${}^{\;\,4}_{\Lambda\Lambda}$H in halo effective field theory

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The ${}^{\;\,4}_{\Lambda\Lambda}$H bound state and the S-wave hypertriton(${}^{3}_{\Lambda}$H)-Λ scattering in spin singlet and triplet channels below the hypertriton breakup momentum scale are studied in halo/cluster effective field theory at leading order by treating the ${}^{\;\,4}_{\Lambda\Lambda}$H system as a three-cluster (Λ-Λ-deuteron) system. In the spin singlet channel, the amplitude is insensitive to the cutoff parameter Λ_c introduced in the integral equation, and we find that there is no bound state. In this case, the scattering length of the hypertriton-Λ scattering is found to be $a_0 = 16.0 \pm 3.0$ fm. In the spin triplet channel, however, the amplitude obtained by the coupled integral equations is sensitive to Λ_c, and we introduce the three-body contact interaction $g_1(\Lambda_c)$. After phenomenologically fixing $g_1(\Lambda_c)$, we find that the correlation between the two-Λ separation energy $B_{\Lambda\Lambda}$ from the ${}^{\;\,4}_{\Lambda\Lambda}$H bound state and the scattering length $a_{\Lambda\Lambda}$ of the S-wave Λ-Λ scattering is significantly sensitive to the value of Λ_c.

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I. INTRODUCTION

Light double-Λ hypernuclei are exotic few-body systems that provide opportunities to investigate the flavor SU(3) structure of baryon-baryon interactions in the strangeness $\mathcal{S} = -2$ channel [1–3]. They are also expected to have a key role in resolving the long-standing puzzle of the existence of the H dibaryon [4], which has attracted recent interest triggered by lattice QCD simulations [5,6]. Since the seminal experiments on double-Λ hypernuclei of Refs. [7,8], however, there are only a few reports on the observation of double-Λ hypernuclei and, as a result, our understanding of these systems is still very poor. In the KEK-E373 experiment the $\Lambda\Lambda$ interaction energy[1] inside ${}^{\;\,6}_{\Lambda\Lambda}$He is measured as $\Delta B_{\Lambda\Lambda} \simeq 1.0$ MeV, which suggests a weakly attractive $\Lambda\Lambda$ interaction [9]. In addition, the formation of another double-Λ hypernucleus, ${}^{\;\,4}_{\Lambda\Lambda}$H, is conjectured in the BNL-AGS E906 experiment [10]. Theoretically, although the first Faddeev-Yakubovsky calculation showed a negative result [11], subsequent theoretical studies [12–15] predicted the possibility of the ${}^{\;\,4}_{\Lambda\Lambda}$H bound state based on the phenomenological $\Lambda\Lambda$ potentials which can describe the bound state of ${}^{\;\,6}_{\Lambda\Lambda}$He.

Since the stability of double-Λ hypernuclei depends on the $\Lambda\Lambda$ interaction, more accurate information on this interaction is strongly required. Recently, the scattering length $a_{\Lambda\Lambda}$ of S-wave $\Lambda\Lambda$ scattering is deduced from the ^{12}C$(K^-, K^+\Lambda\Lambda X)$ reaction [16], which leads to $a_{\Lambda\Lambda} = -1.2 \pm 0.6$ fm [17], and the data for the Au+Au collisions at the Relativistic Heavy Ion Collider [18] are analyzed to obtain $a_{\Lambda\Lambda} \geqslant -1.25$ fm in Ref. [19]. These values are consistent with those extracted from the leading order calculations for the $\mathcal{S} = -2$ baryon-baryon interactions in chiral effective theory [2] and in the Nijmengen ESC04d phenomenological potential model [20]. On the other hand, other phenomenological potential model predictions are scattered in values from -0.27 fm to -3.804 fm even though such models could explain the existence of the ${}^{\;\,6}_{\Lambda\Lambda}$He bound state. The present situation is summarized, for example, in Table I of Ref. [17]. This may imply that the parameter space of potential models would be too large to determine unambiguously the parameter values from the currently available experimental data. In such a situation, it would be worth studying the structure of hypernuclei by employing a very low energy effective field theory (EFT) which has a low separation scale, a well defined expansion scheme, and a few parameters to determine.

The methods of EFT have become popular in many fields. (For a review, see, e.g., Refs. [21,22].) In this scheme, a theory is constructed based on a scale which separates low energy and high energy degrees of freedom, and the theory constructed in such a way provides a systematic perturbative expansion in powers of Q/Λ_H, where Q is the typical scale of the reaction in question and Λ_H is the large (or high energy) scale. High energy degrees of freedom above Λ_H are integrated out and their effects are accounted for through the coefficients of contact interactions, so-called low energy constants, in higher order.

In this work, we investigate the relation between the ${}^{\;\,4}_{\Lambda\Lambda}$H bound state and the S-wave hypertriton-Λ scattering below the hypertriton breakup momentum for spin singlet and triplet channels by employing halo/cluster EFT at leading order (LO). In particular, we treat the ${}^{\;\,4}_{\Lambda\Lambda}$H hypernucleus as a three-body $\Lambda\Lambda d$ system, where d stands for a deuteron. Although the scattering experiment with double-Λ systems is not feasible in the near future, qualitative information from the scattering results can be possibly connected to the bound state problem, which is main motivation of the present work.

Below the hypertriton breakup momentum, we can choose the typical momentum (Q) of the reaction as the Λ particle separation momentum from the hypertriton, which is defined by $\gamma_{\Lambda d} = \sqrt{2\mu_{\Lambda d} B_\Lambda} \simeq 13.5 \pm 2.6$ MeV, where $\mu_{\Lambda d}$ is the

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[1]The $\Lambda\Lambda$ interaction energy $\Delta B_{\Lambda\Lambda}$ is defined as $\Delta B_{\Lambda\Lambda}({}^{\;A}_{\Lambda\Lambda}Z) = B_{\Lambda\Lambda}({}^{\;A}_{\Lambda\Lambda}Z) - 2B_\Lambda({}^{A-1}_{\;\;\Lambda}Z)$, where $B_{\Lambda\Lambda}$ and B_Λ are binding energies of the corresponding nuclei.

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reduced mass of the Λd system and B_Λ is the Λ particle separation energy from the hypertriton, $B_\Lambda^{\rm expt.} \simeq 0.13 \pm 0.05$ MeV [23]. On the other hand, the large (high momentum) scale Λ_H is chosen to be the deuteron binding momentum, $\gamma = \sqrt{m_N B_2} \simeq 45.7$ MeV, where m_N is the nucleon mass and B_2 is the deuteron binding energy, $B_2 \simeq 2.22$ MeV. Then our expansion parameter is $Q/\Lambda_H \sim \gamma_{\Lambda d}/\gamma \simeq 1/3$, which supports our expansion scheme. Because the deuteron is not broken up into two nucleons at low momentum below the deuteron binding momentum, we may treat the deuteron field as a cluster field, i.e., like an elementary field.

The $\Lambda\Lambda d$ system can form spin singlet and spin triplet states for the ${}^4_{\Lambda\Lambda}$H channel and we consider only the S-wave case for the relative orbital angular momentum. For the spin singlet channel of the S-wave hypertriton-Λ scattering, we obtain a single integral equation for the scattering amplitude, which is parameterized by the effective range parameters of the S-wave Λ-d scattering in the hypertriton channel, namely, the scattering length $a_{\Lambda d}$ (or equivalently the hypertriton binding momentum $\gamma_{\Lambda d}$) and the effective range $r_{\Lambda d}$. The integral is regularized by introducing a sharp momentum cutoff Λ_c in the integral equation. We find that when the cutoff Λ_c is larger than Λ_H, there is no cutoff dependence in the results, which implies that the system is insensitive to the short range mechanism [24]. This then suggests that introducing a three-body contact interaction at LO is not necessary. In addition, here we employ the standard Kaplan-Savage-Wise (KSW) counting rules [25], where the effective range, $r_{\Lambda d}$, is treated as a higher order term. This shows that the scattering length a_0 and the phase shift δ_0 of the S-wave hypertriton-Λ scattering are well controlled by $\gamma_{\Lambda d}$.

On the other hand, for the spin triplet channel, coupled integral equations are obtained for the scattering amplitudes. Because of spin statistics these equations consist of two cluster channels. One is the hypertriton-Λ channel of spin-1 and the other is the deuteron and double-Λ system, where we assume that the double-Λ is described by the $\Lambda\Lambda$-dibaryon state and the components in the cluster states are in relative S wave. We find that the coupled integral equations show a sensitivity to the cutoff Λ_c. Thus, as in the case of three-nucleon system in the triton channel within pionless EFT [26], a three-body contact interaction needs to be introduced in order to make the results cutoff independent. In addition, within the standard KSW counting rules [25] the dressed composite propagators of the hypertriton for the Λ-d composite state and of the dibaryon for two Λ particles in ${}^1S_0$ state are expanded in terms of the effective range parameters. Thus the coupled integral equations are represented in terms of only four parameters at LO, namely, $\gamma_{\Lambda d}$, $a_{\Lambda\Lambda}$, the coupling of the three-body contact interaction $g_1(\Lambda_c)$, and the cutoff Λ_c. Unlike the effective range parameters, however, there are no experimental data to constrain $g_1(\Lambda_c)$ for ${}^4_{\Lambda\Lambda}$H.

Because of the paucity of empirical information to constrain the low energy constants it is very hard to draw a robust prediction on the existence of the bound state in the ${}^4_{\Lambda\Lambda}$H channel. Therefore, instead of tackling the problem of the existence of bound states we investigate the effect of the contact term in the ${}^4_{\Lambda\Lambda}$H system. For this purpose we consider two cases. In the first case, we do not include the contact interaction by setting $g_1 = 0$. Then the system is found to have a large negative scattering length at $\Lambda_c \simeq \Lambda_H$, which may imply the formation of a quasi-bound state. Furthermore, if Λ_c is sent to the asymptotic limit, $\Lambda_c \to \infty$, we find that a bound state arises in the system.

In the second case, we turn on the contact interaction. To constrain the value of $g_1(\Lambda_c)$, we employ the results of the potential model calculations of Refs. [11,12] and determine $g_1(\Lambda_c)$ by using the computed double-Λ separation energy $B_{\Lambda\Lambda}$ of ${}^4_{\Lambda\Lambda}$H for given values of $a_{\Lambda\Lambda}$. Then we find that the renormalized $g_1(\Lambda_c)$ exhibits the so-called limit-cycle when Λ_c is sent to the asymptotic limit. In the present work, we also calculate $B_{\Lambda\Lambda}$ as a function of $a_{\Lambda\Lambda}$ for a fixed $g_1(\Lambda_c)$ and a correlation between $B_{\Lambda\Lambda}$ and $1/a_1$ as well, where a_1 is the scattering length of the S-wave hypertriton-Λ scattering in the spin triplet channel at LO. We find that the $a_{\Lambda\Lambda}$ dependence of $B_{\Lambda\Lambda}$ is quite sensitive to the value of Λ_c. For example, $B_{\Lambda\Lambda}$ is found to be almost insensitive to $a_{\Lambda\Lambda}$ when $\Lambda_c \simeq \Lambda_H$. On the other hand, the reported $a_{\Lambda\Lambda}$ dependence of $B_{\Lambda\Lambda}$ in the potential model calculations of Refs. [11,12] is recovered when $\Lambda_c \simeq 6\Lambda_H$. In the present work, we will investigate the implications of the choice on the cutoff Λ_c and the $a_{\Lambda\Lambda}$ and Λ_c dependence of the properties of ${}^4_{\Lambda\Lambda}$H system in the cluster theory.

This paper is organized as follows. We start with the relevant effective Lagrangian in the next section, which defines notations and our basic tools for studying hypernuclei. In Sec. III, the two-body parts of the $\Lambda\Lambda d$ system, i.e., the dressed $\Lambda\Lambda$ dibaryon propagator in ${}^1S_0$ channel and the dressed hypertriton propagator (as a Λd system), are constructed. In Sec. IV, the integral equations of the $\Lambda\Lambda d$ three-body system for the S-wave hypertriton-Λ scattering are constructed in the spin singlet and triplet states. The numerical results are presented in Sec. V, and Sec. VI contains a summary and conclusions of this work.

II. EFFECTIVE LAGRANGIAN

In EFT, effective Lagrangian is constructed on the symmetry requirement with relevant degrees of freedom at low energies being expanded in terms of the number of derivatives order by order [27]. The effective Lagrangian at LO for this work can be written as

$$\mathcal{L} = \mathcal{L}_\Lambda + \mathcal{L}_d + \mathcal{L}_s + \mathcal{L}_t + \mathcal{L}_{\Lambda t}. \tag{1}$$

Here, $\mathcal{L}_\Lambda$ and $\mathcal{L}_d$ are the standard one-body Λ and (elementary) deuteron Lagrangian in the heavy-baryon formalism [28], which read

$$\mathcal{L}_\Lambda = \mathcal{B}_\Lambda^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{2m_\Lambda} \right] \mathcal{B}_\Lambda + \cdots, \tag{2}$$

$$\mathcal{L}_d = d_i^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{2m_d} \right] d_i + \cdots, \tag{3}$$

where $\mathcal{B}_\Lambda$ is the Λ-baryon field of spin-1/2, d_i is the deuteron (vector) field of spin-1, and v^μ is a velocity vector with $v^\mu = (1,\mathbf{0})$ in our case. The Λ and deuteron masses are represented by m_Λ and m_d, respectively. The dots denote the higher order terms that are irrelevant for the LO calculations.

Equation (1) also contains the Lagrangian for the composites containing strangeness. For this purpose, we introduce s and t fields to denote the $\Lambda\Lambda$ dibaryon in the $^1S_0$ state and the Λd composite in the $^2S_{1/2}$ state. Then $\mathcal{L}_s$ and $\mathcal{L}_t$ are the Lagrangians for these fields including $s \leftrightarrow \Lambda\Lambda$ and $t \leftrightarrow \Lambda d$ interactions, which read [29–31]

$$\mathcal{L}_s = \sigma_s s^\dagger \left[i v\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{4m_\Lambda} + \Delta_s \right] s - y_s\left[s^\dagger \left(\mathcal{B}_\Lambda^T P^{(^1S_0)} \mathcal{B}_\Lambda\right) + \text{H.c.}\right] + \cdots, \tag{4}$$

$$\mathcal{L}_t = \sigma_t t^\dagger \left[i v\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{2(m_d + m_\Lambda)} + \Delta_t \right] t + \frac{y_t}{\sqrt{3}}[t^\dagger \vec{\sigma}\cdot\vec{d}\,\mathcal{B}_\Lambda + \text{H.c.}] + \cdots, \tag{5}$$

where σ_s and σ_t are sign factors, Δ_s and Δ_t are the mass differences between the composite states and their constituents, and y_s and y_t are coupling constants. The spin projection operator of the $\Lambda\Lambda$ composite onto the $^1S_0$ state is

$$P^{(^1S_0)} = -\frac{i}{2}\sigma_2, \tag{6}$$

The three-body contact interaction is given by the Lagrangian $\mathcal{L}_{\Lambda t}$, where t and Λ fields are in the $^3S_1$ channel, which reads

$$\mathcal{L}_{\Lambda t} = -\frac{g_1(\Lambda_c)}{\Lambda_c^2}\left(\mathcal{B}_\Lambda^T P_i^{(^3S_1)} t\right)^\dagger \left(\mathcal{B}_\Lambda^T P_i^{(^3S_1)} t\right) + \cdots, \tag{7}$$

with the spin projection operator onto the $^3S_1$ state,

$$P_i^{(^3S_1)} = -\frac{i}{2}\sigma_2\sigma_i. \tag{8}$$

The coupling constant of the three-body contact interaction is given by $g_1(\Lambda_c)$ as a function of the cutoff Λ_c which will be introduced in the integral equations below.

III. TWO-BODY PART

A. *S*-wave $\Lambda\Lambda$ scattering in $^1S_0$ channel

At low energies, we assume that the dominant partial wave of $\Lambda\Lambda$ scattering is the $^1S_0$ state and the scattering process can be described by the effective range parameters. Therefore, this is similar to the low-energy nucleon-nucleon scattering in the $^1S_0$ channel studied, for example, in Ref. [30]. Diagrams for the dressed dibaryon field and for the scattering amplitude are shown in Figs. 1 and 2, respectively.

Referring the details to Ref. [30], we can obtain the scattering amplitude in the center-of-mass (CM) frame as

$$A(E) = \frac{4\pi}{m_\Lambda}\left(-\frac{1}{a_{\Lambda\Lambda}} + \frac{1}{2}r_{\Lambda\Lambda}k^2 - ik\right)^{-1}, \tag{9}$$

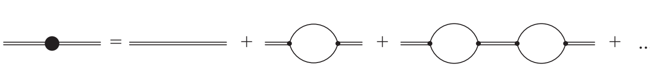

FIG. 1. Diagrams for dressed dibaryon propagator. On the right-hand side, the double solid line represents the bare dibaryon propagator and the single solid line denotes the Λ propagator.

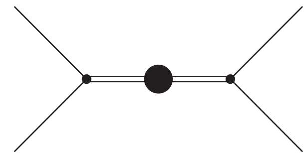

FIG. 2. Diagram for $\Lambda\Lambda$ scattering amplitude. A double line with a filled circle denotes a dressed propagator as explained in Fig. 1.

where $a_{\Lambda\Lambda}$ and $r_{\Lambda\Lambda}$ are the scattering length and effective range of $\Lambda\Lambda$ scattering in the $^1S_0$ channel. The on-shell total energy is $E = k^2/m_\Lambda$ with $k = |\boldsymbol{k}|$.

Thus the renormalized dressed dibaryon propagator can be written as

$$D_s(p_0,\boldsymbol{p}) = \frac{4\pi}{m_\Lambda y_s^2}\left[\frac{1}{a_{\Lambda\Lambda}} + \frac{1}{2}r_{\Lambda\Lambda}\left(-m_\Lambda p_0 + \frac{1}{4}\boldsymbol{p}^2 - i\epsilon\right) - \sqrt{-m_\Lambda p_0 + \frac{1}{4}\boldsymbol{p}^2 - i\epsilon}\right]^{-1} \tag{10}$$

and

$$y_s = -\frac{2}{m_\Lambda}\sqrt{\frac{2\pi}{r_{\Lambda\Lambda}}}. \tag{11}$$

Here, p_0 and $\boldsymbol{p}$ are the off-shell (loop) energy and momentum which do not satisfy the on-shell condition in the CM frame mentioned above. In addition, we have suppressed the cutoff dependence in the effective range parameters from the bubble diagrams. We use the same cutoff value for renormalizing $a_{\Lambda\Lambda}$ and $r_{\Lambda\Lambda}$ in the three-body part, which will be discussed in Sec. IV.

B. *S*-wave Λd system in hypertriton channel

The hypertriton ($^3_\Lambda$H) has the quantum numbers of $J^\pi = 1/2^+$ and $T = 0$, where T stands for isospin, and its Λ separation energy is $B_\Lambda = 0.13 \pm 0.05$ MeV [23]. We refer the readers to Ref. [32] for a study on this state within pionless EFT.

Shown in Fig. 3 are the diagrams for the dressed hypertriton (t field) propagator as a Λd composite state. Then the

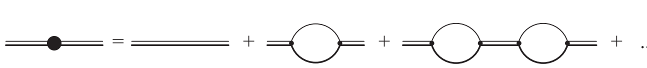

FIG. 3. Diagrams for dressed hypertriton propagator as a Λd system. In the right-hand side, the solid line denotes the Λ hyperon while the thick solid line represents the deuteron. The bare t field as a Λd composite state in hypertriton channel is denoted by the double (thin and thick) solid line.

renormalized dressed hypertriton propagator is obtained as

$$D_t(p_0,\boldsymbol{p}) = \frac{2\pi}{\mu_{\Lambda d} y_t^2}\left\{\frac{1}{a_{\Lambda d}} + \frac{1}{2} r_{\Lambda d}\left[-2\mu_{\Lambda d}\left(p_0 - \frac{1}{2(m_\Lambda + m_d)}\boldsymbol{p}^2 + i\epsilon\right)\right] - \sqrt{-2\mu_{\Lambda d}\left(p_0 - \frac{1}{2(m_\Lambda + m_d)}\boldsymbol{p}^2 + i\epsilon\right)}\right\}^{-1} \quad (12)$$

with

$$y_t = -\frac{1}{\mu_{\Lambda d}}\sqrt{\frac{2\pi}{r_{\Lambda d}}}, \quad (13)$$

where $\mu_{\Lambda d}$ is the reduced mass of the Λd system, i.e., $\mu_{\Lambda d} = m_\Lambda m_d/(m_\Lambda + m_d)$, and $a_{\Lambda d}$ and $r_{\Lambda d}$ are the effective range parameters of the S-wave Λ-d scattering in the hypertriton channel. In Ref. [32], these effective range parameters are estimated as $a_{\Lambda d} = 16.8^{+4.4}_{-2.4}$ fm, and $r_{\Lambda d} = 2.3 \pm 0.3$ fm, which leads to $\gamma_{\Lambda d} = 1/a_{\Lambda d} + r_{\Lambda d}\gamma_{\Lambda d}^2/2 \simeq 12.8$ MeV when we use the central values of the parameters. This value is consistent with the one given in Sec. I within error.

Since there exists a bound state for hypertriton, the propagator should have a pole at $k = i\gamma_{\Lambda d}$ and we may rewrite the on-energy-shell dressed propagator as

$$D_t(E) = \frac{2\pi}{\mu_{\Lambda d} y_t^2}\left[\gamma_{\Lambda d} - \frac{1}{2} r_{\Lambda d}\left(k^2 + \gamma_{\Lambda d}^2\right) + ik\right]^{-1}, \quad (14)$$

where $E = k^2/(2\mu_{\Lambda d})$. Furthermore, near the pole, the propagator can be further simplified as

$$D_t(E) \simeq \frac{Z_{\Lambda d}}{E + B_\Lambda} \quad \text{with} \quad Z_{\Lambda d} = \frac{\gamma_{\Lambda d} r_{\Lambda d}}{1 - \gamma_{\Lambda d} r_{\Lambda d}}, \quad (15)$$

where $Z_{\Lambda d}$ is the wave function normalization factor of the hypertriton as a Λd system. Since the inverse of the effective range has a large scale, $r_{\Lambda d}^{-1} \simeq 86$ MeV, one can see that the KSW counting rules, where the propagator and $Z_{\Lambda d}$ are expanded in terms of $r_{\Lambda d}$, would be a good approximation, which can be seen from the fact that $\gamma_{\Lambda d} r_{\Lambda d} \simeq 0.16 < 1/3$.

IV. THREE-BODY PART

In this section, we construct the integral equations for S-wave scattering of hypertriton and Λ, which has two spin channels, $S = 0$ and 1, because both the hypertriton and Λ have spin-1/2. For $S = 0$ channel, the amplitude $t(p,k;E)$ consists of hypertriton-Λ channel only. In Fig. 4, diagrams of the integral equation for the scattering amplitude are shown, which lead to

$$t(p,k;E) = -3K_{(a)}(p,k;E) + \frac{1}{2\pi^2}\int_0^{\Lambda_c} d\ell\,\ell^2\, 3K_{(a)}(p,\ell;E) \times D_t\left(E - \frac{\ell^2}{2m_\Lambda}, \boldsymbol{\ell}\right) t(\ell,k;E) \quad (16)$$

with the one-deuteron-exchange interaction $K_{(a)}(p,l;E)$,

$$K_{(a)}(p,\ell;E) = \frac{1}{3}\frac{m_d y_t^2}{2p\ell}\ln\left(\frac{\frac{m_d}{2\mu_{\Lambda d}}(p^2+\ell^2) + p\ell - m_d E}{\frac{m_d}{2\mu_{\Lambda d}}(p^2+\ell^2) - p\ell - m_d E}\right), \quad (17)$$

where p and k are relative off-shell and on-shell momenta of hypertriton-Λ scattering in the CM frame, respectively, and E is the total energy,

$$E = -\frac{\gamma_{\Lambda d}^2}{2\mu_{\Lambda d}} + \frac{1}{2\mu_{\Lambda(\Lambda d)}}k^2, \quad (18)$$

with $\mu_{\Lambda(\Lambda d)}$ being the reduced mass of the Λ-(Λd) system so that $\mu_{\Lambda(\Lambda d)} = m_\Lambda(m_\Lambda + m_d)/(2m_\Lambda + m_d)$. A sharp cutoff momentum Λ_c was introduced as before in the integral equation. However, as we shall see below, the integral equation is insensitive to the value of Λ_c, which weakens the necessity of three-body contact interactions.

For the $S = 1$ channel, however, we have two scattering amplitudes, namely, $a(p,k;E)$ for the spin triplet Λt (Λ and hypertriton) cluster channel and $b(p,k;E)$ that connects the Λt cluster channel to the ds (deuteron and the $\Lambda\Lambda$ dibaryon) cluster channel. In Fig. 5, diagrams of coupled integral equations are presented, from which we obtain

$$\begin{aligned} a(p,k;E) =\ & K_{(a)}(p,k;E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2} - \frac{1}{2\pi^2}\int_0^{\Lambda_c} d\ell\,\ell^2\left[K_{(a)}(p,\ell;E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2}\right] D_t\left(E - \frac{\ell^2}{2m_\Lambda}, \boldsymbol{\ell}\right) a(\ell,k;E) \\ & - \frac{1}{2\pi^2}\int_0^{\Lambda_c} d\ell\,\ell^2 K_{(b1)}(p,\ell;E) D_s\left(E - \frac{\ell^2}{2m_d}, \boldsymbol{\ell}\right) b(\ell,k;E), \\ b(p,k;E) =\ & K_{(b2)}(p,k;E) - \frac{1}{2\pi^2}\int_0^{\Lambda_c} d\ell\,\ell^2 K_{(b2)}(p,\ell;E) D_t\left(E - \frac{\ell^2}{2m_\Lambda}, \boldsymbol{\ell}\right) a(\ell,k;E), \end{aligned} \quad (19)$$

with one-Λ-exchange interactions $K_{(b1)}(p,\ell;E)$ and $K_{(b2)}(p,\ell;E)$, which read

$$K_{(b1)}(p,\ell;E) = -\sqrt{\frac{2}{3}}\frac{m_\Lambda y_s y_t}{2p\ell}\ln\left[\frac{p^2 + \frac{m_\Lambda}{2\mu_{d\Lambda}}\ell^2 + p\ell - m_\Lambda E}{p^2 + \frac{m_\Lambda}{2\mu_{d\Lambda}}\ell^2 - p\ell - m_\Lambda E}\right], \quad (20)$$

$$K_{(b2)}(p,\ell;E) = -\sqrt{\frac{2}{3}}\frac{m_\Lambda y_s y_t}{2p\ell}\ln\left[\frac{\frac{m_\Lambda}{2\mu_{d\Lambda}}p^2 + \ell^2 + p\ell - m_\Lambda E}{\frac{m_\Lambda}{2\mu_{d\Lambda}}p^2 + \ell^2 - p\ell - m_\Lambda E}\right]. \quad (21)$$

In Eq. (19), we have introduced the three-body contact interaction that contains the coupling constant

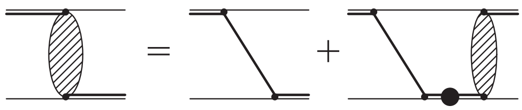

FIG. 4. Diagrams of the integral equation for S-wave scattering of hypertriton and Λ for spin singlet ($S = 0$) channel. See the caption of Fig. 3 as well.

$g_1(\Lambda_c)$.[2] As we shall see below, the integral equations depend on the cutoff Λ_c, and $g_1(\Lambda_c)$ accounts for the high momentum effects above Λ_c.

V. NUMERICAL RESULTS

A. S-wave scattering of hypertriton and Λ in $S = 0$ channel

In the dressed hypertriton propagator D_t given in Eq. (12), there are two singularities at $\ell \simeq 13$ MeV and $\ell \simeq 172$ MeV when $E = 0$ in Eq. (19). The first one corresponds to the binding momentum of the hypertriton in the Λ-d system and the second one to an unphysical deeply bound state. We avoid the effect from the unphysical deeply bound state by expanding the effective range correction, as mentioned above, employing the KSW counting rules.

The on-shell scattering matrix is given by

$$T(k,k) = \sqrt{Z_{\Lambda d}}\, t(k,k;E)\sqrt{Z_{\Lambda d}}\,, \tag{22}$$

and thus the integral equation in terms of the half-off-shell scattering matrix at LO reads

$$\begin{aligned} T(p,k) = &-3\gamma_{\Lambda d} r_{\Lambda d} K_{(a)}(p,k;E) \\ &- \frac{3}{2\pi^2}\mu_{\Lambda(\Lambda d)} r_{\Lambda d}\int_0^{\Lambda_c} d\ell K_{(a)}(p,\ell;E) \\ &\times \left\{\gamma_{\Lambda d} + \sqrt{\gamma_{\Lambda d}^2 + \frac{\mu_{\Lambda d}}{\mu_{\Lambda(\Lambda d)}}(\ell^2 - k^2)}\right\} \frac{\ell^2 T(\ell,k)}{\ell^2 - k^2 - i\epsilon}. \end{aligned} \tag{23}$$

This shows that the integral equation is expressed in terms of two parameters, namely, $\gamma_{\Lambda d}$ and Λ_c, in addition to the deuteron and Λ masses. As mentioned before, this integral

[2]The coupling constant $g_1(\Lambda_c)$ is a dimensionless quantity.

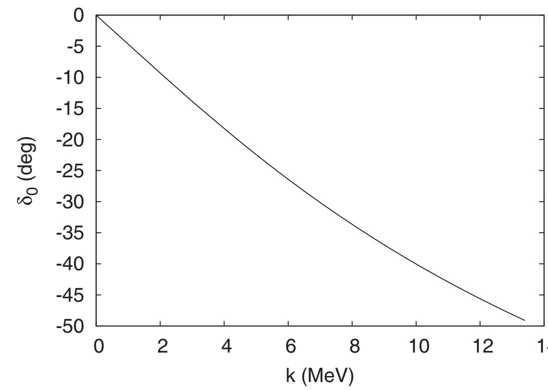

FIG. 6. Phase shift δ_0 (in degrees) of the S-wave hypertriton-Λ scattering in the spin singlet channel as a function of momentum k (in MeV).

equation is insensitive to the value of Λ_c and thus the scattering in the $S = 0$ channel is well controlled by one effective range parameter, $\gamma_{\Lambda d}$.

The scattering length a_0 of the S-wave hypertriton-Λ scattering in the $S = 0$ channel is then computed by taking the limit for the on-shell momentum $k \to 0$, which leads to $T(0,0) = -\frac{2\pi}{\mu_{\Lambda(\Lambda d)}}a_0$. Here, we introduce the half-off-shell scattering length $a(p,0)$ as

$$a(p,0) = -\frac{\mu_{\Lambda(\Lambda d)}}{2\pi}\, T(p,0), \tag{24}$$

so that it reduces to the scattering length as $a_0 = a(0,0)$.

We numerically calculate the off-diagonal part of the scattering length $a_0(p,0)$ with $\Lambda_c \sim 170$ MeV to find that the off-diagonal part of the scattering length becomes indeed very small when the off-shell momentum p is larger than the large scale $\Lambda_H \sim \gamma \simeq 45.7$ MeV. We also calculate the scattering length $a_0(0,0)$ as a function of the cutoff Λ_c to find that $a(0,0)$ is nearly independent of the cutoff if it is relatively small, such as $\Lambda_c \simeq 20$ MeV. Therefore, the S-wave hypertriton-Λ scattering in the spin singlet channel would be well described by considering the cutoff region of $\Lambda_c \simeq \Lambda_H$. From this procedure we obtain

$$a_0 = 16.0 \pm 3.0 \text{ fm}, \tag{25}$$

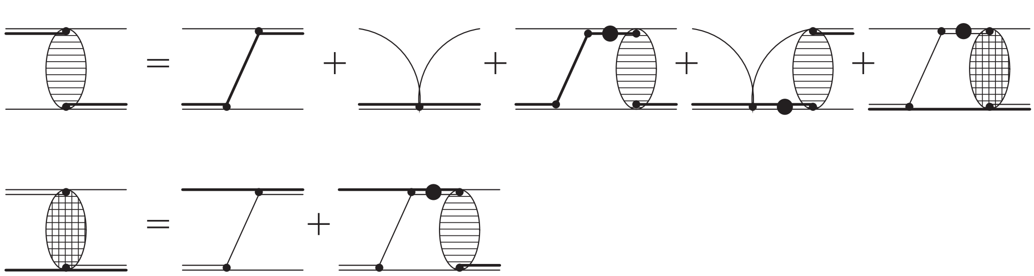

FIG. 5. Diagrams of coupled integral equations for S-wave scattering of the hypertriton and Λ for the spin triplet ($S = 1$) channel. See the captions of Figs. 1 and 3 as well.

which is our prediction on the scattering length, where the error was estimated from the uncertainties in $\gamma_{\Lambda d}$.[3]

In Fig. 6, the calculated phase shift δ_0 of the S-wave hypertriton-Λ scattering in the spin singlet channel is presented as a function of k. The form of the calculated phase shift δ_0 determines the two effective range parameters as $a_0 \simeq 16.0$ fm and $r_0 \simeq 2$ fm. In addition, we find no limit-cycle in the numerical calculation of the integral equation within the range up to $\Lambda_c \sim 10^8$ MeV.

B. $^{\ 4}_{\Lambda\Lambda}$H bound state and S-wave scattering of hypertriton-Λ in the $S = 1$ channel

For the spin triplet channel, the coupled integral equations can be rewritten in terms of the half-off-shell scattering amplitudes $a_1(p,k)$ and $b_1(p,k)$ which are defined by

$$a_1(p,k) = -\frac{Z_{\Lambda d}}{2\pi}\mu_{\Lambda(\Lambda d)}\left[K_{(a)}(p,k;E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2}\right] - \frac{1}{2\pi^2}\int_0^{\Lambda_c} d\ell\,\ell^2\left[K_{(a)}(p,\ell;E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2}\right] D_t\left(E - \frac{\ell^2}{2m_\Lambda},\boldsymbol{\ell}\right) a_1(\ell,k)$$
$$- \frac{1}{2\pi^2}\int_0^{\Lambda_c} d\ell\,\ell^2 K_{(b1)}(p,\ell;E) D_s\left(E - \frac{\ell^2}{2m_d},\boldsymbol{\ell}\right) b_1(\ell,k), \tag{26}$$

$$b_1(p,k) = -\frac{Z_{\Lambda d}}{2\pi}\mu_{\Lambda(\Lambda d)}K_{(b2)}(p,k;E) - \frac{1}{2\pi^2}\int_0^{\Lambda_c} d\ell\,\ell^2 K_{(b2)}(p,\ell;E) D_t\left(E - \frac{\ell^2}{2m_\Lambda},\boldsymbol{\ell}\right) a_1(\ell,k), \tag{27}$$

with the normalizations

$$a_1(k,k) = \sqrt{Z_{\Lambda d}}\,a(k,k)\sqrt{Z_{\Lambda d}}\,,$$
$$b_1(k,k) = \sqrt{Z_{\Lambda d}}\,b(k,k)\sqrt{Z_{\Lambda d}}\,. \tag{28}$$

The scattering length a_1 is then defined as

$$a_1 = -\frac{\mu_{\Lambda(\Lambda d)}}{2\pi}a_1(0,0). \tag{29}$$

Because the effect from the unphysical singularities in the dressed dibaryon and hypertriton propagators (D_s and D_t) on the scattering length a_1 is significant, we employ the KSW counting rules and expand the propagators and the wave function normalization factor $Z_{\Lambda d}$ in terms of the effective ranges $r_{\Lambda d}$ and $r_{\Lambda\Lambda}$, as discussed in Sec. I. Therefore, at LO, the propagators D_t and D_s and the wave function normalization factor $Z_{\Lambda d}$ are written as

$$D_t^{\rm LO}\left(E - \frac{\ell^2}{2m_\Lambda},\boldsymbol{\ell}\right) = -\frac{2\pi\,\mu_{\Lambda(\Lambda d)}}{\mu_{\Lambda d}^2 y_t^2}\left[\gamma_{\Lambda d} + \sqrt{\gamma_{\Lambda d}^2 + \frac{\mu_{\Lambda d}}{\mu_{\Lambda(\Lambda d)}}(\ell^2 - k^2)}\right]\frac{1}{\ell^2 - k^2 - i\epsilon}\,, \tag{30}$$

$$D_s^{\rm LO}\left(E - \frac{\ell^2}{2m_d},\boldsymbol{\ell}\right) = \frac{4\pi}{m_\Lambda y_s^2}\left[\frac{1}{a_{\Lambda\Lambda}} - \sqrt{\frac{m_\Lambda}{2\mu_{\Lambda d}}\gamma_{\Lambda d}^2 - \frac{m_\Lambda}{2}\left(\frac{\ell^2}{\mu_{d(\Lambda\Lambda)}} - \frac{k^2}{\mu_{\Lambda(\Lambda d)}}\right)}\right]^{-1}\,, \tag{31}$$

$$Z_{\Lambda d}^{\rm LO} = \gamma_{\Lambda d}\,r_{\Lambda d}\,, \tag{32}$$

where $\mu_{d(\Lambda\Lambda)}$ is the reduced mass of the d-($\Lambda\Lambda$) system, $\mu_{d(\Lambda\Lambda)} = 2m_\Lambda m_d/(2m_\Lambda + m_d)$.

In addition to the masses, therefore, we have four parameters, namely, $\gamma_{\Lambda d}$, $a_{\Lambda\Lambda}$, $g_1(\Lambda_c)$, and Λ_c.[4] In the present work, we fix $\gamma_{\Lambda d}$ by the hypertriton binding energy. The parameter $a_{\Lambda\Lambda}$ may be determined from other available empirical information. However, there exists no available information from the three-body system to constrain the value of $g_1(\Lambda_c)$. In the present work, therefore, instead of studying the energy levels of the $^{\ 4}_{\Lambda\Lambda}$H hypernucleus, we examine the effect of the coupling $g_1(\Lambda_c)$ in this system.

1. *Scattering length a_1 without three-body contact interaction*

We first consider the case when $g_1(\Lambda_c) = 0$ and calculate the two-Λ separation energy $B_{\Lambda\Lambda}$ in the $^{\ 4}_{\Lambda\Lambda}$H bound state and the scattering length a_1 of the S-wave hypertriton-Λ scattering for the spin triplet channel at LO. In this case we find that there is no bound state formed with the cutoff value in the range of $\Lambda_c = 50$–300 MeV.

In Fig. 7, we present our results for the LO scattering length a_1 with several values of $a_{\Lambda\Lambda}$, namely, $a_{\Lambda\Lambda} = -0.5$, -1.0, -1.5, -2.0 fm, as a function of the momentum cutoff Λ_c.

[3] Alternatively, one may include the effective range $r_{\Lambda d}$ in the dressed propagator, as in Refs. [33,34] for the studies on the S-wave neutron-deuteron scattering in the spin quartet channel within pionless EFT. If we take this procedure, we would obtain $a_0 = 17.3 \pm 2.9$ fm.

[4] In principle, the integral equation depends on the effective ranges $r_{\Lambda d}$ and $r_{\Lambda\Lambda}$ through the coupling constants $y_{t,s}$ and the normalization factor $Z_{\Lambda d}^{\rm LO}$. But this dependence is canceled or included in the normalization of the amplitude b_1, and, therefore, they do not appear in the final expressions.

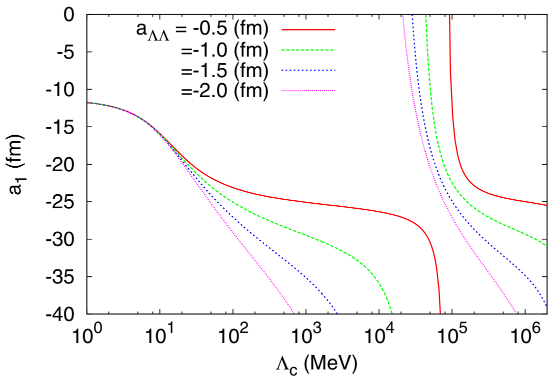

FIG. 7. (Color online) Scattering length a_1 of S-wave hypertriton-Λ scattering in the $S = 1$ channel at leading order as a function of Λ_c for $a_{\Lambda\Lambda} = -0.5, -1.0, -1.5, -2.0$ fm.

This shows that the calculated a_1 curves show a significant dependence on Λ_c as well as on $a_{\Lambda\Lambda}$. The $a_{\Lambda\Lambda}$ dependence of a_1 becomes more significant when Λ_c is larger than Λ_H as shown in Fig. 7. When the cutoff parameter Λ_c is down close to the large scale of the theory, i.e., $\Lambda_c \simeq \Lambda_H \sim 45.7$ MeV, such a dependence becomes mild. We then obtain negative values for the scattering length, namely, $a_1 \simeq -21.7, -22.7, -23.8, -24.8$ fm for $a_{\Lambda\Lambda} = -0.5, -1.0, -1.5, -2.0$ fm, respectively, with $\Lambda_c = 45.7$ MeV. Since a_1 is negative and its magnitude is large, it may imply a formation of a quasibound state.

As Λ_c increases, a_1 decreases until it shows a pole-structure at around $\Lambda_c \sim 80$, 33, 17, 10 GeV depending on the value of $a_{\Lambda\Lambda}$. After passing the pole, a_1 changes the sign as shown in Fig. 7. This corresponds to a formation of a bound state with zero binding energy at such a huge cutoff. In other words, the one-deuteron-exchange interaction has a sensitivity to Λ_c and it becomes attractive enough to make a bound state at the asymptotic limit of the cutoff.

To make the result cutoff-independent, however, one needs to promote the three-body contact interaction at LO so that the cutoff dependence is controlled by the additional coupling constant [26]. We work on in this scheme below.

2. $^{4}_{\Lambda\Lambda}$H bound state with three-body contact interaction

We now consider the case with $g_1(\Lambda_c) \neq 0$ to investigate its role in the $^{4}_{\Lambda\Lambda}$H hypernucleus. Since there is no experimental information to constrain the value of $g_1(\Lambda_c)$, we adopt the values of this coupling constant determined as follows. We first assume a formation of the $^{4}_{\Lambda\Lambda}$H bound state due to the three-body-contact interaction and fit $g_1(\Lambda_c)$ to reproduce the potential model results of Refs. [11,12]. To be specific, we choose the following three sets for $B_{\Lambda\Lambda}$ and $a_{\Lambda\Lambda}$:

$$\begin{aligned} &\text{(I)} \quad B_{\Lambda\Lambda} \simeq 0.2 \text{ MeV and } a_{\Lambda\Lambda} = -0.5 \text{ fm}, \\ &\text{(II)} \quad B_{\Lambda\Lambda} \simeq 0.6 \text{ MeV and } a_{\Lambda\Lambda} = -1.5 \text{ fm}, \\ &\text{(III)} \quad B_{\Lambda\Lambda} \simeq 1.0 \text{ MeV and } a_{\Lambda\Lambda} = -2.5 \text{ fm}. \end{aligned} \tag{33}$$

In Fig. 8, we show the calculated strength of the three-body contact interaction $g_1(\Lambda_c)$ as a function of Λ_c, which can reproduce the three parameter sets of Eq. (33). One can see that the curves of $g_1(\Lambda_c)$ are rather mildly varying

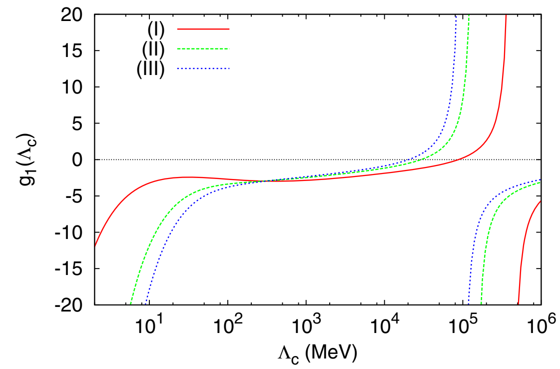

FIG. 8. (Color online) Coupling $g_1(\Lambda_c)$ of three-body contact interaction as a function of the cutoff Λ_c which produces a bound state of $^{4}_{\Lambda\Lambda}$H with three different sets of $B_{\Lambda\Lambda}$ and $a_{\Lambda\Lambda}$. See the text for the parameter sets (I), (II), and (III).

at $\Lambda_c = 10$–10^4 MeV, and each curve has a singularity at $\Lambda_c \sim 10^5$ MeV indicating the possibility of the first cycle of the limit-cycle. This implies that the one-deuteron-exchange interaction for the $S = 1$ channel contains an attractive (singular) interaction at very high momentum, say, $\Lambda_c \sim 10^5$ MeV. This property has also been observed in the calculation of a_1 as shown in Fig. 7. At such a very high momentum, however, the applicability of the present theory, a very low energy EFT, cannot be guaranteed and thus the mechanisms of the formation of a bound state must have different origins. We note, on the other hand, that, if we choose $g_1(\Lambda_c) \simeq -2$ or smaller at $\Lambda_c \sim 50$ MeV in the coupled integral equations, a bound state can be created. Such a value of $g_1(\Lambda_c)$ is in a natural size and may be generated from the mechanisms of high energy such as σ-meson exchange or two-pion exchange near the intermediate range of nuclear force, i.e., $\Lambda_c = 300$–600 MeV.

In order to study the correlation between $B_{\Lambda\Lambda}$ and $a_{\Lambda\Lambda}$, we calculate $B_{\Lambda\Lambda}$ as a function of $a_{\Lambda\Lambda}$ and show the results in Fig. 9 for various cutoff values, i.e., $\Lambda_c = 50$, 150, 300 MeV.

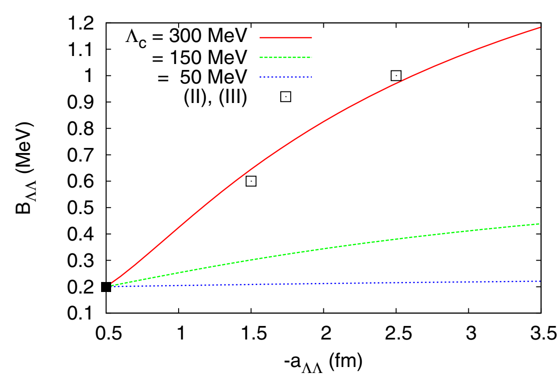

FIG. 9. (Color online) Calculated two-Λ separation energy $B_{\Lambda\Lambda}$ from $^{4}_{\Lambda\Lambda}$H bound state as a function of the scattering length $a_{\Lambda\Lambda}$ of the S-wave $\Lambda\Lambda$ scattering for the $^1S_0$ channel with the cutoff values $\Lambda_c = 50$, 150, 300 MeV. The value of $g_1(\Lambda_c)$ of all three curves is fitted at the point (I): $B_{\Lambda\Lambda} = 0.2$ MeV and $a_{\Lambda\Lambda} = -0.5$ fm, marked by a filled square. The points (II) and (III) are also included as blank squares in the figure.

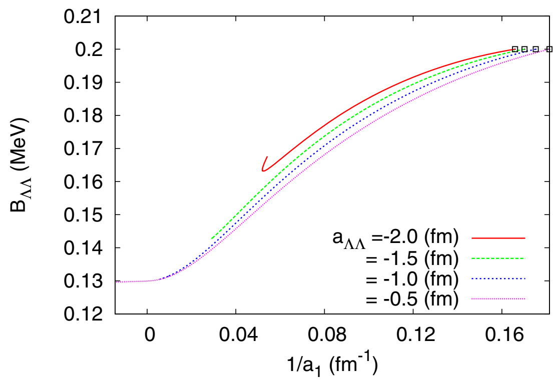

FIG. 10. (Color online) Correlations between $B_{\Lambda\Lambda}$ and $1/a_1$ with $a_{\Lambda\Lambda} = -2.0, -1.5, -1.0, -0.5$ fm. The coupling $g_1(\Lambda_c)$ is fixed by $B_{\Lambda\Lambda} = 0.2$ MeV and $\Lambda_c = 50$ MeV, marked by open squares in the upper-right corner, for each value of $a_{\Lambda\Lambda}$. The curves are obtained by varying Λ_c from 50 MeV to 300 MeV.

Here, the coupling $g_1(\Lambda_c)$ is fixed by using the parameter set (I), i.e., $B_{\Lambda\Lambda} = 0.2$ MeV and $a_{\Lambda\Lambda} = -0.5$ fm, which is marked by a filled square in Fig. 9. This is achieved with $g_1(\Lambda_c) \simeq -2.48, -2.83, -2.96$ for $\Lambda_c = 50, 150, 300$ MeV, respectively. Once the starting values are fixed, we vary the value of $a_{\Lambda\Lambda}$ for a fixed value of Λ_c, which changes the values of $B_{\Lambda\Lambda}$. We then find that the behaviors of the $B_{\Lambda\Lambda}$ curves as functions of $a_{\Lambda\Lambda}$ are quite sensitive to the values of the cutoff Λ_c. For example, when we choose $\Lambda_c \simeq \Lambda_H$, i.e., $\Lambda_c = 50$ MeV, $B_{\Lambda\Lambda}$ is insensitive to the value of $a_{\Lambda\Lambda}$ and makes a nearly flat curve as shown by the dotted line in Fig. 9. However, with a larger cutoff value, $\Lambda_c = 300$ MeV, $B_{\Lambda\Lambda}$ strongly depends on $a_{\Lambda\Lambda}$ and we can fairly well reproduce the $a_{\Lambda\Lambda}$ dependence of $B_{\Lambda\Lambda}$ obtained by Filikhin and Gal [11] or Nemura *et al*. [12].

This may imply that the main part of the correlation between $B_{\Lambda\Lambda}$ and $a_{\Lambda\Lambda}$ in potential model calculations is related to the high momentum part and, when we choose the cutoff $\Lambda_c \simeq \Lambda_H$, the mechanisms with high momentum are integrated out and their effects are absorbed by the renormalized three-body contact interaction $g_1(\Lambda_c)$. Thus we do not have the dynamics that is sensitive to the high momentum regime and this leads to the cutoff-insensitive results. Therefore, when we choose $\Lambda_c \simeq \Lambda_H = 50$ MeV in our cluster EFT, the theory does not to adequately probe the Λ-Λ interactions, but, when we choose $\Lambda_c = 300$ MeV, we can fairly well reproduce the results obtained in the potential model calculations. However, in the latter case, the theory becomes inconsistent because of neglecting other mechanisms relevant in the high momentum region, such as the channels of deuteron breakup into two nucleons and of meson-exchanges among baryons.

In Fig. 10, we present our results on the correlation between $B_{\Lambda\Lambda}$ and $1/a_1$ with four values of $a_{\Lambda\Lambda}$ where $g_1(\Lambda_c)$ is fixed by using the condition that $B_{\Lambda\Lambda} = 0.2$ MeV at $\Lambda_c = 50$ MeV. Thus with $\Lambda_c = 50$ MeV, we have $g_1 = -2.48, -2.45, -2.43, -2.40$ for $a_{\Lambda\Lambda} = -0.5, -1.0, -1.5, -2.0$ fm, respectively. Then the curves are obtained by varying Λ_c from 50 to 300 MeV with the fixed values of g_1 determined at $\Lambda_c = 50$ MeV. We find that, at $\Lambda_c = 50$ MeV, which gives the starting points of the curves at the top right corner (marked by open squares), the calculated scattering length a_1 at LO turned out to be positive due to the existence of the ${}_{\Lambda\Lambda}^{4}$H bound state, and the positions of these points are not sensitive to the value of $a_{\Lambda\Lambda}$, as was seen in Fig. 9 for the case of $B_{\Lambda\Lambda}$ with $\Lambda_c = 50$ MeV. Thus we have $a_1 \sim 5.7$ fm corresponding to $B_{\Lambda\Lambda} \simeq 0.2$ MeV. By increasing the cutoff values, we obtain the lower values of $B_{\Lambda\Lambda}$. When $a_{\Lambda\Lambda} = -0.5$ fm, the ${}_{\Lambda\Lambda}^{4}$H bound state eventually becomes unbound, and when $a_{\Lambda\Lambda} = -2.0$ fm, $B_{\Lambda\Lambda}$ has a minimum and then starts to increase with increasing cutoff. We also find that the correlations do not show the sensitivity to $a_{\Lambda\Lambda}$.

VI. SUMMARY AND DISCUSSION

In the present work, we studied the ${}_{\Lambda\Lambda}^{4}$H bound state and S-wave hypertriton-Λ scattering for spin singlet and triplet channels below the hypertriton breakup momentum in halo EFT at LO by treating the ${}_{\Lambda\Lambda}^{4}$H system as a three-body $\Lambda\Lambda d$ cluster system. In this approach, the hypertriton breakup momentum $\gamma_{\Lambda d} \simeq 13.4$ MeV is chosen to be the typical scale Q of the theory, whereas the deuteron binding momentum $\gamma \simeq 45.7$ MeV is chosen to be the high momentum scale Λ_H. Thus, in such a small typical momentum scale, the deuteron is not broken into two nucleons, which justifies the treatment of the deuteron field as a cluster (elementary) field. Furthermore, our expansion parameter is $Q/\Lambda_H \sim \gamma_{\Lambda d}/\gamma \sim 1/3$.

For the spin singlet channel of the S-wave hypertriton-Λ scattering, the amplitude is nearly independent of the cutoff, thus there is no need to introduce the three-body contact interaction at LO. Consequently, the integral equation at LO is well described by one effective range parameter, $\gamma_{\Lambda d}$. This leads to the value of the scattering length a_0 of the S-wave hypertriton-Λ scattering for the spin singlet channel as $a_0 = 16.0 \pm 3.0$ fm. We also found no bound state in this channel at LO.

For the spin triplet channel of the S-wave scattering of hypertriton and Λ, the scattering amplitudes are obtained through two coupled integral equations. We find that when the cutoff parameter Λ_c is close to the asymptotic limit, the coupling of the three-body contact interaction, i.e., $g_1(\Lambda_c)$, exhibits the limit-cycle, and thus the three-body contact interaction should be included in the spin triplet channel. Consequently the coupled integral equations are represented by four parameters, $\gamma_{\Lambda d}, a_{\Lambda\Lambda}, g_1(\Lambda_c)$, and Λ_c. The value of $\gamma_{\Lambda d}$ can be fixed from the Λ separation energy of the hypertriton and that of $a_{\Lambda\Lambda}$ may be fixed from other experiments or possibly lattice QCD simulations. However, there is no available experimental data to constrain the value of $g_1(\Lambda_c)$.

When we do not introduce $g_1(\Lambda_c)$ in the theory, we obtain $a_1 \simeq -25 \sim -22$ fm with $\Lambda_c \simeq \Lambda_H$. This may imply that the hypertriton-Λ interaction is attractive but it is not strong enough to form a bound state. Thus, if the ${}_{\Lambda\Lambda}^{4}$H bound state is formed, the main binding mechanism should stem from the mechanisms of high momentum region, which is represented by the coupling $g_1(\Lambda_c)$ in the present approach. Therefore, to take into account this effect, we assume a formation of the ${}_{\Lambda\Lambda}^{4}$H bound state and employ the results of the potential model calculations for the two-Λ separation energy $B_{\Lambda\Lambda}$ for several values of $a_{\Lambda\Lambda}$ to constrain the value of $g_1(\Lambda_c)$. Using the fixed $g_1(\Lambda_c)$ we then calculate $B_{\Lambda\Lambda}$ as a function of $a_{\Lambda\Lambda}$. We also calculate the correlations between $B_{\Lambda\Lambda}$ and $1/a_1$, where a_1 is the scattering length of the S-wave hypertriton-Λ scattering for spin triplet channel.

As can be notably seen in the numerical results for the correlation between $B_{\Lambda\Lambda}$ and $a_{\Lambda\Lambda}$ as given in Fig. 9, when the cutoff is chosen to be the large scale of the theory, i.e., $\Lambda_c \simeq \Lambda_H$, $B_{\Lambda\Lambda}$ is insensitive to the value of $a_{\Lambda\Lambda}$. But, when Λ_c is larger than Λ_H, say $\Lambda_c \simeq 6\Lambda_H$, $B_{\Lambda\Lambda}$ is sensitive to $a_{\Lambda\Lambda}$, which gives results similar to the potential model predictions. This would be a natural consequence because $a_{\Lambda\Lambda}^{-1}$ is a quantity of a large scale, $|a_{\Lambda\Lambda}^{-1}| \simeq$ 100–400 MeV, compared to the typical scale of the system, $Q \sim \gamma_{\Lambda d} \simeq 13.4$ MeV. In addition, the dynamics that exhibits the sensitivity to the Λ-Λ interaction above $\Lambda_c \simeq \Lambda_H$ is integrated out and its effect in high momentum is embedded in the contact interaction $g_1(\Lambda_c)$. Meanwhile, although the deuteron cluster theory with a large cutoff value such as $\Lambda_c \simeq 6\Lambda_H$ can reproduce the $a_{\Lambda\Lambda}$ dependence of $B_{\Lambda\Lambda}$ similar to the potential model predictions, this would be inconsistent with the construction principles of EFT and it will miss the important dynamic mechanisms as discussed before. Therefore, the $a_{\Lambda\Lambda}$ sensitivities in the physical observables for the $_{\Lambda\Lambda}^{\ 4}$H hypernucleus, such as $B_{\Lambda\Lambda}$, inevitably depend on the scale of the theory. Investigating the $a_{\Lambda\Lambda}$ sensitivity in more detail at another scale in the $_{\Lambda\Lambda}^{\ 4}$H system requires us to work with a noncluster theory such as the pionless theory for four-body systems [35].

Experimentally, we still do not have enough information to judge whether the $_{\Lambda\Lambda}^{\ 4}$H system is bound or not. This causes the difficulty in studying the energy levels of the $_{\Lambda\Lambda}^{\ 4}$H hypernucleus within EFT since the value of the contact interaction $g_1(\Lambda_c)$ cannot be constrained by other information. Therefore, it would be interesting to apply this approach to other double-Λ hypernuclei where some empirical data are available, such as the $_{\Lambda\Lambda}^{\ 6}$He system. The $_{\Lambda\Lambda}^{\ 6}$He hypernucleus as a $\Lambda\Lambda\alpha$ three-body cluster system can be investigated in the scheme of EFT. Because the binding energy, or equivalently the two-Λ separation energy, of $_{\Lambda\Lambda}^{\ 6}$He is experimentally known, it can be used to determine the strength of the three-body contact interaction in the $\Lambda\Lambda\alpha$ system. Moreover, because the α particle is more tightly bound than the deuteron, the high momentum scale of the cluster theory becomes larger than Λ_H of the present work. Therefore, the study of $_{\Lambda\Lambda}^{\ 6}$He in halo/cluster EFT can provide another tool to study $a_{\Lambda\Lambda}$ in exotic systems and shed light on our understanding of strong interactions in the strangeness sector. Work in this direction is in progress and will be reported elsewhere.

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