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In-medium η' mass and $\eta' N$ interaction based on chiral effective theory

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The in-medium η' mass and the $\eta' N$ interaction are investigated in an effective theory based on the linear realization of the SU(3) chiral symmetry. We find that a large part of the η' mass is generated by the spontaneous breaking of chiral symmetry through the $U_A(1)$ anomaly. As a consequence of this observation, the η' mass is reduced in nuclear matter where chiral symmetry is partially restored. In our model, the mass reduction is found to be 80 MeV at the saturation density. Estimating the $\eta' N$ interaction based on the same effective theory, we find that the $\eta' N$ interaction in the scalar channel is attractive sufficiently to form a bound state in the $\eta' N$ system with a several MeV binding energy. We discuss the origin of attraction by emphasizing the special role of the σ meson in the linear sigma model for the mass generation of η' and N.

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I. INTRODUCTION

The η' meson has a large mass compared to the other pseudoscalar mesons, such as π, K, or η. The mass spectrum of low-lying pseudoscalar mesons has been discussed as the $U_A(1)$ problem [1]. The mass of η' can be explained by the $U_A(1)$ anomaly in quantum chromodynamics (QCD) [2,3]. The quantum anomaly is the phenomenon that symmetries in the classical level are broken by quantum effects. The QCD Lagrangian is invariant under $U_A(1)$ transformation for the quark field, but the symmetry is broken explicitly by the quark loop effect, and the divergence of the $U_A(1)$ current does not vanish [4]. When chiral symmetry is broken spontaneously, the nonzero divergence of the $U_A(1)$ current permits the nonvanishing mass of the pseudoscalar flavor-singlet meson even in the chiral limit.

The medium effect to the η' mass through the effective $U_A(1)$ restoration has been discussed. The effective $U_A(1)$ restoration is caused by the in-medium decrease of the instanton density [5,6]. The reduction of the instanton density in the medium may lead to the suppression of the expectation value of the $U_A(1)$ current divergence in the medium. The vanishing expectation value of the $U_A(1)$ current for the vacuum and η' states forces the η' meson to be massless in the same way as the other pseudoscalar mesons.

Apart from the effective $U_A(1)$ restoration, as we will discuss later in detail, chiral symmetry breaking is indispensable to the mass difference between the pseudoscalar flavor-singlet and flavor-octet mesons in addition to the $U_A(1)$ anomaly. Recently, the reduction of the absolute value of the quark condensate—which is referred to as partial restoration of chiral symmetry—in the nuclear medium has been discussed intensively from the theoretical and experimental points of view, and it is suggested by the analysis of experimental data of pionic atoms that the partial restoration actually does take place in nuclei [7]. If one takes account of the necessity of chiral symmetry breaking in the generation of the η' mass, it is expected that the flavor-singlet meson mass decreases in the nuclear medium, in which chiral symmetry is partially restored [8].

There are many theoretical works [6,9–22] and experimental attempts [23–26] involved in determining the in-medium η' properties from various points of view. Particularly, the effect of chiral symmetry on the η' meson is discussed in Refs. [8,27,28].

The mass reduction of η' in the nuclear medium implies that the η' meson feels attraction in the nuclear medium because the mass modification is represented by the self-energy of the meson in the medium, and the self-energy turns out to be the optical potential in the nonrelativistic limit. The attraction in nuclear matter suggests an attractive $\eta' N$ two-body force as an elementary interaction. If it is enough strong, we expect an $\eta' N$ bound state. This is an analogous state of $\Lambda(1405)$, which is considered as a bound state of $\bar{K}N$.

So far, the interaction between η' and N is not known. We do not know even whether it is attractive or repulsive. There are some experimental suggestions about the $\eta' N$ scattering length and the in-medium η' properties. From the $pp \to pp\eta'$ process, the scattering length of $\eta' p$ has been extracted and its value has been estimated to be about 0.8 fm [23] or 0.1 fm with the sign undetermined [24]. The absorption of η' into nuclei has been extracted by the $\gamma p \to \eta' p$ process in nuclei, and the absorption of η' is relatively small compared to that of the ω meson [25]. These experimental data suggest the weakness of the $\eta' N$ interaction. On the other hand, a large mass reduction of η' has been reported from the analysis of the low-energy pion distribution in relativistic heavy ion collisions [26]. This suggests a strong attraction between $\eta' N$ if one considers that this mass reduction occurs due to the partial restoration of chiral symmetry. The $\eta' N$ interaction and in-medium properties of η' should be understood in a unified manner, and theoretical study concerning the $\eta' N$ interaction is progressing [19].

In this paper, taking partial restoration of chiral symmetry in the nuclear medium as a basis of our argument, we estimate the amount of the expected η' mass reduction in the nuclear medium and the two body interaction strength of $\eta' N$ in vacuum. A preliminary account of this work was reported at a conference proceedings [29]. In this paper, we explain fully the details of the model that we use and the calculation method. We also discuss the dependence of the results on the model. In Sec. II, we explain the relation of the η' meson and the chiral symmetry breaking. In Sec. III, we introduce an effective Lagrangian for the η' meson in the nuclear medium based on the linear sigma model, and evaluate the in-medium mass reduction of the η'. In Sec. V, we show the obtained $\eta' N$ interaction strength and the binding energy and scattering length of the $\eta' N$ in vacuum. The conclusion and some remarks are given in Sec. VI.

II. THE RELATION BETWEEN THE η' MESON AND CHIRAL SYMMETRY

The mass difference between the η and η' mesons has been discussed based on the QCD partition function [27,28] or the SU(3) chiral symmetry [8]. The $U_A(1)$ symmetry is broken explicitly due to the quantum effect. Therefore, with spontaneous chiral symmetry breaking, the η' meson can have a finite mass even in the chiral limit, contrary to the other pseudoscalar Nambu–Goldstone (NG) bosons. But, the $U_A(1)$ anomaly effect lifting the η' meson mass in vacuum cannot affect the pseudoscalar mass spectrum when chiral symmetry is restored. This is because the η and η' mesons masses should degenerate in the SU(3) chiral symmetric phase even if $U_A(1)$ symmetry is explicitly broken by the anomaly effect according to Refs. [8,27,28].

In the following, we explain the mechanism of the degeneracy of the pseudoscalar flavor singlet and octet mesons based on the SU(3) chiral symmetry [8]. We consider the three-flavor chiral symmetry $SU(3)_L \otimes SU(3)_R$, and we assume that the effect of the change of the instanton density near normal nuclear density on the η' mass is small compared to the effect of partial restoration of chiral symmetry.

First, we define the transformation properties of the quark field under the $SU(3)_L \otimes SU(3)_R$ transformation. The left-handed quark q_L and the right-handed quark q_R are defined as

$$q_L = \frac{1-\gamma_5}{2} q, \tag{1}$$

$$q_R = \frac{1+\gamma_5}{2} q. \tag{2}$$

Because the quark fields, q_L and q_R, belong to the fundamental representations of $SU(3)_L$ and $SU(3)_R$ respectively, the transformation properties of the quark fields under $SU(3)_L \otimes SU(3)_R$ are written as

$$q_i \to e^{i\theta_i^a \lambda^a/2} q_i \quad (i = L, R). \tag{3}$$

Here, λ^a ($a = 1,\ldots,8$) is the Gell-Mann matrix.

The QCD Lagrangian is invariant under the $SU(3)_L \otimes SU(3)_R$ transformation in the chiral limit. When $\theta_R = \theta_L \equiv \theta_V$, the transformation for the quark field q is written as

$$q \to e^{i\theta_V^a \lambda^a/2} q. \tag{4}$$

We call this transformation the vector transformation. When $\theta_R = -\theta_L \equiv \theta_A$, the transformation for the quark field q is written as

$$q \to e^{i\theta_A^a \gamma_5 \lambda^a/2} q. \tag{5}$$

We call this transformation the axial transformation. For an infinitesimal transformation, the quark field transforms as

$$q \to \left(1 + i\theta_A^a \gamma_5 \frac{\lambda^a}{2}\right) q. \tag{6}$$

This implies

$$\left[Q_A^a, q\right] = -\tfrac{1}{2}\lambda^a \gamma_5 q \tag{7}$$

and

$$\left[Q_A^a, \bar{q}\right] = -\tfrac{1}{2}\bar{q}\lambda^a \gamma_5 \tag{8}$$

with the generator of the axial transformation Q_A^a.

Under the $SU(3)_L \otimes SU(3)_R$ symmetry, the hadron fields can be classified in terms of the irreducible representation of $SU(3)_L \otimes SU(3)_R$. Assuming that the mesons are composed of the quark bilinear form and that parity invariance is satisfied in vacuum, the meson fields belong to the $(\mathbf{3}_L, \bar{\mathbf{3}}_R) \oplus (\bar{\mathbf{3}}_L, \mathbf{3}_R)$ representation. In terms of the vector transformation, the meson fields belonging to $(\mathbf{3}_L, \bar{\mathbf{3}}_R) \oplus (\bar{\mathbf{3}}_L, \mathbf{3}_R)$ can be decomposed into octet and singlet representations that are the irreducible representation of $SU(3)_V$, with the fact of $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$. Taking the meson fields as the parity eigenstates, one can obtain the parity-even mesons as $\frac{1}{\sqrt{2}}(\bar{q}_L \frac{1}{\sqrt{3}} q_R + \bar{q}_R \frac{1}{\sqrt{3}} q_L) = \frac{1}{\sqrt{6}}\bar{q}q$, $\frac{1}{\sqrt{2}}(\bar{q}_L \frac{\lambda_a}{2} q_R + \bar{q}_R \frac{\lambda_a}{2} q_L) = \frac{1}{\sqrt{2}}\bar{q}\frac{\lambda_a}{2} q$ and the parity-odd mesons as $\frac{i}{\sqrt{6}}(\bar{q}_L q_R - \bar{q}_R q_L) = \frac{1}{\sqrt{6}}\bar{q} i\gamma_5 q$, $\frac{i}{\sqrt{2}}(\bar{q}_L \frac{\lambda_a}{2} q_R - \bar{q}_R \frac{\lambda_a}{2} q_L) = \frac{1}{\sqrt{2}}\bar{q}\frac{\lambda_a}{2} i\gamma_5 q$. We assign the pseudoscalar octet mesons (π, K, η_8) to $\mathbf{8}$ and singlet (η_0) to $\mathbf{1}$, so the 9 pseudoscalar mesons are settled into a part of the same representation of $SU(3)_L \otimes SU(3)_R$. In the real world, the η and η' mesons are mixed states of η_0 and η_8 owing to the flavor SU(3) symmetry breaking, and their masses are obtained by diagonalizing their mass matrix. In the same way, the scalar mesons ($\sigma_0, a_0, \kappa, \sigma_8$) are assigned to the remainder of the $(\mathbf{3}_L, \mathbf{3}_R) \otimes (\mathbf{3}_R, \mathbf{3}_L)$ representation of $SU(3)_L \otimes SU(3)_R$. From these assignments, the 18 scalar and pseudoscalar mesons belong to the same chiral multiplet of $SU(3)_L \otimes SU(3)_R$.

If one considers the $SU(3)_L \otimes SU(3)_R$ transformation, the η_0 meson can be transformed to other pseudoscalar mesons such as π, K, or η_8. The singlet and octet are irreducible representations in $SU(3)_V$, so the vector transformation alone cannot transform the singlet η_0 into pseudoscalar octet mesons. In contrast, the axial transformation can mix the singlet and octet mesons because the axial transformation is an element not of $SU(3)_V$ but of $SU(3)_L \otimes SU(3)_R$. Thus, the decomposition into singlet and octet makes sense when chiral symmetry is broken, while they are transformed into each other with axial transformations in the case that chiral symmetry exists.

Here, we demonstrate the transformation between these nine pseudoscalar mesons with the $SU(3)_L \otimes SU(3)_R$ transformation explicitly. Using Eq. (7), the flavor singlet pseudoscalar meson field $\eta_0 = \bar{q} i \frac{\gamma_5}{\sqrt{6}} q$ is transformed as

$$\delta^a \left(\bar{q} i \frac{\gamma_5}{\sqrt{6}} q \right) = \left[Q_A^a, \bar{q} i \frac{\gamma_5}{\sqrt{6}} q \right] = \bar{q} \left\{ \frac{i\gamma_5}{\sqrt{6}}, -\frac{\lambda^a}{2}\gamma_5 \right\} q$$
$$= -\bar{q} i \frac{\lambda^a}{\sqrt{6}} q, \qquad (9)$$

and the obtained octet-scalar meson field is transformed as

$$\delta^b \left(-\bar{q} i \frac{\lambda^a}{\sqrt{6}} q \right) = \left[Q_A^b, -\bar{q} i \frac{\lambda^a}{\sqrt{6}} q \right] = \bar{q} \left\{ -i \frac{\lambda^a}{\sqrt{6}}, -\frac{\lambda^b}{2}\gamma_5 \right\} q$$
$$= d^{abc} \bar{q} i \gamma_5 \frac{\lambda^c}{\sqrt{6}} q. \qquad (10)$$

Equation (9) shows that the singlet pseudoscalar meson is transformed into a scalar octet meson through the first axial transformation, and the second axial transformation changes the flavor-octet scalar meson into a pseudoscalar octet meson in Eq. (10). Thus, the pseudoscalar flavor singlet and octet mesons are transformed into each other under the $SU(3)_L \otimes SU(3)_R$ transformations. This means that the η_0 meson degenerates to the other pseudoscalar mesons when chiral symmetry exists.

Here, it is notable that the degeneracy of the singlet and the octet mesons does not necessarily happen in the $N_f = 2$ case. The case of $N_f = 2$ corresponds to the limit that the strange quark mass, m_s, goes to infinity in $N_f = 3$. So the $SU(3)_L \otimes SU(3)_R$ symmetry is strongly broken. Hence, the mass degeneracy of the η' and pseudoscalar octet mesons does not necessarily take place. This is consistent with the argument in Ref. [28].

With simple assumptions, we can estimate the amount of the mass reduction of η' in a nuclear medium. Here, we take the chiral limit, so η' and η correspond to η_0 and η_8 respectively.

First, we assume the linear dependence of the mass difference of the η and η' on the flavor singlet combination of chiral condensate. With this assumption, the mass difference of η and η' is written using a constant C as

$$m_{\eta'}^2 - m_\eta^2 = C\,(2\langle \bar{q}q \rangle + \langle \bar{s}s \rangle). \qquad (11)$$

Here, we have taken $\langle \bar{q}q \rangle = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle$. From Eq. (11), C can be written as $C = \frac{m_{\eta'}^2 - m_\eta^2}{2\langle \bar{q}q \rangle + \langle \bar{s}s \rangle}$. We suppose that the strangeness condensate $\langle \bar{s}s \rangle$ and the η mass do not change so much in nuclear matter. Substituting the explicit form of C, we obtain

$$m_{\eta'}^2 - m_{\eta'}^{*2} = \frac{2}{3}\left(m_{\eta'}^2 - m_\eta^2\right)\left(1 - \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle}\right), \qquad (12)$$

where $m_{\eta'}^*$ and $\langle \bar{q}q \rangle^*$ denote the in-medium values of the η' mass and the quark condensate, respectively. With the low-density theorem [30], the reduction of the quark condensate at the leading order of the density is written as

$$\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} = 1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho + O(\rho^{4/3}), \qquad (13)$$

where $\sigma_{\pi N}$ is the πN sigma term. With $m_{\eta'}^* = m_{\eta'} - \Delta m_{\eta'}$ and neglecting $(\Delta m_{\eta'})^2$, we obtain the mass reduction of η' as

$$\Delta m_{\eta'} = \frac{2}{3} \frac{m_{\eta'}^2 - m_\eta^2}{2m_{\eta'}} \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho. \qquad (14)$$

Using the observed values of the masses and the decay constant and the typical value for $\sigma_{\pi N}$, which reproduces the 35% reduction of the quark condensate at normal nuclear density, $\Delta m_{\eta'}$ takes a value around 80 to 100 MeV at normal nuclear density.

III. LINEAR SIGMA MODEL

To study η' in nuclear matter and the $\eta' N$ interaction in vacuum, we use the linear sigma model as a chiral effective theory. The linear sigma model is based on the global symmetry same as QCD, and contains the effects of the finite current quark mass and the $U_A(1)$ anomaly [31–34]. The advantages of the linear sigma model are as follows; It has the mechanism of spontaneous chiral symmetry breaking, and it expresses the physical quantities by the sigma condensate, which is the order parameter of the spontaneous chiral symmetry breaking in the linear sigma model. The sigma condensate is given by minimizing the effective potential calculated from the Lagrangian. The sigma condensate characterizes the vacuum to be realized as the ground state. This means that the linear sigma model is a model which can describe the response of the physical quantities caused by the change of the vacuum. In the case of the nonlinear sigma model, the physical quantities are written by the low-energy constants, which should be, in principle, determined again by the information of the vacuum in the nuclear medium. Therefore, the nonlinear sigma model is not suitable for the present aim to directly connect the η' mass with partial restoration of chiral symmetry. In addition, since the hadron is the fundamental degree of freedom in the linear sigma model, we can introduce the nucleon fields straightforwardly. This is a different point from a quark-based model, such as the Nambu–Jona-Lasinio (NJL) model, in which we have to build up the nucleon within the model.

A. The Lagrangian of the linear sigma model

The Lagrangian of the linear sigma model is constructed to possess the same global symmetry as QCD. The fundamental degree of freedom is the hadron. The hadron fields can be assigned the irreducible representation of $SU(3)_L \otimes SU(3)_R$. As the result, the transformation properties of the hadron fields under the $SU(3)_L \otimes SU(3)_R$ transformation are fixed, and the Lagrangian is constructed so as to be invariant under the transformation. Chiral symmetry is spontaneously broken with certain parameter sets, and then the sigma condensate has a nonzero value. In the following, we explain the Lagrangian of the linear sigma model which we use to calculate the in-medium η' mass and the $\eta' N$ interaction.

1. Meson part

As mentioned above, the meson field M belongs to the $(\mathbf{3}, \bar{\mathbf{3}})$ irreducible representation, which means that the meson field transforms as **3** under the $SU(3)_L$ transformation and $\bar{\mathbf{3}}$ under the $SU(3)_R$ transformation. Thus, the transformation rule of the meson field under $SU(3)_L \otimes SU(3)_R$ is

$$M \to \mathrm{LMR}^\dagger, \tag{15}$$

where $L \in SU(3)_L,\ R \in SU(3)_R$. Here, the scalar and pseudoscalar meson field M is written in terms of the physical meson fields as

$$M = M_s + iM_{ps} = \sum_{a=0}^{8} \frac{\lambda_a \sigma_a}{\sqrt{2}} + i \sum_{a=0}^{8} \frac{\lambda_a \pi_a}{\sqrt{2}}, \tag{16}$$

where λ_a $(a = 1,\dots,8)$ is the Gell-Mann matrix and $\lambda_0 = \sqrt{\frac{2}{3}}\mathbf{1}$ with the unit matrix $\mathbf{1}$, which are normalized as

$$\mathrm{tr}\,(\lambda_a \lambda_b) = 2\delta_{ab} \quad (a,b = 0,\dots,8). \tag{17}$$

The explicit form of the pseudoscalar meson field is given as

$$\begin{aligned} M_{ps} &= \sum_{a=0}^{8} \frac{\lambda_a \pi_a}{\sqrt{2}} \\ &= \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 + \frac{\eta_0}{\sqrt{3}} \end{pmatrix}. \end{aligned} \tag{18}$$

To include the effect of the finite current quark mass, we give the quark mass χ a fictitious transformation rule under the $SU(3)_L \otimes SU(3)_R$ transformation to maintain chiral symmetry. If one assumes the transformation rule of χ as

$$\chi \to L\chi R^\dagger, \tag{19}$$

the QCD lagrangian is invariant under the $SU(3)_L \otimes SU(3)_R$ transformation. Here, we take the explicit form of χ as

$$\chi = \sqrt{3}\begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} = \sqrt{3}\begin{pmatrix} m_q & & \\ & m_q & \\ & & m_s \end{pmatrix}, \tag{20}$$

where m_u, m_d, m_s are the up, down, strange quark masses, respectively. Taking $m_u = m_d \equiv m_q$, we introduce the isospin symmetry, and we break the SU(3) flavor symmetry with $m_q \neq m_s$. Owing to the flavor symmetry breaking, $\langle\sigma_8\rangle$ has a nonzero value as does $\langle\sigma_0\rangle$.

The Lagrangian constructed so as to have the same global symmetry as that of QCD is

$$\begin{aligned} \mathcal{L}_{\mathrm{meson}} = &\frac{1}{2}\mathrm{tr}(\partial_\mu M \partial^\mu M^\dagger) - \frac{\mu^2}{2}\mathrm{tr}(MM^\dagger) \\ &- \frac{\lambda}{4}\mathrm{tr}[(MM^\dagger)^2] - \frac{\lambda'}{4}[\mathrm{tr}(MM^\dagger)]^2 \\ &+ A\,\mathrm{tr}\left(\chi M^\dagger + \chi^\dagger M\right) + \sqrt{3}B(\det M + \det M^\dagger). \end{aligned} \tag{21}$$

This Lagrangian has five parameters, $\mu^2,\ \lambda,\ \lambda',\ A,\ B$, which cannot be fixed only by the symmetry. We determine them to reproduce the physical quantities. In this Lagrangian, the fifth term with χ represents the current quark mass contribution (or the flavor symmetry breaking) as mentioned above. The last term proportional to B represents the effect of the $U_A(1)$ anomaly. This term corresponds to the Kobayashi-Maskawa–'t Hooft term [35,36].

When chiral symmetry is broken spontaneously, the sigma condensates, $\langle\sigma_0\rangle$ and $\langle\sigma_8\rangle$, are nonzero. In the broken chiral symmetry phase, the meson masses are written in terms of the sigma condensate because the meson masses are defined as the curvature mass in vacuum where the sigma condensate is a nonzero value. The explicit forms of the meson masses in the $\rho = 0$ vacuum at tree level are shown in Appendix A.

We obtain the relation between the sigma condensate and the meson decay constants from the axial current and the definition of the meson decay constants. The octet axial current A_a^μ $(a = 1,\dots,8)$ is calculated with the Noether theorem as

$$\begin{aligned} A_a^\mu &= \frac{\partial\mathcal{L}}{\partial(\partial_\mu M)}\delta M_a \\ &= \mathrm{tr}[\partial^\mu M_{ps}\{\lambda_a, M_s\} - \partial^\mu M_s\{\lambda_a, M_{ps}\}], \end{aligned} \tag{22}$$

where $\delta M_a = \frac{i}{2}\{\lambda_a, M\}$ is the infinitesimal variation of the meson field under the axial transformation of $SU(3)_L \otimes SU(3)_R$. The definition of the meson decay constant is

$$\langle 0|A_\mu^a|\pi^b(p)\rangle = -ip_\mu f_a \delta^{ab} e^{-ip\cdot x}. \tag{23}$$

Thus, calculating the matrix element of the axial current with Eq. (23), we obtain the relation between the sigma condensates and the meson decay constants as

$$f_\pi = \sqrt{\frac{2}{3}}\langle\sigma_0\rangle + \frac{1}{\sqrt{3}}\langle\sigma_8\rangle, \tag{24}$$

$$f_K = \sqrt{\frac{2}{3}}\langle\sigma_0\rangle - \frac{\langle\sigma_8\rangle}{2\sqrt{3}}. \tag{25}$$

We discuss the relation of the order parameter of the spontaneous chiral symmetry breaking in the linear sigma model and QCD. The quark and hadron quantities can be related by the ansatz that the symmetry property should be shared by both QCD and the linear sigma model. In the linear sigma model parameter, χ represents the quark mass. Equating the derivatives of the partition functions of QCD and the linear sigma model with respect to the quark mass, we obtain the relation between the quark and sigma condensates at the tree level as

$$\langle\bar{q}q\rangle = -2A\left(\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}}\right), \tag{26}$$

$$\langle\bar{s}s\rangle = -2A(\langle\sigma_0\rangle - \sqrt{2}\langle\sigma_8\rangle). \tag{27}$$

The parameters in the Lagrangian are determined so as to reproduce the physical values of the meson masses, the meson decay constants, and the u, d quark mass m_q. The details of parameter fixing are given in Appendix B.

In this paper, we do not consider the density dependence of the parameters. The dependence of parameter B, which represents the effect of the $U_A(1)$ anomaly, is also responsible for the mass reduction of η'. The density dependence of the parameter B is discussed using the instanton-liquid model, and the effect of the anomaly decreases in nuclear matter [37]. Thus, the calculation in this paper gives a lower bound of the η' mass reduction.

2. Baryon part

To consider the change of the meson properties in the nuclear medium, we introduce the nucleon field to the Lagrangian of the meson fields of the SU(3) linear sigma model. The transformation property of baryons is not unique even if one regards the baryon as a composite object of three quarks. The baryon representations which are allowed within the symmetry are $(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$ and $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$ [38]. Here, we use the $(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$ representation. The Lagrangian is written as

$$\mathcal{L}_{\text{baryon}} = \bar{\psi}(i\partial\!\!\!/ - m_N)\psi - g\bar{\psi}\left(\frac{\tilde{\sigma}_0}{\sqrt{3}}\mathbf{1} + \frac{\tilde{\sigma}_8}{\sqrt{6}}\mathbf{1}\right)\psi - g\bar{\psi}i\gamma_5\left(\frac{\vec{\pi}\cdot\vec{\tau}}{\sqrt{2}} + \frac{\eta_0}{\sqrt{3}}\mathbf{1} + \frac{\eta_8}{\sqrt{6}}\mathbf{1}\right)\psi, \quad (28)$$

where $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$, τ_i $(i = 1, \ldots, 3)$ are Pauli matrices, $\sigma_i = \langle\sigma_i\rangle + \tilde{\sigma}_i$, $\mathbf{1}$ is 2×2 unit matrix in the flavor space, and m_N is the nucleon mass. The nucleon fields are represented as

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix}, \quad (29)$$

and the nucleon mass m_N is given by the spontaneous breaking of chiral symmetry as

$$m_N = \frac{g}{\sqrt{3}}\left(\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}}\right). \quad (30)$$

Here, we have shown only the terms relevant for the following calculation.

The free parameter involved in the Lagrangian of the baryon part is the coupling constant g. This parameter g can be determined from the observation that the quark condensate reduces by about 35% at normal density [7].

In the following, we mention the nucleon mass in the linear sigma model. The parameter g determined by the magnitude of partial restoration of chiral symmetry is so small that the nucleon mass in vacuum cannot be reproduced. On the other hand, if we determine g so as to reproduce the in-vacuum nucleon mass, g is too large to restore chiral symmetry fully at densities lower than the saturation density. This problem is known as the Lee-Wick singularity [39]. This inconsistency can be solved, for instance, by introducing the parity doublet baryon [40–43], where a part of the nucleon mass comes from a chiral invariant mass term rather than the spontaneous breaking of chiral symmetry. According to the lowenergy theorem, the interaction between the pseudoscalar meson and baryon is not dependent on the representation of the baryon in $\text{SU}(3)_L \otimes \text{SU}(3)_R$ when chiral symmetry is spontaneously broken. So, we assume that the following calculations are not affected by how we introduce the baryon in the linear sigma model as long as we keep chiral symmetry.

B. The vacuum condition and the medium effect

In the linear sigma model, the vacuum is defined by the minimum point of the effective potential. In this paper, we evaluate the effective potential with the nucleon one-loop approximation. The one-loop diagrams considered in this work are given in Fig. 1. To include the medium effect, we calculate these one-loop diagrams using the nucleon propagator with the Pauli blocking effect. The nucleon propagator is given as

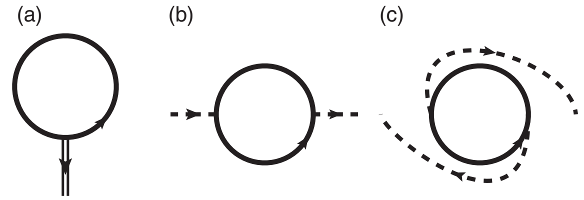

FIG. 1. The medium effect of the nucleon one-loop approximation on the meson mass. The solid line, double-solid line, and dashed line denote a nucleon, scalar meson, and pseudoscalar meson, respectively. Diagram (a) contributes to the determination of the vacuum. Diagrams (b) and (c) are used in the calculation of the in-medium meson mass.

$$P_{\text{med}}(p) = (p\!\!\!/ + m_N)\left\{\frac{i}{p^2 - m_N^2 + i\epsilon} - 2\pi\delta\big(p^2 - m_N^2\big)\theta(p_0)\theta(k_f - |\vec{p}|)\right\}. \quad (31)$$

In the calculation, we regard the nucleon mass as very large and take the leading term of $1/m_N$. Diagram (a) of Fig. 1 contributes to the determination of the vacuum and diagrams (b) and (c) give the in-medium self-energy of the meson and the explicit ρ dependence on the meson mass. We write the contribution to the effective potential from the first diagram of Fig. 1 as $V_{MF}(\rho)$ and the contribution to the meson mass from the second and third diagrams as $\Sigma_{\text{ph}}(\rho)$. Using the propagator including the Pauli blocking effect, $V_{MF}(\rho)$ is calculated as

$$V_{MF}(\rho) = \frac{g\rho}{\sqrt{3}}\left(\sigma_0 + \frac{\sigma_8}{\sqrt{2}}\right), \quad (32)$$

which corresponds to the contribution from the mean-field approximation of the nucleon field. The one-nucleon loop contribution $\Sigma_{\text{ph}}(\rho)$ is obtained as

$$\Sigma_{\text{ph}}(\rho) = C_i\frac{g^2\rho}{m_N}, \quad (33)$$

where $i = \pi,\ \eta_0,\ \eta_8,\ \eta_0\eta_8$ and $C_\pi = \frac{1}{2},\ C_{\eta_0} = \frac{1}{3},\ C_{\eta_8} = \frac{1}{6},\ C_{\eta_0\eta_8} = \frac{1}{3\sqrt{2}}$. These factors C_i are obtained from the meson-baryon coupling constant in the vacuum shown in Eq. (28). The contribution from $\Sigma_{\text{ph}}(\rho)$ corresponds to the nucleon particle-hole excitation. The details of these calculations are shown in Appendix C. In the following, we assume that nuclear matter does not contain the strangeness component.

The value of the sigma condensate is determined by minimizing the effective potential obtained from the linear sigma model Lagrangian. As a result of the introduction of the medium effect, the effective potential for σ_0 and σ_8 of the linear sigma model with the one-loop approximation is given as

$$V_\sigma = \frac{\mu^2}{2}(\sigma_0^2+\sigma_8^2) + \frac{\lambda}{12}\left(\sigma_0^4 + 6\sigma_0^2\sigma_8^2 - 2\sqrt{2}\sigma_0\sigma_8^3 + \frac{3}{2}\sigma_8^4\right) + \frac{\lambda'}{4}(\sigma_0^2+\sigma_8^2)^2 - 2Am_0\sigma_0 - 2Am_8\sigma_8 - \frac{2}{3}B\left(\sigma_0^3 - \frac{3}{2}\sigma_0\sigma_8^2 - \frac{\sigma_8^3}{\sqrt{2}}\right) + \frac{g\rho}{\sqrt{3}}\left(\sigma_0 + \frac{\sigma_8}{\sqrt{2}}\right), \tag{34}$$

where we have defined

$$m_0 = 2m_q + m_s, \tag{35}$$

$$m_8 = \sqrt{2}(m_q - m_s). \tag{36}$$

The term proportional to $g\rho$ comes from the medium effect from the one-loop diagram of the nucleon, Eq. (32). If the nuclear density ρ changes, the potential also changes. Consequently, the vacuum, which is the minimum point of the potential, changes. The minimum conditions of the potential are given as

$$\frac{\partial V_\sigma}{\partial\sigma_0} = \mu^2\sigma_0 + \frac{\lambda}{6}(2\sigma_0^3 + 6\sigma_0\sigma_8^2 - \sqrt{2}\sigma_8^3) + \lambda'\sigma_0(\sigma_0^2+\sigma_8^2) - 2Am_0 - 2B\left(\sigma_0^2 - \frac{\sigma_8^2}{2}\right) + \frac{g\rho}{\sqrt{3}} = 0, \tag{37}$$

$$\frac{\partial V_\sigma}{\partial\sigma_8} = \mu^2\sigma_8 + \lambda\sigma_8\left(\sigma_0^2 - \frac{\sigma_0\sigma_8}{\sqrt{2}} + \frac{\sigma_8^2}{2}\right) + \lambda'\sigma_8(\sigma_0^2+\sigma_8^2) - 2Am_8 + 2B\sigma_8\left(\sigma_0 + \frac{\sigma_8}{\sqrt{2}}\right) + \frac{g\rho}{\sqrt{6}} = 0. \tag{38}$$

The solution for $\sigma_0 = \langle\sigma_0\rangle$ and $\sigma_8 = \langle\sigma_8\rangle$ of these equation is the vacuum at nonzero ρ. The in-medium meson masses are obtained from

$$m^2(\rho) = m_0^2(\langle\sigma\rangle^*) + \Sigma_{\rm ph}(\rho). \tag{39}$$

The first term $m_0^2(\langle\sigma\rangle^*)$ is the same expression as in vacuum but evaluated with the in-medium sigma condensate $\langle\sigma\rangle^*$. The in-vacuum meson masses are shown in Appendix A. $m_0^2(\langle\sigma\rangle^*)$ contains only the contribution from diagram (a). The contribution from diagram (a) to the meson mass can be seen explicitly by using the vacuum condition. The in-medium masses of the pseudoscalar mesons π, η_0, η_8, which are denoted as $m_\pi(\rho)$, $m_{\eta_0}(\rho)$, $m_{\eta_8}(\rho)$, and the mixing term of η_0 and η_8, $m^2_{\eta_0\eta_8}(\rho)$, are

$$\begin{aligned} m_\pi^2(\rho) &= \mu^2 + \frac{\lambda}{3}\left(\langle\sigma_0\rangle^2 + \sqrt{2}\langle\sigma_0\rangle\langle\sigma_8\rangle + \frac{\langle\sigma_8\rangle^2}{2}\right) + \lambda'(\langle\sigma_0\rangle + \langle\sigma_8\rangle) - 2B(\langle\sigma_0\rangle - \sqrt{2}\langle\sigma_8\rangle) + \frac{\sqrt{3}g\rho}{2\left(\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}}\right)} \\ &= \frac{6Am_q}{\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}}}, \end{aligned} \tag{40}$$

$$\begin{aligned} m_{\eta_0}^2(\rho) &= \mu^2 + \frac{\lambda}{3}(\langle\sigma_0\rangle^2 + \langle\sigma_8\rangle^2) + \lambda'(\langle\sigma_0\rangle^2 + \langle\sigma_8\rangle^2) + 4B\langle\sigma_0\rangle + \frac{g\rho}{\sqrt{3}\left(\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}}\right)} \\ &= 6B\frac{\left(\langle\sigma_0\rangle - \frac{\langle\sigma_8\rangle}{\sqrt{2}}\right)^2}{\langle\sigma_0\rangle - \sqrt{2}\langle\sigma_8\rangle} + 2A\left(\frac{2m_q}{\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}}} + \frac{m_s}{\langle\sigma_0\rangle - \sqrt{2}\langle\sigma_8\rangle}\right), \end{aligned} \tag{41}$$

$$\begin{aligned} m_{\eta_8}^2(\rho) &= \mu^2 + \frac{\lambda}{3}\left(\langle\sigma_0\rangle^2 - \sqrt{2}\langle\sigma_0\rangle\langle\sigma_8\rangle + \frac{3\langle\sigma_8\rangle^2}{2}\right) + \lambda'(\langle\sigma_0\rangle^2 + \langle\sigma_8\rangle^2) - 2B(\langle\sigma_0\rangle + \sqrt{2}\langle\sigma_8\rangle) + \frac{g\rho}{2\sqrt{3}\left(\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}}\right)} \\ &= 6B\frac{\langle\sigma_8\rangle^2}{\langle\sigma_0\rangle - \sqrt{2}\langle\sigma_8\rangle} + 2A\left(\frac{m_q}{\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}}} + \frac{2m_s}{\langle\sigma_0\rangle - \sqrt{2}\langle\sigma_8\rangle}\right), \end{aligned} \tag{42}$$

$$\begin{aligned} m_{\eta_0\eta_8}^2(\rho) &= \frac{\sqrt{2}}{3}\lambda\langle\sigma_8\rangle\left(\sqrt{2}\langle\sigma_0\rangle - \frac{\langle\sigma_8\rangle}{2}\right) - 2B\langle\sigma_8\rangle + \frac{g\rho}{\sqrt{6}\left(\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}}\right)} \\ &= -\frac{6B\langle\sigma_8\rangle\left(\langle\sigma_0\rangle - \frac{\langle\sigma_8\rangle}{\sqrt{2}}\right)}{\langle\sigma_0\rangle - \sqrt{2}\langle\sigma_8\rangle} + 2\sqrt{2}A\left(\frac{m_q}{\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle}{\sqrt{2}}} - \frac{m_s}{\langle\sigma_0\rangle - \sqrt{2}\langle\sigma_8\rangle}\right). \end{aligned} \tag{43}$$

Here we have used the vacuum condition, Eqs. (37) and (38), to obtain the second expressions. It is interesting that in the second expressions for the in-medium meson masses the explicit density dependence disappears. This is a consequence of chiral symmetry in meson-nucleon interaction, in which the sigma exchange and Born contributions are canceled away. The physical masses of η and η' are obtained by

$$\begin{aligned} m_\eta^2 &= \tfrac{1}{2}\left(m_{\eta_0}^2 + m_{\eta_8}^2 - \sqrt{\left(m_{\eta_0}^2 - m_{\eta_8}^2\right)^2 + 4m_{08}^{P\,4}}\right), \\ m_{\eta'}^2 &= \tfrac{1}{2}\left(m_{\eta_0}^2 + m_{\eta_8}^2 + \sqrt{\left(m_{\eta_0}^2 - m_{\eta_8}^2\right)^2 + 4m_{08}^{P\,4}}\right), \end{aligned} \tag{44}$$

so as to resolve the off-diagonal mass term $m^2_{\eta_0\eta_8}$.

From these explicit forms of the η_0 and η_8 meson mass, the mass difference of these mesons in the SU(3) flavor symmetric limit ($m_q = m_s$, $\langle\sigma_8\rangle = 0$) is written as

$$m_{\eta_0}^2 - m_{\eta_8}^2 = 6B\langle\sigma_0\rangle. \tag{45}$$

This expression is consistent with the discussion in Sec. II, where we have shown both effects of the $U_A(1)$ anomaly and the chiral symmetry breaking are necessary for the mass difference of η_0 and η_8. In addition, since η_8 is the Nambu-Goldstone boson associated with the spontaneous breaking of chiral SU(3) symmetry, the mass of the η_8 meson comes from the explicit breaking of chiral symmetry. Assuming that the η_0 and η_8 masses are of orders 1000 and 500 MeV, respectively, one finds from Eq. (45) that almost half of the η_0 mass is generated by the spontaneous chiral symmetry breaking through the $U_A(1)$ anomaly.

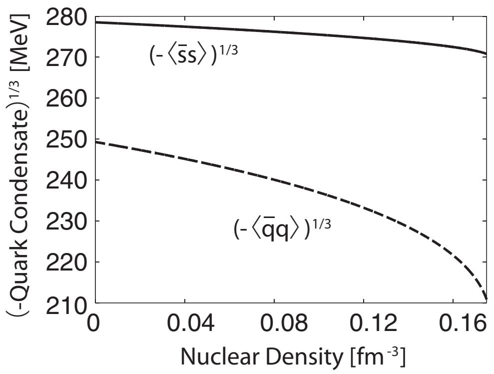

FIG. 2. The chiral condensates in the nuclear medium. The dashed and solid lines denote $(-\langle\bar{q}q\rangle)^{1/3}$ and $(-\langle\bar{s}s\rangle)^{1/3}$, respectively.

In the following, we show the in-medium meson masses calculated with the medium effect including the $SU(3)_V$ breaking owing to the quark mass difference. The parameters are determined by the method shown in Appendix B. As the input parameters, we used $f_\pi, f_K, m_\pi, m_K, m_\sigma$, the sum of m_η^2 and $m_{\eta'}^2$, and the degenerate u, d quark mass m_q. All the used and determined parameters are shown in Appendix B. We determine the meson-baryon coupling parameter g by the reduction of the chiral condensate.

First, we show the density dependence of the chiral condensate in Fig. 2. Since the parameter g is determined to reproduce the 35% reduction of the quark condensate at normal nuclear density, the quark condensate at the saturation density is the input value here. As mentioned above, we assume that the nuclear medium contains no explicit strange component. So, the strange condensate is insensitive to the nuclear density. Nevertheless, the strange condensate does change slightly through the SU(3) flavor breaking of nuclear matter.

Next, we show the result of the in-medium meson masses including the SU(3) breaking by the current quark mass in Fig. 3. From this calculation, we find that the η' mass reduces by about 80 MeV at normal nuclear density. In contrast, the masses of the other pseudoscalar octet mesons are enhanced. Especially for the η case, the enhancement is about 50 MeV. This is because under the partial restoration of chiral symmetry the magnitude of the spontaneous breaking is suppressed and

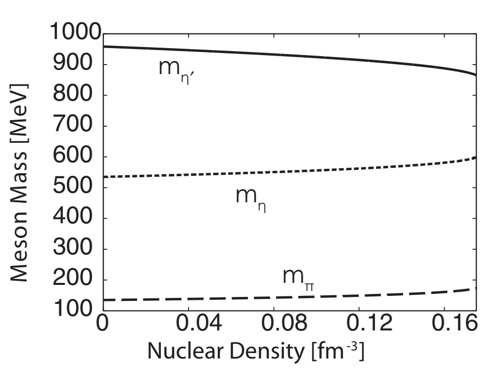

FIG. 3. The mass shift of the η' meson in the nuclear medium. The solid, dotted, and dashed lines represent the η', η, and π meson masses in the nuclear medium, respectively.

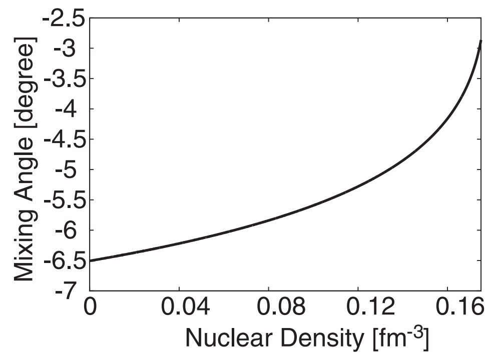

FIG. 4. The η_0-η_8 mixing angle in the nuclear medium.

consequently the Nambu-Goldstone boson nature of the octet pseudoscalar mesons declines.

Finally, we show the density dependence of the mixing angle of η_0-η_8 in Fig. 4. We defined the mixing angle θ with

$$\tan 2\theta = \frac{2m^2_{\eta_0\eta_8}}{m^2_{\eta_0} - m^2_{\eta_8}}. \tag{46}$$

The density dependence of the mixing angle θ is shown in Fig. 4. As we can see in Fig. 4, the absolute value of the mixing angle becomes smaller when the nuclear density become larger. One can understand this behavior as follows: When chiral symmetry is being restored partially with the reduction of the magnitude of the sigma condensates, the first terms of Eqs. (41)–(43) are getting suppressed. In the limit where the first terms vanish, the mixing angle is obtained by $\tan 2\theta = 2\sqrt{2}$ and has a positive large value. Therefore, the mixing angle is approaching to a positive value with the partial restoration.

IV. THE LOW-ENERGY $\eta' N$ INTERACTION IN VACUUM

Let us discuss the $\eta' N$ two-body interaction in vacuum. In the following, we estimate the $\eta' N$ interaction strength with the linear sigma model developed in the previous section. We evaluate the invariant amplitude of the meson and nucleon V_{ab} in the tree level by the scalar meson exchange and Born terms shown in Fig. 5:

$$\begin{aligned} -iV_{ab} &= g_{\sigma_0 NN} C^{(0)}_{ab} \frac{i}{(k-k')^2 - m^2_{\sigma_0}} + g_{\sigma_8 NN} C^{(8)}_{ab} \frac{i}{(k-k')^2 - m^2_{\sigma_8}} \\ &\quad + C_a\gamma_5 \frac{i}{\not{p} + \not{k} - m_N} C_b\gamma_5 + C_b\gamma_5 \frac{i}{\not{p} - \not{k}' - m_N} C_a\gamma_5, \end{aligned} \tag{47}$$

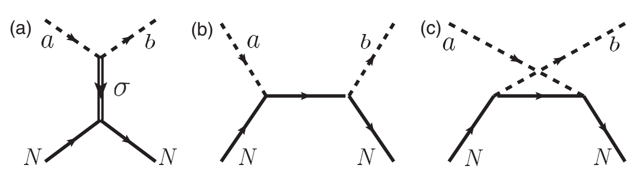

FIG. 5. The diagrams that contribute to the $\eta' N$ interaction. The dashed, single, and double lines mean the pseudoscalar meson, nucleon, and scalar meson propagation, respectively.

where k and k' are incoming and outgoing meson momenta, respectively, and p is the incoming nucleon momentum. The labels a, b correspond to the incoming and outgoing mesons, $C^{(0)}_{ab}, C^{(8)}_{ab}$ are the coupling constant of the pseudoscalar mesons and σ_0 and σ_8 mesons, $g_{\sigma_0 NN}, g_{\sigma_8 NN}$ are σ_0,σ_8 and nucleon couplings, respectively, and C_a is the coupling constant between the pseudoscalar meson and the nucleon. The first term is the contribution from the scalar meson exchange shown in diagram (a) of Fig. 5, while the second and third terms are the Born terms shown as diagrams (b) and (c) of Fig. 5.

With the meson momentum expansion, the Lorentz scalar part of the sum of the amplitude for the NG boson and nucleon scattering is canceled out, while the vector part remains the contribution. This interaction is known as the Weinberg-Tomozawa (WT) low-energy theorem stemming from the spontaneous chiral symmetry breaking. In the flavor SU(3) limit and $a, b \neq \eta_0$, the vacuum condition in the chiral limit is given as

$$\mu^2 + \frac{\lambda}{3}\langle\sigma_0\rangle^2 + \lambda'\langle\sigma_0\rangle^2 - 2B\langle\sigma_0\rangle = 0, \tag{48}$$

and the scalar and pseudoscalar meson coupling are

$$C^{(0)}_{ab} = -i\delta^{ab}\left(\frac{2}{3}\lambda\langle\sigma_0\rangle + 2\lambda'\langle\sigma_0\rangle - 2B\right), \tag{49}$$

$$C^{(8)}_{ab} = -i\delta^{ab}\left(\frac{\sqrt{2}}{3}\lambda\langle\sigma_0\rangle + 2\sqrt{2}B\right). \tag{50}$$

The σ_0 and σ_8 and nucleon couplings are

$$g_{\sigma_0 NN} = -i\frac{g}{\sqrt{3}}, \tag{51}$$

$$g_{\sigma_8 NN} = -i\frac{g}{\sqrt{6}}. \tag{52}$$

The meson-baryon coupling C_a is given as

$$C_a = \frac{g}{\sqrt{2}}\tau_a \quad (a = 1, 2, 3, 8) \tag{53}$$

from Eq. (28). Here, we define $\tau_8 \equiv \frac{1}{\sqrt{3}} \cdot \mathbf{1}$. The masses are

$$\begin{aligned} m^2_{\sigma_0} &= \mu^2 + \lambda\langle\sigma_0\rangle^2 + 3\lambda'\langle\sigma_0\rangle^2 - 4B\langle\sigma_0\rangle \\ &= \frac{2}{3}\lambda\langle\sigma_0\rangle^2 + 2\lambda'\langle\sigma_0\rangle^2 - 2B\langle\sigma_0\rangle, \end{aligned} \tag{54}$$

$$\begin{aligned} m^2_{\sigma_8} &= \mu^2 + \lambda\langle\sigma_0\rangle^2 + \lambda'\langle\sigma_0\rangle^2 + 2B\langle\sigma_0\rangle \\ &= \frac{2}{3}\lambda\langle\sigma_0\rangle^2 + 4B\langle\sigma_0\rangle, \end{aligned} \tag{55}$$

$$m_N = \frac{g}{\sqrt{3}}\langle\sigma_0\rangle, \tag{56}$$

where we used the vacuum condition Eq. (48). Substituting Eqs. (49)–(56) for Eq. (47) and expanding the amplitude in terms of the incoming and outgoing meson momenta k, k', we can obtain the s-wave amplitude of the NG boson ($a,\ b \neq 0$) and baryon scattering as

$$V_{ab} = -\frac{g^2\omega}{8m_N^2}[\tau_a, \tau_b] + O(k^2), \tag{57}$$

where ω is the meson energy. Here, we have used the Dirac equation $(\not{p} - m_N)u(p) = 0$ and we take only the s-wave contribution for low-energy scattering. In Eq. (57), we omitted the unit matrix of the spinor space.

In the case of the $\eta' N$ interaction, the interaction strength in the chiral limit is derived as follows. From Eq. (28), the $\eta_0\sigma_0$ coupling and the $\eta_0\sigma_8$ coupling in the SU(3) symmetric limit can be obtained from the Lagrangian Eq. (28) as

$$C^{(0)}_{\eta_0\eta_0} = -i\left(\tfrac{2}{3}\lambda\langle\sigma_0\rangle + 2\lambda'\langle\sigma_0\rangle + 4B\right), \tag{58}$$

$$C^{(8)}_{\eta_0\eta_0} = 0, \tag{59}$$

and the η_0 and nucleon coupling $C_{\eta_0 N}$ can be written as

$$C_{\eta_0 N} = \frac{g}{\sqrt{3}} \cdot \mathbf{1}. \tag{60}$$

Substituting $C^{(0,8)}_{\eta_0\eta_0}$ and $C_{\eta_0 N}$ for Eq. (47), the $\eta_0 N$ interaction in the linear sigma model in the chiral limit and at low energy compared to the meson and nucleon mass is calculated as

$$\begin{aligned} -iV_{\eta_0 N} &= -\frac{ig}{\sqrt{3}}i\left(\frac{2}{3}\lambda\langle\sigma_0\rangle + 2\lambda'\langle\sigma_0\rangle + 4B\right)\frac{i}{m^2_{\sigma_0}} \\ &\quad + \left(\frac{g}{\sqrt{3}}\right)^2\gamma_5\left(\frac{i}{\not{p} + \not{k} - m_N} + \frac{i}{\not{p} - \not{k} - m_N}\right)\gamma_5 \\ &= \frac{ig}{\sqrt{3}\langle\sigma_0\rangle}\frac{m^2_{\sigma_0} + 6B\langle\sigma_0\rangle}{m^2_{\sigma_0}} \\ &\quad -\frac{i}{2}\left(\frac{g}{\sqrt{3}}\right)^2\left(\frac{\not{k}}{p\cdot k} + \frac{\not{k}'}{p\cdot k'}\right) + O(k^2) \\ &= \frac{ig^2}{3m_N}\left(1 + \frac{6B\langle\sigma_0\rangle}{m^2_{\sigma_0}}\right) - i\frac{g^2}{3m_N} + O(k^2) \\ &= \frac{ig}{\sqrt{3}}\frac{6B}{m^2_{\sigma_0}} + O(k^2). \end{aligned} \tag{61}$$

From the first line to the second line, we have kept the leading contribution in the meson momentum expansion and replaced $\not{p}$ with m_N as well as in the case of NG boson and nucleon scattering, and from the third line to the fourth line we take only s-wave amplitude for the low-energy scattering. As a result, the leading contribution to the $\eta_0 N$ interaction is induced by the B term, which comes from the $U_A(1)$ anomaly. In contrast, the Weinberg-Tomozawa interaction is canceled due to the $U_A(1)$ symmetry. This is because only the terms including B (and the quark mass) break the $U_A(1)$ chiral symmetry and the other terms keep the symmetry. Thanks to the chiral symmetry in these terms, we have the cancellation between the σ exchange and Born terms.

Substituting the values of the parameters into Eq. (61), we find the $\eta' N$ interaction to be attractive with strength $-0.0534\ \mathrm{MeV}^{-1}$. This attraction is strong comparable to the $\bar{K}N$ system with $I = 0$, in which it is conceivable that there exists a $\bar{K}N$ quasibound state regarded as $\Lambda(1405)$. In the following, we use this value as the $\eta' N$ coupling constant.

V. THE $\eta' N$ BOUND STATE

In the previous section, we obtained the tree-level amplitude for the $\eta' N$ scattering in the linear sigma model. Making use of this amplitude as an interaction kernel, we solve a scattering equation for the $\eta' N$ two-body system. Because the $\eta' N$ interaction is attractive with a strength comparable to the $\bar{K}N$ system with $I = 0$, we expect that the $\eta' N$ system forms a bound state similar to $\Lambda(1405)$, which is a bound state in the $\bar{K}N$ channel. In this section, we evaluate the scattering length of the $\eta' N$ system and binding energy if an $\eta' N$ bound state is formed.

To solve the $\eta' N$ scattering system, we make use of the same machinery for the $\Lambda(1405)$ in the $\bar{K}N$ channel with $I = 0$ [44,45], in which the $\bar{K}N$ scattering amplitude obtained with the chiral perturbation theory at the tree level is used as the interaction kernel of the scattering equation and the loop function is regularized so that the scattering amplitude can be described in terms of hadronic objects. As a result one finds a quasibound state in the $\bar{K}N$ channel. The T matrix is calculated by the single-channel Lippmann-Schwinger equation. Here we take the $\eta' N$ interaction evaluated in Eq. (61) as the interaction kernel. We denote the interaction kernel as $V_{kk'}$, where the indices k and k' are incoming and outgoing meson momenta respectively. Now, we are in the case that the interaction kernel $V_{kk'}$ is independent of the external momentum, and the T-matrix can be obtained in an algebraic way:

$$\begin{aligned} T_{kk'} &= V_{kk'} + \int dl\, V_{kl} G_l T_{lk'} \\ &= V_{kk'} + \int dl\, V_{kl} G_l V_{lk'} \\ &\quad + \int dl \int dl' V_{kl} G_l V_{ll'} G_{l'} V_{l'k'} + \cdots \\ &= V \sum_{n=0}^{\infty} \left(V \int dl\, G_l \right)^n = \frac{V}{1 - V \int dl\, G_l}, \end{aligned} \tag{62}$$

where G_l is the two-body Green function of the η' and nucleon. From the second line to the third line, we used the fact that the interaction kernel $V_{kk'}$ is independent of the external momentum, $V_{kk'} = V$. Because we take the momentum-independent contact interaction Eq. (61), the integral of G_l diverges. With dimensional regularization, the integral of G_l is calculated with

$$\begin{aligned} G(W) &\equiv \int dl G_l \\ &= i \int \frac{d^4 l}{(2\pi)^4} \frac{2m_N}{l^2 - m_N^2 + i\epsilon} \frac{1}{(P-l)^2 - m_{\eta'}^2 + i\epsilon} \\ &= \frac{2m_N}{16\pi^2} \Bigg\{ a(\mu) + \ln \frac{m_N^2}{\mu^2} + \frac{m_{\eta'}^2 - m_N^2 + W^2}{2W^2} \ln \frac{m_{\eta'}^2}{m_N^2} \\ &\quad + \frac{\bar{q}}{W} \{ \ln [W^2 - (m_N^2 - m_{\eta'}^2) + 2\bar{q}W] \\ &\quad + \ln [W^2 + (m_N^2 - m_{\eta'}^2) + 2\bar{q}W] \\ &\quad - \ln [-W^2 - (m_N^2 - m_{\eta'}^2) + 2\bar{q}W] \\ &\quad - \ln [-W^2 + (m_N^2 - m_{\eta'}^2) + 2\bar{q}W] \} \Bigg\}, \end{aligned} \tag{63}$$

where μ is the scale of dimensional regularization, and the center-of-mass momentum is given by

$$\bar{q} = \frac{\sqrt{[W^2 - (m_N + m_{\eta'})^2][W^2 - (m_N - m_{\eta'})^2]}}{2W}, \tag{64}$$

From the second line to the third line of Eq. (63), we have supposed that the divergent part could be absorbed in interaction vertices in the renormalization procedure, and the remaining finite constant is denoted as $a(\mu)$. The subtraction constant $a(\mu)$ has to be determined in some way. Here we take the natural renormalization scheme proposed in Ref. [46] in which the Castillejo-Dalitz-Dyson (CDD) pole contribution are excluded from the scattering amplitude in a way consistent with chiral counting. This means that the scattering amplitude is described by dynamics of η' and N. In our calculation, we use $a(\mu) = -1.838$ and the renormalization point $\mu = m_N$.

Using the T matrix calculated with the above method, we evaluate the binding energy and scattering length of the $\eta' N$ system. The mass m_B of the bound state is obtained as the pole position of the T matrix. The binding energy E_B is calculated by $E_B = m_N + m_{\eta' N} - m_B$.

With Eqs. (62) and (63), the $\eta' N$ binding energy E_B is obtained as 6.2 MeV. The scattering length and effective range are obtained as -2.7 and 0.25 fm with the definition in Appendix D. We show the scattering amplitude with $m_{\sigma_0} = 700$ MeV in Fig. 6.

In this calculation, we used the mass of the sigma meson m_{σ_0} as an input to fix the parameter of the Lagrangian of the linear sigma model. In the previous calculations, we used $m_{\sigma_0} =$ 700 MeV. The sigma meson mass dependence of the binding energy, scattering length, and effective range is given in Table I. The parameters of the Lagrangian are determined for each m_{σ_0} with the procedure shown in Sec. III B. Within the wide range of m_{σ_0}, we found that the existence of the $\eta' N$ bound state and the binding energy have a somewhat m_{σ_0} dependence. From Table I, we find that the larger binding energy accompanies the smaller scattering length. This behavior can be understood because the scattering length can be roughly evaluated with $1/\sqrt{2mE_B}$, where m is the reduced mass of η' and nucleon. We find the scattering length is about 1 fm if a $\eta' N$ bound state exists with binding energy of a few MeV.

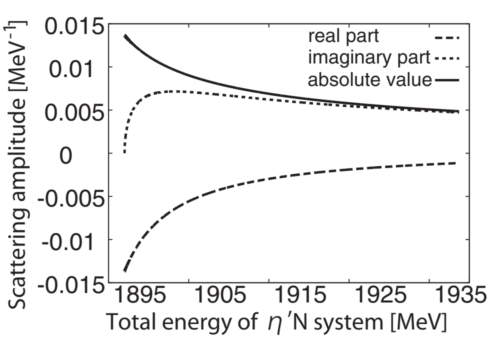

FIG. 6. The value of the scattering amplitude above the threshold. The dashed, dotted, and solid lines represent the real part, the imaginary part, and the absolute value of the scattering amplitude of the $\eta' N$ system with $m_{\sigma_0} = 700$ MeV, respectively.

TABLE I. The m_{σ_0} dependence of the $\eta' N$ bound state.

m_{σ_0} (MeV)	Binding energy (MeV)	Scattering length (fm)	Effective range (fm)
500	3.5	−3.5	0.25
600	6.2	−2.7	0.25
700	6.2	−2.7	0.25
800	4.6	−3.1	0.25
900	2.4	−4.2	0.26
1000	0.6	−8.1	0.33

The result in the low-energy limit depends on the choice of the subtraction constant $a(\mu)$, and we determined $a(\mu)$ to exclude dynamics other than those η' and N here. The other degree of freedom, for example the ω meson exchange interaction or the microscopic quark dynamics, may spoil such a description [46].

VI. CONCLUSION AND REMARKS

In this paper, we have constructed a chiral effective Lagrangian for mesons based on the linear realization of the SU(3) chiral symmetry in symmetric nuclear matter and estimated the mass reduction of η' in the medium. The Lagrangian contains the explicit breaking of the chiral symmetry and flavor symmetries and the determinant type $U_A(1)$ breaking term which introduces the effect of the $U_A(1)$ anomaly. We find that a substantial part of the η' mass is generated by the spontaneous breaking of chiral symmetry through the $U_A(1)$ anomaly. Nuclear matter is taken into account as a mean field by calculating one nucleon loop in the Fermi gas. The parameters of the Lagrangian have been fixed by the observed quantities, such as the meson decay constants and the meson masses. In the determination of the coupling strength of nucleon and the sigma meson, we make use of partial restoration of chiral symmetry, that is, the experimental suggestion of a 30% reduction of the quark condensate as the basic assumption. In our calculation, we have obtained an 80 MeV reduction of the η' meson mass at the normal nuclear density.

Based on the effective Lagrangian used for the calculation of the in-medium properties of the mesons, we have also estimated the two-body $\eta' N$ interaction in vacuum. Using the interaction of $\eta' N$ as the kernel of the scattering equation, we have evaluated the T matrix of the $\eta' N$ system. As a result, we have obtained an $\eta' N$ bound state, which is a state analogous to $\Lambda(1405)$ in the $\bar{K} N$ system. The binding energy of the system is found to be several MeV, which is comparable to the typical value of the hadronic bound state; for example, that of $\Lambda(1405)$ or the deuteron. We have also evaluated the scattering length and the effective range of the $\eta' N$ system, having obtained a few fm with the repulsive sign for the scattering length, which is a consequence of the existence of the bound state, and a quarter fm of the effective range.

In the linear sigma model, the $\eta' N$ interaction originates from sigma meson exchange with the $\eta_0\eta_0\sigma$ coupling coming from the $U_A(1)$ breaking determinant term. The Weinberg-Tomozawa type vector interaction is canceled away by the scalar-meson-exchange and Born terms thanks to chiral symmetry. In contrast, the interactions of the octet pseudoscalar meson and nucleon are expressed by the Weinberg-Tomozawa interaction at low energies as a consequence of the spontaneous breaking of chiral symmetry, and there is no sigma exchange term, which is canceled away with the Born term and turns into the Weinberg-Tomozawa interaction. This implies that the difference comes from the fact that the η' meson is not a Nambu-Goldstone boson due to the $U_A(1)$ anomaly.

Actually, the $\sigma\eta_0\eta_0$ coupling from the explicit $U_A(1)$ breaking induces the mass of the η' meson when chiral symmetry is broken spontaneously with finite σ condensate. In this way, the $\sigma\eta_0\eta_0$ coupling plays an important role for the mass generation of the η' meson. This is the case also for the nucleon. The nucleon mass is generated by the sigma condensate through the σNN coupling. In addition, the strong σNN coupling induces a strong attraction in the scalar-isoscalar channel for the NN interaction with the σ meson exchange, Thus, we conclude that the $\eta' N$ interaction in the scalar channel is entirely analogous to the NN interaction. Since the $\sigma\eta_0\eta_0$ and σNN coupling are necessary for the mass generation of the η' meson and nucleon in the linear sigma model, the $\eta' N$ interaction coming from the σ exchange is inevitable. (In the same manner, one could have a strong attraction also in the $\eta'\eta'$ system.) This attraction may open the possibility to have bound states in $\eta' N$ and η'-nucleus systems. Nevertheless, there could be such repulsive interactions in other channels as to spoil the bound states. It should be noted that chiral symmetry says that there is no Weinberg-Tomozawa interaction in the $\eta_0 N$ channel.

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APPENDIX A: THE LINEAR SIGMA MODEL IN VACUUM

In this section, we show the application of the linear sigma model in vacuum. From the meson Lagrangian Eq. (21), we obtain the effective potential for σ_0 and σ_8, $V_\sigma(\sigma_0,\sigma_8)$, using the tree approximation as follows:

$$\begin{aligned} V_\sigma(\sigma_0,\sigma_8) = {} & \frac{\mu^2}{2}\left(\sigma_0^2+\sigma_8^2\right) \\ & + \frac{\lambda}{12}\left(\sigma_0^4 + 6\sigma_0^2\sigma_8^2 - 2\sqrt{2}\sigma_0\sigma_8^3 + \frac{3}{2}\sigma_8^4\right) \\ & + \frac{\lambda'}{4}\left(\sigma_0^2+\sigma_8^2\right)^2 - 2A(m_0\sigma_0 + m_8\sigma_8) \\ & - \frac{2}{3}B\left(\sigma_0^3 - \frac{3}{2}\sigma_0\sigma_8^2 - \frac{\sigma_8^3}{\sqrt{2}}\right). \end{aligned} \tag{A1}$$

In Eq. (A1), we have omitted σ_3 because we assume isospin symmetry and trivially $\langle\sigma_3\rangle = 0$. The vacuum expectation values, $\langle\sigma_0\rangle$, $\langle\sigma_8\rangle$, are obtained as the minimum point of the potential. The minimum point is obtained by solving the vacuum condition

$$\frac{\partial V_\sigma}{\partial \sigma_0} = \mu^2\sigma_0 + \frac{\lambda}{6}\left(2\sigma_0^3 + 6\sigma_0\sigma_8^2 - \sqrt{2}\sigma_8^3\right) + \lambda'\sigma_0\left(\sigma_0^2+\sigma_8^2\right) - 2Am_0 - 2B\left(\sigma_0^2 - \frac{\sigma_8^2}{2}\right) = 0, \tag{A2}$$

$$\frac{\partial V_\sigma}{\partial \sigma_8} = \mu^2\sigma_8 + \frac{\lambda}{2}\left(2\sigma_0^2\sigma_8 - \sqrt{2}\sigma_0\sigma_8^2 + \sigma_8^3\right) + \lambda'\sigma_8\left(\sigma_0^2+\sigma_8^2\right) - 2Am_8 + 2B\left(\sigma_0\sigma_8 + \frac{\sigma_8^2}{\sqrt{2}}\right) = 0, \tag{A3}$$

where we have defined

$$m_0 = 2m_q + m_s, \tag{A4}$$

$$m_8 = \sqrt{2}(m_q - m_s). \tag{A5}$$

The meson masses are obtained as the second-order derivative of the full effective potential V at the vacuum point $\frac{\partial V_\sigma}{\partial \sigma} = 0$:

$$m_{ab}^2 = \left.\frac{\partial^2 V}{\partial \pi^a \partial \pi^b}\right|_{\pi^{a,b}=0}. \tag{A6}$$

Here, π^a is the meson field and $m_a \equiv m_{aa}$ stands for the mass of the meson π^a and m_{ab} $(a \neq b)$ means the mixing term between π^a and π^b. Using the vacuum expectation values, $\langle\sigma_0\rangle$, $\langle\sigma_8\rangle$, we obtain the meson masses as follows:

$$m_{\sigma_0}^2 = \mu^2 + \lambda(\langle\sigma_0\rangle^2 + \langle\sigma_8\rangle^2) + \lambda'(3\langle\sigma_0\rangle^2 + \langle\sigma_8\rangle^2) - 4B\langle\sigma_0\rangle, \tag{A7}$$

$$= \frac{2}{3}\lambda\left(\langle\sigma_0\rangle^2 + \frac{\langle\sigma_8\rangle^3}{2\sqrt{2}\langle\sigma_0\rangle}\right) + 2\lambda'\langle\sigma_0\rangle^2 - 2B\left(\langle\sigma_0\rangle + \frac{\langle\sigma_8\rangle^2}{2\langle\sigma_0\rangle}\right) + \frac{2Am_0}{\langle\sigma_0\rangle}, \tag{A8}$$

$$m_{\sigma_8}^2 = \mu^2 + \lambda(\langle\sigma_0\rangle^2 - \sqrt{2}\langle\sigma_0\rangle\langle\sigma_8\rangle + 3\langle\sigma_8\rangle^2/2) + \lambda'(\langle\sigma_0\rangle^2 + 3\langle\sigma_8\rangle^2) + 2B(\sigma_0 + \sqrt{2}\sigma_8) \tag{A9}$$

$$= \lambda\left(\frac{2}{3}\langle\sigma\rangle^2 - \sqrt{2}\langle\sigma_0\rangle\langle\sigma_8\rangle + \frac{\langle\sigma_8\rangle^3}{3\sqrt{2}\langle\sigma_0\rangle} + \frac{\langle\sigma_8\rangle^2}{2}\right) + 2\lambda'\langle\sigma_8\rangle^2 + \frac{2Am_0}{\langle\sigma_0\rangle} + B\left(4\langle\sigma_0\rangle + 2\sqrt{2}\langle\sigma_8\rangle - \frac{\langle\sigma_8\rangle^2}{\langle\sigma_0\rangle}\right), \tag{A10}$$

$$m_{\sigma_0\sigma_8}^2 = \frac{\lambda}{2}\left(4\sigma_0\sigma_8 - \sqrt{2}\sigma_8^2\right) + 2\lambda'\sigma_0\sigma_8 + 2B\sigma_8, \tag{A11}$$

$$m_\pi^2 = \mu^2 + \frac{\lambda}{3}(\langle\sigma_0\rangle^2 + \sqrt{2}\langle\sigma_0\rangle\langle\sigma_8\rangle + \langle\sigma_8\rangle^2/2) + \lambda'(\langle\sigma_0\rangle^2 + \langle\sigma_8\rangle^2) - 2B(\sigma_0 - \sqrt{2}\sigma_8) \tag{A12}$$

$$= \frac{2\sqrt{6}Am_q}{f_\pi}, \tag{A13}$$

$$m_K^2 = \mu^2 + \frac{\lambda}{3}(\langle\sigma_0\rangle^2 - \langle\sigma_0\rangle\langle\sigma_8\rangle/\sqrt{2} + 7\langle\sigma_8\rangle^2/2) + \lambda'(\langle\sigma_0\rangle^2 + \langle\sigma_8\rangle^2) - 2B(\langle\sigma_0\rangle + \langle\sigma_8\rangle/\sqrt{2}) \tag{A14}$$

$$= \frac{\sqrt{6}A\left(m_q + m_s\right)}{f_K} \tag{A15}$$

$$m_{\eta_0}^2 = \mu^2 + \frac{\lambda}{3}(\langle\sigma_0\rangle^2 + \langle\sigma_8\rangle^2) + \lambda'(\langle\sigma_0\rangle^2 + \langle\sigma_8\rangle^2) + 4B\langle\sigma_0\rangle \tag{A16}$$

$$= \sqrt{\frac{2}{3}}B\frac{(4f_K - f_\pi)^2}{2f_K - f_\pi} + \frac{2\sqrt{2}}{\sqrt{3}}A\left(\frac{2m_q}{f_\pi} + \frac{m_s}{2f_K - f_\pi}\right) \tag{A17}$$

$$m_{\eta_8}^2 = \mu^2 + \frac{\lambda}{3}(\langle\sigma_0\rangle^2 - \sqrt{2}\langle\sigma_0\rangle\langle\sigma_8\rangle + 3\langle\sigma_8\rangle^2/2) + \lambda'(\langle\sigma_0\rangle^2 + \langle\sigma_8\rangle^2) - 2B(\langle\sigma_0\rangle + \sqrt{2}\langle\sigma_8\rangle) \tag{A18}$$

$$= \frac{8\sqrt{2}}{\sqrt{3}}B\frac{(f_\pi - f_K)^2}{2f_K - f_\pi} + \frac{2\sqrt{2}}{\sqrt{3}}A\left(\frac{m_q}{f_\pi} + \frac{2m_s}{2f_K - f_\pi}\right), \tag{A19}$$

$$m_{\eta_0\eta_8}^2 = \frac{\sqrt{2}}{3}\lambda\langle\sigma_8\rangle\left(\sqrt{2}\langle\sigma_0\rangle - \frac{\langle\sigma_8\rangle}{2}\right) - 2B\langle\sigma_8\rangle, \tag{A20}$$

where $m_{\sigma_0\sigma_8}^2$ and $m_{\eta_0\eta_8}^2$ are the mixing terms of $\sigma_0\sigma_8$ and $\eta_0\eta_8$, respectively. The physical mass is defined so as to diagonalize the mass term. For η and η', we have

$$m_\eta^2 = \tfrac{1}{2}\left(m_{\eta_0}^2 + m_{\eta_8}^2 - \sqrt{\left(m_{\eta_0}^2 - m_{\eta_8}^2\right)^2 + 4m_{\eta_0\eta_8}^4}\right), \tag{A21}$$

$$m_{\eta'}^2 = \tfrac{1}{2}\left(m_{\eta_0}^2 + m_{\eta_8}^2 + \sqrt{\left(m_{\eta_0}^2 - m_{\eta_8}^2\right)^2 + 4m_{\eta_0\eta_8}^4}\right). \tag{A22}$$

APPENDIX B: THE DETERMINATION OF PARAMETERS

We determine the parameters in the linear sigma model from the physical values in vacuum. The sigma condensates can be determined from the meson decay constants through Eqs. (24) and (25):

$$\langle\sigma_0\rangle = \frac{1}{\sqrt{6}}(f_\pi + 2f_K), \tag{B1}$$

$$\langle\sigma_8\rangle = \frac{2}{\sqrt{3}}(f_\pi - f_K). \tag{B2}$$

Once the sigma condensates are fixed, Am_q and Am_s can be determined by the π and K meson masses, Eqs. (A13) and (A15):

$$Am_q = \frac{f_\pi}{2\sqrt{6}}m_\pi^2, \tag{B3}$$

$$A(m_q + m_s) = \frac{f_K}{\sqrt{6}}m_K^2. \tag{B4}$$

This fixes the ratio of m_q and m_s. In the linear sigma model, the quark masses appear always with the parameter A. For independent determination of m_q, m_s, and A, we introduce an explicit value $m_q = 5$ MeV to fix A and m_s.

TABLE II. Input values.

f_π (MeV)	f_K (MeV)	m_π (MeV)	m_K (MeV)	$m^2_{\eta'}+m^2_\eta$ (MeV^2)	m_{σ_0} (MeV)	m_q (MeV)
92.2	110.4	135	495	550^2+958^2	700	5.0

Noticing $m^2_{\eta_0}+m^2_{\eta_8}=m^2_{\eta'}+m^2_\eta$ from Eqs. (A21) and (A22), we can determine B from $m^2_{\eta'}+m^2_\eta$ with Eqs. (A17) and (A19):

$$B=\frac{1}{\sqrt{6}}\frac{2f_K-f_\pi}{3f_\pi^3-8f_\pi f_K+8f_K^2}\times\left[\left(m_\eta^2+m_{\eta'}^2\right)^2-2\sqrt{6}A\left(\frac{m_q}{f_\pi}+\frac{m_s}{2f_K-f_\pi}\right)\right]. \tag{B5}$$

From Eqs. (A12) and (A14), we can fix λ from

$$\lambda=\frac{m_K^2-m_\pi^2}{(f_K-f_\pi)(2f_K-f_\pi)}-\frac{2\sqrt{6}B}{2f_K-f_\pi}. \tag{B6}$$

Finally from the vacuum conditions Eqs. (A2) and (A3), we can fix μ^2 and λ' as

$$\mu^2=a_1\lambda+a_2\lambda'+a_3B \tag{B7}$$

with

$$a_1=-\frac{m_8(2\langle\sigma_0\rangle^3+6\langle\sigma_0\rangle\langle\sigma_8\rangle^2-\sqrt{2}\langle\sigma_8\rangle^3)/6}{\langle\sigma_0\rangle m_8-\langle\sigma_8\rangle m_0}+\frac{m_0\langle\sigma_8\rangle(2\langle\sigma_0\rangle^2-\sqrt{2}\langle\sigma_0\rangle\langle\sigma_8\rangle+\langle\sigma_8\rangle^2)/2}{\langle\sigma_0\rangle m_8-\langle\sigma_8\rangle m_0},$$

$$a_2=-(\langle\sigma_0\rangle^2+\langle\sigma_8\rangle^2), \tag{B8}$$

$$a_3=\frac{2\left\{m_8\left[\langle\sigma_0\rangle^2-\frac{\langle\sigma_8\rangle^2}{2}+m_0\langle\sigma_8\rangle\left(\langle\sigma_0\rangle+\frac{\langle\sigma_8\rangle}{\sqrt{2}}\right)\right]\right\}}{\langle\sigma_0\rangle m_8-\langle\sigma_8\rangle m_0}, \tag{B9}$$

and

$$\lambda'=\frac{m_\sigma^2-\lambda(a_1+\langle\sigma_0\rangle^2+\langle\sigma_8\rangle^2)-B(a_3-4\langle\sigma_0\rangle)}{2\langle\sigma_0\rangle^2}. \tag{B10}$$

We show the input values to determine the parameters of the Lagrangian and the determined parameters in Tables II and III.

APPENDIX C: THE CALCULATION OF THE IN-MEDIUM NUCLEON LOOP DIAGRAMS

In this section, we show the explicit calculation of the nucleon one-loop contribution to the sigma effective potential and the meson masses. Here we assume the chiral limit. We use the in-medium nucleon propagator defined as

$$P_{\rm med}(p)=(\not{p}+m_N)\left\{\frac{i}{p^2-m_N^2+i\epsilon}-2\pi\delta\left(p^2-m_N^2\right)\theta(p_0)\theta(k_f-|\vec{p}|)\right\}. \tag{C1}$$

First, we evaluate the tadpole diagram for the σ effective action denoted as $V_{MF}(\rho)$ in Sec. III B. The effective potential for σ_0 coming with the nucleon-tadpole diagram $V^0_{MF}(\rho)$ is calculated as

$$-iV^0_{MF}(\rho)=-2\sigma_0 g_{\sigma_0 NN}\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}P_{\rm med}(p) \tag{C2}$$

with the $\sigma_0 N$ coupling

$$g_{\sigma_0NN}=-\frac{ig}{\sqrt{3}} \tag{C3}$$

obtained from the Lagrangian. The factor 2 comes from the isospin degeneracy and the minus sign comes from the fermion loop. Removing the pure vacuum contribution, which is divergent and should be renormalized into physical quantities, we have obtained

$$V^0_{MF}(\rho)=\frac{g\rho}{\sqrt{3}}\sigma_0. \tag{C4}$$

Here we have used

$$-\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}P_{\rm med}(p)=\frac{4m_N}{(2\pi)^3}\int d^4p\frac{\delta(p_0-E_N)}{2E_N(\vec{p})}\theta(k_f-|\vec{p}|)=\frac{k_f^3}{3\pi^2}=\frac{\rho}{2}, \tag{C5}$$

where $\rho=\frac{2k_f^3}{3\pi^2}$ and $E_N(\vec{p})=\sqrt{|\vec{p}|^2+m_N^2}$. Here we have approximated $E_N=m_N$. In the same way, the effective potential for σ_8 is obtained as

$$-iV^8_{MF}(\rho)=-2g_{\sigma_8NN}\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}P_{\rm med}(p)=-i\frac{g\rho}{\sqrt{6}}\sigma_8 \tag{C6}$$

with the $\sigma_8 N$ coupling

$$g_{\sigma_8N}=-\frac{ig}{\sqrt{6}}. \tag{C7}$$

Summing up Eqs. (C4) and (C6), we obtain Eq. (32).

TABLE III. Determined quantities.

$\langle\sigma_0\rangle$ (MeV)	$\langle\sigma_8\rangle$ (MeV)	μ^2 (MeV^2)	λ	λ'	B (MeV)	A (MeV^2)	m_s (MeV)	g	m_η (MeV)	$m_{\eta'}$ (MeV)	m_{σ_8} (GeV)	$(-\langle\bar{q}q\rangle)^{1/3}$ (MeV)	$(-\langle\bar{s}s\rangle)^{1/3}$ (MeV)
128	−21.0	1.16×10^5	59.4	−2.4	984	6.86×10^4	156	7.67	535	959	1.23	249	279

Next, we calculate the particle-hole contribution to the meson self-energy $\Sigma_{\rm ph}(\rho)$. The particle-hole contribution to the in-medium self-energy of mesons written as $\Sigma_{\rm ph}(\rho)$ is

$$-i\Sigma_{\rm ph}(\rho) = -g^2 C_i \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\{\gamma_5 P_{\rm med}(p+q)\gamma_5 P_{\rm med}(p)\}. \tag{C8}$$

The coefficient C_i is dependent on the channel: $C_\pi = \frac{1}{2}$, $C_{\eta_0} = \frac{1}{3}$, $C_{\eta_8} = \frac{1}{6}$, $C_{\eta_0\eta_8} = \frac{1}{3\sqrt{2}}$, which are obtained by the meson-nucleon couplings $g_{\pi NN} = g/\sqrt{2}$, $g_{\eta_0 NN} = g/\sqrt{3}$, $g_{\eta_8 NN} = g/\sqrt{6}$. Denoting the part of the nucleon loop integral in $\Sigma_{\rm ph}(\rho)$ as $\Pi(\rho)$ and removing the divergent vacuum contribution, we evaluate $\Pi(\rho)$ as follows:

$$\begin{aligned} -i\Pi(\rho) &= -\int \frac{d^4p}{(2\pi)^4} \mathrm{tr}[\gamma_5(\not{p}+\not{q}+m_N)\gamma_5(\not{p}+m_N)] \\ &\quad \times \frac{i}{(p+q)^2 - m_N^2 + i\epsilon}(-2\pi)\delta\left(p^2 - m_N^2\right) \\ &\quad \times \theta(p_0)\theta(k_f - |\vec{p}|) \\ &= -\frac{i}{(2\pi)^3}\int d^3\vec{p}\, \frac{4p\cdot q}{2p\cdot q + q^2}\frac{1}{2E_N(\vec{p})}\theta(k_f - |\vec{p}|) \\ &= -\frac{i}{(2\pi)^3}\int d^3\vec{p}\, \frac{\theta(k_f - |\vec{p}|)}{m_N} = -\frac{i}{4m_N}\rho. \end{aligned} \tag{C9}$$

From the first line to the second line, we used the Dirac equation and from the second line to the third line, we have taken the soft limit, $q^2 = 0$. Here, we have omitted the contribution from the pure medium contribution, which contains the two step functions, because the contribution vanishes in the soft limit. Multiplying the symmetry factor and the isospin degeneracy and adding the contribution from another cross term of the particle-hole diagram and the contribution from crossed diagram, which gives the same contribution as the noncrossed diagram in the soft limit, we obtain finally

$$\Sigma_{\rm ph}(\rho) = C_i \frac{g^2\rho}{m_N} \tag{C10}$$

APPENDIX D: DEFINITION OF SCATTERING LENGTH

Based on Ref. [47], the scattering length a and effective range r_e are given by the scattering amplitude $f(k)$ as

$$f(k) = -\frac{M}{4\pi W} t(k), \tag{D1}$$

$$a = f(k)|_{k\to 0}, \tag{D2}$$

$$r_e = \frac{d^2}{dk^2}\left(\frac{1}{f(k)}\right)\bigg|_{k\to 0}, \tag{D3}$$

where $t(k)$ is the T matrix defined in Eq. (62). The relation of the center-of-mass momentum k and the total energy W is

$$k = \frac{\sqrt{[W^2 - (M+m)^2][W^2 - (M-m)^2]}}{2W} \tag{D4}$$

with the baryon mass M and the meson mass m.

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