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Tentative probe into the nuclear charge radii of superheavy odd-mass and odd-odd nuclei

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The root-mean-square (rms) nuclear charge radii of superheavy odd-A and odd-odd nuclei are tentatively pursued by the deduction of experimental α decay data. The framework of calculating α decay half-lives is constructed via the combination of the improved two-potential approach with the density-dependent cluster model. In this procedure, the charge distribution of daughter nuclei is determined to exactly reproduce the measured α decay half-lives. Next, the rms charge radius of daughter nuclei is obtained by using the corresponding charge distribution. For comparison, the previously proposed formula of our group is employed to estimate the rms charge radii as well. Besides the reasonable agreement between the extracted nuclear charge radii and the available experimental values, the nuclear radii of heaviest odd-A and odd-odd nuclei are extracted from the α decay energies and half-lives. This can be considered as an effective attempt in terms of the nuclear size in the superheavy mass region.

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I. INTRODUCTION

The detection of the number of chemical elements is always pursued as a basic topic in natural science. The heaviest nuclei have correspondingly received special attention for a long time, in both chemistry and physics. Chemically, it is of particular interest whether the superheavy elements behave in a similar way as their lighter homologs [1]. On the other hand, in view of the nuclear liquid-drop model, nuclei beyond $Z \approx 106$ were supposed to be unstable with the increase of atomic number according to the pioneer prediction by C. F. von Weizsäcker in 1935 [2]. Later on, superheavy elements with long enough lifetime and even the island of stability were expected to be observed due to the strong influence of deformed and spherical shell closures [3–5]. Ever since then, besides the attempts to probe heavier elements in natural samples [6–10], a world-wide effort was inspired to synthesize new heaviest elements in laboratories. Significant progress on this topic has been achieved during the last 30 years with the help of "cold" and "hot" fusion reactions [11–21]. Within the production of the new element 117 under the ^{48}Ca induced hot fusion [14], the atomic number of nuclei reached $Z = 118$. Furthermore, extensive experiments have been proposed to detect new superheavy isotopes [16–21] and aim at the heavier $Z = 119$ and 120 systems. Impressively, α decay chains actually play a key role in the identification of new heaviest elements and isotopes. Following the pioneering quantum explanation of α decay as a tunneling effect [22], various theoretical models have been established to pursue a reasonable curve of α decay [23–45]. Among these studies, our group proposed a new formula to evaluate the half-lives of cluster radioactivity [31], and a unified law was sequently presented for α decay and cluster radioactivity [32]. Very recently, a new Geiger-Nuttall relation was proposed for α decay including the effects of the quantum numbers of α-core relative motion [33]. There are also other studies on α decay and changing the concept of heavier particle radioactivity, based on the analytical superasymmetric fission model [29,40–43]. Along with the discovery of radioactivity, α decay has always been an important and attractive subject due to the wide and deep impact of α decay on nuclear physics.

Despite the push all over the world to create nuclei with bigger and bigger Z number, information on the structural character of superheavy nuclei (SHN) is still limited from the experimental side. As one fundamental feature of a nucleus, the size of SHN (i.e., the radius of the nucleus) could not be measured up to now. Although there are available methods for measuring the nuclear charge radius such as particles (like e, p, $\pi^{\pm}$, and μ), scattering on target nuclei, measurements of transitions energies in muonic atoms, $K\alpha$ x rays, and optical isotope shifts [46,47], it appears that the probe into SHN radii is still extremely difficult, resulting from their short lifetimes and tiny cross sections. Recently, the rms charge radii of nuclei were self-consistently obtained within nuclear mass models [48,49]. In fact, α decay was taken as a tool to recognize the nuclear radii at an early stage in the study of nuclear structure [50,51], regardless of the rough estimations. In the wake of the development of experimental facilities and theoretical frameworks, α decay data have been accumulated with improved accuracy and α decay calculations have made considerable progress. It is believed that α decay may, at present, be a possibly effective tool in the research of the size of nuclei in the heavier mass region. In a tentative attempt, our group recently gave the first result on nuclear charge radii of superheavy even-even nuclei based on the experimental α decay data [52]. Besides this study, we tried to explore the rms radii of lighter nuclei and neutron-deficient nuclei around the proton drip line via the cluster and proton

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emissions [53]. Based on the double-folding procedure, the density distribution of target nuclei is naturally involved in the connection between the rms nuclear charge radius and the α decay half-life and energy. The aim of this article is not only to extend the range of studied nuclei, but also to check the validity of the model again and pursue valuable information on nuclear sizes of heaviest odd-A and odd-odd nuclei. It is to be noted that the above attention is paid to the properties of ground states of focused nuclei.

This article is organized as follows. Section II presents a detailed description of the correlation of nuclear radii with measured α decay data. The extracted rms charge radii are displayed in Sec. III, including the comparison of these values with the available data and other empirical evaluations. A brief summary is given in Sec. IV.

II. THEORETICAL FRAMEWORK

On the basis of the Gamow picture, the parent nucleus involved in the α decay is usually considered as a two-body interacting system consisting of a daughter nucleus and an α particle. There is no doubt that the α-daughter potential is a crucial input for the depiction of α decay. It is obvious that the density distribution of the daughter nucleus, as a key point in the α-core interacting process, is naturally linked with the α decay data. Subsequently, the density distribution (i.e., the rms radius of daughter nuclei) can be expected to be deduced from the corresponding experimental α decay data. We initially introduce the framework of calculating α decay half-lives. Provided that an α particle interacts with an axially symmetric deformed core nucleus, the total potential of the α-core system comprises of the nuclear and Coulomb potentials plus the centrifugal part,

$$V(r,\theta) = \lambda V_N(r,\theta) + V_C(r,\theta) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2}, \tag{1}$$

where θ is the orientation angle of the emitted α cluster with respect to the symmetric axis of the daughter nucleus. The λ factor is the depth of nuclear potential, μ is the reduced mass of the α-core system measured in units of the nucleon mass $\mu = A_\alpha A_d/(A_\alpha + A_d)$, and the angular momentum ℓ is carried by the emitted particle. Within the density-dependent cluster model (DDCM), the nuclear and Coulomb potentials are numerically constructed via the double-folding integral of the effective nucleon-nucleon interactions with the density distributions of α particles and daughter nuclei,

$$V_{N or C}(\mathbf{r},\theta) = \iint d\mathbf{r_1} d\mathbf{r_2} \rho_1(\mathbf{r_1}) \upsilon(\mathbf{s} = |\mathbf{r_2} + \mathbf{r} - \mathbf{r_1}|) \rho_2(\mathbf{r_2}), \tag{2}$$

where $\upsilon(\mathbf{s})$ represents the realistic M3Y-Reid-type nucleon-nucleon interaction and the standard proton-proton Coulomb interaction for the nuclear potential and the Coulomb potential, respectively. Besides the widely used Gaussian density distribution ρ_1 of the spherical α particle, the mass and charge density distributions of the core daughter nucleus are assumed to behave in the same way,

$$\rho_2(r_2,\theta) = \frac{\rho_0}{1 + \exp\left[\frac{r_2 - r_0 A_d^{1/3}[1+\beta_2 Y_{20}(\theta)+\beta_4 Y_{40}(\theta)]}{a}\right]}. \tag{3}$$

Here the ρ_0 value is determined by integrating the density distribution equivalent to the mass or atomic number of the target daughter nucleus, and β_2 and β_4 are, respectively, the quadrupole and hexadecapole deformation parameters of the residual daughter nucleus [5]. As compared with the available experimental information on nuclear charge radii [54,55], the nuclear neutron distributions appear to be quite ambiguous. Considering this, we assume that the density distribution of neutrons has the same form as that of protons in heavy nuclei [52,53].

Given one certain orientation angle θ, the total potential $V(r,\theta)$ is reduced into a one-dimensional case, namely $V(r)$. Within the two-potential approach, $V(r)$ is then divided into two parts: the "inner" term and the "outer" term by a separation radius R_0 reasonably taken inside the potential barrier. The Schrödinger equation is numerically solved in the inner potential $U(r)$ for the bound state wave function $\phi_{n\ell j}(r)$, which vanishes sharply exponentially from the separation radius R_0 [44]. In the above procedure, the depth λ is adjusted to the experimental decay energy Q for each decay. The quantum number n of the solution (the number of nodes of the wave function $\phi_{n\ell j}$) is determined by the Wildermuth condition taking into account the main effect of the Pauli principle [56]. The decay width is then evaluated as

$$\Gamma(\theta) = \frac{\hbar^2 k}{\mu} \left[\frac{\phi_{n\ell j}(\bar{r})}{G_\ell(k\bar{r})}\right]^2, \tag{4}$$

where the wave number k is $\sqrt{2\mu Q}/\hbar$, and G_ℓ is the irregular Coulomb wave function. The value of $\bar{r}$ is chosen in such a way that the potential V can be well approximated by the repulsive part (the attractive part disregards) for $r \geqslant \bar{r}$ (see Refs. [44,53] and references therein). By a careful averaging of $\Gamma(\theta)$ in all directions [26,27,44], the final decay width is ultimately given by

$$\Gamma = \int_0^{\pi/2} \Gamma(\theta) \sin(\theta) d\theta. \tag{5}$$

The α decay half-life is related as

$$T_{1/2} = \frac{\hbar \ln 2}{P_\alpha \Gamma}, \tag{6}$$

where P_α is an indispensable quantity inscribing the preformation probability of an α particle in the parent nucleus. The above sequential description can result in the final production of α decay half-lives. Previously, the parameters r_0 and a of the density distribution are suggested at $r_0 = 1.07$ fm and $a = 0.54$ fm in the opinion of a nuclear textbook [50], which could lead to the final results of α decay half-lives. In the present study, the mentioned parameters are in turn fixed to exactly reproduce the experimental α decay half-lives. According to the detailed analysis [52,53], the parameter a is still chosen as the suggested value, and the more sensitive quantity r_0 is considered as the bridge between rms nuclear radii and α decay half-lives. The rms charge radius of ρ_2 is then conveniently written as

$$R \equiv \sqrt{\langle r^2\rangle} = \left[\frac{\iint \rho_2(r,\theta) r^4 \sin\theta \, dr \, d\theta}{\iint \rho_2(r,\theta) r^2 \sin\theta \, dr \, d\theta}\right]^{1/2}. \tag{7}$$

Before the extraction of rms radii, the α-preformation factor should be paid special attention. One can take the constant P_α for one kind of nuclei [23,27,32], to keep the number of free parameters to a minimum, which is consistent with experimental facts [57]. Based on the measured data of rms charge radii [related to the key quantity r_0 by Eq. (7)] [46,47], it is found that the experimental α decay half-lives can be well reproduced with the following case: $P_\alpha^{\text{odd}-A} = 0.0752$ for odd-A nuclei and $P_\alpha^{\text{odd}-\text{odd}} = 0.0643$ for odd-odd nuclei. It is obvious that the P_α value of odd-odd nuclei is smaller than that of odd-A nuclei, and the two values are both smaller in comparison with that of even-even nuclei obtained in our previous work [52]. This is expected and reasonable due to the structural effect of unpaired protons or neutrons and proton-neutron coupling, which is identical with previous studies [23,27,32] as well.

III. NUMERICAL RESULTS AND DISCUSSIONS

Besides the above theoretical procedure, our group has proposed a simple relation of rms charge radii with α decay data [52],

$$\sqrt{R} = (X_1 - X_2 \log_{10} T_{1/2})/\xi_1\xi_2 + X_3\xi_1 Q_\alpha^{-1/2}. \tag{8}$$

Here $\xi_1 = \sqrt{Z_c Z_d e^2}$, $\xi_2 = \sqrt{2\mu}/\hbar$, and X_1, X_2, X_3 are the parameters to be fitted by matching the experimental data. We have initially concentrated on the available experimental cases, in which the α transitions involve the ground states of daughter nuclei and the charge radii of these studied nuclei were meanwhile measured. Significantly, whatever the isomeric or ground state for the α emitters, the spin-parity of parent nuclei was chosen to be the same as that of the ground state for daughter nuclei leading to the favored α decays. In contrast, the situation of the hindered decays is more complicated because of structural effects such as the irregular behavior of the α preformation factor [58]. As a tentative probe into the rms nuclear charge radii from the α decay data, we would pay main attention to the favored decay cases of odd-A and odd-odd nuclei. In our extractions, the used data of α decay energies and half-lives are taken from the AME2012 table [59] and the NNDC database [60], and some data of superheavy nuclei are taken from Refs. [15–17]. The data on rms nuclear charge radii are taken from Ref. [47], where some data were obtained by systematic estimation. As the first step, 19 odd-A α emitters and 2 odd-odd ones are focused on in the following.

Table I represents the comparison of experimental rms charge radii with the extracted values from the available measured α decay data. The first column denotes these target daughter nuclei, and the last two columns, respectively, list the calculated results via the present model and the proposed expression (8). The parameters in Eq. (8) are fixed at $X_1 = -14.93271$, $X_2 = 0.57505$, and $X_3 = 0.7828$ for odd-A α emitters, through a least-squares fit to the available charge radii. Unfortunately, the data of odd-odd nuclei in our studied condition seem to be not enough for the matching process, with the result that the estimated radii [in Eq. (8)] of ^{148,150}Tb are not given here. For the extraction within the DDCM, the nuclear size of the two odd-odd nuclei is also tentatively pursued, added at the end of Table I. To evaluate the overall agreement between experiment and deduction, the standard deviation of the present calculations is obtained as $\sigma = [\sum_{i=1}^{21}(R^i_{\text{expt}} - R^i_{\text{calc}})^2/21]^{1/2} = 0.12$ fm. The deviation for 19 odd-A nuclei by the formula (8) is 0.06 fm. For further insight, we plot the comparison of our evaluations with available data versus the mass number A of target nuclei in Fig. 1. As one can see, the present extracted radii generally match the experimental ones and are consistent with the estimated values given by Eq. (8). The constant P_α factor cannot reflect the reduction in the α formation probability due to the effect of shell closure, and thus the P_α value used in the calculation is larger than it should be for closed-shell nuclei. This can lead to smaller r_0 values and smaller rms nuclear radii, which may be the reason that the rms radii seem to be underestimated for nuclei with $A = 190$–210 involved the $Z = 82$ shell. It should be better that one connects the quantity P_α with structural facts such as the valence nucleon products [39], nucleonic promiscuity factors, and so on. Subsequently, the large deviations between theory and experiment may be reduced. This deserves to be further investigated and it is our pursuit in the following work. At present, experimental facilities are not yet effective for the detection of nuclear sizes of heavier nuclei, especially for superheavy nuclei. Strikingly, α decay may be supposed to be an alternative method for such an objective in addition to the identification of new synthesized superheavy elements [52].

TABLE I. Comparison of extracted rms nuclear charge radii, including the evaluation given by the proposed expression (8), with available experimental data for odd-A nuclei plus a few odd-odd ones.

Element	A	R_{expt} (fm)	R_{calc} (fm)	R_{form} (fm)
Hg	187	5.40	5.35	5.44
Tl	191	5.42	5.39	5.38
Pb	189	5.42	5.27	5.31
Pb	191	5.42	5.32	5.37
Pb	193	5.43	5.33	5.40
Pb	195	5.44	5.39	5.47
Pb	197	5.44	5.37	5.48
Pb	201	5.46	5.38	5.51
Pb	203	5.47	5.29	5.44
Pb	209	5.51	5.48	5.47
Pb	211	5.53	5.65	5.66
Bi	203	5.49	5.38	5.51
Bi	205	5.50	5.26	5.42
Bi	209	5.52	5.55	5.48
Fr	213	5.60	5.73	5.64
Tb	147	4.92	5.07	4.95
Tb	149	4.94	4.95	4.90
Ho	151	5.04	5.25	5.16
Tm	153	5.06	5.10	5.02
Tb	148	4.93	5.00	
Tb	150	4.95	4.84	

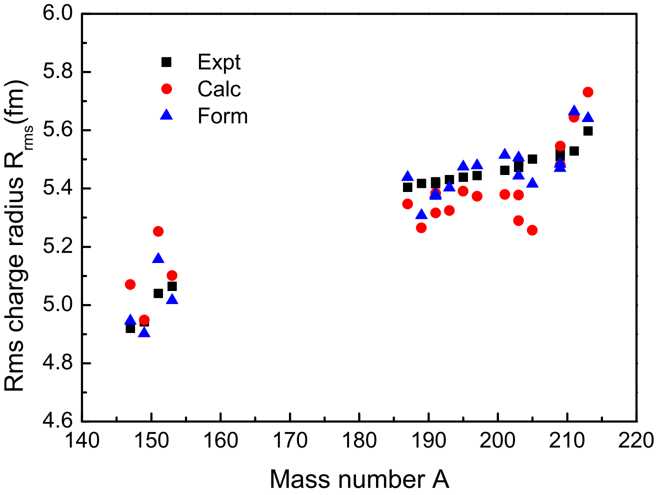

FIG. 1. (Color online) Comparison of extracted rms radii with the experimental data for odd-A nuclei with $Z = 65$–87. Black squares denote the experimental rms charge radii, red circles denote the calculated values within the DDCM, and blue triangles denote the estimations by Eq. (8).

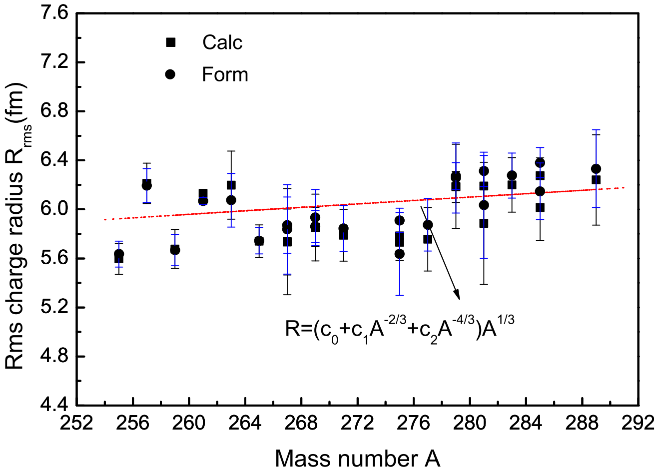

FIG. 2. (Color online) Deduced rms charge radii versus the mass number for heaviest odd-A isotopes with $Z = 102$–115, where the experimental rms radii are still not available. Along with the error bars coming from the large uncertainties of the experimental α decay data, squares (black bars) indicate the extracted rms radii based on the DDCM and circles (blue bars) indicate the evaluated values via the formula (8). The red dotted line denotes the simple relation between rms nuclear radii and mass numbers.

TABLE II. List of deduced rms nuclear charge radii for superheavy odd-A nuclei with $Z = 102$–115. Note that the error bars come from the large uncertainties of experimental α decay half-lives and energies.

Nucleus	R_{calc} (fm)	ΔR_{calc} (fm)	R_{form} (fm)	ΔR_{form} (fm)
^{257}No	6.21	0.16	6.19	0.14
^{255}Lr	5.60	0.13	5.64	0.11
^{267}Rf	5.73	0.27	5.87	0.23
^{267}Db	5.74	0.43	5.84	0.36
^{259}Sg	5.68	0.16	5.67	0.13
^{261}Sg	6.13	0.03	6.07	0.03
^{269}Sg	5.85	0.27	5.93	0.23
^{271}Bh	5.79	0.21	5.85	0.19
^{263}Hs	6.20	0.28	6.07	0.22
^{265}Hs	5.74	0.13	5.74	0.11
^{269}Hs	5.86	0.16	5.86	0.13
^{275}Hs	5.78	0.12	5.91	0.10
^{275}Mt	5.73	0.15	5.64	0.34
^{277}Ds	5.76	0.26	5.88	0.22
^{279}Ds	6.18	0.13	6.27	0.11
^{281}Ds	6.19	0.15	6.31	0.13
^{279}Rg	6.19	0.34	6.26	0.29
^{281}Rg	5.89	0.50	6.03	0.43
^{285}Cn	6.28	0.15	6.38	0.12
^{283}113	6.20	0.22	6.28	0.18
^{285}113	6.01	0.27	6.15	0.23
^{289}115	6.24	0.37	6.33	0.32

Next, we make the attempt to probe into rms nuclear charge radii of superheavy odd-A and odd-odd nuclei. The detailed results of odd-A nuclei are presented in Table II and Fig. 2. Considering that the uncertainties of experimental α decay data are usually quite large due to the few observed events, the error bars of the deduced radii are correspondingly given in the table (denoted by ΔR). For a given nucleus, the minimum of rms radii is obtained from the maximum α decay data ($T^{\alpha}_{1/2} + \Delta T^{\alpha}_{1/2}$) and ($Q_\alpha + \Delta Q_\alpha$), while the maximum is obtained from the minimum data ($T^{\alpha}_{1/2} - \Delta T^{\alpha}_{1/2}$) and ($Q_\alpha - \Delta Q_\alpha$). Different from Fig. 1, there are no measured radii values and the deduced radii are accompanied by large error bars as mentioned. In addition, the simple relationship [46,47]

$$R = (c_0 + c_1 A^{-2/3} + c_2 A^{-4/3}) A^{1/3} \tag{9}$$

is displayed as a similar benchmark, and the parameters are $c_0 = 0.9071$ fm, $c_1 = 1.105$ fm, and $c_2 = -0.548$ fm [47]. It is seen that the extracted points of nuclear radii lie in the vicinity of the line predicted by the relationship between nuclear radius and mass number. Interestingly, the underestimation of extracted rms radii for ^{255}Lr and ^{259}Sg in comparison with the evaluation given by the simple relation (9) may imply the existence of the subshell $N = 152$, while the underestimation of ^{267}Db, ^{271}Bh, and ^{275}Mt may be caused by the effect of the subshell $N = 162$ or 164, as analyzed previously in Sec. II. Although the available data for odd-odd target nuclei appear to be relatively-few, we also tentatively extend the study to the superheavy odd-odd nuclei to seek certain information on nuclear sizes. In Table III, the extracted radii of superheavy odd-odd nuclei within the DDCM are listed in the second and third columns including the error bars, and the results obtained by the expression (9) in the last column are given for comparison [the proposed formula (8) is not applied for these odd-odd nuclei due to the lack of data in the fitting process]. There seems to be an abnormal situation for some odd-odd nuclei such as ^{266}Db, ^{266}Bh, and ^{268}Mt, which may imply that the nonzero angular momenta of the emitted α particle need to be introduced in the calculations. Up to now, there have been few structure researches on superheavy nuclei in

TABLE III. Tentative results of rms nuclear charge radii for superheavy odd-odd nuclei with $Z = 105$–115. For comparison, the evaluated values obtained by the simple relation (9) between nuclear radii and mass numbers are listed in the last column as well.

Nucleus	$R_{\rm calc}$ (fm)	$\Delta R_{\rm calc}$ (fm)	$R_{\rm ref.}$ (fm)
^{262}Db	5.90	0.15	5.97
^{266}Db	5.59	0.34	6.00
^{268}Db	6.00	0.34	6.02
^{270}Db	5.94	0.20	6.03
^{266}Bh	6.40	0.11	6.00
^{272}Bh	6.10	0.55	6.05
^{274}Bh	5.73	0.23	6.06
^{268}Mt	5.68	0.09	6.02
^{274}Mt	5.93	0.24	6.06
^{276}Mt	6.16	0.56	6.07
^{278}Mt	6.17	0.24	6.09
^{278}Rg	5.74	0.24	6.09
^{280}Rg	6.19	0.22	6.10
^{282}Rg	5.99	0.21	6.12
^{284}113	6.12	0.19	6.13
^{286}113	6.55	0.45	6.14
^{290}115	6.01	0.20	6.17

experiments due to the present constraint of instruments and the unique characteristic of these artificial elements. It will be of physical significance to compare the present results with future observations. More importantly, we hope that future experiments can provide valuable guidance, such as a more detailed α decay scheme, in the improvement of the attempt to extract nuclear radii from α decay data.

IV. SUMMARY

To conclude, we have employed the density-dependent cluster model to extract rms nuclear charge radii of daughter nuclei from α decay data. In the DDCM combined with the two-potential approach, the density distribution of target nuclei is adjusted in the double-folding model so as to reproduce the experimental α decay half-lives, including the nuclear deformation effect. The rms nuclear radius of the studied nucleus is then obtained using the corresponding density distribution. In addition, the proposed formula from our group, connecting rms radii with α decay energies and half-lives, is also used to evaluate the rms radii for comparison. Encouraged by the reasonable agreement between theory and experiment for odd-A nuclei plus a few odd-odd nuclei, we make the tentative extension to detect the rms radii in the superheavy mass region. These deduced radii are expected to be helpful in future research in the regime of superheavy odd-A and odd-odd nuclei.

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