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Lowest negative-parity states in ^{12}Be

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A very simple model is applied to the first four negative-parity states of ^{12}Be. Energies of the corresponding four states in ^{14}C are used to validate the model and to determine the doublet splitting parameters. Predictions for ^{12}Be are in remarkable agreement with excitation energies of the known 1^- at 2.70 MeV and the suspected (3^-) at 4.56 MeV. Predicted excitation energies of 0^- and 2^- are 3.59 and 5.12 MeV, respectively.

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Introduction. In ^{12}Be, adding $2s_{1/2}$ and $1d_{5/2}$ neutrons to the first-excited state of ^{11}Be at 0.320 MeV [1] produces four negative-parity states, with $J = 0$ to 3. Of these, only the 1^- state at $E_x = 2.702$ MeV [2] has been definitely identified. A candidate for the 3^- state is at 4.56 MeV. This state was initially suggested as 2^+ [3], but it now appears likely [4,5] to be (3^-) or a $(3^-/2^+)$ doublet. Millener [4] noted the excellent agreement between the 1.86 MeV energy difference between the 1^- and the probable (3^-) in ^{12}Be and the $1/2^+$-$5/2^+$ splitting of 1.78 MeV in ^{11}Be. Comparison of heavy-ion-induced reactions leading to ^{12}Be [6,7] and ^{14}C [8] (where the 3^- is known) significantly strengthens the 3^- suggestion for the 4.56-MeV state. The suggestion [9] that the (3^-) state might instead be 0^+ has been addressed recently [10,11]. Nothing is known about possible 0^- and 2^- states, whose unnatural parity would have caused them to be very weak in the ^{10}Be(t,p) reaction. They were also not observed in two different investigations [12,13] of the ^{11}Be(d,p) reaction (in reverse kinematics), even though the 0^- spectroscopic factor is predicted [12] to be reasonably large. The possibility that the 0^- might be an isomer [14] depends on its energy relative to the neutron breakup threshold at 3.171 MeV [15].

Various calculations have predicted the energy of the 0^- state. Romero-Redondo *et al.* [14,16] produced a 0^- excitation energy of about 2.5 to 2.8 MeV. They had adjusted their potential to reproduce the ground state (g.s.) energy. They then found that they missed the energies of the second 0^+ and first 2^+ states by 0.84 and 0.92 MeV, respectively, which they fixed by adjusting a three-body force. They found they needed no three-body force for the 1^- state and hence used none for 0^-. Kanada-En'yo and Horiuchi [17] had predicted the 0^- to be very unbound, with $E_x \sim 8$–9 MeV, but they had the first 1^- above 5 MeV, and it is known at 2.7 MeV. Blanchon *et al.* [18] obtained an excitation energy of 2.91 MeV. Garrido *et al.* [9] have it at about 3.19 MeV. Kanungo *et al.* [12] calculated a 0^- energy at 5.59–5.91, but their energy of the 1^- state was 3.38–3.71 MeV. These predictions are summarized in Table I. A no-core shell-model calculation [19] considered only the first few states of ^{12}Be and thus provided no prediction for 0^-. Descouvement and Baye [20] calculated only natural-parity states and therefore had no 0^- prediction.

TABLE I. Earlier predictions of 0^- excitation energy (MeV) in ^{12}Be. The single-neutron separation energy is 3.171 MeV.

Source	E_x	Ref.
Kanada-En'yo and Horiuchi	8–9	17
Romero-Redondo *et al.*	2.71	14
Romero-Redondo *et al.*	2.50–2.78	16
Blanchon *et al.*	2.91	18
Garrido *et al.*	3.19	9
Kanungo *et al.*	5.59–5.91	12

In ^{14}C, four states with $J^\pi = 0^-$ to 3^- are known [21] and can be thought of as s and d neutrons coupled to the $1/2^-$ ground state (g.s.) of ^{13}C. Our aim here is to analyze these lowest negative-parity states of ^{14}C in the simplest possible model, and then use that model to predict the energies of similar states in ^{12}Be.

Calculations and results. We are interested in making reliable estimates of the energies of the lowest negative-parity states in ^{12}Be. We look first at similar states in ^{14}C. The simplest model for the energies of these states is

$$E((J_c j)J^-) = E(J_c) + E(j) + \alpha_\ell \mathbf{J_c} \cdot \mathbf{j},$$

where $J_c{}^\pi = 1/2^-$, j is $2s_{1/2}$ or $1d_{5/2}$, ℓ is s or d, and J^- is the negative-parity state in ^{14}C.

For each doublet, the first two terms provide a parameter-free estimate of their energy centroids. The last term describes the doublet splittings with a single parameter for each doublet. This equation is standard [22,23] for the energies of states constructed by coupling single particles to cores with $J_c \neq 0$. In Ref. [22], one of the examples involves the two ^{14}C doublets that I am treating here. Note that I am not treating the $1/2^-$ states of ^{11}Be and ^{13}C (Table II) as $p_{1/2}$ single particles, but

TABLE II. Relevant energies (MeV) in ^{13}C and ^{11}Be.

Nucleus	J^π	E_x	E_n
^{13}C[a]	$1/2^-$	0.00	−4.946
	$1/2^+$	3.089	−1.857
	$5/2^+$	3.854	−1.092
^{11}Be[b]	$1/2^-$	0.320	−0.183
	$1/2^+$	0.00	−0.503
	$5/2^+$	1.778	1.275

[a] Reference [21].
[b] Reference [1].

TABLE III. Experimental and calculated energies (MeV) in ^{14}C.

J^π	Experimental[a]			Calculated[b]	
	E_x	E_{2n}	E_{2n} (centroid)	E_{2n} (centroid)[c]	α
1^-	6.094	−7.028			
0^-	6.902	−6.220			
$1^-, 0^-$			−6.826	−6.803	−0.808
3^-	6.728	−6.394			
2^-	7.341	−5.781			
$3^-, 2^-$			−6.139	−6.038	−0.204

[a] Reference [21].
[b] Present.
[c] E_{2n} (centroid) $= E(\text{core}) + E(j)$.

TABLE IV. Calculated and experimental energies (MeV) in ^{12}Be.

J^π	Calculated			Experimental
	E_{2n} (centroid)[a]	E_x (centroid)	E_x[b]	E_x
$1^-, 0^-$	−0.686	2.987		
1^-			2.785, 2.751	2.702[c]
0^-			3.593, 3.694	—
$3^-, 2^-$	1.092	4.765		
3^-			4.51, 4.47	(4.56)[d]
2^-			5.12, 5.18	—

[a] E_{2n} (centroid) $= E(\text{core}) + E(j)$.
[b] First value uses α from ^{14}C, second uses $\alpha' = (14/12)\,\alpha$.
[c] Reference [2].
[d] References [3–8].

only as $1/2^-$ cores to which I add s and d single neutrons. The only assumption is that for these cores the sd occupancy is small enough to be ignored. This expression should be reasonably precise if the states are single-particle (sp) s or d coupled to the p-shell core, even if the structure of the core is complicated. A similar assumption has worked well for several positive-parity states in several nuclei considered as arising from two sd-shell neutrons coupled to p-shell cores [24–26].

With this energy expression, the $(2J+1)$-weighted energy centroid should be equal to the sum of core and sp s or d energies, and the splitting within a doublet is just $\alpha_\ell\,(2j+1)/2$. As mentioned above, this treatment for the ^{14}C doublets is not new. I merely use ^{14}C to illustrate the procedure, and then apply the same procedure to ^{12}Be. For the 0^-, 1^- pair in ^{14}C, we see from Table III that the calculated centroid is $E_{2n} = -6.803$ MeV, very close to the experimental value of -6.826 MeV, only a 23-keV difference. For 2^-, 3^-, the calculation gives -6.038 MeV, and the experimental value is -6.139, a 101-keV difference. From the observed splittings within the doublets, values of α are $\alpha_s = -0.808$ MeV, $\alpha_d = -0.204$ MeV.

We now apply this very simple model to ^{12}Be. In ^{11}Be, the $1/2^+$ and $5/2^+$ contain the majority of the sp strength, but only about 74% and 60%, respectively, somewhat smaller than in ^{13}C. Nevertheless, we shall see what ensues. The centroid prediction for ^{12}Be is straightforward, with no room for adjustment. For splitting within the doublets, the possible dependence of α on core mass A and/or isospin T is not obvious. For now, we first assume no A or T dependence, and return to this point later. The simplest results are listed in Table IV.

We see that this simple model predicts a 1^- excitation energy of 2.785 MeV in ^{12}Be, very close to the known energy of 2.702 MeV. (I note that this is an absolute energy prediction.) The calculated result for the 0^- state is 3.593 MeV. The predicted 3^- energy is 4.51 MeV, extremely close to the (3^-) candidate [4–8] at $E_x = 4.56$ MeV. The 2^- prediction is 5.12 MeV.

If instead of using the α_ℓ values from ^{14}C, I had scaled them by $1/A$, the predicted energies would not have changed very much, as can be seen from Table IV, where I also list the predictions with these scaled α_ℓ values. I thus consider these predictions of a very simple model to be reasonably robust. Given the simplicity of the model, the agreement with the known 1^- and suspected 3^- states is remarkable. I therefore expect the 0^- and 2^- states in ^{12}Be will be found near the energies listed in Table IV. Of course, both the 0^- and 2^- energies are unbound to single-neutron emission. And, unbound s-wave neutron states are notoriously difficult to locate. So, it might be better to look for the 2^- state. Possible reactions might be ^{13}C(^{9}Be,^{10}C) or ^{13}C(^{13}C, ^{14}O).

Summary. A very simple model considers the first four negative-parity states of ^{14}C and ^{12}Be to consist of $2s_{1/2}$ and $1d_{5/2}$ neutrons coupled to $J^\pi = 1/2^-$ cores. Given measured energies in ^{13}C and ^{11}Be, the model is parameter free for the doublet centroids, for which calculations agree with experimental values to within 25–100 keV in ^{14}C. Observed doublet splittings in ^{14}C are used to compute doublet splittings in ^{12}Be. The 1^- energy in ^{12}Be is calculated to be 2.78 MeV, very close to the known value of 2.70 MeV. The 3^- prediction is 4.51 MeV, remarkably close to the suspected (3^-) at 4.56 MeV. Predicted energies for 0^- and 2^- are 3.59 and 5.12 MeV, respectively.

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