Tunneling times and bremsstrahlung in α decay

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A semiclassical model based on quantum time concepts is presented for the evaluation of bremsstrahlung emission probabilities in α decay of nuclei. The contribution to the bremsstrahlung emission from the different regions in tunneling is investigated using realistic double-folded nuclear and Coulomb potentials. Within this model, the contribution from the radiation emitted in front of the barrier before tunneling is much larger than that while leaving the barrier. A comparison with the data on ^{210}Po shows that the results are sensitive to the nuclear potential, and the rectangular well used in many of the quantum mechanical approaches can even give qualitatively different results.

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I. INTRODUCTION

The emission of photons accompanying the Coulomb interaction of charged particles is well explained by classical electrodynamics. The strength of the electromagnetic radiation is proportional to the acceleration which the charged particle experiences in an external field. To study bremsstrahlung emission accompanying α decay in nuclei, however, one needs to go beyond the classical picture where an α particle is accelerated in the Coulomb field of the daughter nucleus. In contrast to the photon emission accompanying nuclear β decay, the photons in α decay can also be emitted during the quantum tunneling process. The natural question that arises is therefore, Do the α particles emit radiation during tunneling or do they emit only in their acceleration outside the barrier? This curiosity gave rise to experiments measuring the emission probabilities of photons in the α decay of ^{214}Po [1,2], ^{210}Po [3–5], ^{226}Ra [1,6], and ^{244}Cu [7]. However, with the emission probabilities being small and the experiments difficult to perform, there remained discrepancies in the data. The theoretical calculations trying to explain these data also saw a similar fate. For example, the authors in [3] used an existing theoretical approach [8] based on a semiclassical calculation of the tunneling motion through the barrier and found very good agreement with their data. A repetition of the same calculation in a different manner [9], however, generated qualitatively different results. In [10], within a fully quantum mechanical approach, the authors found that the main contribution to photon emission arose from Coulomb acceleration and the under barrier tunneling contribution was tiny. The authors in [11], however, concluded that the total contribution results from a subtle interference of the tunneling, mixed, and classical regions. Different aspects of this process, such as a time-dependent description [12], the “interference of space regions” [13], analysis of angular bremsstrahlung spectra [14], the dynamic characteristics such as the position, velocity, and acceleration of the α particle [15], contribution of quadrupole radiation [16], etc. have also been studied. However, with the lack of data, the discrepancies in the understanding of the bremsstrahlung emission in α decay remain. The present work attempts to analyze some of the issues with a new semiclassical approach based on tunneling times.

In the next section, after a brief introduction to the time concepts used in the present work, we shall present a semiclassical model to evaluate the photon emission probabilities in α decay. In particular we consider the case of α decay in ^{210}Po. Though some of the theoretical approaches in the literature perform a fully quantum mechanical treatment of the problem, not much attention has been paid to the details of the nuclear potential. We present results displaying the sensitivity of the calculations to the nuclear potential used, the necessity of including an α cluster preformation factor, and the role of the under barrier and outside the barrier acceleration of the α particle. Finally, before summarizing our results, we present a section with a critical view of the various theoretical approaches available.

II. TUNNELING TIMES

Tunneling is one of the most remarkable phenomena of quantum physics. Interesting is also the question of how long a particle takes to traverse the barrier. The latter indeed gave rise to several quantum time concepts such as the phase, dwell, traversal, and Larmor time [17]. With the availability of so many definitions (which some times even include complex times [18,19]), it is of interest to inspect which of these times could correspond to physically measured quantities. The stationary concepts of dwell time and traversal time do find a connection with measurable quantities, with the former giving the half-life of radioactive nuclei and the latter the inverse of the assault frequency in α particle tunneling [20]. It is these two concepts which we shall use below in developing a semiclassical model for bremsstrahlung in α decay. Before discussing the model, we briefly introduce the two concepts.

Given an arbitrary potential barrier $V(x)$ in one dimension (a framework which is also suitable for spherically symmetric problems), confined to an interval (x_1,x_2), the dwell time is given by the number of particles in the region divided by the incident flux j:

$$\tau_D = \frac{\int_{x_1}^{x_2} |\Psi(x)|^2 \, dx}{j}. \tag{1}$$

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Here $\Psi(x)$ is the time-independent solution of the Schrödinger equation in the given region. The dwell time is usually defined as the time spent in the region (x_1,x_2) regardless of how the particle escaped (by reflection or transmission) and $j = \hbar k_0/\mu$ (where $k_0 = \sqrt{2\mu E}/\hbar$ with E being the kinetic energy of the tunneling particle and μ the reduced mass) for a free particle. When one defines the dwell time for a particle bound in a region which either got transmitted or reflected later, the flux j gets replaced by the transmitted or reflected fluxes, $j_T = \hbar k_0|T|^2/\mu$ and $j_R = \hbar k_0|R|^2/\mu$ [20,22], respectively. Here $|T|^2$ and $|R|^2$ are the transmission and reflection coefficients (with $|T|^2 + |R|^2 = 1$ due to conservation of probability). The traversal time defined by Büttiker [21] is somewhat different and is given as

$$\tau_{\rm trav}(E) = \int_{x_1}^{x_2} \frac{\mu}{\hbar k(x)}\, dx, \tag{2}$$

where $k(x) = \sqrt{2\mu[|V(x) - E|]}/\hbar$.

III. BREMSSTRAHLUNG EMISSION IN α DECAY

Given the number of theoretical works which have appeared on this subject over the years (as listed in the Introduction, too) the question that probably comes to the reader's mind here is, Why are we proposing yet another model? We therefore begin by stating the reasons for such an undertaking. To start with, (i) the quantum time concepts were successfully applied to realistic examples in nuclear and particle physics such as locating particle resonances [23], η-mesic nuclear states [24], and half-lives of heavy nuclei, and even in other branches like atomic and semiconductor physics, chemistry, and biology (see [20] and references therein). It is certainly interesting to extend these concepts to an intriguing phenomenon in nuclear physics. (ii) The quantum mechanical treatments are based on the evaluation of the transition matrix involving integrals where a separation of the space regions before, within, and after the barrier where the photon could have been emitted is not so obvious. Besides, while some papers simply use a rectangular well nuclear potential [10,11], others exclude the inner (nuclear potential) region from the integration [2,14]. The present work will use a realistic nuclear potential (with a double-folding model of nuclear densities and the M3Y nucleon-nucleon interaction [25]) and verify the role of emission in the various spatial regions. (iii) Another new input is that the α-daughter cluster preformation probability is incorporated in the calculation and found to be important.

A. Semiclassical model

We begin by defining an average velocity of the particle between points b and a as

$$\langle v\rangle = \frac{\int_a^b |\Psi(x)|^2\, v(x)\, dx}{\int_a^b |\Psi|^2\, dx}. \tag{3}$$

With the wave function being stationary and hence the density $\rho = |\Psi|^2$ being time independent, the continuity equation is $\vec{\nabla}\cdot\vec{j} = 0$ and the current density j is constant in the one-dimensional problem. Identifying $j = \rho v$ in the above equation,

$$\langle v\rangle = \frac{j(b-a)}{\int_a^b |\Psi|^2\, dx} = \frac{b-a}{\tau_D}. \tag{4}$$

Given the fact that we are interested in only those events where the α particle was transmitted through the barrier, we choose the constant flux j to be the transmitted flux $j_T = \hbar k_0|T|^2/\mu$. In a semiclassical picture one could consider $b - a$ as the distance traveled by the particle while it spent the time τ_D in that region. Coming back to the α-nucleus potential, one could then write this distance as the one between the classical turning points times the number of assaults, $\mathcal{N}$, made by the particle before leaving that region. For example, for the potential with the classical turning points r_1, r_2, and r_3 defined by $V(r) = E$ (where E is the energy of the tunneling particle), the frequency of assaults at the barrier, ν, can be written as the inverse of the time required to traverse the distance back and forth between the turning points r_1 and r_2 as [26]

$$\nu = \frac{\hbar}{2\mu}\left[\int_{r_1}^{r_2}\frac{dr}{k(r)}\right]^{-1}. \tag{5}$$

which is the inverse of twice the traversal time [Eq. (2)] from r_1 to r_2. The number of assaults made by the α in region I is then, $\mathcal{N}_{\rm I} = \nu_{\rm I}\,\tau_D$. With $\nu_{\rm I} = 1/(2\tau^{\rm I}_{\rm trav})$,

$$\mathcal{N}_{\rm I} = \frac{\tau_D^{\rm I}}{2\tau^{\rm I}_{\rm trav}}. \tag{6}$$

Replacing for $b - a$ with $\mathcal{N}_{\rm I}(r_2 - r_1)$ in Eq. (4) for region I and similarly with $\mathcal{N}_{\rm II}(r_3 - r_2)$ for region II, the average velocity in regions I and II can be finally written as

$$v_{\rm I} = \frac{r_2 - r_1}{2\tau^{\rm I}_{\rm trav}}, \quad v_{\rm II} = \frac{r_3 - r_2}{2\tau^{\rm II}_{\rm trav}}. \tag{7}$$

The velocity in region III, $v_{\rm III}$, is simply the free velocity and is given by $\sqrt{2E_\alpha/\mu}$. Defining the times at the turning points r_2 and r_3 as t_2 and t_3, respectively, the velocity function can be written as

$$v(t) = v_{\rm I}\Theta(t_2 - t) + v_{\rm II}\Theta(t_3 - t)\,\Theta(t - t_2) + v_{\rm III}\Theta(t - t_3), \tag{8}$$

where the step function $\Theta(t_0 - t)$ is unity for all $t < t_0$ and zero otherwise.

The classical formula for the photon emission probability in α decay is given as [8,10]

$$\frac{dP}{dE_\gamma} = P_\alpha\frac{2\alpha Z^2_{\rm eff}}{3\pi E_\gamma}|a_\omega|^2, \tag{9}$$

where

$$a_\omega = \int_{-\infty}^{\infty} dt\,\frac{dv}{dt}e^{-i\omega t}, \tag{10}$$

and we have introduced a factor P_α in order to account for the α-cluster preformation probability. $Z_{\rm eff}$ is the effective charge for dipole transitions and is given as $Z_{\rm eff} = (2A - 4Z)/(A + 4)$, where A and Z are the mass and atomic numbers of the daughter nucleus. For example, $Z_{\rm eff} = 0.4$ for ^{210}Po decay. Replacing for the velocity from Eq. (8) in Eq. (10) we obtain

$$a_\omega = [v_{\rm II}(Q-\hbar\omega) - v_{\rm I}(Q)]\, e^{-i\omega t_2} + [v_{\rm III}(Q-\hbar\omega) - v_{\rm II}(Q)]\, e^{-i\omega t_3}, \quad (11)$$

where we have written the energy dependence of the velocities explicitly. Q is the Q value of the decay and $\hbar\omega$ is the energy of the emitted photon. This dependence appears due to the fact that energy conservation has to be respected (neglecting, however, the tiny recoil of the nucleus). The energy in $v_{\rm III}$ should actually be $E_\alpha - \hbar\omega$; however, for all practical purposes, this does not lead to a big difference in the results. t_3 and t_2 define the times at which the particle enters and leaves the barrier. We choose $t_3 - t_2$ in the interference term to be the traversal time in the barrier. Thus for a given α-nucleus potential, the velocities and hence a_ω can be calculated from the traversal times. Evaluating the dwell times (and hence half-life) [20], the preformation factor is fixed (see the discussion below) and finally the emission probability is determined from Eq. (9).

B. Potential and cluster preformation factor

Starting with the standard definition of the WKB decay width [27],

$$\Gamma(E) = P_\alpha \frac{\hbar^2}{2\mu}\left[\int_{r_1}^{r_2} \frac{dr}{k(r)}\right]^{-1} e^{-2\int_{r_2}^{r_3}\kappa(r)\,dr}, \quad (12)$$

where $k(r) = \sqrt{2\mu[E-V(r)]}/\hbar$ and $\kappa(r) = \sqrt{2\mu[V(r)-E]}/\hbar$, the half-life of the nucleus can be evaluated to be $\tau_{1/2} = \hbar \ln 2/\Gamma$. The factor P_α is determined by comparing the experimental half-life of the nucleus with the theoretical one. The potential $V(r) = V_n(r) + V_c(r) + \frac{\hbar^2 (l+1/2)^2}{\mu r^2}$, where $V_n(r)$ and $V_c(r)$ are the nuclear and Coulomb parts of the α-nucleus (daughter) potential, r the distance between the centers of mass of the daughter nucleus and α and μ their reduced mass. The last term represents the Langer modified centrifugal barrier [28]. With the WKB being valid for one-dimensional problems, the above modification from $l(l+1) \to (l+1/2)^2$ is essential to ensure the correct behavior of the WKB scattered radial wave function near the origin as well as the validity of the connection formulas used [29]. Another requisite for the correct use of the WKB method is the Bohr-Sommerfeld quantization condition, which for an α with energy E is given as

$$\int_{r_1}^{r_2} K(r)\,dr = (n+1/2)\pi, \quad (13)$$

where $K(r) = \sqrt{\frac{2\mu}{\hbar^2}|V(r)-E|}$ and n is the number of nodes of the quasibound wave function of α-nucleus relative motion. The number of nodes are re-expressed as $n = (G-l)/2$, where G is a global quantum number obtained from fits to data [30,31]. We choose $G = 22$ for the ^{210}Po calculations. The folded nuclear potential is written as

$$V_n(r) = \lambda \int d\mathbf{r}_1\, d\mathbf{r}_2\, \rho_\alpha(\mathbf{r}_1)\, \rho_d(\mathbf{r}_2)\, v(\mathbf{r}_{12} = \mathbf{r} + \mathbf{r}_2 - \mathbf{r}_1, E), \quad (14)$$

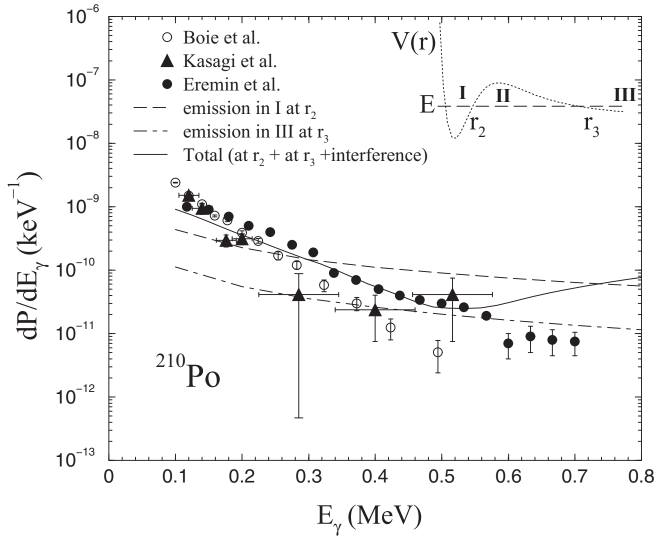

FIG. 1. Emission probabilities for bremsstrahlung accompanying the α decay of ^{210}Po. The data are from Refs. [3–5].

where ρ_α and ρ_d are the densities of the α and the daughter nucleus in a decay, and $v(\mathbf{r}_{12},E)$ is the nucleon-nucleon interaction. $|\mathbf{r}_{12}|$ is the distance between a nucleon in the α and a nucleon in the daughter nucleus. $v(\mathbf{r}_{12},E)$ is written using the M3Y nucleon-nucleon (NN) interaction as in [25]. The Coulomb potential is obtained using a similar double-folding procedure [32] with the matter densities of the α and the daughter replaced by their respective charge density distributions ρ_α^c and ρ_d^c.

C. Photon emission probabilities

The photon emission probabilities evaluated within the semiclassical tunneling time model are presented in Fig. 1 for the α decay of the nucleus ^{210}Po. One can see that the contribution to the results from the acceleration at the beginning of the Coulomb barrier (dashed line) is much larger than the acceleration while leaving the barrier (dot-dashed line). The shape of the total emission probability (solid line), however, gets decided by the sum and interference of the two terms. The disagreement with the data (which as such also disagree with each other having three different slopes) at high energies could either be caused by a limitation of the semiclassical model or by the energy dependence of the cluster preformation factor (which in the present work has been chosen to be constant). It is also important to note that we obtain $P_\alpha = 0.03$ on comparing the experimental and theoretical half-lives of ^{210}Po and this factor is essential for reproducing the right order of magnitude of the photon emission probability.

To test the sensitivity of the results to the potential used, we display in Fig. 2 the results evaluated using the realistic potential $V(r)$ mentioned in the previous section and a simpler potential of the form $V(r) = [2Z\alpha/r]\Theta(r-r_0) - V_0\Theta(r_0-r)$, where V_0 and r_0 are chosen to take the values used in [10] for ^{210}Po. Using $V_0 = 16.7$ MeV and $r_0 = 8.76$ fm as in [10] and the Q value of 5.407 MeV, the experimental half-

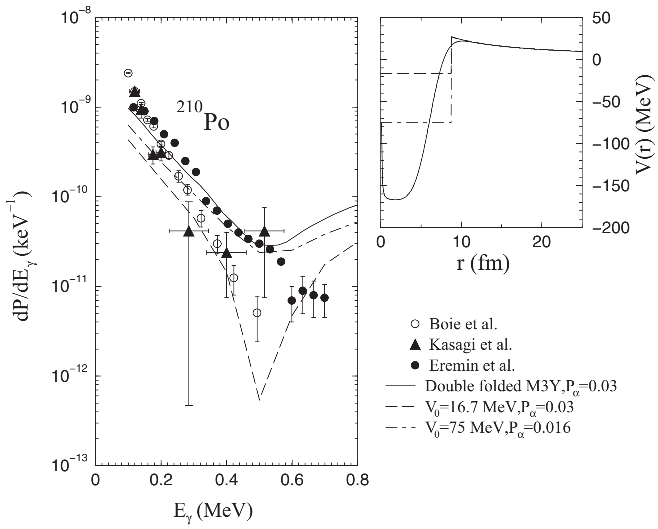

FIG. 2. Sensitivity of the photon emission probability to the nuclear potential.

life in Eq. (12) can be reproduced only after the inclusion of $P_\alpha = 0.03$. One can also rewrite the rectangular potential as $V(r) = [2Z\alpha/r]\Theta(r - r_0) - \lambda\,\tilde{V}_0\Theta(r_0 - r)$ and adjust λ in order to satisfy the Bohr-Sommerfeld condition. This leads to $V_0 = \lambda\tilde{V}_0 = 75$ MeV. It is interesting to see that such a rectangular well brings the results closer to those with the realistic potentials. The preformation factor, however, changes to $P_\alpha = 0.016$.

The semiclassical tunneling time model could in principle be applied to other existing data on the decay of ^{214}Po, ^{226}Ra, and ^{244}Cu. These results are not presented here since the qualitative behavior of the emission probabilities remains the same. The magnitude of the results is sensitive to the input of the preformation factor which in turn gets decided by the strength of the nuclear potential (which is decided by the global quantum number input). For an input $G = 24$, for example, the probabilities for ^{226}Ra and ^{214}Po are slightly overestimated as compared to data in the present approach.

IV. CRITICAL VIEW OF THE THEORETICAL APPROACHES

Apart from the fact that the data on bremsstrahlung emission in α decay are sparse, there exist contradictory conclusions from theoretical approaches in the literature. In the present section we try to give an overview of the results from different approaches and a comparison of their conclusions.

A. Semiclassical approaches

One of the first papers which appeared on this topic was that by Dyakonov and Gornyi [8], who considered the tunneling motion of a charged particle using the semiclassical WKB wave functions. They derived a classical formula for the radiation spectral density in terms of the quantum mechanical traversal time delay Δt which was given by

$$\frac{\partial E}{\partial \omega} = \frac{2}{3\pi}\frac{e^2}{c^3}\,\omega^2\,v_0^2\,|\Delta t|^2, \tag{15}$$

where the traversal time delay $\Delta t = \Delta t(-\infty)$ was defined as the difference of the traversal time under the barrier and the free traversal time in the same region. The above spectral density is related to the experimentally measured emission probability by a factor proportional to $(4\pi E_\gamma)^{-1}$ [9]. The acceleration obtained in [8], $|a_\omega^{\rm DG}|^2 = \omega^2 v_0^2 |\Delta t|^2$ can be rewritten in terms of the average velocities appearing in the present work. Considering the fact that the authors in [8] consider a free α particle tunneling the barrier, the only contribution to the "delay" is finite for the region within the barrier, and elsewhere $\Delta t = 0$. Thus, Δt of Eq. (10) in [8] can be rewritten as

$$\Delta t = \int_{r_2}^{r_3} \frac{1}{v(z)}\,dz - \frac{r_3 - r_2}{v_{\rm III}}, \tag{16}$$

leading to $|a_\omega^{\rm DG}|^2 = \omega^2(\tau_{\rm trav}^{\rm II})^2\,(v_{\rm III} - 2v_{\rm II})^2$. This appears somewhat similar to our expression where if we were to retain the contribution only from the acceleration at the end of the barrier, we would obtain $|a_\omega|^2 = (v_{\rm III} - v_{\rm II})^2$. One would, however, expect $|a_\omega^{\rm DG}|^2$ to grow with increasing photon energy as compared to $|a_\omega|^2$ of the present work. Working within the approach of [8] but with a different formalism [9] to evaluate $|a_\omega|^2$, Dyakonov obtained exponentially falling emission probabilities in reasonably good agreement with the ^{210}Po data.

The discrepancy to be noted here is that (i) Kasagi *et al.* [3] obtained an almost perfect agreement with the data (with a dip around $E_\gamma = 300$ MeV) using the model proposed in [8], (ii) the arguments presented above for $|a_\omega^{\rm DG}|^2$ seem to suggest that it would be difficult to expect steeply falling probabilities with the expression in [8], and (iii) the author of [8] using an apparently similar formalism did obtain exponentially falling probabilities in [9], however, with the absence of the dip and in disagreement with the result in (i) [3]. The author mentioned a possible reason for the disagreement to be the use of different cutoffs of the Coulomb potential chosen in [9] and [3].

B. Quantum mechanical treatments

A fully quantum mechanical description [10] of the photon emission accompanying α decay followed the early experiments and the semiclassical theoretical approaches in [8,9]. The authors expressed the emission probability in terms of a transition matrix involving the radial wave functions Φ_i and Φ_f of the initial and final α, respectively, and treating the photon field in the dipole approximation. The matrix element $\langle\Phi_f|\partial_r V|\Phi_i\rangle$ was evaluated using the following potential: $V(r) = [2Z\alpha/r]\Theta(r - r_0) - V_0\Theta(r_0 - r)$. The parameters V_0 and r_0 were fitted to obtain a half-life consistent with an expression obtained from wave function matching. The authors found that the main contribution to photon emission stems from Coulomb acceleration and only a small contribution arises from the tunneling wave function under the barrier. This is in contrast to the findings of [11] where the authors (in a similar kind of quantum mechanical approach involving the calculation of the transition matrix elements with a rectangular nuclear potential) found the total spectrum to be a result of the interplay between different regions. The authors in [11] replaced the quantum mechanical Coulomb wave functions by semiclassical ones and divided the integral into different regions. They defined classical turning points and thus obtained semiclassical integral expressions for the tunneling, mixed, and outside regions. Whereas Ref. [10] concluded that the soft-photon limit agrees with the classical results, Ref. [11] found classical theories inadequate in reproducing the subtle interference effects. In another quantum mechanical treatment [13] of the interference of the different space regions in tunneling, the results seemed to be in agreement with Ref. [10].

A revived interest in the topic was seen by some more recent works [2,6,14] which studied the experimental spectra for photon emission accompanying the ^{210,214}Po and ^{226}Ra α decay. The authors in [14], for example, employed a multipole expansion of the vector potential of the electromagnetic field of the daughter nucleus and also took into account the dependence on the angle between the directions of the α-particle propagation and photon emission. They found the contribution of the photon emission during tunneling to be small. In their investigation of ^{226}Ra they took into account the deformation of the nucleus and found the results to be different from those of the spherically symmetric case. Even if they agreed in general with [10] that the tunneling motion contributes little, using the potential parameters of [10] they could not, however, reproduce the slope of the ^{210}Po spectra.

C. Time-dependent formalisms

Finally, before ending this section we discuss two time-dependent descriptions of the bremsstrahlung emission. In contrast to the stationary descriptions of quantum tunneling described so far, the authors in [12] resort to numerically solving the time-dependent Schrödinger equation. The emission probability involves the radial momentum which is evaluated using the time-dependent wave function. Apart from finding the time-dependent modification of the wave function to be important, the authors notice that the usual assumption of a preformed α cluster in a well leads to sharp peaks at high frequencies in the bremsstrahlung emission. These peaks are interpreted as the manifestation of the fact that the initial localized state has some overlap from neighboring resonant states. Though the importance of these peaks would reduce if the initial state were a sharp resonance (as is the case for ^{210}Po), the authors express the need for more experimental data on bremsstrahlung radiation by a tunneling particle in order to understand better the preformation of clusters and the above phenomenon of "quantum beats."

In [15] the authors propose a numerical algorithm based on the Crank-Nicolson method to solve the time-dependent Schrödinger equation and thereby evaluate average position, momentum, and acceleration in α decay. They conclude that a big effect of the tunneling motion should be expected in the region of hard photons. Though the authors do not compare their results with data, they find that the contribution coming from the tunneling motion is an order of magnitude smaller than that from Coulomb acceleration.

V. SUMMARY

To summarize the findings of the present work, we can say the following:

(i) We have presented a new semiclassical model based on the concept of quantum tunneling times in order to evaluate the photon emission probabilities in α decay of nuclei. Special attention was paid to the use of realistic nuclear and Coulomb potentials and the results were found sensitive to the type of nuclear potential used.

(ii) A review of the existing theoretical literature shows that the opinion regarding the contribution of the photon emission during tunneling is divided among some who consider this motion as well as subtle interference effects between regions to be important and others who consider the Coulomb acceleration to be the dominant one.

(iii) The existing data on ^{210}Po are not consistent with each other and for other nuclei are few. We emphasize here the need for new reliable data in order to resolve the intriguing question which we started with: Does the α particle emit radiation during tunneling?

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