see the Appendix (Sec. 2)]. We obtain

$$U = U_{nA} + U_{pA} + U_{nA} \frac{Q_A}{e} U_{pA} + U_{pA} \frac{Q_A}{e} U_{nA} + \dots, \quad (7)$$

where

$$U_{NA} = \Delta v_{NA} + \Delta v_{NA} \frac{Q_A}{e} U_{NA} \quad (8)$$

and the dots in Eq. (7) are terms of third or higher order in U_{nA} and/or U_{pA} , always with an excited target (though not necessarily excited deuteron) as an intermediate state.

We emphasize that although the target ground-state matrix element $U_n^0 + U_p^0 + \langle \phi_A | U | \phi_A \rangle$ is independent of the U_N^0 , this is not true of the matrix elements $U_N^0 + \langle \phi_A | U_{NA} | \phi_A \rangle$. The underlying reason for this is that the denominator e in U_{NA} depends both on U_n^0 and U_p^0 and, therefore, U_{NA} is not symmetric in n and p .

We notice that the operators U_{nA} and U_{pA} in Eqs. (7) and (8) are strongly reminiscent of Feshbach's [11] operator, which gives the nucleon optical potential when sandwiched between target ground-state vectors. The exact expressions for this operator are given below.

B. Two-body optical potential operator

According to Feshbach [11] the optical model operator for a nucleon with kinetic energy E_N satisfies the integral equation

$$U_{NA}^{\text{opt}}(E_N) = v_{NA} + v_{NA} \frac{Q_A}{e_N} U_{NA}^{\text{opt}}(E_N), \quad (9)$$

where $e_N = E_N + i0 - T_{NA} - (H_A - E_A)$. In terms of the solution of this equation, the optical potential, V_{NA}^{opt} , for nucleon scattering by A at energy E_N is given by

$$V_{NA}^{\text{opt}}(E_N) = \langle \phi_A | U_{NA}^{\text{opt}}(E_N) | \phi_A \rangle = \langle \phi_A | v_{NA} + v_{NA} Q_A \times \frac{1}{e_N - Q_A v_{NA} Q_A} Q_A v_{NA} | \phi_A \rangle. \quad (10)$$

It should be noted that in the proton case we include Coulomb interactions in the NN potential v_{pA} and, therefore, our proton optical potential includes a Coulomb potential $V_c(p) = \langle \phi_A | \sum_{i=1}^A v_{pi}^{\text{Coul}} | \phi_A \rangle$.

To make a connection with the $n + p + A$ case of the last section, we introduce a slightly modified definition of nucleon optical potential,

$$\begin{aligned}\bar{U}_{NA}^{\text{opt}} &= \Delta v_{NA} + \Delta v_{NA} \frac{Q_A}{\bar{e}_N} Q_A \bar{U}_{NA}^{\text{opt}} \\ &= \Delta v_{NA} + \Delta v_{NA} Q_A \\ &\quad \times \frac{1}{\bar{e}_N - Q_A \Delta v_{NA} Q_A} Q_A \Delta v_{NA},\end{aligned}\quad (11)$$

where

$$\bar{e}_N = E_N + i0 - T_{NA} - U_N^0 - (H_A - E_A) \quad (12)$$

differs from e_N by inclusion of the potential U_N^0 . The operators $\bar{U}_{NA}^{\text{opt}}$ and U_{NA}^{opt} have different matrix elements, in general, but their ground-state expectation values are related by $\langle \phi_A | \bar{U}_{NA}^{\text{opt}} | \phi_A \rangle = V_{nA}^{\text{opt}} - U_N^0$. The Feshbach optical potential V_{NA}^{opt} is now given by

$$V_{NA}^{\text{opt}}(E_N) = U_N^0(N) + \langle \phi_A | \bar{U}_{NA}^{\text{opt}}(E_N) | \phi_A \rangle. \quad (13)$$

Using the same basic relation between the nucleon optical potential and the underlying many-body theory that we use here, Buck and Lipperheide [17, 18] showed that a nonlocal and energy-independent potential operator can be formally defined that generates the same projection of the many-body wave function on to the target ground state as does the explicitly energy-dependent Eq. (13). However, the potential they derive, as well as being non-Hermitian, as is also Eq. (13), does not satisfy a standard transformation under time reversal. When the underlying two-body interactions v_{Ni} are invariant under time reversal, Feshbach's (13) satisfies

$$\mathcal{K} V_{NA}^{\text{opt}} \mathcal{K}^{-1} = (V_{NA}^{\text{opt}})^\dagger, \quad (14)$$

where \mathcal{K} is the nonlinear time-reversal operator for the $N + A$ system. The operator defined by Buck and Lipperheide does not satisfy this relation. This results in a very clumsy discussion of reciprocity in reaction theories that make use of optical potentials and we therefore choose not to make use of the Buck-Lipperheide definition here.

C. Relation between the three-body and two-body optical potentials

The three-body operator U has contributions from the U_{NA} as a leading term in the three-body Hamiltonian through Eqs. (2) and (7):

$$\begin{aligned}\langle \phi_A | U_{NA}(E_3) | \phi_A \rangle &= \langle \phi_A | \Delta v_{NA} | \phi_A \rangle \\ &\quad + \langle \phi_A | \Delta v_{NA} Q_A \frac{1}{e - Q_A \Delta v_{NA} Q_A} Q_A \Delta v_{NA} | \phi_A \rangle.\end{aligned}\quad (15)$$

These quantities have a similar structure to the Feshbach form for the nucleon optical potential discussed in the last section. Note, however, that the energy denominator $e = E_3 + i0 - T_3 - V_{np} - U_n^0(n) - U_p^0(p) - (H_A - E_A)$, which appears everywhere in the $n + p + A$ case, is not the denominator

$\bar{e}_N = E_N + i0 - T_{NA} - U_N^0 - (H_A - E_A)$ that appears in the neutron operator (12).

To deal with this difference when calculating $\langle \phi_A | U | \phi_A \rangle$ we proceed as follows.

- (i) We neglect the higher-order multiple scattering terms that appear on the right-hand side of Eq. (7). These neglected terms involve n - A and p - A excitations that cannot be expressed in any obvious way as a sum of terms depending separately on the n and p coordinates. They remain to be investigated in future work. These contributions are neglected in all three-body models of the $n + p + A$ system with heavy targets. They are not discussed further. This means our approximation U_{eff} for the effective interaction in the three-body Hamiltonian (2) in addition to V_{np} is

$$\begin{aligned}U_{\text{eff}}(n, p) &= U_{\text{eff}}^n(n) + U_{\text{eff}}^p(p), \\ U_{\text{eff}}^N(N) &= U_N^0(N) + \langle \phi_A | U_{NA} | \phi_A \rangle,\end{aligned}\quad (16)$$

where N is n or p and the U_{nA} and U_{pA} are given by Eq. (8).

- (ii) We consider only the limit of the infinite target mass so that the three-body kinetic-energy operator, T_3 , can be separated into terms that depend on the individual neutron and proton coordinates relative to the target, $T_3 = T_p + T_n$, where $T_p \equiv T_{pA}$ and $T_n \equiv T_{nA}$. This approximation seems to be essential if we are to obtain a three-body Hamiltonian that involves a sum of n - A and p - A potentials each of which only depend on the coordinates of one nucleon. The energy denominator e is now

$$\begin{aligned}e &= E_d + i0 - \epsilon_0 - T_n - T_p - V_{np} \\ &\quad - U_n^0(n) - U_p^0(p) - (H_A - E_A).\end{aligned}\quad (17)$$

IV. DISTORTING POTENTIALS FOR ADIABATIC MODEL OF $A(d, p)B$ REACTIONS

It has been shown [3] that for a range of incident deuteron energies of current experimental interest the $A(d, p)B$ amplitude (1) is dominated by the first Weinberg component of $\Psi_{k_d}^{(+)}(n, p)$, i.e.,

$$\langle \phi_1 | \Psi_{k_d}^{(+)}(n, p) \rangle, \quad (18)$$

where ϕ_1 is defined in terms of the n - p interaction V_{np} by

$$|\phi_1\rangle = \frac{V_{np}|\phi_0\rangle}{\langle \phi_0 | V_{np} | \phi_0 \rangle}, \quad (19)$$

in which ϕ_0 is the deuteron ground-state wave function. In Eq. (18), the integration is performed over the n - p relative coordinate, $\mathbf{r} = \mathbf{r}_n - \mathbf{r}_p$, resulting in a function of the n - p center-of-mass coordinate, $\mathbf{R} = \frac{1}{2}(\mathbf{r}_n + \mathbf{r}_p)$, only. The state ϕ_1 restricts the n - p relative coordinate to be less than the range of the n - p interaction V_{np} .

In the ADWA approximation [6] the first Weinberg component of $\Psi_{k_d}^{(+)}(n, p)$ is determined by a distorted wave $\chi_{k_d}^{(+)}(\mathbf{R})$

satisfying the equation

$$(E_d - T_R - \langle \phi_1 | U_{\text{eff}} | \phi_0 \rangle) \chi_{k_d}^{(+)}(\mathbf{R}) = 0, \quad (20)$$

where U_{eff} is given by Eq. (16). We first consider the U_{nA} term in the expression for U_{eff} .

The dependence of the U_{nA} on the proton coordinates is solely through the presence in e of the operators $T_p + U_p^0$ and V_{np} . The values taken by $T_p + U_p^0 + V_{np}$ influence the net neutron energy parameter in e and hence determine the appropriate energy at which to choose the neutron optical potential. Our method here is to replace $T_p + U_p^0 + V_{np}$ with an average value consistent with the dominance of the first Weinberg component to the (d, p) reaction. We deduce this value within the ADWA. Similarly, we replace the $T_n + U_n^0 + V_{np}$ operator in the U_{pA} term for U_{eff} with an average value.

The ADWA potential in Eq. (20) has energy dependence that comes through the energy denominators in $\langle \phi_1 \phi_A | \Delta v_{NA} Q_A (e - Q_A \Delta v_{NA} Q_A)^{-1} Q_A \Delta v_{NA} | \phi_0 \phi_A \rangle$, which contains the matrix elements $\langle \phi_A | P_A \Delta v_{NA} Q_A | \phi_A^* \rangle$ that are the functions of $\mathbf{R} \pm \mathbf{r}/2$. The range of variable \mathbf{r} is restricted by the range of $\phi_1 \phi_0$ which is about 0.45 fm. Therefore, for most \mathbf{R} relevant for evaluating $\chi_{k_d}^{(+)}(\mathbf{R})$ we have $r/2 \ll R$. Therefore, to a first approximation the operators $Q_A \Delta v_{NA} P_A$ in the numerator can be replaced by their form at $\mathbf{r} = 0$. Then the only matrix element involving integration over \mathbf{r} is $\langle \phi_1 | (e - Q_A \Delta v_{NA} Q_A)^{-1} | \phi_0 \rangle$.

We show in the Appendix (Sec. 3) that within the ADWA we can consistently use the approximation

$$\langle \phi_1 | (e - Q_A \Delta v_{NA} Q_A)^{-1} | \phi_0 \rangle \approx \frac{1}{\langle \phi_1 | e - Q_A \Delta v_{NA} Q_A | \phi_0 \rangle}. \quad (21)$$

We consider the contribution from $N = n$ first. To evaluate the denominator in the right-hand side of Eq. (21) we use definition (17) for e and the result from the Appendix (Sec. 4),

$$\langle \phi_1 | T_p + V_{np} | \phi_0 \rangle = \frac{1}{2}(T_R - \langle T_r \rangle) - \epsilon_0, \quad (22)$$

where T_R and T_r are the kinetic energy operators associated with the coordinates \mathbf{R} and \mathbf{r} , respectively, and

$$\langle T_r \rangle = \langle \phi_1 | T_r | \phi_0 \rangle. \quad (23)$$

We determine a reasonable value for the operator T_R in Eq. (22) by recalling that it acts on the first Weinberg component $\chi_{k_d}^{(+)}(\mathbf{R})$. Using Eq. (20) we get

$$\begin{aligned} & \langle \phi_1 | e - Q_A \Delta v_{nA} Q_A | \phi_0 \rangle \\ &= \frac{1}{2} [E_d + \langle \phi_1 | U_{\text{eff}} | \phi_0 \rangle - 2 \langle \phi_1 | U_p^0(p) | \phi_0 \rangle + \langle T_r \rangle] \\ & \quad - [\langle \phi_1 | T_n | \phi_0 \rangle + \langle \phi_1 | U_n^0(n) | \phi_0 \rangle] \\ & \quad - \langle \phi_1 | Q_A \Delta v_{nA} Q_A | \phi_0 \rangle - (H_A - E_A). \end{aligned} \quad (24)$$

We now recall that the potentials U_n^0 and U_p^0 are arbitrary provided that they commute with Q_A . We are therefore at liberty to choose

$$\langle \phi_1 | U_p^0(p) | \phi_0 \rangle = \frac{1}{2} \langle \phi_1 | U_{\text{eff}} | \phi_0 \rangle. \quad (25)$$

With this step Eq. (21) reduces to

$$\begin{aligned} & \langle \phi_1 | (e - Q_A \Delta v_{nA} Q_A)^{-1} | \phi_0 \rangle \\ &= \langle \phi_1 | \frac{1}{\mathcal{E}_{\text{eff}} - T_n - U_n^0(n) - Q_A \Delta v_{nA} Q_A} | \phi_0 \rangle, \end{aligned} \quad (26)$$

where

$$\mathcal{E}_{\text{eff}} = \frac{1}{2} E_d + \frac{1}{2} \langle T_r \rangle. \quad (27)$$

The neutron contribution to the ADWA effective interaction (16) is given by

$$\begin{aligned} & \langle \phi_1 | U_{\text{eff}}^n(n) | \phi_0 \rangle \\ &= \langle \phi_1 | U_n^0(n) + \langle \phi_A | U_{nA} | \phi_A \rangle | \phi_0 \rangle \\ &= \langle \phi_1 | U_n^0(n) | \phi_0 \rangle + \langle \phi_1 \phi_A | \Delta v_{nA} + \Delta v_{nA} Q_A \\ & \quad \times \frac{1}{\mathcal{E}_{\text{eff}} - T_n - U_n^0(n) - Q_A \Delta v_{nA} Q_A} Q_A \Delta v_{nA} | \phi_A \phi_0 \rangle. \end{aligned} \quad (28)$$

We see that expression (28) is just the neutron contribution to the ADWA distorting potential calculated with a nonlocal neutron optical potential taken at energy \mathcal{E}_{eff} . A similar conclusion can be made for the proton contribution $\langle \phi_1 | U_{\text{eff}}^p(n, p) | \phi_0 \rangle$ to the ADWA distorting potential. With the choice

$$\langle \phi_1 | U_n^0(n) | \phi_0 \rangle = \frac{1}{2} \langle \phi_1 | U_{\text{eff}} | \phi_0 \rangle, \quad (29)$$

we find in identical fashion that this contribution is obtained using the p - A optical potential, including Coulomb terms, taken at the proton energy E_p also equal to $\mathcal{E}_{\text{eff}} = \frac{1}{2}(E_d + \langle T_r \rangle)$. We note that with the choices given by Eqs. (29) and (25) the neutron and proton contributions to the effective interaction of the three-body model in the single scattering approximation are independent of the U_N^0 . The choice (29) for U_n^0 will influence higher-order multiple scattering terms in Eq. (7). We do not consider these terms further here.

V. COMMENTS

We have given arguments that suggest that, for the purpose of calculating (d, p) , cross sections in the ADWA approximation when the nucleon optical potentials are explicitly energy dependent as well as nonlocal the neutron and proton kinetic energies used in the incident channel should be $\frac{1}{2} E_d + \frac{1}{2} \langle T_r \rangle$. This is achieved by averaging the proton kinetic energy in the incoming deuteron and adding the extra kinetic energy the neutron has because it must be close to a proton to contribute to the (d, p) amplitude. Thus, the problem of calculating the ADWA amplitude for $A(d, p)B$ with energy-dependent nonlocal potentials is reduced to the problem of calculating this amplitude with energy-independent nonlocal potentials, the solution of which is found in Refs. [1,2]. If a phenomenological nonlocal, explicitly energy-dependent nucleon potential is available, the prescription of present paper together with that from Refs. [1,2] can be used unambiguously.

The energy-independent nonlocal potential that replaces the original energy-dependent nonlocal potential is obtained from the latter by evaluating it at the energy shifted from the intuitively assumed $E_d/2$ value by one half of the average

n - p kinetic energy within the range of the n - p potential, approximately equal to 57 MeV [1]. This value is close to, but not equal to, the energy shift $\Delta E \sim 40$ MeV, identified in Refs. [1,2] as the shift to the $E_d/2$ value that provides the appropriate energy of energy-dependent equivalents of nonlocal potentials to be used in ADWA.

For all known phenomenological optical potentials the underlying energy-dependent nonlocal potentials are not known. It may be tempting to use the conclusion of the previous section for choosing in the ADWA calculations the phenomenological local potentials taken at energies $\mathcal{E}_{\text{eff}} = \frac{1}{2}E_d + \frac{1}{2}\langle T_r \rangle$. This, however, would be a source of confusion and may lead to a wrong result. Indeed, if $\mathcal{U}(E, r, r')$ is the underlying energy-dependent nonlocal potential for the phenomenological local energy-dependent equivalent $U_{\text{phen}}(E, r)$, then the ADWA needs to use the energy-independent nonlocal potentials $\mathcal{U}(\mathcal{E}_{\text{eff}}, r, r')$ that would give different equivalent energy-dependent potential $\tilde{U}(E, r)$ which then should be taken at the energy $E_d/2 + \Delta E$. The two potentials $\tilde{U}(E_d/2 + \Delta E, r)$ and $U_{\text{phen}}(\mathcal{E}_{\text{eff}}, r)$ are not the same. We show a link between them for a particular class of $\mathcal{U}(E, r, r')$ in the next section. However, the knowledge of energy-dependent nonlocal potentials is crucial.

It is interesting to compare our treatment of explicit energy-dependent optical potentials with that of Ref. [8]. In that work the Faddeev approach was used to solve the three-body problem. Energy dependence is treated in Ref. [8] by taking the neutron (proton) optical potential at the energy equal to the variable neutron (proton) energy parameter in evaluating the neutron (proton) t matrix as it appears in the Faddeev equations. This seems physically plausible at first sight, but to what kind of Schrödinger equation such an approach corresponds is not clear. The Faddeev equations have been derived from the Schrödinger equation with energy-independent nonlocal pairwise potentials. We are not aware of a formal derivation of the Faddeev equations for explicitly energy-dependent pair potentials of the type that arise from many-body effects as discussed in Sec. II.

In our view, explicit energy dependence is an effect that can only be understood by explicitly recognizing the internal degrees of freedom of the target, as is done here and in Refs. [7,19]. It should be added that the approximate prescription proposed here is intended for use only in the calculation of the incident channel distorting potential in a particular expression for the $A(d, p)B$ transition matrix and is not expected to be useful in any other context.

VI. APPLICATIONS TO THE $^{16}\text{O}(d, p)^{17}\text{O}$, $^{36}\text{Ar}(d, p)^{37}\text{Ar}$, AND $^{40}\text{Ca}(d, p)^{41}\text{Ca}$ REACTIONS WITH GLOBAL ENERGY-DEPENDENT NONLOCAL OPTICAL POTENTIAL

In this section we apply the formalism developed above to the ADWA calculation of the cross sections of the $^{16}\text{O}(d, p)^{17}\text{O}$, $^{36}\text{Ar}(d, p)^{37}\text{Ar}$, and $^{40}\text{Ca}(d, p)^{41}\text{Ca}$ reactions at $E_d = 15, 9$, and 11.8 MeV, respectively. The cross sections of these reactions have been measured in Refs. [20–22]. These are the same reactions that have been considered in Ref. [2] using energy-independent nonlocal nucleon optical potentials given by Giannini and Ricco (GR) systematics in Ref. [23]. The

choice of targets made in Ref. [2] was to justify the use of GR which is valid for $N = Z$ nuclei only. In the present work, we first use the Giannini-Ricco-Zucchiatti (GRZ) nonlocal potential [24], which has an energy-dependent imaginary part. Then we make a suggestion of how to account for nonlocality when only energy-dependent local phenomenological potentials are known without knowing the underlying energy-dependent nonlocal potentials. As an example, we use a widely used CH89 systematics [12].

A. Giannini-Ricco-Zucchiatti potential

For $N = Z$ targets, the GRZ potential is given by

$$U_{NA}(E, \mathbf{r}, \mathbf{r}') = H(\mathbf{r} - \mathbf{r}')U_{NA}[E, (\mathbf{r} + \mathbf{r}')/2], \quad (30)$$

with the nonlocality factor

$$H(\mathbf{x}) = \pi^{-3/2} \beta^{-3} e^{-\left(\frac{x}{\beta}\right)^2} \quad (31)$$

determined by the nonlocality range $\beta = 2\hbar^2\alpha/\mu$, where $\alpha = 0.0116$ MeV $^{-1}$ and μ is the nucleon reduced mass. The potential form factor U_{NA} is defined as

$$U_{NA}(E, y) = -V_N f_N(y) - 4i W_N(E) f_{NI}(y) [1 - f_{NI}(y)], \\ f_i(y) = \{1 + \exp[(y - R_N)/a_i]\}^{-1}. \quad (32)$$

It has one energy-dependent parameter, the depth of the imaginary part,

$$W_N(E) = 17.5[1 - \exp(-0.05E)] \text{ MeV}. \quad (33)$$

All other parameters are energy-independent and have the following values: the depth of the real potential $V_N = 85$ MeV, the radius of both the real and imaginary potentials $R_N = 1.16A^{1/3}$ fm, the diffuseness $a_N = 0.57$ fm and $a_{NI} = 0.54 + 0.0032A$ fm for the real and the imaginary potentials, respectively. We neglect the spin-orbit term because its contributions to the (d, p) cross sections at the chosen incident deuteron energies is small.

According to the prescription of Sec. IV, we solve the three-body Schrödinger equation in the adiabatic approximation using an energy-independent imaginary potential (33) evaluated at $\mathcal{E}_{\text{eff}} = \frac{1}{2}E_d + \frac{1}{2}\langle T_r \rangle$. For $\frac{1}{2}\langle T_r \rangle$ we use the value of 57 MeV obtained in Refs. [1,2] for the Hultén n - p potential. We also use $\bar{V}_c = -1.08 + 1.35Z/A^{1/3}$ MeV from Refs. [23,24]. As explained in Refs. [1,2], in the adiabatic approximation the deuteron distorted wave is found from a local-equivalent two-body model,

$$[E_d - T_R - U_c(R) - U_{\text{loc}}(R)] \chi_{k_d}^{(+)}(\mathbf{R}) = 0, \quad (34)$$

where $U_c(R)$ is the d - A Coulomb potential and the local potential U_{loc} in the local-energy approximation is a solution of the transcendental equation

$$U_{\text{loc}}(R) = M_0^{(0)} [U_{NA}(\mathcal{E}_{\text{eff}}, R) + U_{pA}(\mathcal{E}_{\text{eff}}, R)] \\ \times \exp\left[-\frac{\mu_d \beta_d^2}{2\hbar^2} (E_d - U_{\text{loc}}(R) - \bar{V}_c)\right]. \quad (35)$$

In this equation μ_d is the deuteron reduced mass and the constants $M_0^{(0)}$ and β_d depend on V_{np} and the nucleon nonlocality range β only. These constants are tabulated

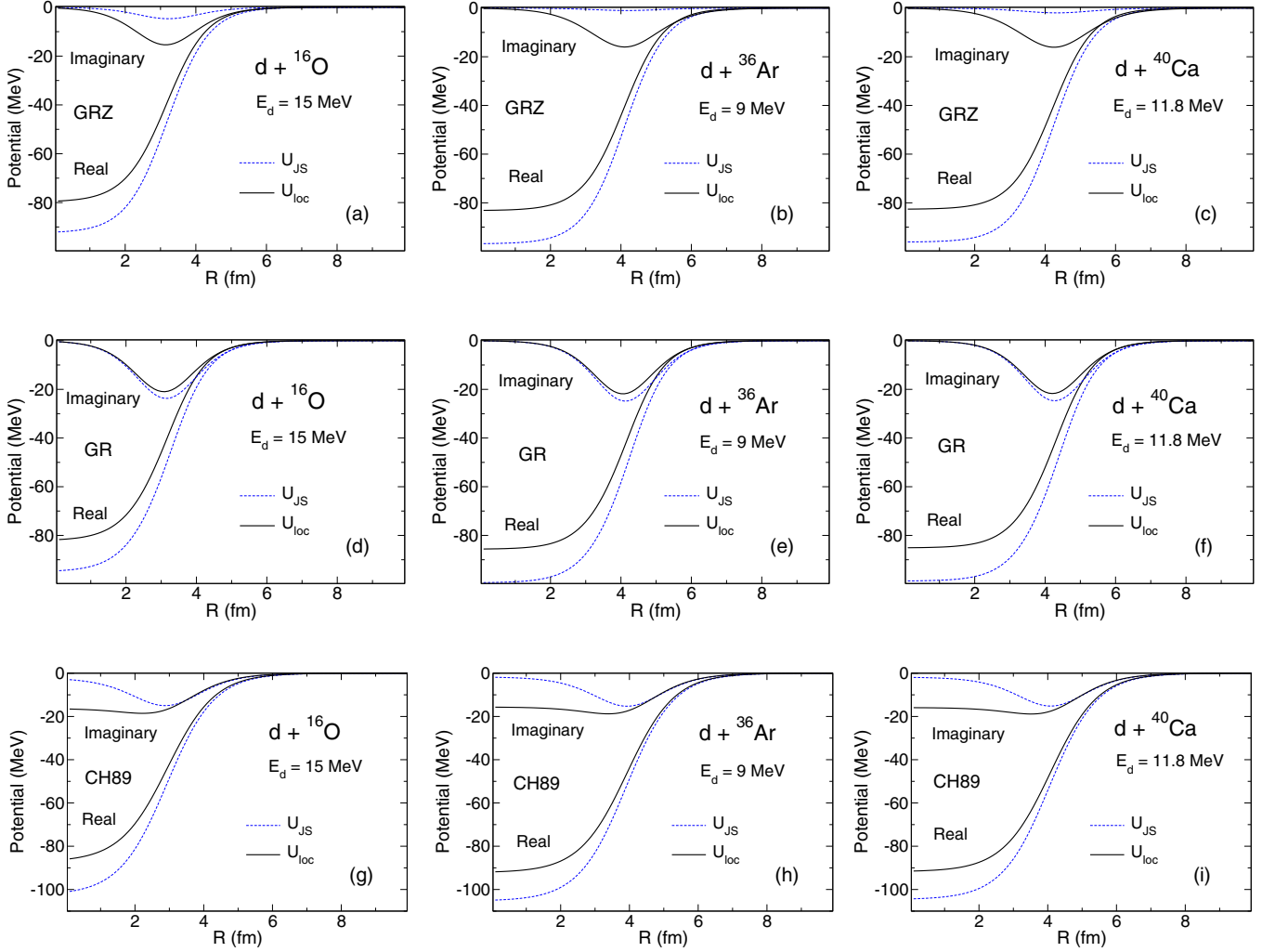


FIG. 1. (Color online) Local adiabatic deuteron potentials U_{loc} for energy-dependent Giannini-Ricco-Zucchiatti (GRZ) (a),(b),(c), energy-independent Giannini-Ricco (GR) (d),(e),(f), nonlocal optical nucleon potentials and energy-dependent local potential CH89 (g),(h),(i), in comparison to the corresponding Johnson-Soper potentials U_{JS} for ^{16}O , ^{36}Ar , and ^{40}Ca targets.

in Ref. [2] for the Hultén n - p potential. The β_d has the meaning of effective nonlocality range for the deuteron-target interaction and is approximately equal to 0.46, which is close to $\beta/2$. The moment $M_0^{(0)}$ is about 0.75 and its deviation from one determines the shift of energy at which the local equivalents of $U_{NA}(\mathcal{E}_{\text{eff}}, R, R')$ should be evaluated to be used in Johnson-Tandy potential calculation. In this paper, we determine U_{loc} by solving the transcendental equation (35) directly without constructing the local equivalent of $U_{NA}(\mathcal{E}_{\text{eff}}, R, R')$.

The local potentials U_{loc} obtained for $d + ^{16}\text{O}$, $d + ^{36}\text{Ar}$, and $d + ^{40}\text{Ca}$ are shown in Figs. 1(a)–1(c). They are compared to the widely used Johnson-Soper potentials U_{JS} constructed from the energy-dependent local-equivalents of $U_{NA}(E, R, R')$ taken at $E_d/2$:

$$U_{JS}(R) = U_{loc}^n(E_d/2, R) + U_{loc}^p(E_d/2, R), \quad (36)$$

$$U_{loc}^n(E_d/2, R) = U_{nA}(E_d/2, R) \exp \left\{ -\frac{\mu\beta^2}{2\hbar^2} \times \left[\frac{E_d}{2} - U_{loc}^n(E_d/2, R) \right] \right\},$$

$$U_{loc}^p(E_d/2, R) = U_{pA}(E_d/2 - \bar{V}_c, R) \exp \left\{ -\frac{\mu\beta^2}{2\hbar^2} \left[\frac{E_d}{2} - \bar{V}_c - U_{loc}^p(E_d/2, R) \right] \right\}. \quad (37)$$

One can see that the real parts of the deuteron local potentials U_{loc} obtained with GRZ are shallower than the real parts of the corresponding Johnson-Soper potential U_{JS} . This is exactly what has been already seen for energy-independent nonlocal potential GR in Refs. [1,2] and is shown again Figs. 1(d)–1(f). This reflects the fact that the real part of GRZ is energy independent. The imaginary parts of U_{loc} obtained with GRZ

are much deeper than those of U_{JS} , which is completely opposite to what happens for the imaginary parts of U_{loc} and U_{JS} obtained with energy-independent potential GR. This happens because the imaginary part of GRZ increases with E and at $\mathcal{E}_{\text{eff}} = E_d/2 + 57$ MeV it takes the largest values while the imaginary parts of U_{JS} is constructed from nucleon potentials taken at 4–8 MeV, where $W_N(E)$ is small. On the contrary, for energy-independent potential GR the imaginary part of its local equivalent slowly decreases with E . Taking this potential at $E_d/2 + \Delta E$ leads to a smaller imaginary part. The different behavior of the imaginary parts of U_{loc} with respect to that of U_{JS} for GRZ and GR results in the difference manifestation of the nonlocal effects in the corresponding differential cross sections of (d, p) reactions.

To calculate the angular distributions of (d, p) reactions we used the framework (1) in which the remnant term is absent and the p - B channel is described by the p - B distorted wave obtained with the p - A optical potential, thus neglecting recoil excitation and breakup effects (note that in Refs. [1,2] the p - B optical potential was used, which means that the remnant term was neglected). These optical potentials were obtained from the local model corresponding to either GRZ or GR potential. The local proton-scattering wave function has been then multiplied by the Perey factor [25] with the proton nonlocality range given by GRZ or GR. No Perey effect was considered in the deuteron channel. Such an effect would arise from the linear terms in the expansion of the nucleon optical potentials near $\mathbf{r} = 0$ for which no averaging procedure has been developed. These terms will influence the shape of the deuteron-target effective potential. The associated Perey effect will also modify the deuteron distorted wave, but only in the nuclear interior which does not contribute significantly for the (d, p) considered here where peripheral contributions dominate.

Both the proton equivalent local potentials and the effective deuteron potentials U_{loc} were read into the TWOFNR code [26], which was used to calculate the (d, p) cross sections in the zero-range approximation. The overlap functions for ^{17}O , ^{37}Ar , and ^{41}Ca were represented by neutron single-particle wave functions obtained by fitting the Woods-Saxon potential wells to reproduce their corresponding separation energies. The standard radius $r_0 = 1.25$ fm and diffuseness $a = 0.65$ fm were used both for central and spin-orbit potentials while the depth of the spin-orbit potential was 5 MeV. No Perey effect was used for the neutron wave functions because we study only the relative change in the cross sections caused by replacing the Johnson-Soper potentials with our new adiabatic distorting deuteron potentials.

The (d, p) cross sections calculated using deuteron distorting adiabatic potentials $U_{loc}(R)$ derived with GRZ and GR are shown in Figs. 2(a)–2(c) and Figs. 2(d)–2(f), respectively, where they compared with the calculations, performed with the Johnson-Soper potentials derived with GR and GRZ, and with experimental data. The (d, p) cross sections are plotted with the spectroscopic factor $S = 1$ for ^{16}O and ^{40}Ca targets and $S = 0.5$ for ^{36}Ar . One can see that the influences of nonlocality for GRZ and GR are completely different. While for the energy-dependent potential GR the cross sections increase and their shapes change insignificantly with respect to Johnson-

Soper calculations, for the energy-dependent potential GRZ the cross sections decrease and the changes in the shapes of the angular distributions are more noticeable. This happens because the Johnson-Soper potential corresponding to GRZ has a much smaller imaginary part than that of U_{loc} so that the reduction of absorption leads to large cross sections. For GR potential, the U_{loc} has less absorption than U_{JS} resulting in larger cross sections in comparison to the Johnson-Soper ones. While the difference between the Johnson-Soper cross sections calculated with GRZ and GR can almost reach a factor of three (for ^{36}Ar), the difference of the cross sections obtained in the non-local model with GRZ and GR is much smaller, $\sim 20\%$ – 30% in the main peak.

B. Estimation of deuteron distorting potential from phenomenological energy-dependent local nucleon optical potentials

Suppose we know some phenomenological energy-dependent optical potential $U_{\text{phen}}(E, r)$. We assume that its underlying energy-dependent nonlocal potential $\tilde{U}_{NA}(E, r, r')$ has the Perey-Buck form

$$\tilde{U}_{NA}(E, r, r') = H(|\mathbf{r} - \mathbf{r}'|)U_{NA}\left(\frac{\mathbf{r} + \mathbf{r}'}{2}, E\right), \quad (38)$$

where H is given by Eq. (31) with some nonlocality range β . We suppose also that $U_{NA}(E, r)$ is related to $U_{\text{phen}}(E, r)$ by the Perey-Buck transformations given by (37). Then, according to Sec. IV, we need to use form factors $U_{NA}(E, r)$ taken at the energy $E = \mathcal{E}_{\text{eff}} = E_d/2 + \langle T_r \rangle/2$, treating them as energy-independent. The local d - A distorting potential U_{loc} is obtained from the transcendental equation (35), which can be rewritten as

$$U_{loc}(E_d, r) \exp\left[-\frac{\mu_d \beta_d^2}{2\hbar^2} U_{loc}(E_d, r)\right] = \mathcal{V}(E_d, r), \quad (39)$$

where

$$\begin{aligned} \mathcal{V}(E_d, r) = & \exp\left[\frac{\mu\beta^2}{2\hbar^2} \mathcal{E}_{\text{eff}} - \frac{\mu_d \beta_d^2}{2\hbar^2} (E_d - \bar{V}_c)\right] \\ & \times M_0 \left(U_{\text{phen}}^n(\mathcal{E}_{\text{eff}}, r) \exp\left[\frac{\mu\beta^2}{2\hbar^2} U_{\text{phen}}^n(\mathcal{E}_{\text{eff}}, r)\right] \right. \\ & \left. + U_{\text{phen}}^p(\mathcal{E}_{\text{eff}}, r) \exp\left\{\frac{\mu\beta^2}{2\hbar^2} [U_{\text{phen}}^p(\mathcal{E}_{\text{eff}}, r) - \bar{V}_c]\right\} \right). \end{aligned} \quad (40)$$

Thus, knowing phenomenological proton and neutron optical potentials we can calculate $\mathcal{V}(E_d, r)$ and find U_{loc} as a solution of Eq. (39). In Figs. 1(g)–1(i) we have plotted these solutions for ^{16}O , ^{36}Ar , and $^{40}\text{Ca} + d$ using the CH89 systematics for U_{phen}^n and U_{phen}^p in comparison with the Johnson-Soper potential obtained also with CH89. We used $\beta = 0.85$ fm because the real part of the CH89 potential seems to be consistent with Perey-Buck nonlocal potential of this range. The chosen value $\beta = 0.85$ fm corresponds to $\beta_d = 0.4$ fm and $M_0 = 0.78$ [2]. We have also solved Eq. (39) by expanding the exponential in its left-hand side up to second-order terms but such solutions were not accurate enough.

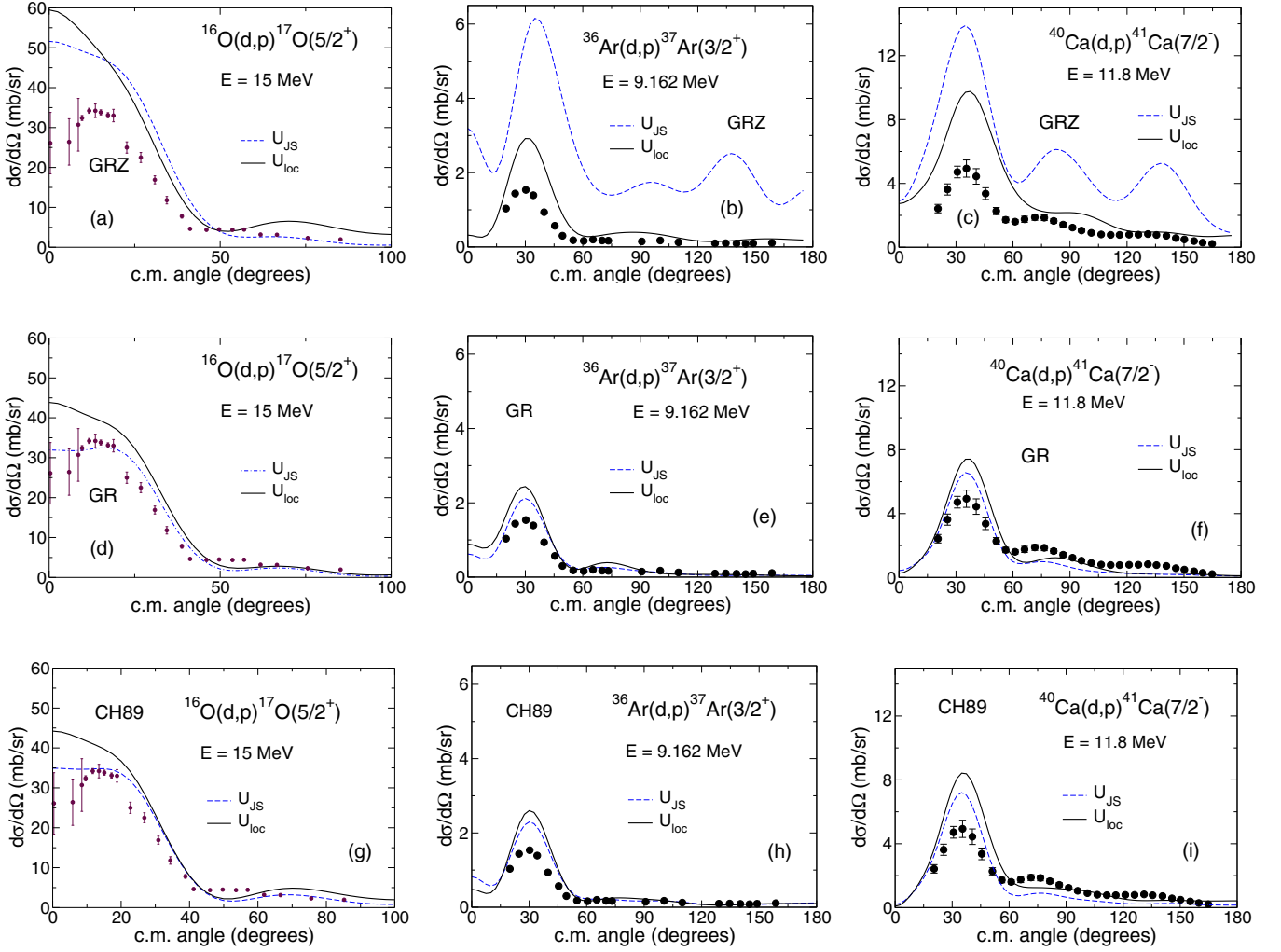


FIG. 2. (Color online) Angular distributions for the $^{16}\text{O}(d,p)^{17}\text{O}$, $^{36}\text{Ar}(d,p)^{37}\text{Ar}$, and $^{40}\text{Ca}(d,p)^{41}\text{Ca}$ reactions calculated with local adiabatic deuteron potentials U_{loc} obtained with energy-dependent GRZ (a),(b),(c), energy-independent GR (d),(e),(f), nonlocal optical nucleon potentials and energy-dependent local potential CH89 (g),(h),(i), in comparison to the corresponding Johnson-Soper calculations.

Figures 1(g)–1(i) show that the distorting d - A potential U_{loc} (CH89) generated from energy-dependent phenomenological local potential has a smaller real part than U_{JS} (CH89), similar to the case of GR and GRZ. However, the imaginary part of U_{JS} (CH89) has more absorption in the internal region than U_{loc} (CH89) does while being similar outside the nucleus. This is the consequence of the interplay between the surface and volume absorption in CH89 and the fact that the imaginary part has to be evaluated at higher energies where the volume absorption grows. Nevertheless, the differential cross sections, plotted in Figs. 2(g)–2(i) show a similar influence of non-locality to that observed with the energy-independent potential GR. The cross sections increase in the main peak by 16%, 10%, and 18% for the ^{16}O , ^{36}Ar , and ^{40}Ca targets, respectively. In comparison, the increase by 15% is obtained with GR. The absolute cross sections in the area of the main experimental peak obtained in nonlocal calculations with three different nucleon optical potentials differ by 20% in the case of ^{16}O and ^{36}Ar and by 30% in the case of ^{40}Ca . This is comparable to the typical dependence

on optical potentials in distorted-wave Born approximation calculations.

VII. DISCUSSION AND CONCLUSIONS

We have suggested an approximate practical way of dealing with explicitly energy-dependent nonlocal nucleon optical potentials when calculating the (d, p) reactions within the $A + n + p$ model space. We have shown that within an approximation consistent with the adiabatic model of (d, p) reactions the problem of using energy-dependent nonlocal nucleon potentials is reduced to the problem of calculating the (d, p) cross sections with energy-independent nonlocal potentials, the solution of which has been found in Refs. [1,2]. It is important to note that in obtaining our results we have assumed that any explicit energy dependence of optical potentials arises from many-body effects. In our treatment we start from an approximate three-body Hamiltonian with nonlocal nucleon optical potential operators at an energy fixed by the incident deuteron energy. Our prescription is

based on the identification of the particular feature of the (d, p) amplitude (small n - p separations) that is relevant to the calculation of a particular form of the many-body (d, p) transition operator. It is not a prescription that is intended to be useful for the whole of configuration space; for example, it cannot be used in calculations of d - A elastic scattering.

It is also important to note that in deriving the three-body effective Hamiltonian we have ignored the $U_{pA} \frac{Q_A}{e} U_{nA} + \dots$ terms in Eq. (7). These terms reflect the fact that the many-body problem associated with the $n + p + A$ system cannot be mapped exactly onto a three-body model with Hamiltonian $T_3 + V_{np} + U_p + U_n$. The neglected terms are nonlocal energy-dependent three-body forces that are a necessary consequence of the underlying many-body problem. They describe physical processes in which the target is excited by the incident neutron and deexcited by the proton and so on in higher orders. Terms in which successive target excitations are caused by the same nucleon are taken into account by the U_n and U_p operators. The quantitative significance of the $U_{pA} \frac{Q_A}{e} U_{nA} + \dots$ terms is unknown.

The phenomenological optical potentials are assumed to take important antisymmetrization effects into account. These effects are believed to make important contributions to nonlocality and be taken into account by the treatment of Timofeyuk and Johnson in Refs. [1,2]. However, for the $d + A$ system there are additional antisymmetrization effects that cannot be taken into account in this way. For a discussion of their quantitative significance, see Tostevin *et al.* [27].

The prescription of the present paper is simple enough to be widely used in analysis of (d, p) experiments. However, it requires knowledge of the energy-dependence of nonlocal optical potentials. At present, we are not aware of any potential of such a kind (see “Note added” just above the Acknowledgment section below), except for the GRZ potential. While this potential generates reasonable d - A effective local potentials, it clearly gives an unphysical local Johnson-Soper potential that strongly overestimates (d, p) cross sections at the low incident deuteron energies frequently used at several modern radioactive-beam facilities. This shows that more studies, from both phenomenological and microscopic models, are needed to pin down the energy dependence of nucleon optical potentials.

We have also suggested a simple way of correcting the d - A potentials for nonlocality when the energy dependence of nonlocal nucleon optical potentials is unknown but energy-dependent local systematics are available. We have shown that by making a specific assumption about the form of energy-dependent nonlocal potentials the local distorting deuteron potentials for adiabatic (d, p) calculations are obtained as solutions of a transcendental equation. While we believe that this assumption should be valid for the targets considered here, in general, it may not be correct. For example, according to GRZ, the asymmetry term of the optical potential has much wider nonlocality. Therefore, we can expect that for $Z \neq N$ targets Eqs. (35) and (40) could be modified.

Finally, we did not intend to provide new values for the spectroscopic factors and asymptotic normalization coefficients obtained from the analysis of (d, p) reactions using our new prescription. We were aiming to clarify how the explicit energy

dependence of nonlocal optical potentials affects known results of adiabatic (d, p) calculations. We do have preliminary ideas of what changes in spectroscopic factors and asymptotic normalization coefficients could be expected, but the details of any change will depend on how well we understand the energy dependence of non-local optical potentials. On the other hand, our study is not comprehensive and many issues remain outstanding. These include the influence of the deuteron d -state on the effective d - A potential, corrections to the approximate treatment of non-locality in [1,2], non-adiabatic corrections, additional antisymmetrization effects, multiple scattering, etc. These questions present a challenge to the development of the (d, p) reaction theory and answering them is important for the correct interpretation of measured (d, p) cross sections in terms of the nuclear structure quantities.

Note added. Recently, two preprints have appeared on the arXiv web site [28], where a phenomenological nonlocal energy-dependent potential for ^{40}Ca has been found for the first time.

ACKNOWLEDGMENT

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APPENDIX

1. The operator U

Here we derive the formula (3) from Sec. II for the operator U whose matrix elements can be used to give formally exact expressions for projections of the many-body scattering state $|\Psi_{k_d, A}^{(+)}\rangle$ corresponding to a deuteron of momentum k_d in its ground state $|\phi_0\rangle$ incident on a target in its ground state $|\phi_A\rangle$, which we assume has spin zero for simplicity here. The relevant projection operators P_A and Q_A project onto the ground and excited states of nucleus A , respectively, and satisfy

$$\begin{aligned} P_A + Q_A &= 1, & P_A|\phi_A\rangle &= |\phi_A\rangle, & Q_A|\phi_A\rangle &= 0, \\ P_A^2 &= P_A, & Q_A^2 &= Q_A, & P_A Q_A &= 0. \end{aligned} \quad (\text{A1})$$

The state $|\Psi_{k_d, A}^{(+)}\rangle$ is obtained by taking the limit $\epsilon \rightarrow +0$ of the state $|\Psi_{k_d, A}^{(\epsilon)}\rangle$ that satisfies the equation

$$(E + i\epsilon - H)|\Psi_{k_d, A}^{(\epsilon)}\rangle = i\epsilon|\mathbf{k}_d, \phi_0\phi_A\rangle, \quad (\text{A2})$$

where H is the Hamiltonian and E is the total energy of the $A + n + p$, which in the notations of Sec. II is

$$H = T_3 + V_{np} + U_n^0(n) + U_p^0(p) + H_A + \Delta v_{nA} + \Delta v_{pA}. \quad (\text{A3})$$

For $\epsilon \neq 0$ the function $|\Psi_{k_d, A}^{(\epsilon)}\rangle$ is uniquely defined by the inhomogeneous Eq. (A2). No further statements of boundary conditions are required. It follows from Eq. (A2) that $|\Psi_{k_d, A}^{(\epsilon)}\rangle - |\mathbf{k}_d, \phi_0\phi_A\rangle$ is square integrable. As discussed in Goldberger and Watson (Ref. [29], Chap. 5, “Formal Scattering Theory”), the function $|\Psi_{k_d, A}^{(\epsilon)}\rangle$ is used as a stepping stone to the calculation of transition matrices for all final channels using appropriate

expressions in which the limit $\epsilon \rightarrow +0$ can be taken without ambiguity. They show that their methods can be justified by wave-packet arguments.

Following Feshbach [11] we obtain equations coupling the P_A and Q_A components of $|\Psi_{\mathbf{k}_d, A}^{(\epsilon)}\rangle$, defined as

$$\Psi_P = P_A |\Psi_{\mathbf{k}_d, A}^{(\epsilon)}\rangle, \quad \Psi_Q = Q_A |\Psi_{\mathbf{k}_d, A}^{(\epsilon)}\rangle, \quad (\text{A4})$$

by operating with P_A and Q_A in turn on both sides of Eq. (A2) to obtain

$$\begin{aligned} (E + i\epsilon - P_A H P_A) \Psi_P - P_A H Q_A \Psi_Q &= i\epsilon |\mathbf{k}_d, \phi_0 \phi_A\rangle, \\ (E + i\epsilon - Q_A H Q_A) \Psi_Q - Q_A H P_A \Psi_P &= 0. \end{aligned} \quad (\text{A5})$$

From the second equation we deduce

$$\Psi_Q = \frac{1}{(E + i\epsilon - Q_A H Q_A)} Q_A H P_A \Psi_P, \quad (\text{A6})$$

and substituting this result into the first of Eq. (A5) we get an equation for Ψ_P alone,

$$\begin{aligned} (E + i\epsilon - P_A H P_A) \Psi_P \\ - P_A H Q_A \frac{1}{(E + i\epsilon - Q_A H Q_A)} Q_A H P_A \Psi_P \\ = i\epsilon |\mathbf{k}_d, \phi_0 \phi_A\rangle, \end{aligned} \quad (\text{A7})$$

where $|\mathbf{k}_d, \phi_0 \phi_A\rangle$ describes a plane-wave deuteron in its ground state incident on A in its ground state $|\phi_A\rangle$. For a spin-zero target A the projection Ψ_P will have the form $|\Psi_{\mathbf{k}_d}^{(\epsilon)}(n, p)\rangle |\phi_A\rangle$. We obtain an equation for the function of n and p coordinates, $|\Psi_{\mathbf{k}_d}^{(\epsilon)}(n, p)\rangle$, by taking the inner product of Eq. (A7) with $|\phi_A\rangle$. Using notations employed in Eqs. (3) and (4) we get

$$\begin{aligned} [E_3 + i\epsilon - T_3 - V_{np} - U_n^0(n) - U_p^0(p) \\ - \langle \phi_A | (\Delta v_{nA} + \Delta v_{pA}) | \phi_A \rangle \\ - \langle \phi_A | (\Delta v_{nA} + \Delta v_{pA}) Q_A \frac{1}{e - Q_A (\Delta v_{nA} + \Delta v_{pA}) Q_A} \\ \times Q_A (\Delta v_{nA} + \Delta v_{pA}) | \phi_A \rangle] |\Psi_{\mathbf{k}_d}^{(\epsilon)}(n, p)\rangle = i\epsilon |\mathbf{k}_d, \phi_0\rangle, \end{aligned} \quad (\text{A8})$$

where $E_3 = E - E_A$ and we have used the fact that $E + i\epsilon - T_3 - V_{np} - U_n^0(n) - U_p^0(p)$ commutes with P_A and Q_A so that

$$\begin{aligned} P_A [E + i\epsilon - T_3 - V_{np} - U_n^0(n) - U_p^0(p)] P_A \\ = [E + i\epsilon - T_3 - V_{np} - U_n^0(n) - U_p^0(p)] P_A, \\ Q_A [E + i\epsilon - T_3 - V_{np} - U_n^0(n) - U_p^0(p)] Q_A \\ = [E + i\epsilon - T_3 - V_{np} - U_n^0(n) - U_p^0(p)] Q_A, \\ P_A [E + i\epsilon - T_3 - V_{np} - U_n^0(n) - U_p^0(p)] Q_A = 0. \end{aligned} \quad (\text{A9})$$

We have also used the fact that because the operator $1/(\text{denominator})$ in Eq. (A8) appears with Q_A on either side then (i) only excited-state eigenvalues of H_A ever appear and (ii) H_A is diagonal in the basis of states of A . Therefore, we can replace $Q_A (H_A - E_A) Q_A$ with $H_A - E_A$.

It is convenient here to define an operator U in the complete $n + p + A$ many-body space by the equation

$$\begin{aligned} U &= (\Delta v_{nA} + \Delta v_{pA}) + (\Delta v_{nA} + \Delta v_{pA}) Q_A \\ &\times \frac{1}{e - Q_A (\Delta v_{nA} + \Delta v_{pA}) Q_A} Q_A (\Delta v_{nA} + \Delta v_{pA}). \end{aligned} \quad (\text{A10})$$

In terms of U , Eq. (A8) can be written as

$$\begin{aligned} (E_3 + i\epsilon - T_3 - V_{np} - U_n^0(n) - U_p^0(p) \\ - \langle \phi_A | U | \phi_A \rangle) |\Psi_{\mathbf{k}_d}^{(\epsilon)}(n, p)\rangle = i\epsilon |\mathbf{k}_d, \phi_0\rangle. \end{aligned} \quad (\text{A11})$$

We see that the wave function $|\mathbf{k}_d, \phi_0\rangle$ of the three-body model is determined by the target ground-state matrix element of U , $\langle \phi_A | U | \phi_A \rangle$. By using the identity

$$\begin{aligned} \frac{1}{e - Q_A (\Delta v_{nA} + \Delta v_{pA}) Q_A} \\ = \frac{1}{e} + \frac{1}{e} Q_A (\Delta v_{nA} + \Delta v_{pA}) Q_A \\ \times \frac{1}{e - Q_A (\Delta v_{nA} + \Delta v_{pA}) Q_A}, \end{aligned} \quad (\text{A12})$$

we can show that U satisfies Eq. (3) of Sec. II.

2. Derivation of a multiple scattering expansion for U

The operator U is the unique solution of the equation

$$U = \Delta v_{nA} + \Delta v_{pA} + (\Delta v_{nA} + \Delta v_{pA}) \frac{Q_A}{e} U. \quad (\text{A13})$$

We derive the multiple scattering expansion of U using the techniques of Goldberger and Watson [29]. We introduce two new operators W_n and W_p defined in terms of U by

$$W_N = \Delta v_{NA} + \Delta v_{NA} \frac{Q_A}{e} U, \quad N = n, p. \quad (\text{A14})$$

By adding the formulas for $N = n$ and $N = p$ it can be seen that $W_n + W_p$ satisfies Eq. (A13) and, therefore,

$$U = W_n + W_p. \quad (\text{A15})$$

We next show how these operators are related to the operators U_{NA} used in the text and defined by

$$U_{NA} = \Delta v_{NA} + \Delta v_{NA} \frac{Q_A}{e} U_{NA}. \quad (\text{A16})$$

A useful way of expressing the solution of this equation is to take the second term on the right over to the left and deduce

$$U_{NA} = \frac{1}{(1 - \Delta v_{NA} \frac{Q_A}{e})} \Delta v_{NA}. \quad (\text{A17})$$

Performing the analogous step in Eq. (A14) with $N = n, p$ in turn and with U replaced with $W_n + W_p$ we obtain

$$\begin{aligned} W_n &= U_{nA} + U_{nA} \frac{Q_A}{e} W_p, \\ W_p &= U_{pA} + U_{pA} \frac{Q_A}{e} W_n. \end{aligned} \quad (\text{A18})$$

Substituting the W_p formula into the right-hand side of the W_n equation and the W_n formula into the right-hand side of the W_p we obtain the uncoupled equations

$$\begin{aligned} W_n &= U_{nA} + U_{nA} \frac{Q_A}{e} U_{pA} + U_{nA} \frac{Q_A}{e} U_{pA} \frac{Q_A}{e} W_n, \\ W_p &= U_{pA} + U_{pA} \frac{Q_A}{e} U_{nA} + U_{pA} \frac{Q_A}{e} U_{nA} \frac{Q_A}{e} W_p. \end{aligned} \quad (\text{A19})$$

In the standard way we iterate both of these equations and then construct $U = W_n + W_p$. The first few terms give the multiple scattering series discussed in Sec. III.

3. Derivation of the inversion formula, Eq. (21)

To understand the origin of the approximate inversion of the operator $(e - Q_A \Delta v_{NA} Q_A)$ given in Eq. (21) it is first necessary to look at the ADWA from a different point of view from that customarily used.

In Sec. II, Eq. (1), we introduced $\Psi_{k_d}^{(+)}(n, p)$, the projection of the full $d + A$ many-body wave function onto the ground state of A . It is the limit $\epsilon \rightarrow +0$ of the function $\Psi_{k_d}^{(\epsilon)}(n, p)$ that satisfies

$$(E_3 + \iota\epsilon - H_{\text{eff}}) \Psi_{k_d}^{(\epsilon)}(n, p) = \iota\epsilon |\phi_0 \mathbf{k}_d\rangle, \quad (\text{A20})$$

where H_{eff} is the effective three-body Hamiltonian,

$$H_{\text{eff}} = T_R + H_{np} + \mathcal{U}(n, p), \quad (\text{A21})$$

with $H_{np} = T_{np} + V_{np}$ and some arbitrary interaction $\mathcal{U}(n, p)$ which can be an operator in the \mathbf{r} and \mathbf{R} degrees of freedom. A formal solution to Eq. (A20) is

$$\Psi_{k_d}^{(\epsilon)}(n, p) = \frac{\iota\epsilon}{(E_3 + \iota\epsilon - H_{\text{eff}})} |\phi_0 \mathbf{k}_d\rangle. \quad (\text{A22})$$

From this point of view we see that the solution of the three-body problem for $\Psi_{k_d}^{(\epsilon)}(n, p)$ is equivalent to inverting the operator $(E_3 + \iota\epsilon - H_{\text{eff}})$. We now show that the ADWA can be regarded as an approximate inversion of this operator within the space of Weinberg states.

The Weinberg basis $\{\phi_i^W\}$ associated with H_{np} was introduced by Johnson and Tandy [6] as being well adapted to the evaluation of the components of $\Psi_{k_d}^{(\epsilon)}(n, p)$ that dominate the $A(d, p)B$ matrix element, Eq. (1) [3]. The Weinberg states ϕ_i^W satisfy the orthonormality relation

$$\langle \phi_i^W | V_{np} | \phi_k^W \rangle = -\delta_{ik}, \quad (\text{A23})$$

and the components $|\chi_i^{(\epsilon)}\rangle$ of $|\Psi_{k_d}^{(\epsilon)}\rangle$ in this basis are

$$|\chi_i^{(\epsilon)}\rangle = -\langle \phi_i^W | V_{np} | \Psi_{k_d}^{(\epsilon)} \rangle. \quad (\text{A24})$$

The ADWA is obtained by multiplying Eq. (A20) by $-\langle \phi_1^W | V_{np}$ on the left, integrating over \mathbf{r} , and neglecting all couplings between $|\chi_1^{(\epsilon)}\rangle$ and all other $i \neq 1$ Weinberg components. This results in

$$-\langle \phi_1^W | V_{np} (E_3 + \iota\epsilon - H_{\text{eff}}) | \phi_1^W \rangle |\chi_1^{(\epsilon)}\rangle = \iota\epsilon C_1^{-1} |\mathbf{k}_d\rangle, \quad (\text{A25})$$

where C_1 is the constant relating the first Weinberg state ϕ_1^W to the deuteron wave function ϕ_0 by $\phi_1^W = C_1 \phi_0$ and satis-

fies $|C_1|^2 \langle \phi_0 | V_{np} | \phi_0 \rangle = -1$ [6]. However, after multiplying Eq. (A22) by $-\langle \phi_1^W | V_{np}$ on the left and integrating over \mathbf{r} we get

$$|\chi_1^{(\epsilon)}\rangle = -\iota\epsilon C_1^{-1} \langle \phi_1^W | V_{np} \frac{1}{E_3 + \iota\epsilon - H_{\text{eff}}} | \phi_1^W \rangle |\mathbf{k}_d\rangle. \quad (\text{A26})$$

Combining Eqs. (A26) into (A25) we get

$$\begin{aligned} &\iota\epsilon \langle \phi_1^W | V_{np} (E_3 + \iota\epsilon - H_{\text{eff}}) | \phi_1^W \rangle \\ &\times \langle \phi_1^W | V_{np} \frac{1}{E_3 + \iota\epsilon - H_{\text{eff}}} | \phi_1^W \rangle |\mathbf{k}_d\rangle = \iota\epsilon |\mathbf{k}_d\rangle, \end{aligned} \quad (\text{A27})$$

which is satisfied if

$$\begin{aligned} &\langle \phi_1^W | V_{np} \frac{1}{E_3 + \iota\epsilon - H_{\text{eff}}} | \phi_1^W \rangle \\ &= \frac{1}{\langle \phi_1^W | V_{np} (E_3 + \iota\epsilon - H_{\text{eff}}) | \phi_1^W \rangle}. \end{aligned} \quad (\text{A28})$$

Taking Eq. (19) and $\phi_1^W = C_1 \phi_0$ into account in Eq. (A28), we see that the approximation (21) follows from the same approximate inversion idea as used in the ADWA.

We now show how this approach can be made the basis for making systematic improvements to the ADWA. In the Weinberg basis an arbitrary operator \hat{O} is represented by the matrix

$$\hat{O}_{ij}^W = -\langle \phi_i | V_{np} \hat{O} | \phi_j \rangle. \quad (\text{A29})$$

Equation (A22) tells us that these components are given by

$$|\chi_i^{(\epsilon)}\rangle = -\sum_k \iota\epsilon G_{ik}^W \langle \phi_k^W | V_{np} | \phi_0 \mathbf{k}_d \rangle, \quad (\text{A30})$$

where

$$G_{ik}^W = -\langle \phi_i^W | V_{np} \frac{1}{[E_3 + \iota\epsilon - H_{\text{eff}}(n, p)]} | \phi_k^W \rangle. \quad (\text{A31})$$

Note that each matrix element G_{ik}^W is an operator in the space of \mathbf{R} . Using the relation $|\phi_1^W\rangle = C_1 |\phi_0\rangle$, we have

$$-\langle \phi_k^W | V_{np} | \phi_0 \mathbf{k}_d \rangle = \delta_{k1} \frac{1}{C_1} |\mathbf{k}_d\rangle, \quad (\text{A32})$$

and Eq. (A30) reduces to

$$|\chi_i^{(\epsilon)}\rangle = \frac{\iota\epsilon}{C_1} G_{i1}^W |\mathbf{k}_d\rangle. \quad (\text{A33})$$

It was shown in Ref. [3] that for (d, p) reactions in a range of incident energies of current interest the (d, p) transition matrix, Eq. (1), is dominated by the first Weinberg component $|\chi_1^{(\epsilon)}\rangle$. According to Eq. (A33) an exact expression for this is

$$|\chi_1^{(\epsilon)}\rangle = \frac{\iota\epsilon}{C_1} G_{11}^W |\mathbf{k}_d\rangle, \quad (\text{A34})$$

which only involves the single matrix element G_{11}^W .

The evaluation of G_{ik}^W requires the inversion of the matrix

$$D_{lm}^W = -\langle \phi_l^W | V_{np} (E_3 + \iota\epsilon - H_{\text{eff}}) | \phi_m^W \rangle, \quad (\text{A35})$$

so that

$$D^W G^W = 1. \quad (\text{A36})$$

By writing $D_{ij}^W = D_{ii}^W \delta_{ij} + D_{ij}^W (1 - \delta_{ij})$ it is straightforward to show that an exact alternative to the condition (A36) is

$$G^W = \frac{1}{D^{W\text{diag}}} + \frac{1}{D^{W\text{diag}}} D^{W\text{nondiag}} G^W, \quad (\text{A37})$$

where

$$\begin{aligned} D_{ij}^{W\text{diag}} &= D_{ii}^W \delta_{ij}, \\ D_{ij}^{W\text{nondiag}} &= D_{ij}^W (1 - \delta_{ij}). \end{aligned} \quad (\text{A38})$$

In particular,

$$G_{11}^W = \frac{1}{D_{11}^W} + \frac{1}{D_{11}^W} \sum_{j \neq 1} D_{1j}^W G_{j1}^W. \quad (\text{A39})$$

The essence of the ADWA is that it ignores all couplings between the first Weinberg component and all others [6]. In this limit we deduce from Eq. (A39) that

$$G_{11}^W = \frac{1}{D_{11}^W} = -\frac{1}{\langle \phi_1^W | V_{np}(E_3 + \iota\epsilon - H_{\text{eff}}) | \phi_1^W \rangle}, \quad (\text{A40})$$

in agreement with Eq. (A28). The error is of second order in the off-diagonal elements $\langle \phi_1^W | V_{np}[E_3 + \iota\epsilon - T_R - H_{\text{eff}}(n, p)] | \phi_i^W \rangle$ with $i \neq 1$.

This approach can be developed by iteration of Eq. (A37) to give corrections to the ADWA.

4. Averaging $T_p + V_{np}$ and $T_n + V_{np}$

In Sec. IV we have shown that the expression for the ADWA distorting potential requires the evaluation of matrix element (21). The denominator in the neutron contribution contains the operator T_n , as appears in the expression for neutron

optical potential, and the operator $T_p + V_{np}$ for which we want to deduce an average value consistent with ADWA. Here we evaluate the average $\langle \phi_1 | T_p + V_{np} | \phi_0 \rangle$ acting on $\chi_{k_d}^{(+)}(\mathbf{R})$. Because

$$T_p = \frac{1}{2}(T_R + T_r) - \frac{\hbar^2}{2M_N} \nabla_r \cdot \nabla_R, \quad (\text{A41})$$

we obtain

$$\begin{aligned} &\langle \phi_1 | T_p | \phi_0 \rangle \chi_{k_d}^{(+)}(\mathbf{R}) \\ &= \frac{1}{2}(T_R + \langle T_r \rangle) \chi_{k_d}^{(+)}(\mathbf{R}) \\ &\quad - \frac{\hbar^2}{2M_N} \left[\int d\mathbf{r} \phi_1^*(\mathbf{r}) \nabla_r \phi_0(\mathbf{r}) \right] \cdot \nabla_R \chi_{k_d}^{(+)}(\mathbf{R}) \\ &= \frac{1}{2}(T_R + \langle T_r \rangle) \chi_{k_d}^{(+)}(\mathbf{R}), \end{aligned} \quad (\text{A42})$$

where the last line contains the quantity

$$\langle T_r \rangle = \langle \phi_1 | T_r | \phi_0 \rangle \quad (\text{A43})$$

introduced by Timofeyuk and Johnson in Ref. [1]. In obtaining the last line in Eq. (A42) we have used that fact that ϕ_0 and ϕ_1 have the same parity and that $\langle \phi_1 | \phi_0 \rangle = 1$. Using the exact result,

$$\langle \phi_1 | V_{np} | \phi_0 \rangle = -\epsilon_0 - \langle T_r \rangle, \quad (\text{A44})$$

we obtain

$$\begin{aligned} &\langle \phi_1 | T_p + V_{np} | \phi_0 \rangle \chi_{k_d}^{(+)}(\mathbf{R}) \\ &= \left[\frac{1}{2}(T_R - \langle T_r \rangle) - \epsilon_0 \right] \chi_{k_d}^{(+)}(\mathbf{R}). \end{aligned} \quad (\text{A45})$$

The same results is obtained for the average $\langle \phi_1 | T_n + V_{np} | \phi_0 \rangle$ that is relevant to the calculation of the proton contribution to the distorting potential in the deuteron channel.

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