

The $e^+e^- \rightarrow \eta(\eta')2\pi$ process in the extended Nambu–Jona-Lasinio model

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The process of electron-positron annihilation into $\eta(\eta')2\pi$ is described within the extended Nambu–Jona-Lasinio model in the energy range up to about 2 GeV. Contributions of intermediate vector mesons $\rho(770)$ and $\rho(1450)$ are taken into account. Results for the $\eta 2\pi$ channel are found to be in a reasonable agreement with experimental data. Predictions for production of $\eta'2\pi$ are given. The corresponding estimations for decays $\tau \rightarrow \eta(\eta')2\pi\nu$ are given in the Appendix.

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I. INTRODUCTION

The description of hadronic interactions at low energies is problematic. Indeed, perturbative QCD hardly works at energies below 2 GeV. So in this domain various phenomenological models are used, most of them based on the chiral symmetry of strong interactions. One of the most popular models of such a kind is the Nambu–Jona-Lasinio (NJL) model [1–8]. This model describes spectra of light mesons in the ground states and their interactions using a rather small number of parameters.

Recently in the framework of the extended NJL model, a series of processes of meson production in electron-positron annihilation was described [9–13]. In the corresponding calculations, we took into account contributions of intermediate vector mesons $\rho(770)$, $\omega(782)$, $\phi(1020)$ and the radial excited states $\rho(1450)$, $\omega(1420)$. The radial excited states are treated with the help of the extended NJL model suggested in papers [14–18]. It was demonstrated that the extended NJL model provides a reasonably good description of a wide class of strong-interaction processes at energies up to about 1.5 GeV.

In the present paper we finalize a series of studies by consideration of the reaction $e^+e^- \rightarrow \eta(\eta')2\pi$. Both $\rho(770)$ and $\rho(1450)$ intermediate states are taken into account. The applicability of our calculation is limited to the domain of the center-of-mass energies up to about 2 GeV.

The process $e^+e^- \rightarrow \eta 2\pi$ has been studied experimentally at several facilities: DM1 [19], DM2 [20], ND [21,22], CMD-2 [23], and BaBar [24]. From the theoretical point of view they were also discussed within several different phenomenological approaches [23,25,26]. Comparison of our results with those presented in those papers will be given in the conclusion. Below we present the corresponding description in the framework of the extended NJL model and give a comparison with experimental data.

II. EXTENDED NAMBU–JONA-LASINIO MODEL

The Lagrangian of quark-meson interactions in the extended NJL model was given in Refs. [12,13,15,16,18]. After bosonization and diagonalization of the free-field Lagrangian, the relevant quark-meson interactions take the form¹

$$\begin{aligned} \Delta\mathcal{L}_2^{\text{int}} &= \bar{q}(k')(L_f + L_\gamma + L_V + L_{\pi,\hat{\pi}} + L_\eta)q(k), \\ L_f &= i\hat{\partial} - m, \quad L_\gamma = \frac{e}{2} \left(\tau_3 + \frac{1}{3} \right) \hat{A}, \\ L_V &= A_\rho \tau_3 \hat{\rho}(p) - A_{\rho'} \tau_3 \hat{\rho}'(p), \\ L_{\pi,\pi'} &= A_\pi \tau_\pm \gamma_5 \pi(p) - A_{\pi'} \gamma_5 \tau_\pm \pi'(p), \\ L_\eta &= i\gamma_5 \mathbb{I} \sum_{\eta=\eta,\eta',\hat{\eta},\hat{\eta}'} A_\eta \eta(p), \end{aligned} \quad (1)$$

where $\bar{q} = (\bar{u}, \bar{d})$ with u and d quark fields; $m = \text{diag}(m_u, m_d)$, $m_u = m_d = 280$ MeV are the constituent quark masses; e is the electron charge; \hat{A} is the photon field; ρ , ω (ρ' , ω'), π (π'), η , η' ($\hat{\eta}$, $\hat{\eta}'$) are meson fields (hats over η and η' mean excited states); $\tau^\pm = (\tau_1 \mp i\tau_2)/\sqrt{2}$, τ_1 , τ_2 , τ_3 are Pauli matrices; and \mathbb{I} is the unit matrix. Quantities A_i read

$$\begin{aligned} A_\rho &= g_{\rho_1} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + g_{\rho_2} f(k^\perp{}^2) \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)}, \\ A_{\rho'} &= g_{\rho_1} \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} + g_{\rho_2} f(k^\perp{}^2) \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)}, \\ A_\pi &= g_{\pi_1} \frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^\perp{}^2) \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)}, \\ A_{\pi'} &= g_{\pi_1} \frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^\perp{}^2) \frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)}, \\ A_\eta &= g_{\pi_1} \varphi_\eta^1 + g_{\pi_2} \varphi_\eta^2 f(k^\perp{}^2). \end{aligned} \quad (2)$$

Radially excited states are described in the extended NJL model using the following form factor in the quark-meson

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¹Note that for η and η' mesons we will use only the part of Lagrangian that contains interactions with u and d quarks; the full Lagrangian can be found in Refs. [18,27].

TABLE I. Mixing coefficients for isoscalar pseudoscalar meson states ($\eta = \eta, \eta', \hat{\eta}, \hat{\eta}'$).

	η	$\hat{\eta}$	η'	$\hat{\eta}'$
φ_η^1	0.71	0.62	-0.32	0.56
φ_η^2	0.11	-0.87	-0.48	-0.54

interaction:

$$f(k^\perp{}^2) = (1 - d|k^\perp{}^2|)\Theta(\Lambda_3^2 - |k^\perp{}^2|),$$

$$k^\perp = k - \frac{(kp)p}{p^2}, \quad d = 1.788 \text{ GeV}^{-2}, \quad (3)$$

where k and p are the quark and meson momenta, respectively; $\Lambda_3 = 1.03 \text{ GeV}$ is the cutoff parameter. The coupling constants are defined in the extended NJL model by the integrals containing given form factors

$$g_{\pi_1} = \left(4 \frac{I_2^{(0)}}{Z}\right)^{-1/2} = 3.01, \quad g_{\pi_2} = (4I_2^{(2)})^{-1/2} = 4.03,$$

$$g_{\rho_1} = \left(\frac{2}{3}I_2^{(0)}\right)^{-1/2} = 6.14, \quad g_{\rho_2} = \left(\frac{2}{3}I_2^{(2)}\right)^{-1/2} = 9.87, \quad (4)$$

where the Z factor appeared after taking into account pseudoscalar-axial-vector transitions, $Z \approx 1.2$. Note that $g_{\pi_1} \approx m_u/F_\pi$, where $F_\pi \approx 93 \text{ MeV}$ is the pion decay constant. The quark loop integrals are defined as

$$I_m^{(n)} = -iN_c \int \frac{d^4k}{(2\pi)^4} \frac{[f(k^\perp{}^2)]^n}{(m_u^2 - k^2)^m} \Theta(\Lambda_3^2 - \vec{k}^2), \quad (5)$$

where $N_c = 3$ is the number of colors. The mixing angles for pseudoscalar and vector mesons are: $\alpha_0 = 58.39^\circ$, $\alpha = 58.70^\circ$, $\beta_0 = 61.44^\circ$, $\beta = 79.85^\circ$. One can find the definition of mixing angles for π and ρ mesons in Refs. [15,16]. The mixing coefficients for the isoscalar pseudoscalar meson states given in Table I were derived in Refs. [17,18,27].

For the simplification of the presentation we define

$$\varphi_\pi = \frac{1}{\sin(2\alpha_0)} \begin{pmatrix} \sin(\alpha + \alpha_0) \\ \sin(\alpha - \alpha_0) \end{pmatrix},$$

$$\varphi_\eta = \begin{pmatrix} 0.71 \\ 0.11 \end{pmatrix}, \quad \varphi_{\eta'} = \begin{pmatrix} -0.32 \\ -0.48 \end{pmatrix},$$

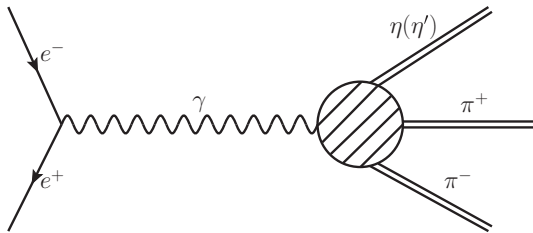


FIG. 1. The Feynman diagram with an intermediate photon. The dashed circle represents the sum of the two subdiagrams given in Figs. 3 and 4.

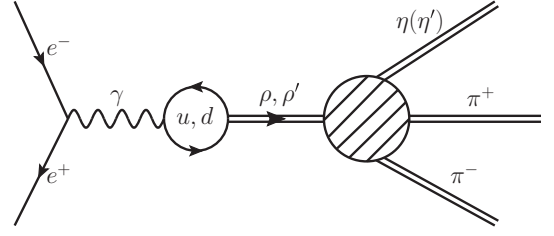


FIG. 2. The Feynman diagram with intermediate $\rho(770)$ and $\rho(1450)$ vector mesons. The dashed circle represents the sum of the two subdiagrams given in Figs. 3 and 4.

$$\varphi_\rho = \frac{1}{\sin(2\beta_0)} \begin{pmatrix} \sin(\beta + \beta_0) \\ \sin(\beta - \beta_0) \end{pmatrix},$$

$$\varphi_{\rho'} = -\frac{1}{\sin(2\beta_0)} \begin{pmatrix} \cos(\beta + \beta_0) \\ \cos(\beta - \beta_0) \end{pmatrix}. \quad (6)$$

III. PROCESS AMPLITUDES AND CROSS SECTION

The total amplitude of the given process has the form

$$T = -\frac{4\pi\alpha}{q^2} \bar{e}\gamma^\mu e \mathcal{H}_\mu, \quad (7)$$

where $q = p_{e^+} + p_{e^-}$ in the center-of-mass system. The hadronic part of the amplitude takes the form²

$$\mathcal{H}_\mu = V_\mu \left(T_\gamma(q^2, s) + \sum_{V=\rho, \rho'} T_V(q^2, s) \right),$$

$$V_\mu = p_\eta^\alpha p_{\pi^+}^\beta p_{\pi^-}^\gamma \varepsilon_{\mu\alpha\beta\gamma}. \quad (8)$$

Electron-positron annihilation with $\eta\pi\pi$ production is described by Feynman diagrams with virtual photons (Fig. 1), and with intermediate vector $\rho(770)$ and $\rho(1450)$ mesons (Fig. 2). In the following calculations we took into account the ground state $\rho(770)$ and the first radial-excited state $\rho(1450)$:

$$T_\gamma(q^2, s) = \sum_{i=1}^2 g_\pi \varphi_\eta^i (T_\square^{(i-1)}(s) + T_\Delta^{(i-1)}(s)),$$

$$T_V(q^2, s) = \frac{(C_{\gamma V}/g_{V_1})q^2}{m_V^2 - q^2 - i\sqrt{q^2}\Gamma_V(q^2)}$$

$$\times \sum_{i=1}^2 \sum_{j=1}^2 g_\pi \varphi_\eta^i g_{V_j} \varphi_V^j (T_\square^{(i+j-2)}(s) + T_\Delta^{(i+j-2)}(s)). \quad (9)$$

As was shown in previous calculations (see, for example, Refs. [12,13]), vector meson dominance could be directly obtained in the standard NJL model. In the present calculations we use only the extended version of the model, which describes the γV transition only with an accuracy about 10%. Thus, for the $\rho(770)$ resonance we apply vector meson dominance

²Hereafter $\eta = \eta, \eta'$.

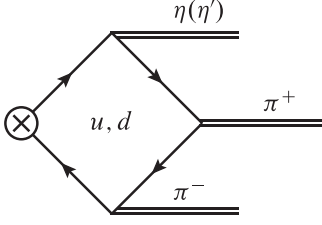


FIG. 3. The $V\eta\pi\pi$ vertex with quark loop of the box type. Interchange of pseudoscalar meson lines gives factor 3!

directly:

$$T_\gamma(q^2, s) + T_\rho(q^2, s) \rightarrow T_\rho^{\text{VMD}}(q^2, s) = \frac{m^2}{q^2} T_\rho(q^2, s). \quad (10)$$

The vertices $\gamma\eta\pi\pi$ and $V\eta\pi\pi$ contain the sum of two terms

$$\begin{aligned} T_\square^{(n)}(s) &= -24F_\pi g_\pi^3 I_4^{(n)}, \\ T_\Delta^{(n)}(s) &= 16F_\pi g_\pi \sum_{V=\rho, \rho'} \frac{g_{V \rightarrow \pi\pi}}{m_V^2 - s - i\sqrt{s}\Gamma_V(s)} \\ &\quad \times \sum_{i=1}^2 g_{\rho_i} \varphi_V^i I_3^{(n+i-1)} \\ &\approx 16F_\pi g_\pi \frac{g_{\rho \rightarrow \pi\pi}}{m_\rho^2 - s - i\sqrt{s}\Gamma_\rho(s)} \sum_{i=1}^2 g_{\rho_i} \varphi_\rho^i I_3^{(n+i-1)}. \end{aligned} \quad (11)$$

The term $T_\square^{(n)}(s)$ corresponds to the contribution of the quark box-diagram; see Fig. 3. The term $T_\Delta^{(n)}(s)$ comes from two triangle quark loops connected by a virtual vector meson; see Fig. 4. We neglected the contribution of the $\rho(1450)$ in the $T_\Delta^{(n)}(s)$ term, since it is very much suppressed with respect to the $\rho(770)$ contribution by kinematics and also due to the small partial decay width of $\rho(1450) \rightarrow 2\pi$; see Ref. [16].

Since $g_{\pi_1} \varphi_\pi^1 \gg g_{\pi_2} \varphi_\pi^2 \approx 0$, we replaced all terms contain vertices with π mesons (see discussion in Refs. [12,16]):

$$\prod_{i=1}^n \sum_{j=1}^2 g_{\pi_j} \varphi_\pi^j T_{\text{non-}\pi}^{(k)} I_{n+k}^{(k+i-j)} \Big|_{g_{\pi_2} \varphi_\pi^2 \rightarrow 0} = g_{\pi_1}^n T_{\text{non-}\pi}^{(k)} I_{n+k}^{(k)}. \quad (12)$$

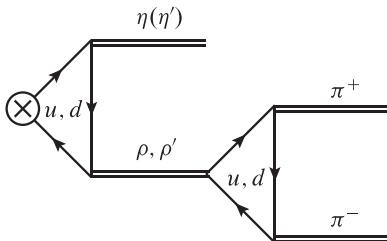


FIG. 4. The $V\eta\pi\pi$ vertex with two triangle quark loops connected by a virtual vector meson.

The second triangle diagram describing decay $V \rightarrow \pi\pi$ was computed in the framework of the extended NJL model in Refs. [12,16]; it gives

$$g_{V \rightarrow \pi\pi} \approx g_{\rho_1} \varphi_V^1 + g_{\rho_2} \varphi_V^2 \frac{I_2^{(1)}}{I_2^{(0)}}. \quad (13)$$

The transitions of a photon into the vector mesons (ρ, ρ') denoted by the terms

$$C_{\gamma V} = \varphi_V^1 + \varphi_V^2 \frac{I_2^{(1)}}{\sqrt{I_2^{(0)} I_2^{(2)}}}. \quad (14)$$

We chose the fixed width for $\rho(770)$ and the running one [10,12] for $\rho(1450)$:

$$\begin{aligned} \Gamma_\rho(s) &= \Gamma_\rho, \\ \Gamma_{\rho'}(s) &= \Theta(2m_\pi - \sqrt{s}) \Gamma_{\rho' \rightarrow 2\pi} \\ &\quad + \Theta(\sqrt{s} - 2m_\pi) \left(\Gamma_{\rho' \rightarrow 2\pi} + \Gamma_{\rho' \rightarrow \omega\pi} \frac{\sqrt{s} - 2m_\pi}{m_\omega - m_\pi} \right) \\ &\quad \Theta(m_\omega + m_\pi - \sqrt{s}) \\ &\quad + \Theta(m_{\rho'} - \sqrt{s}) \Theta(\sqrt{s} - m_\omega - m_\pi) \\ &\quad \times \left(\Gamma_{\rho' \rightarrow 2\pi} + \Gamma_{\rho' \rightarrow \omega\pi} + (\Gamma_{\rho'} - \Gamma_{\rho' \rightarrow 2\pi} - \Gamma_{\rho' \rightarrow \omega\pi}) \right. \\ &\quad \times \left. \frac{\sqrt{s} - m_\omega - m_\pi}{m_{\rho'} - m_\omega - m_\pi} \right) + \Theta(\sqrt{s} - m_{\rho'}) \Gamma_{\rho'}(m_{\rho'}^2), \end{aligned} \quad (15)$$

where $\Gamma_\rho = 147.8$ MeV and $\Gamma_{\rho'}(m_{\rho'}^2) = 400$ MeV are taken from Particle Data Group (PDG) [28]. The values $\Gamma(\rho' \rightarrow 2\pi) = 22$ MeV and $\Gamma(\rho' \rightarrow \omega\pi^0) = 75$ MeV were calculated in Ref. [16].

The total cross section takes the form

$$\sigma(q^2) = \frac{\alpha^2}{192\pi q^6} \int_{s_-}^{s_+} ds \int_{t_-}^{t_+} dt |T(q, s, t)|^2, \quad (16)$$

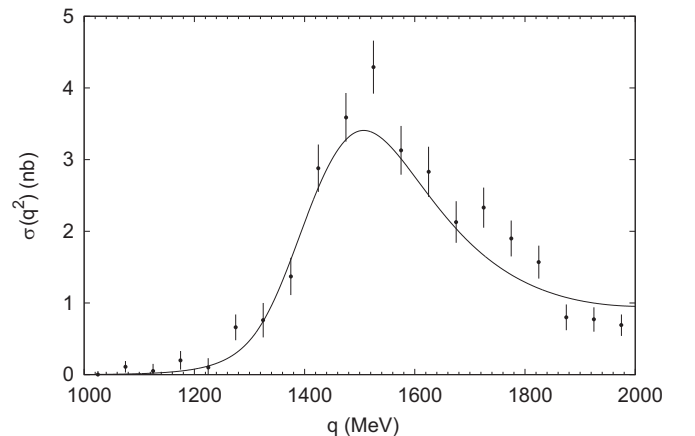


FIG. 5. Comparison of the extended NJL model predictions with the BaBar experiment data [24] for $e^+e^- \rightarrow \eta 2\pi$ process.

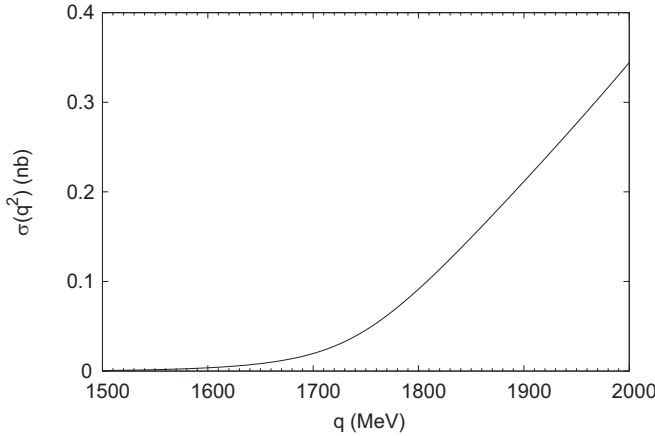


FIG. 6. Prediction of the extended NJL model for $e^+e^- \rightarrow \eta'2\pi$ process.

where variables are defined as $s = (p_\eta + p_{\pi^+})^2$, $t = (p_\eta + p_{\pi^-})^2$, and the limits are

$$t_{\mp} = \frac{1}{4s} \left([q^2 + m_\eta^2 - 2m_\pi^2]^2 - [\lambda^{1/2}(q^2, s, m_\pi^2) \pm \lambda^{1/2}(m_\eta^2, m_\pi^2, s)]^2 \right),$$

$$s_- = (m_\eta + m_\pi)^2, \quad s_+ = (\sqrt{q^2} - m_\pi)^2, \quad (17)$$

$$\lambda(a, b, c) = (a - b - c)^2 - 4bc.$$

The masses of all particles were taken from the PDG [28]: $m_{\pi^\pm} = 139.57$ MeV, $m_\eta = 547.86$ MeV, $m_{\eta'} = 957.78$ MeV, $m_\rho = 775.49$ MeV, $m_{\rho'} = 1465$ MeV. One can see the final results in Figs. 5 and 6.

IV. DISCUSSION AND CONCLUSIONS

The presented calculation shows that the extended NJL model allows us to describe the energy dependence of the total cross section of e^+e^- annihilation into $\eta 2\pi$ in satisfactory agreement with experimental data in the energy region up to 2 GeV. This allows us to expect that our predictions for the channel $e^+e^- \rightarrow \eta'2\pi$ are also reasonable.

One of the first attempts to provide a theoretical interpretation of the experimental data for $e^+e^- \rightarrow \eta 2\pi$ was presented in Ref. [23]. The vector meson dominance model was used taking into account intermediate vector mesons $\rho(770)$, $\rho(1450)$, and $\rho(1700)$. A number of free parameters was fit from the experimental data on the same process. Only one structure which corresponds in our case to the $T_\Delta(s)$ in the amplitude of the process was considered.

In Ref. [25] a resonance chiral theory was used. This model contains a very large number of free parameters. However, the contributions of intermediate radial-excited ρ mesons were not taken into account there, while the one due to $\rho(1450)$ meson is certainly important in the description of this process in the energy region under consideration. An advanced application of the same model was presented in Ref. [26], where intermediate vector mesons $\rho(770)$, $\rho(1450)$, and $\rho(1700)$ were included by means of introduction of additional free parameters. Note that

the contribution of $\rho(1700)$ was found to be not very important numerically.

The main difference of our results from the previous ones is that we work in the extended NJL model where all the parameters have been fixed from the beginning. Moreover, for the width of $\rho(1450)$ we take the PDG value 400 MeV, while in the alternative approaches considerably lower values were used; namely, 211 MeV in Ref. [23] and 238 MeV in Ref. [26]. As a result our model has a certain predictive power, contrary to the alternative approaches. We would like to underline that the extended NJL model was extensively tested by description of a large series of different processes with strong, weak, and electromagnetic interactions of mesons [9–13, 17, 29–32].

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APPENDIX: THE DECAYS $\tau \rightarrow \eta(\eta')2\pi\nu$

To proceed with the calculation of the decay $\tau \rightarrow \eta(\eta')2\pi\nu$ we complement the Lagrangian with terms contain W boson and charged ρ and ρ' mesons:

$$\Delta\mathcal{L}_2 = \frac{g_{EW}}{\sqrt{2}} \hat{W} + A_\rho \tau_\pm \hat{\rho}(p) - A_{\rho'} \tau_\pm \hat{\rho}'(p), \quad (A1)$$

where $\tau_\pm = (\tau_1 \mp i\tau_2)/\sqrt{2}$ and g_{EW} is the electroweak coupling constant.

To get an amplitude for the present process we replaced the e^+e^- current to $\tau\nu$ and intermediate γ to W^+ gauge boson in the amplitude (7) (Figs. 1 and 2). One can obtain the expression for τ decay from electron-positron annihilation after applying the corresponding phase volume transformation

$$\begin{aligned} \Gamma(\tau \rightarrow \eta(\eta')2\pi\nu) &= \frac{3|V_{ud}|^2}{2\pi\alpha^2 m_\tau^8} \Gamma(\tau \rightarrow e\nu_e\nu_\tau) \\ &\times \int_0^{m_\tau} \sigma(q^2) q^2 (m_\tau^2 - q^2)^2 (m_\tau^2 + 2q^2) dq^2, \quad (A2) \end{aligned}$$

where $\sigma(q)$ is the same as in Eq. (16), and $\Gamma(\tau \rightarrow e\nu_e\nu_\tau)$ takes the form

$$\Gamma(\tau \rightarrow e\nu_e\nu_\tau) = \frac{G_F^2 m_\tau^5}{192\pi^3}. \quad (A3)$$

To compare with theoretical predictions which take into account only $\rho(770)$ resonance [25, 33] we also give a prediction with $T_{\rho'} = 0$. Our estimations are in good agreement with current experimental data [28] (see Table II).

TABLE II. Branching ratios for the processes $\tau \rightarrow \eta(\eta')2\pi\nu$.

Process	Full amplitude	Only $\rho(770)$	PDG [28]
$\mathcal{B}(\tau \rightarrow \eta 2\pi\nu) \times 10^3$	1.46	1.01	1.39 ± 0.10
$\mathcal{B}(\tau \rightarrow \eta' 2\pi\nu) \times 10^5$	0.09	0.12	< 1.2

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